Cost-Effective Traffic Grooming in WDM Rings

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***Abstract*—We provide network designs for *optical add–drop wavelength-division-multiplexed* (OADM) rings that minimize overall network cost, rather than just the number of wavelengths needed. The network cost includes the cost of the transceivers required at the nodes as well as the number of wavelengths. The transceiver cost includes the cost of terminating equipment as well as higher-layer electronic processing equipment, which in practice can dominate over the cost of the number of wavelengths in the network. The networks support dynamic (i.e., time-varying) traffic streams that are at lower rates (e.g., OC-3, 155 Mb/s) than the lightpath capacities (e.g., OC-48, 2.5 Gb/s). A simple OADM ring is the *point-to-point* ring, where traffic is transported on WDM links optically, but switched through nodes electronically. Although the network is efficient in using link bandwidth, it has high electronic and opto-electronic processing costs. Two OADM ring networks are given that have similar performance but are less expensive. Two other OADM ring networks are considered that are *nonblocking*, where one has a *wide-sense nonblocking* property and the other has a *rearrangeably nonblocking* property. All the networks are compared using the cost criteria of number of wavelengths and number of transceivers.**

***Index Terms*—Electronic traffic grooming, nonblocking networks, optical networks, wavelength division multiplexing.**

I. Introduction

An OPTICAL add–drop wavelength-division-multiplexed (WDM) ring network (OADM ring), shown in Fig. 1, consists of N nodes labeled $0, 1, \ldots, N-1$ in the clockwise direction, interconnected by fiber links. Each link carries high-rate traffic on optical signals at many wavelengths. The network has a fixed set of wavelengths for all links which we denote by $\{w_0, w_1, \ldots, w_{W-1}\}$, where W denotes the number of wavelengths. OADM ring networks are being developed as part of test-beds and commercial products, and are expected to be an integral part of telecommunication backbone networks. Although mesh topology WDM networks will be of greater importance in the future, at least in the near term, ring topologies are viable because SONET/SDH self-healing architectures are ring oriented.

OADM rings support *lightpaths*, which are all-optical communication connections that span one or more links. We will consider networks where each lightpath is full duplex, and its signals in the forward and reverse direction use the same wavelength and route. Since each lightpath is full duplex, it is terminated by a pair

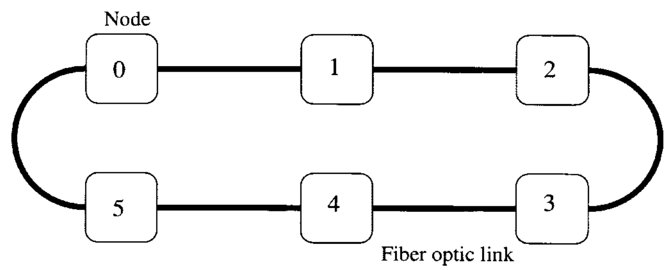

Fig. 1. Optical WDM ring.

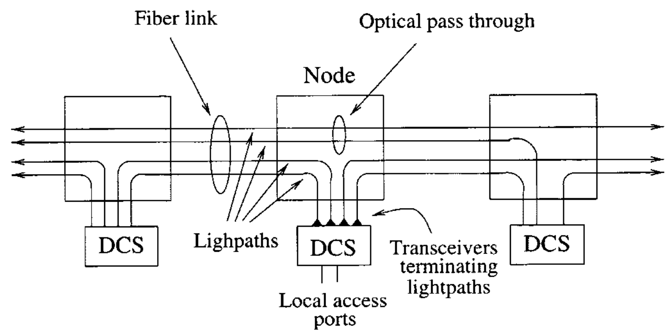

Fig. 2. Optical node.

of *transceivers*. Here, a transceiver is generic for such systems as *line terminating equipment* (LTE) and *add/drop multiplexers* (ADM) (or more accurately, half an ADM). All lightpaths have the same transmission capacity, e.g., OC-48 (2.5 Gb/s) rates.

A node in a OADM ring is shown in Fig. 2. Note that some of the lightpaths pass through the node in optical form. They carry traffic not intended for the node. The remaining lighpaths are terminated at the node by transceivers, and their traffic is converted to electronic form, and processed electronically. The electronic processing (and switching) includes systems such as SONET/SDH ADMs, IP routers, and *digital crossconnect systems* (DCSs) that crossconnect traffic streams. To simplify the presentation, we shall assume DCS systems in the sequel, but the very same discussion holds for the other type of electrical nodes. In Fig. 2, the DCS is shown representing all the electronic processing, and the transceivers are located at the interface of the DCS and lightpaths. Now some of the received traffic may be intended for the node, in which case it is switched to a local entity through local access ports. The rest of the traffic is forwarded on other lightpaths via the transceivers. In our model, the cost of transceivers is a dominant cost.

A special case of an OADM ring network is the *point-to-point* WDM ring network (PPWDM ring) shown in Fig. 3. Here, each link in the network has one-hop lightpaths on each of its wavelengths. The network is called a point-to-point ring because each lightpath implements a point-to-point connection between neighboring nodes. For the network, each node has a single DCS

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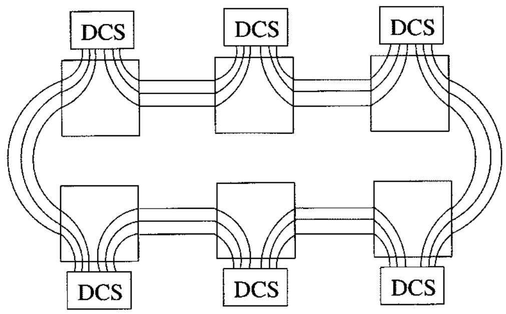

Fig. 3. Point-to-point OADM ring with three wavelengths.

that cross connects traffic from all the lightpaths. The DCS is *wide-sense nonblocking*, which means that a traffic stream may be routed through it without disturbing existing traffic streams. Note that this network does not have a true optical node because lightpaths do not pass through nodes, i.e., traffic at each node is processed electronically.

The PPWDM ring has the advantage of being able to efficiently use the link bandwidth for time-varying traffic. The network can route a traffic stream through it without disturbing other traffic streams as long as there is enough spare capacity along each link of the route. Hence, due to its capability to switch traffic streams between spare capacity on different wavelengths, it will be wavelength efficient. Its disadvantage is that its nodes do not have optical pass-through, resulting in maximum transceiver cost. For instance, in a typical carrier network, each link may have 16 wavelengths, each carrying OC-48 data. Suppose an OADM ring node needs to terminate only one lightpath worth of traffic. In this case, the node would ideally pass through the remaining 15 lightpaths in optical form without "processing" them. On the other hand, a PPWDM ring would require the traffic from all 16 wavelengths to be received, possibly switched through an electronic DCS, and retransmitted.

In practice, however, the situation is somewhat more complicated. Each lightpath typically carries many multiplexed lower-speed traffic streams (e.g., OC-3 streams, which are at 155 Mb/s). An OADM ring node cannot extract an individual lower-speed stream from a wavelength without first receiving the entire wavelength. Thus, in the example above, if we had to extract an individual OC-3 stream from each of the 16 wavelengths at a node, and all the remaining traffic were not intended for that node, all 16 wavelengths must be received. Note that the problem of designing networks that efficiently *grooms traffic* (i.e., multiplex/demultiplex lower-speed traffic streams onto and off of higher capacity lighpaths) is nontrivial, and its solution can have a great impact on network cost.

A. Design Assumptions and Approach

In this paper, we will address the problem of designing OADM rings for cost-effective traffic grooming. Our approach will be to propose and analyze a collection of OADM ring networks under the following assumptions and criteria:

1) **Network costs will be dealt with explicitly.** The costs of interest are as follows:
   a) *Number of Wavelengths* W.
   b) *Transceiver Cost* Q*:* The cost Q is defined to be the average number of transceivers per node in the network. As it turns out, transceiver cost may reflect actual costs better than the number of wavelengths. Note that Q is equal to twice the average number of lightpaths per node since two transceivers terminate each lightpath.
   c) *Maximum Number of Hops* H*:* The cost H is defined to be the maximum number of hops of a lightpath. It is desirable to minimize H since it leads to simpler physical layer designs.

   While most of the previous work on WDM networks dealt with minimizing the number of wavelengths, our work, which first appeared in [11], is the first to consider transceiver costs. In addition, our cost analyses give formulas that quantitatively relate network resources with traffic parameters.
2) **Lightpaths are fixed, although their placement may be optimized at start up.** This is a reasonable assumption for practical WDM networks at least in the near term because: a) the traffic in a lightpath is an aggregation of many traffic streams, making it less likely to fluctuate significantly; b) automatic network switching for lightpaths is not yet cost effective; and c) rerouting lightpaths may cause disruption of service.
3) **The networks are circuit-switched and support lower-speed full-duplex end-to-end connections, all at the same rate.** For example, the lightpaths may be at the OC-48 rate and support only OC-3 circuit-switched connections. We will refer to these connections as *traffic streams*. We will let c denote the number of traffic streams that can be supported in a lightpath, i.e., c traffic streams $=$ 1 lightpath. For example, for OC-48 lighpaths and OC-3 traffic streams, $c = 16$.
4) **Each node has a wide-sense nonblocking DCS that is large enough to crossconnect all traffic between its transceivers and local ports.** This assumption is realistic for many practical situations, and will simplify our subsequent discussion. Notice that the cost of the DCS is not considered in this paper. This is reasonable assuming that the interface-ports rather than switch-fabric dominate DCS costs because then total DCS cost is proportional with total transceiver cost.

The overall network design problem comprises of two phases: first the lower-speed traffic must be aggregated on to lightpaths, so as to minimize transceiver costs as well as wavelength costs. This is the focus of our paper. The second phase may incorporate constraints in organizing the lightpaths. For instance, an OADM network may be called upon to realize multiple SONET rings. This phase of network design is treated in a follow-up paper that also includes transceiver (ADM) costs [9]. Here, an OADM network must realize multiple SONET rings (one ring per wavelength). However, the lightpaths are already assumed to be given and the focus is on arranging them in rings. Besides [9], the only other studies that consider transceiver costs are [14], [19], which focus on ring networks without DCSs and for specific *static* (i.e., fixed over time) traffic, e.g., uniform static traffic. Typically, researchers have concentrated on numbers of wavelengths, congestion, delay, or probability of blocking. We should mention that there is previous work on WDM network design for lower-speed traffic streams [2], [4], [8], [17], [18], assuming static traffic. There are also a number of papers on WDM networks with *dynamic* (i.e., time-varying) traffic (e.g., [3], [1], [10], [13], [16]), but where lightpaths are switched and lower speed traffic streams are not considered. The study of (nonstatistical) dynamic traffic and fixed lightpaths for OADM networks seems to be unique to this paper.

B. Traffic Models

When considering a network architecture, the traffic time dependent behavior, distribution, and routing are of paramount importance. We consider three traffic types insofar as their time dependency is concerned: *static*, *dynamic*, and *incremental*. *Static* traffic means that lower-speed traffic streams are set up all at once, at some initial time, and fixed thereafter. *Dynamic* traffic means that traffic streams are set up and terminated at arbitrary times. *Incremental* traffic is dynamic traffic, but the traffic streams never terminate. This models the situation when traffic streams are expected to have a long holding times, as is usually the case with provisioning of high-speed connections today.

The traffic distribution will be represented by a traffic matrix $T = [T(i, j)]$, where $c \cdot T(i, j)$ equals the number of traffic streams between nodes i and j. Thus, $T(i, j)$ is the number of "lighpaths of traffic" between nodes i and j. Note that $T(i, j)$ can be fractional. For example, if 24 OC-3 connections (1 OC-48 = 16 OC-3s) are to be supported between i and j, then $T(i, j) = 1.5$. If the traffic is static, then T is fixed for all time, while if the traffic is dynamic then T is time-varying. Note that placement of transceivers is dependent on the traffic pattern. For example, for each node i, $\sum_{j=0}^{N-1} T(i, j)$ is a lower bound on the number of transceivers it requires.

The routing of traffic affects the traffic loads on links, which in turn affect bandwidth requirements. We consider traffic that either requires routing or are *pre-routed*, i.e., they come with their own pre-computed routes. In addition, pre-routed traffic are assumed to have *simple* routes, which means that they visit a node at most once. In this sense, they are routed efficiently in the network. Note that the pre-routed traffic model holds for many practical scenarios, such as when traffic is routed according to shortest paths or traffic loads. It allows us to define a maximum "traffic load" over links, which is a lower bound on the number of wavelengths to accommodate the traffic.

We consider three different traffic assumptions (i.e., scenarios), given below. The first assumes dynamic traffic that only has restrictions on the amount of traffic that terminates at the nodes. The next assumption has pre-routed traffic and a maximum traffic load parameter. The parameter is a measure of the required bandwidth (wavelengths) on the links. This model may be more appropriate when wavelengths are limited because then the load parameter value can be chosen appropriately. The final assumption is a uniform traffic assumption used in the literature as a benchmark to compare different architectures.

Traffic Assumption A: Traffic is dynamic, i.e., T is time-varying. The traffic has integer parameters $t_A(i)$: $i = 0, 1, \cdots, N-1$). At any time, each node i can terminate at most $c \cdot t_A(i)$ traffic streams. Thus, at any time, for each node i, $t_A(i) \geq \sum_{j=0}^{N-1} T(i, j)$ and $t_A(i) \geq \sum_{j=0}^{N-1} T(j, i)$. □

Traffic Assumption B: The traffic is dynamic, with traffic streams being pre-routed and having simple routes. The traffic has integer parameters L and $(t_B(i)$: $i = 0, 1, \cdots, N-1)$.

At any time, the number of traffic streams over any link is at most $c \cdot L$, assuming no blocking. In addition, each node i may terminate at most $c \cdot t_B(i)$ traffic streams from the clockwise or counter-clockwise direction along the ring. Thus, node i can terminate up to $2c \cdot t_B(i)$ traffic streams, but then half must come from the clockwise direction and the other half must come from the counter-clockwise direction. [Note that $2t_B(i)$ is a lower bound on the number of transceivers at node i to insure no blocking.] □

Traffic Assumption C: This is the *static uniform traffic*. It has an integer parameter g, and has exactly g traffic streams between every pair of nodes. Thus, $T(i, j) = g/c$ if $i \neq j$, and $T(i, j) = 0$ if $i = j$. This traffic is commonly used to compare networks in the theoretical literature because it requires good network connectivity since all nodes are connected to one another, and its uniformity simplifies analysis.

C. Proposed Network Architectures

In this paper, we will consider six OADM ring networks. To define these networks we need to specify the placement of lightpaths (and the corresponding transceivers) and a routing algorithm for the traffic streams onto lightpaths. Note that the placement of a lightpath requires finding a route and wavelength for it.

The six networks assume different constraints on the traffic. The following are brief descriptions of each OADM ring and their corresponding traffic constraints. In Section II, we will provide a more detailed description of the networks and their costs.

The following network assumes static traffic.

Fully Optical Ring: For this network, between each pair of nodes i and j there are $\lceil T(i, j) \rceil$ lightpaths between them. Traffic streams between the nodes are carried directly by these connecting lightpaths. We consider this network because it has no electronic traffic grooming (which is why it is called "fully optical"). It is therefore the opposite of the PPWDM ring which has maximal traffic grooming capability. Note that it becomes bandwidth efficient if the traffic is high enough to fill the lightpaths.

The next two networks are for Traffic Assumption A. Under the assumption, they are *nonblocking*, which means that they will not block any arriving traffic stream. Note that $t_A(i)$ is a lower bound on the number of transceivers at node i to insure no blocking.

Single-Hub: This network has a unique node designated as a *hub*, which has lightpaths directly connecting it to all other nodes. It is *wide-sense* nonblocking, i.e., traffic streams may be added without disturbing existing ones.

Double-Hub: This network has two hubs, which have lightpaths connecting them to all other nodes. This network is *rearrangeably* nonblocking, which means that it may have to rearrange existing traffic streams to make way for new ones. Note that rearranging existing traffic streams is undesirable in practical networks. However, the double-hub network is reasonably efficient in W and Q, so it could be used for static traffic.

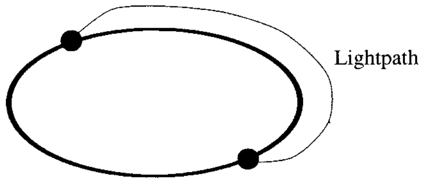

Fig. 4. Setting up a lightpath between the first two nodes.

The single-hub and double-hub networks are nonblocking for dynamic traffic, with the only constraint that each node i can terminate at most $c \cdot t_A(i)$ traffic streams. They result in large W to accommodate worst-case traffic distributions that lead to high traffic loads on links. The next three ring networks are for Traffic Assumption B. They require $W \geq L$ to insure no traffic blocking. Notice that the requirement is necessary for any network to be nonblocking. However, the inequality by itself is insufficient because traffic cannot use spare bandwidth at different wavelengths if they cannot be switched at intermediate nodes.

PPWDM Ring: This is the PPWDM ring network described earlier. For Traffic Assumption B and $W = L$, it is wide-sense nonblocking.

Hierarchical Ring: This is a simple network composed of two PPWDM subrings, and is wide-sense nonblocking for Traffic Assumption B. The network uses more wavelengths than a PPWDM ring, but it often uses less transceivers.

Incremental Ring: This a ring network that is organized (recursively) from sections of the ring. For Traffic Assumption B, the network is wide-sense nonblocking for incremental traffic. It requires the same L wavelengths as the PPWDM ring, but a smaller number of transceivers. Since it is wide-sense nonblocking for incremental traffic, it is rearrangeable nonblocking when the traffic is fully dynamic and satisfies Traffic Assumption B. Here, rearrangeably nonblocking means that traffic streams may change wavelengths, but not their routes, to make way for a new traffic stream.

These six OADM rings are for different traffic models, but they can all support static traffic. Hence, in Section III, we compare their costs W, Q, and H under Traffic Assumption C. Our conclusions are given in Section IV.

II. Optical WDM Ring Architectures

A. Fully Optical Ring

Consider a network where traffic must be routed on a single lightpath from its source to its destination. This will require setting up lightpaths between each source and destination node between which there is any traffic. We will compute the costs for the ring assuming the static uniform traffic with parameter $g \leq c$. Then we need to set up one lightpath between each pair of nodes. This type of a network has been considered in [7].

Next, the lightpath set-up will be described using a recursive definition, that appeared in [11] and was also independently discovered in [6]. Other definitions of fully optical rings can be found in [21].

We first consider the case when N is even.

1) Start with two nodes on the ring (see Fig. 4). The sole lightpath that needs be set up will require one wavelength.

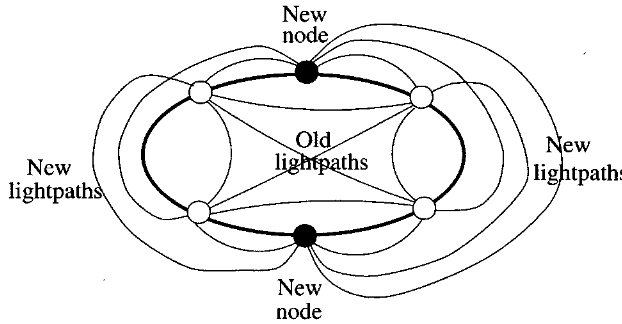

Fig. 5. Setting up the lightpaths for two new nodes.

2) (Recursive step) Let k denote the number of nodes currently in the ring. While $k \leq N-2$, add two more nodes to the ring such that they are diametrically opposite to each other, i.e., separated by the maximum possible number of hops (see Fig. 5). The two new nodes divide the ring in half, where each half has $k/2$ old nodes. In one half, each old node sets up a lightpath to each new node. This requires one wavelength per old node since each old node can fit its two lightpaths in a wavelength (since the lightpaths use disjoint routes). Thus, a total of $k/2$ new wavelengths are required. The old nodes in the other half of the ring can do the same thing and use the same $k/2$ wavelengths. Finally, the two new nodes require an additional wavelength to set up a lightpath between them. Thus, we need to add a total of $(k/2)+1$ new wavelengths.

So the number of wavelengths needed to do the assignment is

$$W = 1+2+3+\cdots+\frac{N}{2} = \frac{N^2}{8}+\frac{N}{4}.$$

For arbitrary g the wavelength assignment can be done with

$$W = \left\lceil \frac{g}{c} \right\rceil \left(\frac{N^2}{8}+\frac{N}{4}\right)$$

wavelengths, where N is even.

When N is odd, we start the procedure above with three nodes and add two nodes each time. The number of wavelengths in this case can be calculated to be

$$W = \left\lceil \frac{g}{c} \right\rceil \frac{N^2-1}{8}.$$

Clearly, the number of transceivers required per node is given by

$$Q = \left\lceil \frac{g}{c} \right\rceil (N-1).$$

The maximum hop length is

$$H = \left\lfloor \frac{N}{2} \right\rfloor. \quad (1)$$

B. Single-Hub Ring

The *single-hub* ring network is for Traffic Assumption A. It has a node designated as the *hub*, which will be referred to as node h. An example of a single-hub network is shown in Fig. 6. The hub node is chosen such that it achieves the maximum $\max_{0 \leq i < N} t_A(i)$. As we shall see, this choice for the hub

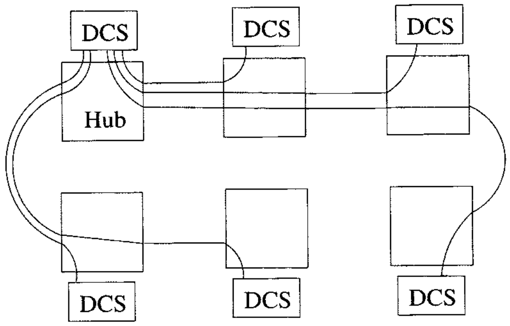

Fig. 6. Single-hub network for the case when $t_A(i) = 1$ for all nodes i.

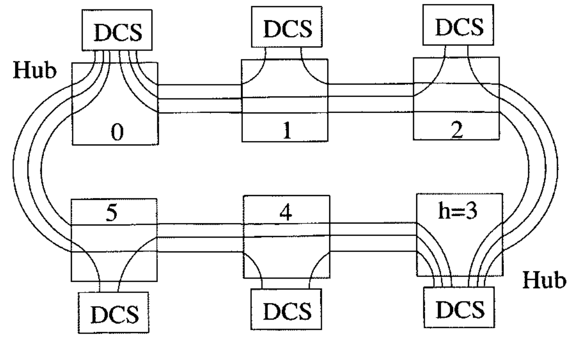

Fig. 7. Double-hub network when $t_A(i) = 2$ for all nodes i.

minimizes the required number of wavelengths. For simplicity, we will denote $\max_{0 \le i < N} t_A(i)$ by $t_{\max}$.

Each node i is directly connected to the hub by $t_A(i)$ lightpaths. Thus, the logical topology of the network is a star topology. Traffic streams are routed between nodes by going through the hub. The network is wide-sense nonblocking because the DCS at the hub is wide-sense nonblocking, and for each node i there are enough lightpaths provisioned to the hub to accommodate all of its traffic. Thus, we have the following theorem.

Theorem 1: For Traffic Assumption A, the single-hub ring is wide-sense nonblocking.

The number of wavelengths required is $\lceil (1/2) \sum_{i \ne h} t_A(i) \rceil$ because there are $\sum_{i \ne h} t_A(i)$ lightpaths, and we can fit two lightpath connections into a wavelength (the lightpaths on the same wavelength use disjoint routes along on the ring).

We have the following properties of the single hub ring:

- $W = \lceil (1/2) [\sum_{i=0}^{N-1} t_A(i) - t_{\max}] \rceil$.
- $Q = 2(\sum_{i=0}^{N-1} t_A(i) - t_{\max})/N$ since there are $\sum_{i=0}^{N-1} t_A(i) - t_{\max}$ lightpaths.
- $H = N - 1$ since lightpath routes may be forced to circumvent the ring to minimize wavelengths.

Now, note that since the single-hub ring is *wide-sense nonblocking*, it is also rearrangeably nonblocking. The following theorem gives a simple lower bound on the number of wavelengths required for such a OADM ring, and we give its proof for completeness. Notice that the number of wavelengths for the single-hub ring is about twice as much as the lower bound. However, in the next subsection, a rearrangeably nonblocking OADM ring is given that almost meets the lower bound.

Theorem 2: Consider a rearrangeably nonblocking OADM ring network under Traffic Assumption A. Suppose N is even, and for each node $i = 0, 1, \cdots, N-1$, $t_A(i) = \tau$, where τ is some integer. Then the number of wavelengths W is at least $\lceil \tau(N/4) \rceil$.

Proof: Consider the case where for $i = 0, 1, \cdots, (N/2) - 1$, there is $c \cdot \tau$ traffic streams between the pair of nodes i and $i + (N/2)$. Since each traffic stream must traverse $N/2$ links, there are $N/2$ pairs of nodes, and there are N links, the average number of traffic streams going through a link must be at least

$$\frac{c \cdot \tau \cdot \dfrac{N}{2} \cdot \dfrac{N}{2}}{N} \tag{2}$$

which is equal to $(c \cdot N \cdot \tau)/4$. The theorem is implied. □

C. Double-Hub Ring

The *double-hub* ring network is for Traffic Assumption A. It has two nodes that are *hubs*. An example of a double-hub ring is shown in Fig. 7. Without loss of generality, assume one of the hubs is node 0, and denote the other hub by h. Each node i has communication connections to each hub, and the aggregate capacity to each hub is equivalent to $(t_A(i)/2)$ lightpaths. This allows node i to send (and sink) up to $c(t_A(i)/2)$ traffic streams to (and from) each hub.

We will now describe how the communication connections are realized by lightpaths. We will use the following terminology and definitions. The nodes $0, 1, 2, \cdots, h-1$ will be referred to as *side 1* of the ring. The rest of the nodes $h, h+1, \cdots, N-1$ will be referred to as *side 2* of the ring. We will also use the notation $fr(t_A(i)/2)$ to denote the fractional part of $(t_A(i)/2)$. Note that $fr(t_A(i)/2)$ is zero if $t_A(i)$ is even and $1/2$ if $t_A(i)$ is odd. We will refer to nodes that have odd $t_A(i)$ as *odd traffic* nodes.

We will now describe how nodes in side 1 connect to the hubs. (Note that the nodes in side 2 are connected to the hubs in a similar way.) Each node i in side 1 uses $\lfloor (t_A(i)/2) \rfloor$ wavelengths to carry $\lfloor (t_A(i)/2) \rfloor$ lightpaths directly to each hub. The lightpaths are routed only using links on side 1 of the ring. Note that it is possible to use only $\lfloor (t_A(i)/2) \rfloor$ wavelengths because lightpaths going to the two hubs have disjoint routes.

Note that if $t_A(i)$ is odd then node i must have an additional $1/2 \, (= fr(t_A(i)/2))$ worth of lightpath connection to each hub. These "half-a-lightpath" connections are realized by having two odd-traffic nodes share a wavelength. For example, if u and v are odd-traffic nodes sharing a wavelength and $u < v$ then there would be lightpaths between the pairs $(0, u)$, (u, v), and (v, h). Thus, if $u \ne 0$ then there would be three lightpaths, and if $u = 0$ then there would be two lightpaths. Now nodes u and v can use half the bandwidth of a lightpath to carry $c/2$ traffic streams to and from each hub.

It is straightforward to check that number of wavelengths required for side 1 of the ring is $\lceil (1/2) \sum_{i=0}^{h-1} t_A(i) \rceil$. Note that nodes 0 and h each have $\lceil (1/2) \sum_{i=0}^{h-1} t_A(i) \rceil$ transceivers to terminate lightpaths on side 1. Each node $i = 1, 2, \cdots, h-1$ have $2\lfloor (t_A(i)/2) \rfloor$ transceivers to terminate lightpaths with a "full lightpath worth" of connection, and has $2\lceil fr(t_A(i)/2) \rceil$ transceivers to terminate lightpaths with "half a lightpath worth" of connection. Thus, each node $i = 1, 2, \cdots, h-1$

has $2\lceil (t_A(i)/2) \rceil$ transceivers. Hence, the total number of transceivers that terminate lightpaths on side 1 is

$$2\left\lceil \frac{1}{2}\sum_{i=0}^{h-1} t_A(i) \right\rceil + \sum_{i=1}^{h-1} 2\left\lceil \frac{t_A(i)}{2} \right\rceil.$$

Similar calculations can be done for side 2. Thus, we have

$$W = \max\left\{ \left\lceil \tfrac{1}{2}\sum_{i=0}^{h-1} t_A(i) \right\rceil, \left\lceil \tfrac{1}{2}\sum_{i=h}^{N-1} t_A(i) \right\rceil \right\}$$

$$Q = \frac{1}{N}\left(2\left\lceil \frac{1}{2}\sum_{i=0}^{h} t_A(i) \right\rceil + 2\left\lceil \frac{1}{2}\sum_{i=h+1}^{N-1} t_A(i) \right\rceil + \sum_{i=1}^{h-1} 2\left\lceil \frac{t_A(i)}{2} \right\rceil + \sum_{i=h+1}^{N-1} 2\left\lceil \frac{t_A(i)}{2} \right\rceil \right)$$

and

$$H = \max\{h, N-h\}.$$

To compare W with the simple lower bound in Theorem 2, let N be even, $h = N/2$, and for all nodes i, $t_A(i) = \tau$, where τ is some integer. Then $W = \lceil \tau N/4 \rceil$, which equals the lower bound.

Theorem 3: Consider Traffic Assumption A and the double-hub ring network. Suppose c is even and, for each node i, $c \cdot t_A(i)$ is divisible by four. Then the double-hub ring network is rearrangeably nonblocking.

Proof: The double-hub ring can be viewed as a switching network where lower-speed traffic streams are routed between nodes via hub nodes. Note that the traffic streams are full duplex (i.e., bidirectional) so they do not have distinct source and destination nodes typically used to define connections in switching networks. We will artificially give each traffic stream a *direction*, so that it will have a source and destination. Note that the directions are used for routing purposes only, and the traffic streams are still full duplex. Also note that the directions for traffic streams may change over time which may be necessary for rerouting.

We can assume that any collection of traffic streams, can be directed so that at each node i, at most $c(t_A(i)/2)$ streams are directed into it or out of it. This assignment can be done as follows. Since each node i has an even value for $c \cdot t_A(i)$, we may assume that each node i terminates exactly $c \cdot t_A(i)$ traffic streams. Otherwise, the assumption can be made true by greedily adding dummy streams, where streams that have both their ends terminating at a single node are allowed. Since there is an even number of traffic streams incident to any node, we can find a *Eulerian walk* [15] where the streams are treated as *edges* in a *multigraph* (i.e., a graph that can have multiple edges between pairs of nodes), and *self-loop* edges (i.e., edges from a vertex to itself) are possible. Recall that such a walk is a tour that visits each edge exactly once and then returns to the starting node. The traversal of such a tour gives directions to the streams such that at each node i, exactly $c(t_A(i)/2)$ (real or dummy) streams are directed into and out of it.

With the traffic streams directed, the double-hub ring can be viewed as emulating a three-stage switching network, as shown in Fig. 8, that supports directed traffic streams. The first stage has N vertices denoted by $s_0, s_1, \cdots, s_{N-1}$, where s_i represents node i in the ring network. The second stage has two vertices representing the two hubs. The third stage has N vertices denoted by $d_0, d_1, \cdots, d_{N-1}$, where d_i also represents node i in the ring network. Hence, node i in the ring is represented by two vertices s_i and d_i in the three-stage switching network. (It will be shown shortly that if i is a hub, then it is also represented by a vertex in the second stage of the three stage switching network.)

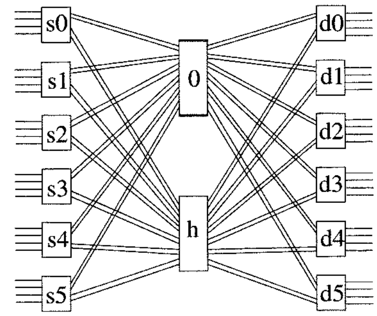

Fig. 8. Three-stage switch for $N = 6$, $c = 4$, and $t_A(i) = 2$ for all nodes i.

Each vertex s_i in the first stage has $c(t_A(i)/2)$ input links which represents the fact that node i in the ring can source $c(t_A(i)/2)$ directed traffic streams. Similarly, each vertex d_i in the third stage has $c(t_A(i)/2)$ output links which represents the fact that node i in the ring can be the destination of $c(t_A(i)/2)$ directed traffic streams.

Each vertex s_i in the first stage has $c(t_A(i)/4)$ links to each vertex in the second stage, and each vertex d_i in the third stage has $c(t_A(i)/4)$ links from each vertex in the second stage. Thus, vertices s_i and d_i together have $c(t_A(i)/2)$ links to each hub. These links represent the fact that node i in the ring network can have $c(t_A(i)/2)$ traffic streams to each hub.

The three-stage switching network is rearrangeably nonblocking. This can be shown by first transforming it into a three-stage Clos network (see [12] for a description of a Clos network). In particular, each vertex s_i in the first stage is transformed into $c(t_A(i)/2)$ vertices, each having two input links and one link to each second-stage vertex. Similarly, each vertex d_i in the third stage is transformed into $c(t_A(i)/2)$ vertices, each having two output links, and one link from each second stage vertex. The Clos network is rearrangeably nonblocking because there are two input links at each first-stage vertex, two output links at each third-stage vertex, and two vertices in the second stage [20], [5]. The original three-stage network is rearrangeably nonblocking because it can emulate the Clos network. Hence, the double-hub ring is rearrangeably nonblocking. □

D. Point-to-Point WDM Ring

We will present the costs of the PPWDM ring network assuming Traffic Assumption B. The network has $W = L$. Therefore, the network is wide-sense nonblocking. For the ring, obviously, the number of transceivers per node is

$$Q = 2W \tag{3}$$

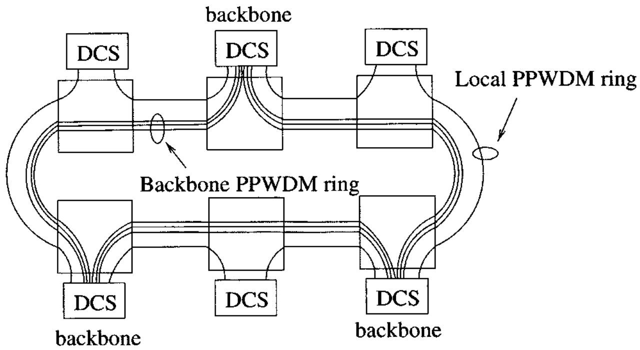

Fig. 9. Hierarchical ring with parameter $\alpha = 2$, $\tau = 1$, and $L = 3$.

and the maximum hop length is

$$H = 1. \tag{4}$$

E. Hierarchical Ring

In this section, we will describe the *hierarchical ring*. Throughout the section, we will assume Traffic Assumption B. To simplify the discussion, we will also assume that for each node i, $t_B(i) = \tau$, where τ is some integer parameter.

The network has an integer $\alpha > 0$, and has a total of $W = L + \tau \cdot (\alpha - 1)$ wavelengths. L of the wavelengths are referred to as *backbone* wavelengths, and the other $\tau(\alpha - 1)$ wavelengths are referred to as *access* wavelengths. The nodes of the ring are also classified into *access* or *backbone* types. Note that the backbone nodes are $0, \alpha, 2\alpha, \ldots, (\lceil N/\alpha \rceil - 1)\alpha$, while the other nodes are access nodes. Thus, the nodes are arranged such that there are at most $\alpha - 1$ access nodes between any consecutive pair of backbone nodes. The backbone wavelengths are used to form a PPWDM ring among the backbone nodes. In other words, lightpaths are formed between consecutive backbone nodes using the backbone wavelengths. We will refer to this as the *backbone PPWDM ring*. Note that lightpaths at the backbone wavelengths have at most α hops.

The access wavelengths are used to form a PPWDM ring among all nodes, both access and backbone types. In other words, lightpaths are formed between consecutive nodes using access wavelengths. We will refer to this as the *access PPWDM ring*. The lightpaths at the access wavelengths have one hop.

Fig. 9 shows an example hierarchical ring. It is straightforward to show the following properties of the hierarchical ring.

$$\begin{aligned} W &= L + (\alpha - 1)\tau \\ Q &= 2(\alpha - 1)\tau + 2(L/N)\lceil (N/\alpha) \rceil \\ H &= \alpha. \end{aligned}$$

Note that the value of α can be chosen to minimize Q. For large N, the optimal value of α is approximately $\sqrt{L/\tau}$.

The lightpath assignment algorithm for the ring network is as follows. Consider an arriving traffic stream with its route. Suppose we follow the route in the clockwise direction around the ring. Let u_0 and v_1 be the first and last nodes of the route. Let v_0' and v_1' be the first and last backbone nodes, respectively, along the route. The traffic stream from v_0 to v_0' and then from v_1' to v_1 will be assigned lightpaths from the access PPWDM ring. Between nodes v_0' and v_1' the traffic stream will be assigned lightpaths from the backbone PPWDM ring. Thus, the access PPWDM ring serves as an access network to the backbone PPWDM ring, and the backbone PPWDM ring transports traffic streams across the network. The traffic stream will not be blocked because enough wavelengths have been provisioned. In particular, the L wavelengths of the backbone PPWDM ring are sufficient to transport traffic across the network. Also, the $(\alpha - 1)\tau$ wavelengths of the access PPWDM ring are sufficient for traffic streams to access neighboring backbone nodes because between backbone nodes there are at most $(\alpha - 1)$ access nodes and each of then can terminate at most $c \cdot \tau$ traffic streams from the clockwise or counter-clockwise direction along the ring. Thus, we have the following result.

Theorem 4: Consider Traffic Assumption B. The hierarchical ring network is wide-sense nonblocking.

F. Incremental Ring

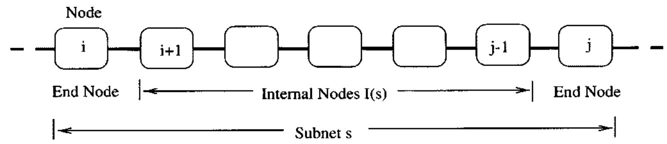

Fig. 10. Segment of subnet s.

In this section, we will consider the *incremental ring network*. Throughout the section, we will assume Traffic Assumption B and incremental traffic. The network has $W = L$ wavelengths which is the same number as for the PPWDM ring. We will first describe the network architecture, and then show that it is nonblocking for incremental traffic. At the end of the section, we will present a strategy to configure the architecture to minimize transceiver cost Q.

The incremental ring network is organized with respect to pieces of it, called *subnets*. A subnet s is composed of a sequence of links (and nodes) along the ring in the clockwise direction. This sequence will be referred to as its *segment*. The subnet's *length* is the length of its segment. The subnet's *end nodes* are the nodes at the end of its segment. The rest of its nodes are referred to as the *internal nodes* and denoted by $I(s)$. Fig. 10 shows a segment and its end and internal nodes.

A subnet s is also composed of a collection of wavelengths and lightpaths. The wavelengths are $w_0, w_1, \ldots, w_{r(s)-1}$, where $r(s)$ denotes the number of wavelengths for s. The lightpaths only use the wavelengths and links of s.

The incremental ring architecture is organized as a *tree* of subnets. The *root* subnet has a segment that includes all the links of the ring. The segment starts and ends at some node, which we refer to as the *root node*. The number of wavelengths for this subnet is W, i.e, $r(\text{root}) = W$.

If a subnet s has length greater than one (i.e., it has at least one internal node) then it is referred to as a *parent* subnet. Note that the *root* subnet is a parent if $N > 1$. A parent subnet has two *children* subnets, say s_0 and s_1. The segments of the children are defined by bisecting the segment of s at some internal node denoted by $b(s) \in I(s)$. The resulting subsegments are the segments for s_0 and s_1. The number of wavelengths assigned to

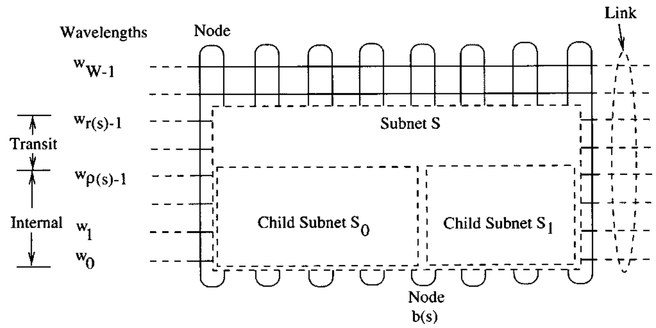

Fig. 11. Parent subnet s composed of its two children subnets s_0 and s_1.

each of the children is $r(s_0) = r(s_1) = \rho(s)$, where $\rho(s)$ is defined to be

$$\rho(s) = \min\left\{ r(s), \sum_{i \in I(s)} t_B(i) \right\}. \tag{5}$$

Note that $\rho(s)$ is equal to

$$\min\left\{ W, \sum_{i \in I(s)} t_B(i) \right\} \tag{6}$$

since $r(\text{root}) = W$ and for an arbitrary parent subnet s' and its child s^*, $\sum_{i \in I(s')} t_B(i) \geq \sum_{i \in I(s^*)} t_B(i)$. Fig. 11 shows an example subnet s, its bisecting node $b(s)$, and two children s_0 and s_1. As shown in the figure, the $r(s)$ wavelengths of s are categorized into two types for s: *internal* and *transit*. The internal wavelengths of s are $\{w_0, w_1, \ldots, w_{\rho(s)-1}\}$, and the transit wavelengths of s are $\{w_{\rho(s)}, w_{\rho(s)+1}, \ldots, w_{r(s)-1}\}$. Thus, the $\rho(s)$ internal wavelengths of s are the wavelengths of its children. This categorization of wavelengths will be used later to define where lightpaths are placed in the subnet, and how lightpaths are assigned to traffic streams.

If a subnet s has only one link, then it has no children and is called a *leaf* subnet. Leaf subnets only have transit wavelengths. Now note that the collection of leaf and parent subnets form a tree of subnets, where the root and leaf subnets are the root and leaves, respectively, of the tree. Also note that a subnet can be both a child (of some subnet) and a parent (of two other subnets).

Next, we define how transceivers are placed at each node, which will in turn define the placement of lightpaths. First, the root node has $2W$ transceivers terminating each wavelength on both of its links. For all other nodes, the placement is as follows. A node must be a bisecting node $b(s)$ for some parent subnet s. The node $b(s)$ has $2\rho(s)$ transceivers, which terminate wavelengths $w_0, \ldots w_{\rho(s)-1}$ on each of the node's links. Thus, for subnet s, its end and bisecting nodes terminate lightpaths on its internal wavelengths, while *only* its end nodes terminate lightpaths on its transit wavelengths (refer to Fig. 11). Thus, lightpaths on its transit wavelengths directly connect its end nodes, and cannot be used for traffic streams that terminate at its internal nodes.

To complete the description of the incremental ring architecture, we define its *lightpath assignment algorithm* (LAA). The LAA takes an incoming traffic stream with a route, and finds a contiguous sequence of lightpaths along the route that can carry the stream. To define the LAA for the ring, we will define LAAs for the subnets.

The LAA for a subnet takes a route and assigns a contiguous sequence of its lightpaths along the route that have spare capacity. Thus, a traffic stream following the route can be carried by the lightpaths. The route is assumed to be entirely in the segment of the subnet. Thus, the route may actually be part of some larger traffic stream route that goes through the subnet. There are two kinds of routes: *internal* and *transit*. An internal route ends at least one internal node of the subnet, while a transit route does not, i.e., it ends at the end nodes.

We will describe the LAA for a subnet s by first assuming that s is a parent subnet, as shown in Fig. 11. How the LAA assigns lightpaths to a route depends on whether the route is internal or transit. If the route is internal then it is first split at the bisecting node $b(s)$. (Of course, it is not split if it does not pass through $b(s)$.) The resulting subroutes fit into the segments of the children subnets of s. The LAAs of the children are used to assign lightpaths to the subroutes. These assignments combine to provide the assignment for the original route. However, if the route is transit, then the LAA will first attempt to assign a lightpath on a transit wavelength of s to the route. If all such lightpaths have no spare capacity then the LAA assigns lightpaths to the route as if it were an internal route, i.e., it uses the LAAs of the children subnets.

The LAA for a leaf subnet s is simpler because all routes and wavelengths are transit. A route is assigned to a lightpath of s with spare capacity.

We have completed the definition of the LAAs for subnets. The LAA for the root subnet is basically the LAA for the incremental ring. In particular, the LAA for the incremental ring will assign lightpaths to an incoming traffic stream as follows. It first splits the route of the stream at the root node (if the route passes through the root). The resulting subroutes are entirely in the root subnet and can be assigned lightpaths by the subnet's LAA. This lightpath assignment is an assignment for the traffic stream since the lightpaths that meet at the root node can be crossconnected.

Next we will prove the following property.

Theorem 5: Consider an incremental ring network under Traffic Assumption B and incremental traffic. Then the incremental ring network is wide-sense nonblocking.

The theorem is implied by the next lemma, which uses the following definitions. Consider a subnet s. The subnet is said to have *spare capacity* in a link if one or more of its wavelengths have spare capacity in the link. The subnet is said to be *wide-sense nonblocking* if given a route, the LAA of the subnet will find a lightpath assignment for the route as long as the subnet has spare capacity in each link along the route. Note that having the spare capacity does not preclude a valid lightpath assignment because in order for a set of lightpaths to carry a traffic stream they must terminate at the ends of the route, and at common intermediate nodes. Note that the next lemma implies Theorem 5 because it is applicable to the root subnet.

Lemma 1: Consider an incremental ring network under Traffic Assumption B and incremental traffic. Then each subnet s of the ring is wide-sense nonblocking.

Proof: To prove the lemma for leaf subnets is trivial since the subnet has only one link. Thus, we will only consider parent subnets. The proof of the lemma is by induction. Consider a parent subnet s and its two children subnets s_0 and s_1. We will assume that the children subnets are nonblocking, and proceed to show that s is also nonblocking.

Let us see how subnet s assigns lightpaths to traffic streams over time. Initially, there are no traffic streams so all lightpaths in the subnet are empty. As traffic streams arrive they are assigned lightpaths so that internal traffic (i.e., streams that terminate at internal nodes of s) are assigned to lightpaths with internal wavelengths, and transit traffic (i.e., streams that do not terminate at internal nodes of s) are assigned to lightpaths with transit wavelengths.

Note that this can continue while transit traffic does not completely fill lightpaths with transit wavelengths. This is partly due to the traffic model which assumes that each internal node $i \in I(s)$ can terminate at most $c \cdot t_B(i)$ traffic streams from the clockwise or counter-clockwise direction. This implies that for any link of the subnet, the number of internal traffic streams that go over it is at most $c \cdot \rho(s)$. The internal traffic can be accommodated by the lightpaths with internal wavelengths because:

1) children subnets are assumed to be nonblocking and occupy the $\rho(s)$ internal wavelengths; and
2) bisecting node $b(s)$ (which is between the children subnets) terminates all the internal wavelengths with transceivers, and can therefore crossconnect the spare capacity in the two children subnets.

Thus, while transit streams do not completely fill the transit wavelengths, internal traffic streams will be accommodated by lightpaths at internal wavelengths. Hence, there will be no blocking.

Now, at some point, the lightpaths with transit wavelengths may become completely filled. Then the only spare capacity left in the subnet s is in its internal wavelengths. Arriving traffic streams will not be blocked from this spare capacity because the spare capacity resides in the nonblocking children subnets, and the bisecting node $b(s)$ can crossconnect the spare capacity in the two children subnets. □

Finally, we will describe a strategy to place transceivers (and lightpaths) to minimize the transceiver cost. Notice that the placement of transceivers is dependent on the choice of root and bisecting nodes. Thus, the strategy is to find an optimal choice of these nodes. Before presenting the strategy, we give some definitions.

1) Let $\sigma(i, k)$ denote a segment of the ring that starts at node i and goes clockwise over k links.
2) Let $I'(i, k)$ denote the *internal* nodes of $\sigma(i, k)$, i.e., $I'(i, k) = \{(i+j) \bmod N: 0 < j < k\}$.
3) Consider all possible ways of configuring the incremental ring, i.e., all possible ways of choosing the root and bisecting nodes. Of these, consider all configurations that have a subnet with segment $\sigma(i, k)$. Let $\mathcal{C}(i, k)$ denote these configurations of the incremental ring.
4) For each configuration $\gamma \in \mathcal{C}(i, j)$, let $q_\gamma(i, k)$ be the number of transceivers placed at the internal nodes $I'(i, k)$ assuming the incremental ring is configured as γ. Let $q(i, k) = \min_{\gamma \in \mathcal{C}(i, j)} q_\gamma(i, k)$. Thus, $q(i, k)$ is the minimum number of transceivers at the internal nodes $I'(i, k)$ assuming that $\sigma(i, k)$ is the segment of a subnet.

Notice that for each node i, $2W + q(i, N)$ is the minimum number of transceivers in the incremental ring, assuming i is the root node because there are $2W$ transceivers at i and a minimum of $q(i, N)$ transceivers at all other nodes. Thus, the minimum number of transceivers for the incremental ring is $2W + \min_i q(i, N)$.

We now describe the strategy to determine the root and bisecting nodes. It consists of two steps. The first step is to compute the values for $q(i, k)$ for all nodes i and all $k = 1, 2, \cdots, N$ by using

$$q(i, k) = \begin{cases} 0, & \text{if } k = 1 \\ \min_{i<j<k} \\ \quad (q(i, j) + q((i+j) \bmod N, k-j) \\ \quad +2\min\left\{W, \sum_{n \in I'(i,k)} t_B(n)\right\}, & \text{if } k > 1. \end{cases} \tag{7}$$

The equation is true for $k = 1$ because $\sigma(i, 1)$ corresponds to a leaf subnet, which has no internal nodes. The equation is also true for $k > 1$ because: 1) the minimum is over all bisecting nodes $(i+j) \bmod N$ for a subnet with segment $\sigma(i, k)$; 2) the terms $q(i, j)$ and $q(i+j, k-j)$ are the minimum number of transceivers at internal nodes of the two children of the subnet; and 3) the term

$$2\min\left\{W, \sum_{n \in I'(i,k)} t_B(n)\right\}$$

is the number of transceivers at the bisecting node which is twice (6). The values for $\{q(\cdot, \cdot)\}$ are determined by first computing $q(i, 2)$ for all nodes i, then computing $q(i, 3)$ for all nodes i, and so forth.

The second step is to determine the root and bisecting nodes that minimize the number of transceivers in the network from the values of $\{q(\cdot, \cdot)\}$. This is straightforward. In particular, the node i that minimizes $\min_i q(i, N)$ is the optimal root node. The node $(i+j) \bmod N$ that minimizes the right-hand side of (7) is the optimal bisecting node for the corresponding subnet. It is straightforward to show that this two-step strategy has time complexity $O(N^3)$.

III. Comparisons

In this section, we will compare the six OADM ring networks in the previous section. Note that the networks operate under different traffic models. Table I lists the networks with their traffic models and switching capabilities. To compare their costs, we will use the uniform static traffic model with parameter g because all six networks can operate under this model. Recall that the traffic has g traffic streams between every pair of nodes, i.e.

$$T(i, j) = \begin{cases} g/c & \text{if } i \neq j \\ 0 & \text{otherwise.} \end{cases} \tag{8}$$

TABLE I
COMPARISON OF THE TRAFFIC MODELS AND SWITCHING CAPABILITIES OF THE SIX OADM RINGS

Network	Traffic Assumption	Switching Capability
Fully Optical	C, static	not applicable
Single-Hub	A, dynamic	wide-sense nonblocking
Double-Hub	A, dynamic	rearrangeably nonblocking
PPWDM	B, dynamic	wide-sense nonblocking
Hierarchical	B, dynamic	wide-sense nonblocking
Incremental	B, incremental	wide-sense nonblocking

To simplify the comparison we will also assume that N is a power of two.

Note that the costs of the fully optical ring have already been presented for the static uniform traffic. We will proceed to present the costs of the other five of the OADM rings in the following order: single-hub, double-hub, PPWDM, hierarchical, and incremental.

For the single-hub and double-hub rings, the costs W and Q are a function of $(t_A(i)\colon i=0, 1, \cdots, N-1)$. For the static uniform traffic, $t_A(i)=u_A(g)$ for all nodes i, where

$$u_A(g)=\left\lceil \frac{g(N-1)}{c} \right\rceil \tag{9}$$

because each node terminates $g(N-1)$ traffic streams. Thus, we have the following costs.

Single-Hub Ring:

$$\begin{aligned} W &= \left\lceil \frac{u_A(g)\cdot(N-1)}{2} \right\rceil \\ Q &= 2u_A(g)(1-(1/N)) \\ H &= (N/2). \end{aligned}$$

Double-Hub Ring: Assuming the hubs are 0 and $N/2$,

$$\begin{aligned} W &= \lceil u_A(g)(N/4)\rceil \\ Q &= \frac{1}{N}\left(4\left\lceil \frac{N}{4}u_A(g)\right\rceil + (N-2)2\left\lceil \frac{u_A(g)}{2}\right\rceil\right) \\ H &= (N/2). \end{aligned}$$

For the PPWDM, hierarchical, and incremental rings, the traffic streams should satisfy Traffic Assumption B. We will assume that the pre-routes of the traffic streams are shortest hop paths. Next we determine appropriate values for $\{t_B(i)\colon i=0, 1, \cdots, N-1\}$ and L.

Note that since N is even, nodes at opposite ends of the ring (e.g., arbitrary node i and node $(i+(N/2))\bmod N$) have two shortest paths, each on opposite sides of the ring. We will assume that the traffic streams between them are split as evenly as possible among the two paths. Thus, one path will have $\lceil g/2\rceil$ streams, while the other will have $\lfloor g/2\rfloor$ streams. Then the node requires $u_B(g)$ transceivers on each of its links just to terminate its traffic streams, where

$$u_B(g)=\left\lceil \frac{1}{c}\left(\frac{g\cdot(N-2)}{2}+\lceil g/2\rceil\right)\right\rceil. \tag{10}$$

Therefore, we set $t_B(i)=u_B(g)$ for all nodes i.

Also note that the average number of hops of a traffic stream is $\mathcal{H}_{\rm Avg}=(N+1)/4+(1/4(N-1))$. Note that for arbitrary N

$$\mathcal{H}_{\rm Avg}=\begin{cases} \dfrac{N+1}{4}, & N \text{ odd} \\ \dfrac{N+1}{4}+\dfrac{1}{4(N-1)}, & N \text{ even.} \end{cases}$$

Thus, the average number of traffic streams going through a link is at least

$$\begin{aligned} \ell(g) &= \frac{\mathcal{H}_{Avg}\times \text{ Total traffic}}{\text{Number of links}} \\ &= \frac{\mathcal{H}_{Avg}\times \frac{1}{2}\sum_i\sum_j c\cdot T(i,j)}{N} \\ &= g(N-1)\left(\frac{N+1}{8}+\frac{1}{8(N-1)}\right). \end{aligned}$$

Therefore, we can set

$$L=\left\lceil \frac{\ell(g)}{c}\right\rceil. \tag{11}$$

Then we have the following costs.

PPWDM Ring:

$$\begin{aligned} W &= \lceil(\ell(g)/c)\rceil \\ Q &= 2\lceil(\ell(g)/c)\rceil \\ H &= 1. \end{aligned}$$

Hierarchical Ring:

$$\begin{aligned} W &= \lceil(\ell(g)/c)\rceil+(\alpha-1)u_B(g) \\ Q &= 2(\alpha-1)u_B(g)+(2/N)\lceil(\ell(g)/c)\rceil\lceil(N/\alpha)\rceil \\ H &= \alpha \end{aligned}$$

Incremental Ring:

$$W=\lceil(\ell(g)/c)\rceil.$$

To determine the other costs Q and H, we assume that the bisecting nodes for each subnet is exactly in the middle because this will minimize Q. Thus, all subnets have *length* (i.e., length of its segment) that are powers of two. If a subnet's length is 2^i (for some i) then its bisecting node terminates $\min\{W, u_B(g)(2^i-1)\}$ wavelengths. Let J be the largest value such that $W > u_B(g)(2^J-1)$. Then a lightpath can pass through a subnet of length 2^J without going through an intermediate transceiver. However, a lightpath cannot make such a pass through a subnet of length 2^{J+1} because the bisecting node terminates all W wavelengths. Thus

$$H=2^J$$

and we also have the simple upper bound

$$H\le\lfloor(W/u_B(g))+1\rfloor.$$

Finally, note that

$$Q=\frac{1}{N}\left(2W+\sum_{i=0}^{\log_2 N-1}2^i\cdot 2\min\left\{W, u_B(g)\left(\frac{N}{2^i}-1\right)\right\}\right)$$

TABLE II
ASYMPTOTIC RELATIVE COSTS FOR DIFFERENT OADM RING NETWORKS ASSUMING THE STATIC UNIFORM TRAFFIC WITH PARAMETER g, AND N IS LARGE AND A POWER OF TWO

	Asymptotic Relative Costs		
	$W/W_{\min}$	$Q/Q_{\min}$	$H/H_{\min}$
Fully Optical	$\frac{\lceil g/c \rceil}{g/c}$	$\frac{\lceil g/c \rceil}{g/c}$	$\frac{N}{2}$
Single-Hub	4	2	$\frac{N}{2}$
Double-Hub	2	2	$\frac{N}{2}$
PPWDM	1	$\frac{1}{4}N$	1
Hierarchical ($\alpha = \sqrt{N/4}$)	1	$N^{0.5}$	$\frac{1}{4}N^{0.5}$
Incremental	1	$2\log_2 N$	$\frac{N}{4}$

since the root node terminates all W wavelengths, and there are 2^i subnets of length $(N/2^i)$. We also have a simple upper bound for Q:

$$Q \leq \frac{1}{N}\left(2W + \sum_{i=0}^{\log_2 N-1} 2^i \cdot 2u_B(g)\frac{N}{2_i}\right) \leq 2\left(\frac{W}{N} + u_B(g)\log_2 N\right). \quad (12)$$

To simplify the comparison of the six OADM ring networks, we provide Table II which has *asymptotic relative costs* assuming N is large. The *relative costs* are a ratios $W/W_{\min}$, $Q/Q_{\min}$, and $H/H_{\min}$, where $W_{\min} = \lceil \ell(g)/c \rceil$, $Q_{\min} = \tau_A(g)$, and $H_{\min} = 1$. Note that $W_{\min}$, $Q_{\min}$, and $H_{\min}$ are simple lower bounds for the costs W, Q, and H, respectively. The asymptotic relative costs given in the table are approximate because small terms are ignored for large N. For example, for the hierarchical ring, $W/W_{\min}$ is larger than one, but the entry in the table is 1 because $\lim_{N\to\infty} W/W_{\min} = 1$. Also note that for the hierarchical ring, we assume that α is approximately $\sqrt{(N/4)}$ $(\approx \sqrt{(L/\tau_B(g))})$, which minimizes Q.

Based upon Table II, we draw the following conclusions:

1) If wavelengths are plentiful, then the single-hub ring is a good choice, since it has low transceiver cost and can support dynamic traffic. The double-hub ring is a good choice if the traffic is static (and not necessarily uniform), since it requires only half the number of wavelengths and has about the same transceiver cost.
2) If wavelengths are precious then the PPWDM, hierarchical, and incremental rings are reasonable choices for OADM ring networks, since they use minimal wavelengths. The PPWDM ring provides the most efficient use of wavelengths for dynamic traffic. If there are some spare wavelengths then the hierarchical ring can potentially reduce the transceiver cost. If the traffic is static (and not necessarily uniform) or incremental, then the incremental ring is a good choice, since it minimizes wavelengths and has low transceiver cost.

An interesting point is that the fully optical network has the smallest transceiver cost in the range $(c/2) < g \leq c$. For this range, each pair of nodes has at least half a lightpath worth of traffic between them. For smaller values of g, the single-hub and double-hub rings have lower transceiver cost.

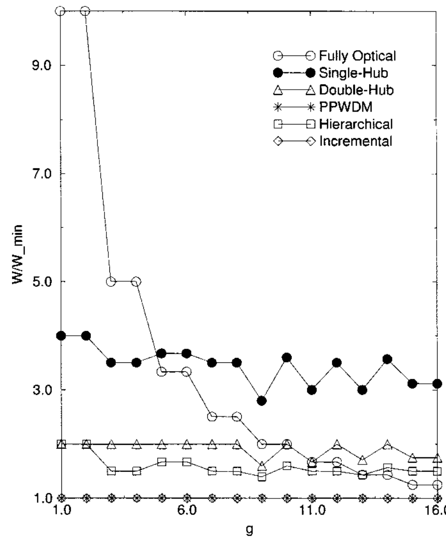

Fig. 12. Relative cost $W/W_{\min}$ for $N = 8$, $c = 16$, and $g = 1, 2, \cdots, 16$.

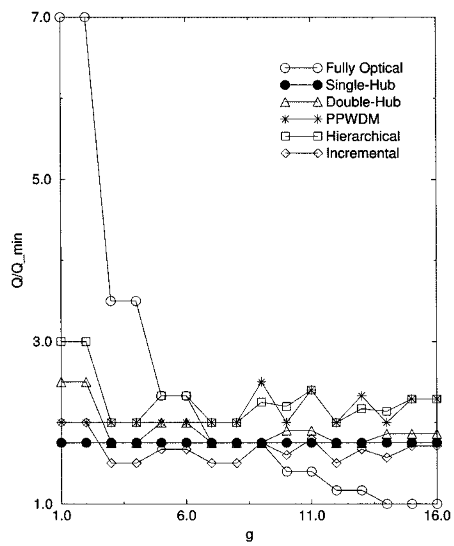

Fig. 13. Relative cost $Q/Q_{\min}$ for $N = 8$, $c = 16$, and $g = 1, 2, \cdots, 16$.

Note that Table II is based on the unrealistic assumption that N is very large. However, Figs. 12 and 13 show $W/W_{\min}$ and $Q/Q_{\min}$ values, respectively, for the more realistic value of $N = 8$. In the figures, $c = 16$ (e.g., traffic stream = OC-3, lightpath = OC-48), g ranges from 1 to $c = 16$, and it is assumed that the hierarchical ring has $\alpha = 2$. Fig. 12 shows that the PPWDM and incremental rings have the optimal W. Fig. 13 shows that minimum Q is attained by different networks for different values of g. For $g = 1$ and 2, the single-hub ring has the smallest Q. For $g > c/2$, the fully optical ring has the smallest Q. Surprisingly, for $2 < g \leq c/2$, the incremental ring has the smallest Q Thus, for small to moderate values of g (which is where traffic grooming is most interesting), the incremental ring attains the minimum or is near minimum for both Q and W. In fact, its improvement in Q over the PPWDM ring can be significant. For example, in the case $g = 4$, the incremental ring has a Q that is 25% smaller than the one for the PPWDM ring. To translate this into dollars, suppose $W = 32$ and the cost

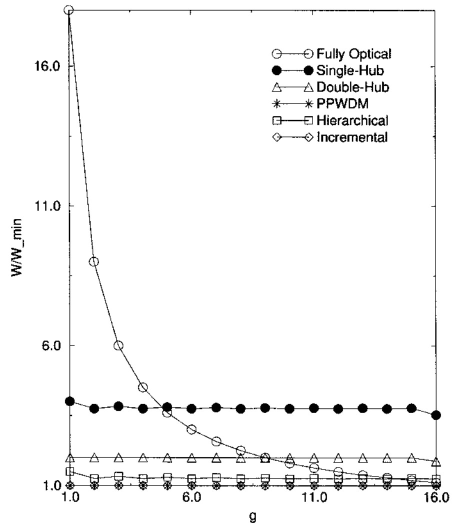

Fig. 14. Relative cost $W/W_{\min}$ for $N = 16, c = 16$, and $g = 1, 2, \cdots, 16$.

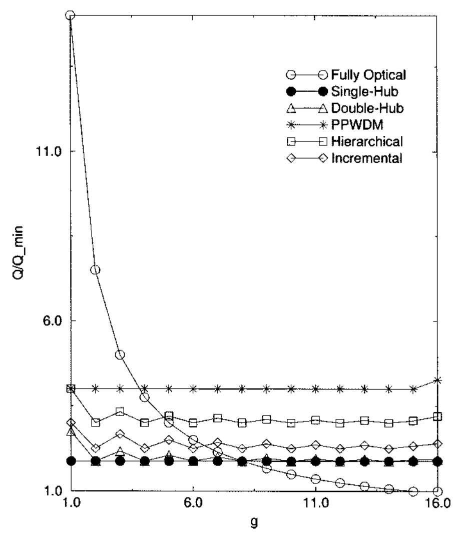

Fig. 15. Relative cost $Q/Q_{\min}$ for $N = 16, c = 16$, and $g = 1, 2, \cdots, 16$.

of a "transceiver" is \$75 K, which includes half-an-ADM (\$50 K), a transponder (\$20 K), and a WDM cost per wavelength per terminal (\$10 K). Then a node in the PPWDM ring will cost $2 \times W \times$ \$75 K = \$4800 K, and a 25% cost savings translates to \$1200 K.

Notice that the hierarchical ring has higher cost than the PPWDM ring for all values of g. This is due to the N being a relatively small value of 8. Small N means small average hop lengths for traffic streams (average hop length of approximately 2.3), which in turn implies low amounts of transit traffic. For larger N, the hierarchical ring will have better cost than the PPWDM ring. Figs. 14 and 15 show $W/W_{\min}$ and $Q/Q_{\min}$ values, respectively, for $N = 16$. Here, $c = 16$ and $\alpha = 2$ as before. Now the transceiver cost Q for the hierarchical ring is smaller by about 25% than for the PPWDM ring. The Q for the incremental ring is 44% smaller than for the PPWDM ring for even values of g. Note if a PPWDM node costs \$4800 K, then a 44% savings translates to \$2112 K.

IV. Conclusion

We have proposed and analyzed a number of OADM ring networks. At one extreme is the single-hub ring that requires large amounts of bandwidth (wavelengths) but has small transceiver cost. At the other extreme is the PPWDM ring that requires minimal bandwidth (wavelengths) but has maximum transceiver cost. In the middle we have the hierarchical ring that provides a trade-off between numbers of wavelengths and transceiver costs. Also in the middle, we have the double-hub and incremental rings. These last two do not support fully dynamic traffic, but seem to be reasonable solutions for static nonuniform traffic, and in the case of the incremental ring can support incremental traffic.

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