

## **TDoA based UAV Localization using Dual-EKF Algorithm**

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### **Abstract**

*Most UAV(unmanned aerial vehicle) systems use GPS signals only to locate the emitter's position. However GPS signals contain unwanted information contaminated by environmental components and many interference signals. In this paper, to obtain TDoA signal, we use two UAVs which are equipped with embedded wireless sensors. Under the real geolocation circumstance, it is very difficult to estimate the emitter's position exactly due to environmental noise. In this paper we use the dual-EKF algorithm to obtain the optimal estimation of state values and unknown parameters. The dual-EKF algorithm overcomes the weakness of EKF algorithm which has been widely used in geolocation problem. The performance of our proposed algorithm will be demonstrated through some simulations of UAVs.*

*Keywords: Time difference of arrival, unmanned aerial vehicles, dual-EKF, localization.*

### **1. Introduction**

The tracking techniques for locating emitter's position in UAV are studied in many approaches these days. The localization technique includes many applications such as robot SLAM, search for missing child and rescue against disaster.[1-2] There are two main methods for acquiring time difference of arrival(TDoA) signal. The first method measures time of arrival(ToA) from each UAV to emitter. Using the ToA values, the TDoA can be represented in the form of time difference. The second method obtains TDoA signals using the cross-correlation of each received signal.

Most techniques that use TDoA require at least three UAVs. However recently there has been a trend towards reduced cost. The UAV needs only very basic sensors which measure the time of arrival of signals at each receiver and can obtain TDoA signal between UAV and emitter. If there is no measurement noise and if there are many adjacent UAVs, the position estimation is easily obtained by hyperbola method. However, there exist unwanted noise components in real circumstances. In this case, the hyperbolic curves will no longer intersect at one precise position. This problem can be solved with least squares method [3-4] calculating all measurement signals. Fang[5] has studied closed-form of hyperbolic fix through augmented TDoA measurements. However Fang's method cannot be applied to overdetermined situation. Moreover this method needs more TDoA signals.

In tracking algorithm with TDoA signals there are nonlinearity components caused by noises.[6] In the geolocation applications, the extended Kalman filter(EKF) algorithm has been widely used.[7] However EKF has the divergence problem in the calculation process and poor performance in serious noise circumstance. In this paper, we propose dual-EKF

algorithm which can obtain optimal estimation of state and system's unknown parameters. Finally some simulation results demonstrate the performance of dual-EKF under the circumstance in which two TDoA signals received from two UAVs are used.

## 1. Hyperbolic curve method for emitter localization

### 2.1. Analytical methods

In this section we show a general hyperbolic curve method which has been widely used under the ideal circumstance. If there exists more than two UAVs, we can receive more associated TDoA signal values and obtain more precise position of unknown emitter. The emitter position is located as the intersection of hyperbolic curves. Figure 1 represents the general hyperbolic curves under the ideal circumstance. Here we consider only three UAVs for simplicity of method.

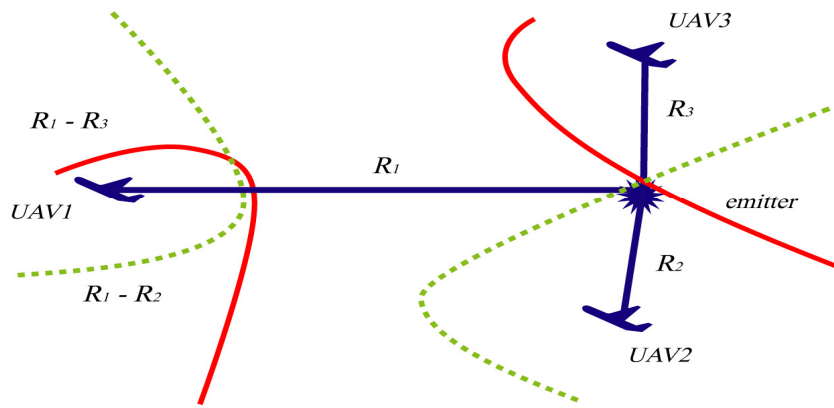


Figure 1. Geometric method using hyperbola

We assume that emitter lying on position  $r^0 = (x, y)$  sends out wireless signals to UAVs during the discrete sampling time. Position of three UAVs are supposed to be known as  $r_i = (x_i, y_i)$ ,  $i = \{0, 1, 2\}$ , respectively and each UAV receives wireless signals from emitter at each discrete sampling time.

$$t_i^0 = \frac{1}{c} r_i^0$$

$$r_i^0 = \sqrt{(x_i - x)^2 + (y_i - y)^2} \quad (1)$$

where  $r_i^0$  means the distance from emitter to UAVs and  $c$  represents the propagation speed. Using the norm of a distance, we can derive the following equation.

$$r_i^0 = \|r_i - r^0\|$$

$$r_{i1}^0 = ct_{i1} = r_i^0 - r_1^0 \quad (2)$$

where  $t_{i1}$  represents TDoA signal between the  $i$ -th receiver and the first one.

In this paper, we use only two receivers(UAVs) for algorithm's simplicity. TDoA signals using two UAVs are represented as follows.

$$TDoA = \frac{1}{c} \left( \|r_2 - r^0\| - \|r_1 - r^0\| + v_{21} \right) \quad (3)$$

where  $v_{21}$  means zero mean Gaussian noise under the real circumstance case and  $r_2, r_1$  mean the position of UAV2, UAV1, respectively.

## 2.2 System modeling

We assume that the moving emitter with constant velocity sends wireless signals and the two UAVs with elliptical orbit movement can obtain the wireless signal from emitter. Through this process, we obtain TDoA measurement values. However TDoA signals contain unwanted nonlinear error components. As a robust solution, the dual-EKF approach using TDoA signals can be more efficient among geolocation methods. To use the dual-EKF algorithm, we need the state equation using the emitter's position and velocity as state variables. We set up the following state equation which represents two dimensional position and time interval.

$$s(k+1) = As(k) + Bu(k) + w(k)$$

$$A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ T & 0 \\ 0 & T \end{bmatrix} \quad (4)$$

where  $s(k) = [x \ y \ \Delta x \ \Delta y]^T$ ,  $(x, y)$  represents the emitter's position,  $(\Delta x, \Delta y)$  means the positional variation at each sampling time,  $u(k) = [v_x \ v_y]^T$  means the known velocity of a moving emitter,  $T$  is sampling time, and  $w(k)$  means the process noise as additive white Gaussian noise(AWGN). The output equation can be expressed by the measurement of TDoA value as

$$z(k) = h(s^0(k), v(k))$$

$$= \frac{1}{c} \left( \|s^0(k) - r_1\| - \|s^0(k) - r_2\| \right) + v(k) \quad (5)$$

where  $s^0(k)$  means the emitter's position and  $v(k)$  is the measurement noise as AWGN.

## 2.3 Dual-extended Kalman filter algorithm

In this section, to apply geolocation problem, we propose dual-EKF algorithm whose plant model is augmented by weight filter(e.g., neural network). The dual-EKF algorithm combines the Kalman state and weight filter. This algorithm can estimate both the state and model parameter through only measurement values. The major process is that two EKF algorithms cooperate concurrently at every time-step. The current model parameter  $\hat{w}_k$  help the EKF state-filter to estimate state values while the EKF weight-filter updates the weight parameters with the current state estimate  $\hat{x}_k$ . [8] During the update process, the weight filter is used as weight training method in neural network. The weight filter has the high convergence than back-propagation in neural network.

To apply the dual-EKF algorithm, we initialize the estimation of state and weight values as

$$\begin{aligned}\hat{w}_0 &= E[w], & P_{w_0} &= E[(w - \hat{w}_0)(w - \hat{w}_0)^T] \\ \hat{x}_0 &= E[x_0], & P_{x_0} &= E[(x_0 - \hat{x}_0)(x_0 - \hat{x}_0)^T]\end{aligned}\quad (6)$$

The time update equations for the weight filter and the state filter are

$$\begin{aligned}\hat{w}_k^- &= \hat{w}_{k-1} \\ P_{w_k}^- &= P_{w_{k-1}} + R_k^r \\ \hat{x}_k^- &= F(\hat{x}_{k-1}, u_k, \hat{w}_k^-) \\ P_{x_k}^- &= A_{k-1} P_{x_{k-1}} A_{k-1}^T + R^v\end{aligned}\quad (7)$$

In equation (7),  $w$  corresponds to values of unknown parameters. The weight parameters( $w$ ) are used in the design process of dynamics system. The innovations covariance( $R_k^r$ ) is set to an arbitrary diagonal matrix in which the elements are close to zeros for weight training.  $R_k^r$  affects the tracking performance and convergence rate. The measurement update equations for the state filter and the weight filter are

$$\begin{aligned}K_k^x &= P_{x_k}^- C^T (C P_{x_k}^- C^T + R^n)^{-1} \\ \hat{x}_k &= \hat{x}_k^- + K_k^x (y_k - C \hat{x}_k^-) \\ P_{x_k} &= (I - K_k^x C) P_{x_k}^- \\ K_k^w &= P_{w_k}^- (C_k^w)^T (C_k^w P_{w_k}^- (C_k^w)^T + R^e)^{-1} \\ \hat{w}_k &= \hat{w}_k^- + K_k^w (y_k - C \hat{x}_k^-) \\ P_{w_k} &= (I - K_k^w C_k^w) P_{w_k}^- \\ C &= \frac{\partial z}{\partial x}, C_k^w = \frac{\partial z}{\partial w}\end{aligned}\quad (8)$$

The noise covariance ( $R^e$ ) is a constant diagonal matrix and can be set arbitrarily as  $0.5I$ . The matrix of  $I$  means an identity matrix. The first order linearization of measurement signal is necessary in order to approximate the nonlinear dynamics. The system applied to geolocation problem is shown schematically in figure 2.

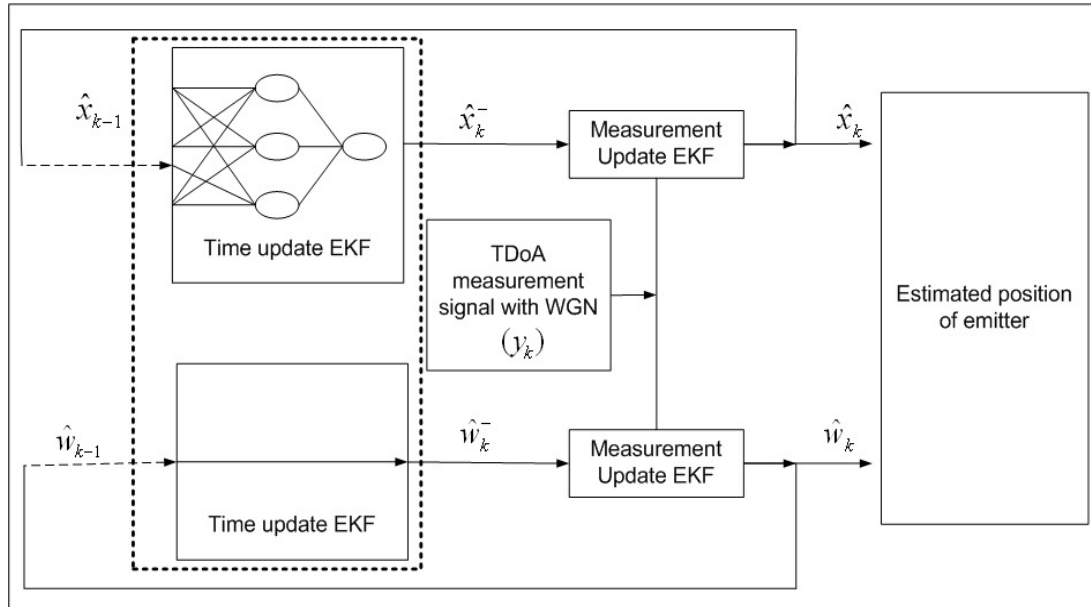


Figure 2. Process of dual-EKF algorithm in UAV localization

### 3. Simulation results

In this section, we demonstrate the effectiveness of the proposed geolocation method with the dual-EKF algorithm through some simulations. In figure 3, we suppose that the emitter moves with a constant speed except that emitter changes its direction every 19 sec. The two UAVs are initially located at the position of (8,6) km and (6,11) km with constant altitude and move following the formation of circular orbits (radius of 1 km). In the simulation, we can obtain 500 TDoA measurement values to apply the dual-EKF algorithm. The initial estimate of emitter location is randomly obtained from a white Gaussian distribution in the ideal emitter. The noise from TDoA signals has the Gaussian distribution of 0.0583 (standard deviation). The standard deviation of process noise is supposed to be 0.0591.

In Fig. 3, the dotted line means the real movement of the emitter while the solid line represents the estimated trajectory using the dual-EKF algorithm. As shown in figure 3, there are some differences between the real path and dual-EKF estimation around the range of 5 km and 6.5 km in the  $x$ -axis. However, after that, the trained weight filter enables the state values to track the real path more accurately. Therefore, we can obtain the estimated path which is similar to real path with updated weight.

Figure 4 shows the performance of the proposed localization algorithm with different path of emitter. The dotted line is the emitter's trajectory and the solid line is the one by dual-EKF with the weight filter and state filter. The weight value of the unknown parameter is updated on each estimation step.

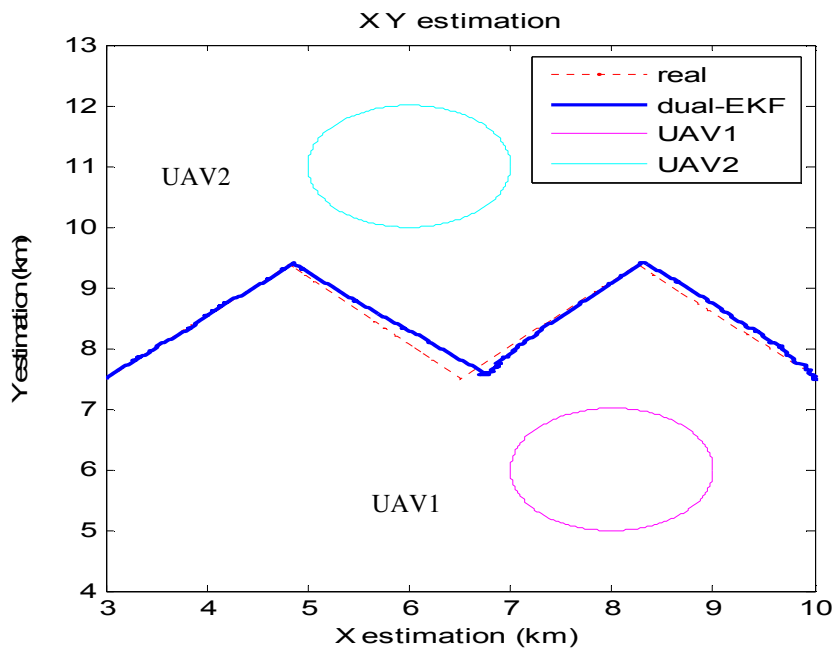


Figure 3. Trajectory estimation of real path

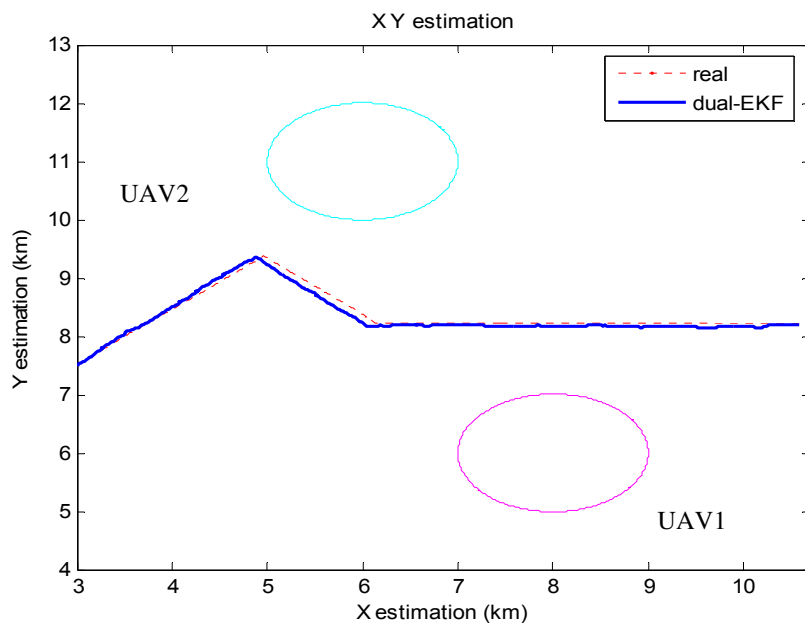


Fig. 4. Trajectory estimation for different path

Figure 5 represents the norm of position error in figure 4, i.e.,  $\|s^0(k) - \hat{s}^0(k)\|$ . The general geolocation method using GPS signals has the limit of 100 m error. In this paper, we supposed that the emitter moves with constant velocity (105 m/s). As shown in Fig. 5, in spite of emitter's fast movement and large unexpected noise, the estimated error values through dual-EKF algorithm is limited within the boundary of  $\pm 100$  m positional error. The

effectiveness of algorithm is confirmed and the estimated trajectory is shown to be close to real path.

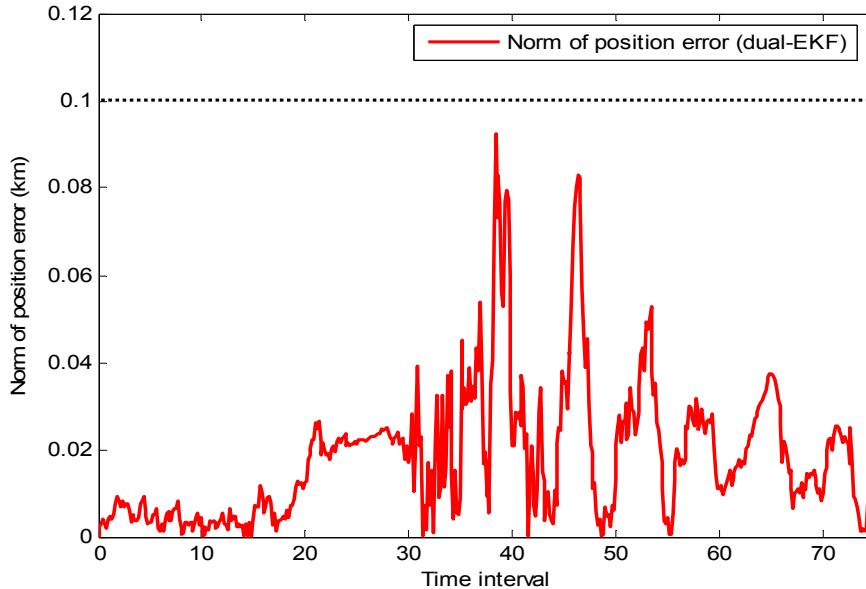


Figure 5. Norm of position error

## 5. Conclusion

In this paper, we introduced a geolocation algorithm using TDoA signals from two UAVs. The dual-EKF method can estimate state value and unknown parameter value of dynamic state model with only measurement TDoA values. In case of position tracking system with TDoA signals, it requires much calculation efforts from the linearization process. Through the simulation results we demonstrated the effectiveness of dual-EKF algorithm. It is confirmed that the position estimation using dual-EKF is close to the trajectory of real emitter. Through the two UAVs simulation, we verified that the positional error of the estimation values resides within the boundary of  $\pm 100m$  error. Furthermore, with only two UAVs for the TDoA measurement values, we realized geolocation algorithm with dual-EKF.

## References

- [1] N. Okello, and D. Musicki, "Emitter Geolocation with two UAVs", Proceedings of Information, Decision & Control Conf., Adelaide, Australia, February, 2007.
- [2] F. Fletcher, and B. Ristic, "Recursive estimation of Emitter location using TDoA measurements from two UAVs", 10th International Conf. Information Fusion, July, 2007, pp.1-8.
- [3] S. Drake, and K. Dogancay, "Geolocation by Time difference of arrival using hyperbolic asymptotes", IEEE International Conf. Acoustics, Speech & Signal Processing 2004 Proceedings, vol. 2, pp. 361-364.
- [4] A. Hashemi-Sakhtsari and K. Dogancay, "Recursive least squares solution to Source tracking using Time difference of arrival". IEEE International Conf. Acoustics, Speech & Signal Processing 2004 Proceedings, vol. 2, May, 2004, pp. 385-388.
- [5] B. T. Fang, "Simple solutions for hyperbolic and related position fixes", IEEE Trans. Aerospace & Electronic Systems, vol 26, no. 5, pp. 748-753.
- [6] K. H. Ho, Y. T. Chan, "Solution and Performance analysis of geolocation by TDoA", IEEE Trans. Aerospace & Electronic Systems, vol. 29, no. 4, 1993, pp. 1311-1322.

- [7] M. Najar, J. Vidal, "Kalman tracking based on TDoA for UMTS mobile location", IEEE International Symp. Personal, Indoor & Mobile Radio Communications, vol. 1, 2001, pp. B45-B49.
- [8] A. Eric Wan, T. Alex, and Nelson, Kalman filtering and neural networks, John Wiley & Sons, Inc. 2001, pp. 123-133.

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