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Adversarial Risk Analysis

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Context

This is the final report for the Institute for Homeland Security Solutions (IHSS)-funded project on Bayesian Game Theory. David Banks of the Department of Statistical Science at Duke University was the principal investigator; the only other person who received support on this grant was Juan Vivar, a postdoctoral research fellow in the same department. The project began on February 26, 2010, and terminated on February 28, 2011.

Study Objectives

The narrow objectives of the research were to

- develop theory and analysis for the two-person first-price sealed bid auction game;
- solve the Borel game in fairly significant generality; and
- provide a template analysis of the smallpox counterterrorism decision of 2003, in sufficient detail to serve as a model for federal contractors in risk management.

The larger objective of this research was to establish an alternative to game theory, and show that the new approach is both practical and better able to exploit realistic information in the decision process.

Research Description

Classical two-person game theory is based on the solution concept of minimaxity, in which one chooses the action that minimizes the maximum loss one can suffer. This leads to equilibria, because one's opponent will choose the action that maximizes the minimum damage. The method extends to mixed strategies when there are multiple equilibria, games with more than two players, and games with certain kinds of random structure (cf. Myerson, 1991). The key computational tool is linear programming.

But classical game theory is a poor guide to human behavior (Camerer, 2003). In particular, humans do not use game-theoretic calculation in most situations, but they often find solutions that give them better payoffs. Also, humans may have access to probabilistic information, and except under stringent and usually unrealistic assumptions (Harsanyi, 1967a,b,c), this cannot be used in game theory.

To address such deficiencies in game theory, several researchers proposed decision analysis (Kadane & Larkey, 1982; Raiffa, 1982). Decision analysis uses an alternative solution concept: one should choose the action that maximizes one's expected utility. The difficulty with this approach is that the analyst must somehow develop a probability distribution on the actions of the opponent to calculate expectations. Previous work in decision analysis provided no guidance on how to obtain such a distribution.

This IHSS-funded research project concerns adversarial risk analysis (ARA), a branch of decision analysis in which the required distribution is found by constructing a model of the decision processes of one's opponent. For a given model, the analyst imposes subjective priors on all unknown quantities, and uses these to derive the distribution on the actions of the opponent. The approach is rather more natural than it sounds; it accords with the kind of level-k thinking studied by Stahl and Wilson (1995), in which opponents think a few moves ahead and choose the action that is optimal with respect to that horizon.

Results

The direct outcomes of this research project are appearing in several publications and in a draft monograph that will soon be submitted for publication. In terms of the research objectives, we have developed the following:

- A new analysis of two-person first-price sealed bid auctions, in which the strategic uncertainties about the opponent's bid are determined entirely by subjective uncertainty about the opponent's value for the item and what the opponent believes is the analyst's value for the item. On fundamental informational grounds, this property is a requirement for a Bayesian solution; it is not a property possessed by the traditional game theory solution. (In traditional game theory, the minimax solution is to bid one's full value.)

  The auction results are being written up as a chapter in the monograph. A shorter account appears in our rejoinder to the discussants of the Borel game paper that has been accepted by *Applied Stochastic Models in Business and Industry.*

- A new solution concept for the Borel game. The Borel game has been addressed by brilliant mathematicians and economists, including Emile Borel, John von Neumann, Oscar Morgenstern, David Blackwell, Richard Bellman, Samuel Karlin, and Thomas Ferguson. These researchers encountered enormous difficulties in extending the simple Borel game to cases that allow multiple players, continuous bet sizes, and any situation other than independent draws from a uniform distribution. Our solution concept handles these extensions easily, while also providing a natural model for the kind of strategic thinking that real players use in sizing up their opponents.

  The primary paper on the Borel game will appear as a discussion paper in the journal *Applied Stochastic Models in Business and Industry*. The discussants are Joseph Kadane at Carnegie Mellon University and Nicholas Polson at the Graduate School of Business of the University of Chicago. The paper contains a full solution to the Borel game, and the rejoinder contains a partial analysis of the two-person first-price sealed bid auction.

- A model solution to the smallpox counterterrorism problem that the United States considered in 2003. That solution uses the adversarial risk analysis perspective to solve the classical normal form game in which the players have asymmetric private information and the outcomes for each pair of actions is random. It provides an alternative to the Harsanyi Bayes Nash equilibrium, and avoids its unreasonable

assumption that all players have the same subjective distribution over all random events.

This solution is a draft chapter for the monograph, and a revised version of it may be written up as a paper that will be submitted to *Statistics, Politics and Policy.*

The ideas developed with IHSS support have also been disseminated through invited presentations on ARA at Georgia Tech, the Naval Postgraduate School, the Probability and Statistics Conference, the King Juan Carlos University, the International Statistics in Business and Industry Conference, the Conference on Algorithmic Decision Making (sponsored by DIMACS at Rutgers), Pennsylvania State University, the Army Conference on Applied Statistics, Yale University, the IHSS Research Summit, the Cincinnati Chapter of the American Statistical Association, Duke University, the INFORMS conference at Monterey, the University of California at Davis, the University of California at Berkeley, and the Los Alamos National Laboratory. Other presentations are scheduled.

The important indirect outcome is that ARA appears to be attracting favorable attention. Besides the publications listed, two papers on ARA have been written by researchers who are unconnected to the group with whom David Banks works. Also, the Banks group has an ARA paper on convoy routing that will appear in *Naval Research Logistics* (funded by a DARPA grant), and there is a monograph on ARA that is about half finished.

Implications

The primary implication is that this grant has helped to launch a new solution concept in the literature on strategic analysis. This concept appears to have substantial advantages in terms of applicability and plausibility for the kinds of scenarios that arise in counterterrorism, and offers interesting alternatives to minimax solutions for well-known games. Whether this ARA approach will ever be adopted as a decision tool by Department of Homeland Security managers is still an open question—it will depend on how credible its logic seems, and how the research community responds to this work.

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