

An analytical model for IEEE 802.15.4 non-beacon enabled CSMA/CA in multihop wireless sensor networks

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ABSTRACT

In this paper we propose a new analytical model for the IEEE 802.15.4 non-beacon enabled CSMA/CA. Previous models often assume that the probability of channel busy assessment is independent of the back-off stage. We show that this condition does not hold for the IEEE 802.15.4 and propose a modeling approximation. In our analysis, we consider multihop topologies and unsaturated traffic. We discuss the challenges imposed by these conditions and propose efficient and effective modeling strategies. The main objective of the model is to derive robust estimates for the probability of channel access failures in multihop topologies. The accuracy of the model is confirmed by comparing the predicted values against the results obtained through extensive network simulations in a wide range of scenarios considering different network topologies, number of nodes, and traffic loads.

1. INTRODUCTION

The IEEE 802.15.4 [1] has been proposed as the standard for low-rate, low-power Wireless Personal Area Networks (PANs). In recent years, its use has become widespread mainly due to the emergence of Wireless Sensor Networks (WSNs). The 802.15.4 standard defines protocols for the physical (PHY) and medium access control (MAC) layer functionalities. The standard defines two channel access mechanisms: *beacon* and *non-beacon* enabled. Both are based on carrier sense multiple access with collision avoidance (CSMA/CA). The beacon-enabled mode requires a *network coordinator*, that at regular intervals sends beacon messages for synchronization and network association. The non-beacon enabled mode is based on an *unslotted* CSMA/CA, which does not require the transmission of beacons. Therefore, in this case, the communication among the nodes occurs in a fully ad-hoc decentralized way. In this paper, we focus on the non-beacon enabled mode, which defines a more general peer-to-peer communication paradigm and does not require any specific network organization.

Typically, nodes using a CSMA-based MAC protocol must perform a clear channel assessment (CCA) before initiating a radio transmission. The wireless medium must be detected as idle in order to proceed to the transmission phase. In general, CSMA protocols can be classified as *non-persistent* or *p-persistent* [14]. If the channel is detected busy, a *p-persistent* CSMA protocol continues to monitor the medium until it becomes idle, while a *non-persistent* CSMA protocol schedules a new retransmission attempt at some point into the future. The IEEE 802.15.4 non-beacon enabled

MAC is a non-persistent CSMA/CA protocol which implements a random backoff scheme to prevent that nodes within the same radio range start their transmissions at the same time. Since a CSMA/CA based MAC requires detecting the medium as idle before initiating a transmission, an implicit competition among the nodes is established to access the wireless channel. In the non-persistent CSMA/CA protocol implemented by the 802.15.4 beaconless MAC, for each packet to send a limited number of retransmission attempts are executed. If the medium is detected as busy in all the attempts, the protocol fails to guarantee access and packet drops may occur. Therefore, since the 802.15.4 MAC protocol can be the main source of communication failures, it represents a factor that strongly affects the effective quality of a wireless link. The availability of an accurate MAC model allows to predict the *probability of channel access failures*, which constitutes in turn an important tool for network designers and users to assess and analyze core network performance metrics such as reliability, throughput, and delay. In this work, we propose a novel analytical model to study the behavior of the 802.15.4 beaconless MAC and derive robust estimations for the probabilities of channel access failure in *multihop networks*. Our model is of general applicability, since no specific assumptions are made regarding traffic generation and network topology.

A large portion of the analytical models proposed so far for CSMA/CA based MAC protocols are derived from the model proposed by Bianchi [2]. In seminal his work, Bianchi performs a *Markov Chain analysis* to estimate the saturation throughput of a network using the IEEE 802.11 DCF MAC protocol. Given the good prediction performance of Bianchi's model and the similarities between IEEE 802.15.4 and IEEE 802.11 DCF access mechanisms, many studies on the IEEE 802.15.4 have followed Bianchi's footsteps. We also follow a similar approach, extending previous work and addressing some of the main weaknesses and limitations of existing models. In the following, we briefly point out these issues and introduce the main contributions of this paper. A detailed discussion of related work is provided in Section 2.

Most of the previous work modeling MAC behavior assumes that the probability of detecting the medium as busy during a CCA is independent of the backoff stage. Unfortunately, this does not hold for the IEEE 802.15.4 [16], and it can result in an important source of inaccuracy in the models that make this assumption. In this paper, we employ a Markov Chain analysis to model the channel access mechanism for a single node. In the model, the probability of channel busy assessment at the end of each backoff stage is

not considered as constant, but is calculated as a function of the network traffic and the backoff window size.

Although the channel access mechanism in the non-beacon enabled 802.15.4 MAC is fully decentralized and can be used in generic ad-hoc multihop networks, most of the previous modeling work has focused on single-hop networks with star topology. In these networks, nodes are inside each other's transmission range. That is, they have a common channel view. In contrast to this, in multihop topologies channel occupation is perceived differently by different nodes, due to the presence of hidden terminals [26]. In our work, we explicitly take into account the multihop nature of the network: for each node, the state transitions of the Markov chain model depend on its local connectivity degree and on the contention among its neighbors.

Another aspect which can be used to differentiate among the various analytical models proposed for CSMA/CA-based protocols regards network traffic assumptions. A common approach is to assume that the network is operating in saturation: at the end of each transmission a node always has a new packet available to transmit. This assumption allows to carry out maximum throughput studies. In unsaturated networks, more realistically, a node can also be in an idle state, during which it is not attempting to transmit a packet nor performing the CSMA/CA algorithm. We consider the case of unsaturated traffic, by modeling nodes' traffic generation by independent Bernoulli processes. Following current literature [8], we let packet generation occurring only during node's idle state, according to a probability that depends on the defined traffic rate.

In summary, in this paper, we propose a novel analytical model for the IEEE 802.15.4 MAC non-beacon enabled version. We consider wireless sensor networks where each node can communicate to any of its neighbors without any assumptions about network topology. Our model also contemplates unsaturated heterogeneous traffic. In the analysis, we focus on the probability of MAC failures, that is, the probability of a failed attempt to get access to the channel for packet transmission. By means of a discrete Markov chain, we perform a decoupled analysis, where we first model the MAC channel access procedure for a single node, including the dependence on the backoff stage of the probability of a channel busy assessment. As a second step, for all the nodes in the network, we consider the relations between the node and its contending neighbors, obtaining a coupled nonlinear multivariate system which allows to compute the probabilities of MAC failure for each single node taking into account network's structure. The proposed model does not consider retransmissions after MAC failures or packet acknowledgements; however, it can be easily extended to include these elements.

The rest of the paper is organized as follows. In Section 2 we discuss related work on analytical modeling of the IEEE 802.15.4 MAC. Section 3 briefly summarizes some of the characteristics of the 802.15.4 that are of interesting for this work. In Section 4 our analytical model is presented, introducing first the model for the individual node, and then discussing the extension to the multihop case. The characteristics of the experimental setup and the results of the experiments for model validation are reported in Section 5. Finally, in Section 6 we summarize our work and the obtained results, and we discuss future extensions and applications of the model.

2. RELATED WORK

A number of different analytical models for the channel access mechanism of the IEEE 802.15.4 have been proposed so far. The large majority of these models consider the beacon enabled version of the protocol, while the non-beacon mode has attracted much less attention. Although significant differences exist between the two versions of protocol, the methodology adopted for the analysis for both modes share some similarities. Therefore, we first briefly overview the modeling assumptions of the most relevant work on the beacon enabled mode and then we discuss relevant work for the non-beacon mode.

In [22, 20, 23] different Markov chain models based on Bianchi's work are proposed to analyze the performance of *beacon-enabled* protocols. In [22], the model assumes saturated traffic conditions, while in [20] the authors consider bidirectional and unsaturated traffic. In [23] both saturation and unsaturated traffic conditions are considered. While previous work mostly considered acknowledged traffic, the analysis in [23] also includes the situation in which acknowledgement messages are not sent out. Also in [25, 24] acknowledgements are not used. Finally, the model proposed in [21] considers unsaturated and acknowledged traffic, as well as packet retransmissions after a channel access failure.

For the *non-beacon mode*, most of the works consider *single hop* networks [15, 4, 5, 13, 12, 7, 16]. In [15], the authors perform a mathematical and simulation analysis to evaluate the maximum achievable bandwidth efficiency of the 802.15.4 for the special case of one single sender-receiver pair. In [12], a Markov chain based model is proposed to study different performance measures. In the model, two consecutive CCAs are used to assess channel status, instead of one, as specified by the standard. The authors have considered unsaturated traffic, and modeled the arrival of new packets as a Bernoulli process occurring during nodes' idle states. No experimental validation of the results is provided. In [13], the model is based on the busy cycle of an M/G/1 queueing system. The approach assumes unsaturated traffic in which packet generation follows a Poisson process. The work in [7] proposes a Markov chain based model to derive formulas for the calculation of throughput and energy consumption. Similarly to [12], the authors assume that the channel must be sensed twice at the end of each backoff stage in order to assess its status, which is not compliant with the standard. The unsaturated traffic load is modeled by a Bernoulli process happening during node's idle state. In [4, 5], the authors assume that nodes only send packets upon a request from a sink. The analysis is based on this assumption. In our model we consider unsaturated traffic, in which nodes are able to transmit packets at any time. We assume that packet arrivals occur only during the idle state, and packet inter-arrival times follow a specific distribution. Similar assumptions have also been taken in [12, 13, 8, 21].

In the large majority of the models proposed so far for the 802.15.4 non beacon, the probability of a channel busy assessment, when a node is performing a CCA, is considered as constant for all backoff stages, which does not reflect the real behavior of the channel access mechanism. This assumption was also present in the original work of Bianchi. In [16], the authors have pointed out that the probability of channel busy assessment varies with the backoff stage and have included this aspect in the model. Their analysis is however limited to networks under saturation throughput.

In our work, we also model the dependence of the probability of busy assessment on the backoff stage but following a different approach. We propose an approximation which takes into account several aspects of the node's local environment.

For *multihop networks*, several analytical studies has been proposed for CSMA-based protocols. One of the first works proposed a general approach for analyzing the throughput performance of a multihop CSMA network [3]. The model is based on a continuous-time Markov chain, where at each time instant the state is represented by the set of currently transmitting nodes. The proposed model is general, in the sense that it cannot be directly applied to most CSMA based protocols: it needs to be adapted to the specific characteristics of the protocol under consideration. The large majority of studies for multihop CSMA-based networks have considered the case of the IEEE 802.11 DCF protocol. However, in order to make the models more amenable to mathematical analysis, many different simplifying assumptions have been adopted. In [28, 29], analytical models for the IEEE 802.11 DCF are proposed assuming that the nodes are spatially distributed according to a particular distribution. In [27], the authors have proposed a throughput model based on the work of [3] using the same assumptions of Poisson distributed packet arrivals and exponential packet lengths. Moreover, some important aspects of the backoff mechanism are omitted. For the IEEE 802.15.4 protocol, only a few works have considered the case of multihop networks. In [19], an analytical model for the beacon enabled version is proposed taking into account the impact of finite size buffers at the senders, and the use of relay nodes to form a cluster-tree network. Relay nodes forward traffic from other nodes but do not generate new packets. To the best of our knowledge, the work of Di Marco et al. [8] is the only study addressing the modeling of the IEEE 802.15.4 non-beacon enabled protocol in multihop networks. Although our analytical model shares some similarities with theirs, there are some key differences between the two approaches: (i) we identify and model the dependency on the backoff stage of the busy assessment probability of the channel access mechanism, while they assumed that this probability is homogeneous for all backoff stages; (ii) our modeling for the multihop characteristics of the network is different from theirs; and (iii) we perform an evaluation of the estimate of the probability of MAC failures under a wide range of scenarios, in contrast their evaluation is quite limited and done is considering performance metrics depending on the probability of MAC failures (e.g., path reliability), which can hide inaccuracies of the 802.15.4 MAC model.

In order to study the performance of an 802.15.4 network, it is important to consider and estimate packet losses after a node has successfully accessed the channel. Packet losses may occur due to interference, radio propagation aspects, and hidden terminals, among others. In most of the cited works focusing on performance analysis, a lossless channel is assumed. More realistic models of packet losses estimation in wireless networks can be found, for instance, in [11, 6]. In our work, we do not attempt to estimate the probability of successful reception of transmitted packets nor the occurrence of packet losses; we focus on the first stage of a transmission process, that is, the probability of the successful completion of the CSMA/CA mechanism at the MAC layer. Therefore, we do not take into account any form of packet losses that may occur during or after a transmission.

3. THE IEEE 802.15.4 MAC

As we stated in the Introduction, in this work we consider the *non-beacon enabled mode* of the IEEE 802.15.4 standard [1], which relies on an *unslotted CSMA/CA* protocol. Two are the main variables regulating the behavior of the channel access algorithm: BE , which is the current *backoff exponent*, and NB , which is used to count the number of backoffs. Each time a node generates a packet for transmission, it waits a random number of *backoff slots* ranging from 0 to $2^{BE} - 1$. Initially, BE is initialized to BE_{min} (by default 3) and its maximum value is BE_{max} (by default 5). The variable NB is set to 0 at the beginning. After waiting the selected amount of time, the node performs a *clear channel assessment (CCA)* to determine whether the channel is busy or not. If the channel is idle for an amount of time T_{CCA} , the procedure terminates, the channel is considered free and the node proceeds with data transmission. When the channel is perceived as busy, BE is increased by one unit (if $BE < BE_{max}$), the backoff counter is also incremented, $NB = NB + 1$, and the procedure is repeated until $NB <= m$. After $m + 1$ unsuccessful attempts to access the channel, the procedure fails and the packet is discarded.

The 802.15.4 MAC protocol can handle the transmission of acknowledgements (*ACK mode*) to indicate the successful reception of a packet. However, this functionality is optional. In many applications, the use of packet acknowledgements may limit network throughput since it determines an increase in communication overhead. Unacknowledged communications (*NACK mode*) can be used to achieve energy conservation, or when the number of sensor nodes is redundant with respect to the sensing task at hand, or when packet losses are tolerable to a certain extent. In our analysis, we consider the behavior of the 802.15.4 MAC protocol operating in *NACK mode*. This modality has been considered in a limited number of other works (e.g., [25]). The extension of the model to include also the *ACK mode* will be part of future work.

4. ANALYTICAL MODEL

As described in the previous section, the CSMA/CA mechanism of a node fails to guarantee access to the wireless channel if the channel is detected busy after a maximum number of backoffs. Our objective is to estimate the probability that these failures occur, denoted in the following with P_{Fail} . Given that our analytical model includes the case of *multihop networks*, we need to take into account that the wireless channel perception might be different for each node in the network. That is, the probability of detecting the channel as busy may vary for each node depending on its local environment. Moreover, the probability of a busy channel assessment for a node n also depends on the *backoff stage* i . In the following, we indicate this probability with $\alpha_i^{(n)}$. Since the CSMA/CA channel access mechanism fails at node n when it senses the channel busy after all the backoff stages, we define $P_{Fail}(n)$, the overall *probability that the channel is detect busy at node n* , as:

$$P_{Fail}(n) = \prod_{i=0}^m \alpha_i^{(n)}. \quad (1)$$

Therefore, if we can estimate the values of $\alpha_i^{(n)}$ we can also estimate the values of the \mathbf{P}_{Fail} vector, for all the nodes n in the network. In a CSMA/CA protocol, a node will assess

the channel as busy when it detects an on-going transmission from any other node in its surroundings. Therefore, a channel busy assessment depends on the probability that other nodes are performing a transmission, or, in another words, on any other node which is in a transmission state.

We build an analytical model to calculate the \mathbf{P}_{Fail} vector following a two-stage approach. As a first modeling stage, we make use of a discrete time Markov chain (DTMC) to model the CSMA mechanism of the 802.15.4 non-beacon MAC for an individual node. Several discrete Markov chain models have been proposed for the IEEE 802.15.4 MAC, for both the beacon and non beacon versions. In comparison to other approaches, DTMC models are characterized by their simplicity and tractability [10]. Our Markov chain analysis is similar to that followed in [21, 8, 23]. However, we extend and enhance the characteristics of the proposed Markov chain models with the aim of relaxing some of the critical assumptions that have been done in these previous works. We consider generic, unsaturated traffic generation at the nodes. Traffic generation is modeled in a way similar to [8], using a Bernoulli process. This approach well suits the discrete time scale of the Markov chain, but assumes that the nodes do not generate new packets during packet transmissions or during the node backoff procedure. Through this first modeling stage, we will obtain an expression for the probability that a node is in a transmission state, defined in terms of the probabilities of channel busy assessment.

In the second modeling stage, we consider the reciprocal influence among all the nodes in the network due to their use of the CSMA/CA mechanism to access the shared wireless channel. The analysis of this reciprocal influence takes into account the different channel perception that different nodes have across the network due to its multihop nature. Contrary to single hop networks, where nodes are not able to transmit simultaneously, in multihop networks simultaneous transmissions can be detected from surrounding nodes. This condition makes the study of multihop networks more challenging compared to single-hop ones. The objective of the second modeling stage, is to define an expression for the channel busy assessment probability of a node n in terms of the transmission state probabilities of its neighbor nodes.

Combining the results of both modeling stages, we will finally obtain a non-linear equation system which we solve to obtain an accurate estimate of the \mathbf{P}_{Fail} vector.

4.1 Markov chain for single node modeling

The states and the transitions of our *Markov chain model* of the CSMA/CA behavior at a given node are shown in the diagram of Figure (1). They are adapted from the model proposed in [21] for the beacon enabled version. We redefined the state transitions to match that we are considering the non-beacon version of the protocol. All probabilities and durations are measured in units of backoff time slots, which correspond to the duration of one single backoff slot.

At any time a node can be in one of the three states: an *idle state*, where the node's MAC layer is waiting for new packets to be sent and is ready to initiate the channel access procedure, a *backoff state*, during which the node is contending for channel access against the nodes in its neighborhood, a *transmission state*, during which the node is performing packet transmission. We use the following notation: W_i is maximum duration of backoff stage $i \in \{0, \dots, m\}$, $W_i = \max(2^{BE_{max}}, 2^{BE_{min}+i})$, m represents the *maximum*

number of backoffs, and P_s is the *time duration of a packet*, expressed in backoff time slot units, $b_{i,j}$ indicates the *backoff state* of a node, where $i \in \{0, \dots, m\}$ represents the *backoff stage*, and $j \in \{0, \dots, W_i - 1\}$ is the *backoff counter*. With t_k we indicate a *transmission state*, where k is the current *packet slot* being transmitted ($k \in \{0, \dots, P_s - 1\}$). Finally, $X(t)$ represents a *stochastic process* such that:

$$X(t) = \begin{cases} b_{i,j} & i \in \{0, \dots, m\}, j \in \{0, \dots, W_i - 1\} \\ idle & \\ t_k & k \in \{0, \dots, P_s - 1\} . \end{cases}$$

$X(t)$ is modeled by the discrete time Markov chain depicted in Figure 1. For sake of notational simplicity, we shorten $P(X(t) = a \mid X(t-1) = b)$ as $P(a \mid b)$. The following transition probabilities are defined:

$$P(b_{i,j} \mid b_{i,j+1}) = 1, \quad j \geq 0 \quad (2)$$

$$P(b_{i+1,j} \mid b_{i,0}) = \frac{\alpha_i}{W_{i+1}}, \quad i < m, j < W_{i+1} \quad (3)$$

$$P(idle \mid t_0) = 1 - q \quad (4)$$

$$P(idle \mid b_{m,0}) = (1 - q)\alpha_m \quad (5)$$

$$P(idle \mid idle) = 1 - q \quad (6)$$

$$P(t_{P_s-1} \mid b_{i,0}) = 1 - \alpha_i \quad (7)$$

$$P(b_{0,j} \mid idle) = \frac{q}{W_0}, \quad j < W_0 \quad (8)$$

$$P(b_{0,j} \mid b_{m,0}) = \frac{q\alpha_m}{W_0}, \quad j < W_0 \quad (9)$$

$$P(b_{0,j} \mid t_0) = \frac{1 - q}{W_0}, \quad j < W_0 \quad (10)$$

$$P(t_i \mid t_{i+1}) = 1, \quad i < P_s - 1 \quad (11)$$

Equation (2) represents the decrement of the backoff counter during a backoff stage. Equation (3) denotes the transition between consecutive backoff stages after a busy channel assessment. Equations (4–6) represent the transition to an idle state, that can happen: (i) after the end of a packet transmission, (ii) following a packet drop due to MAC access failure, or (iii) from a previous idle state, if there are no new packets available for transmission. Equation (7) represents the initiation of a packet transmission after a channel clear assessment. Equations (8–10) indicate the start of the backoff procedure when a new packet is available for transmission. Equation (11) represents the transmission of a packet, whose length is discretized in backoff time slot units.

We denote the *steady state probabilities* of the Markov chain as $\pi_s = P(\{X(t) = s\})$, for any state s .

From the above definitions, it results that:

$$\pi_{b_{i+1,0}} = \pi_{b_{i,0}}\alpha_i, \quad (12)$$

$$\pi_{b_{i+1,0}} = \pi_{b_{0,0}} \prod_{j=0}^i \alpha_j . \quad (13)$$

The transmission states are only reachable after obtaining a clear channel assessment at the end of a backoff stage:

$$\pi_{t_j} = \sum_{i=0}^m (1 - \alpha_i) \pi_{b_{i,0}} \quad \text{for } 0 \leq j < P_s . \quad (14)$$

By applying chain regularity conditions [2]:

$$\pi_{b_{i,k}} = \frac{W_i - k}{W_i} \pi_{b_{i,0}} \quad \text{for } 0 \leq k < W_i . \quad (15)$$

can transmit simultaneously, and n will detect their transmissions. This is an issue which arises in multihop networks. We model this relation using the concept of *independent vertex set*. Let $GCS_n = (V, E)$ be a graph where $V = CS_n$ and $E = \{(i, j) \mid i \in CS_j\}$. An edge (i, j) in this graph implies that nodes i and j are within their carrier sensing ranges. Therefore, no simultaneous transmission from these nodes can happen. However, if $S \subseteq CS_n$ is a subset of nodes in the carrier sense set of node n , the nodes in the set S can simultaneously transmit if and only if S is an independent vertex set in GCS_n . This reasoning is derived from previous works on CSMA analysis in multihop networks [3, 9].

Using the previous notions, we define Sim_n , the *set of possible simultaneous transmissions in CS_n* , as:

$$Sim_n = \{S \in 2^{CS_n} \mid S \text{ is independent set of } GCS_n\}. \quad (22)$$

The set Sim_n also includes single node transmissions, due to the fact that singleton sets of vertexes are independent.

4.2.1 Probability of channel busy assessment

As mentioned before, to derive an accurate model for the channel access mechanism, it is necessary to consider the probability of channel busy assessment for each backoff stage. In fact, for reasonable packet sizes, the backoff windows in the IEEE 802.15.4 standard are not large enough to prevent successive busy assessments. The diagram of Figure 3 illustrates this aspect. After an initial backoff stage, node 1 performs the first CCA (CCA_0), finding the medium busy because of node 2's transmission. In the example, we assume that at the moment of the busy assessment, node 2 was in the middle of its transmission (in reality, it could be at any instant of the transmission). As specified by the protocol, the second backoff duration for node 1 is selected randomly between $(0, W_1)$. We can therefore observe that the second CCA attempt will result again in a busy assessment if the randomly selected duration is less than the remaining time slots of node 2's ongoing transmission. In the general case, the probability of channel busy assessment during the second backoff stage is very likely to be higher than during the first stage, given the fact that there is always an active transmission at the start of the backoff stage. Similar argument applies to all backoff stages. However, in the case of the first backoff stage (α_0), we have no previous evidence about channel status. Therefore, it should be treated differently. In Figure 2, we illustrate this effect. Using the experimental setup described in Section 5, we computed the probability of channel busy assessment after each backoff stage over 1,000 simulation experiments. The figure shows the empirical cumulative distribution resulting from these simulations. As expected, the value for the first CCA is the lowest of the five attempts, while the second CCA has the highest value.

To tackle this issue, we estimate the probability of busy assessment after the initial backoff (α_0), considering only the probabilities that its contending nodes are transmitting. For a channel assessment in other backoff stages, we further approximate this value by taking into account the remaining transmission slots from previous channel busy assessments.

Initial backoff: No evidence about channel status

For a given node n , $\alpha_0^{(n)}$ is equal to the probability that one or more nodes in its carrier sense set are engaged in a transmission. Let T_k represent the event of a packet transmission

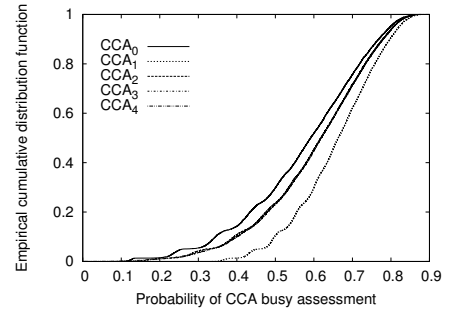


Figure 2: Probability of busy channel assessment after different backoff stages (from stage 0, CCA_0 to stage 4, CCA_4).

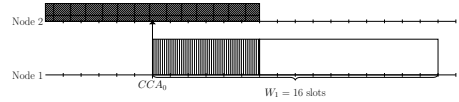


Figure 3: Example of successive channel busy assessments.

performed by node k . Then, we can express $\alpha_0^{(n)}$ as:

$$\alpha_0^{(n)} = \mathbf{P} \left\{ \bigcup_{k \in CS_{(n)}} T_k \right\} \quad (23)$$

Using the inclusion-exclusion principle, and the fact that simultaneous transmissions are only those specified in the set Sim_n , that is, $\mathbf{P} \left\{ \bigcap_{k \in S} T_k \right\} = 0$ if $S \notin Sim(n)$, we obtain:

$$\begin{aligned} \alpha_0^{(n)} &= \sum_{S \in Sim_n} P \left\{ \bigcup_{k \in CS_{(n)}} T_k \right\} \\ &= \sum_{S \in Sim_n} (-1)^{|S|+1} \prod_{k \in S} \tau^{(k)}. \end{aligned} \quad (24)$$

Equation (24) defines the probability of a busy assessment for the initial backoff stage when there is a transmission, from one or more nodes, inside the carrier sense set.

Channel assessment following a busy detection

The statement of a busy channel assessment for node n at backoff stage $j \geq 0$, means that one or more nodes in the carrier sense set are active in packet transmissions. Any further n 's attempt to get access to the channel before all these transmissions are completed will fail. We indicate with *occupation time* the minimum number of time slots that n has to wait before being able to get access to the channel again. We model the occupation time with a random variable $\mathcal{Y}^{(n)}$, $0 < \mathcal{Y}^{(n)} \leq P_s - 1$ given that we only need to take into account active transmissions. The value of $\alpha_i^{(n)}$, for a backoff stage $i > j$ following a busy assessment, depends on the occupation time of the active transmissions. More specifically, if \mathcal{C}_i is a random variable uniformly distributed in $[0, W_i - 1]$ representing the selected backoff counter in backoff stage i , the value of $\alpha_i^{(n)}$ can be approximated in the following way:

$$\alpha_i^{(n)} = \sum_{k=1}^{P_s-1} \mathbf{P} \left(\mathcal{Y}^{(n)} = k \right) \left(\mathbf{P}(\mathcal{C}_i < k) (1) + \mathbf{P}(\mathcal{C}_i \geq k) \alpha_0^{(n)} \right).$$

Given that the variables C_i have a random uniform distribution between 0 and $W_i - 1$, the expression for $\alpha_i^{(n)}$ becomes:

$$\alpha_i^{(n)} = \sum_{k=1}^{P_s-1} \mathbf{P}(\mathcal{Y}^{(n)} = k) \left(\frac{k}{W_i} (1) + \frac{W_i - k}{W_i} \alpha_0^{(n)} \right). \quad (25)$$

Equation (25) reflects that, for any backoff stage $i > 0$, we are certain about the presence of an ongoing transmission from one or more nodes inside the carrier sense set. Therefore, if the selected random backoff duration is less than the remaining time k for current transmissions, then the channel would be sensed busy again (with probability = 1). In the other case, if the selected backoff time is greater or equal to k , we come back to the situation where there is no evidence about channel status, so we consider it in the same way as an initial backoff (probability = α_0).

To define an expression for $\mathbf{P}(\mathcal{Y}^{(n)} = k)$ in (25), we must consider the number of active transmissions. Let $N^{(n)}$ be the expected number of simultaneous transmissions occurring in correspondance of a channel busy assessment at node n . Let us enumerate the transmissions that are active at a busy assessment, from t_1 to $t_{N^{(n)}}$, and let \mathcal{X}_i be a discrete uniform variable in $[0, P_s - 1]$, representing the number of remaining time slots for a single active transmission t_i that has been detected. We can therefore approximate $\mathcal{Y}^{(n)}$ as:

$$\mathcal{Y}^{(n)} = \max\{\mathcal{X}_i\} \quad \text{for } 1 \leq i \leq N^{(n)}, \quad (26)$$

Equation (26) captures the fact that a busy channel assessment may be due to the overlapping of simultaneous transmissions, which increases the probability of having a large number of consecutive time slots during which n senses the channel as busy. $\mathcal{Y}^{(n)}$ is therefore assuming the value of the longest remaining time from all active transmissions. To determine $\mathcal{Y}^{(n)}$, and, in turn, $\alpha_i^{(n)}$, we need an expression for $N^{(n)}$. We use the following approximation to calculate the expected number of simultaneous transmissions at any time slot, given that at least one ongoing transmission is active:

$$N^{(n)} = \left\lfloor \frac{\sum_{S \in Sim_n} |S|}{|Sim_n|} \right\rfloor, \quad (27)$$

which is the average set size over the set of possible combinations of nodes that can simultaneously transmit.

4.3 Calculation of P_{Fail}

Using the definitions (26) and (27) in Equation (25), we obtain expressions of the probability of a busy assessment at any backoff stage, for any node in the network.

Solving numerically the *multivariate non-linear system* determined by grouping the equations (21), (24) and (25) for all nodes in the network, we obtain a unique solution for P_{Fail} , that defines the probabilities of detecting the channel as busy for any node n in the multihop CSMA/CA network.

5. EXPERIMENTAL SETUP

In this section, we evaluate the prediction accuracy of our model by comparing the estimates it provides for P_{Fail} , the probability of nodes' channel access failure, against the empirical measures of the same quantities obtained through *realistic network simulations* using the *TOSSIM simulator*.

In order to provide a thorough validation of the model, we considered a wide set of scenarios in terms of *network sizes*, *topologies*, and *traffic patterns*. The aim is to show the robustness of the model under different network scenarios of interest. In the following, we first describe the characteristics of the experimental setup and then we report and discuss the observed results.

5.1 Network topologies

We selected a large set of network topologies in order to cover a wide range of connectivity scenarios. Each topology was obtained generating uniform random locations for each node. We have classified each topology according to the following characteristics: (i) *total number N of nodes in the network*, (ii) *mean number of contending nodes (μ_{cs})* (average value of the carrier sense set size for all nodes in the network), (iii) *variance of number of contending nodes (σ_{cs}^2)* (variance of the carrier sense set size across the network).

The number of nodes N affects the multihop behavior, while μ_{cs} and σ_{cs}^2 play a major role in all contention mechanisms of the model. Therefore, we included in the test cases topologies with different characteristics regarding these three aspects. Starting from the generation of a very large set of random networks with 50 and 100 nodes, we selected all the connected networks with average carrier sense set size μ_{cs} equal respectively to 5, 7 and 10. Then, based on the measured variance values σ_{cs}^2 , we classified the selected network instances in three groups, with respectively *low*, *medium* and *high* average variance values. At the end of this procedure, we selected a total of $2 \times 3 \times 3 = 18$ groups of connected topologies, each characterized by a different set of values for the triple $(N, \mu_{cs}, \sigma_{cs}^2)$. For each one of the 18 groups of instances, we selected (at random) 100 network topologies. In this way, we could create a test set of $100 \times 18 = 1,800$ different network instances with different topological characteristics to study the robustness of the model predictions to topological features. Figure (4) shows a few examples of the topologies obtained by the procedure described above.

5.2 Traffic loads

In addition to topological features, the model also depends on the characteristics of the input traffic, which we model as a combination of packet sizes and data generation rates.

The IEEE 802.15.4 PHY layer establishes a maximum length of 127 bytes per packet. In our experiments, we consider packet sizes of respectively 60 and 120 bytes.

Together with the packet size, the generation rate determines the amount of traffic across the network. Traffic generation is implemented as a *Bernoulli process*, as discussed in Section 4. That is, geometric packet inter-arrival times are approximated with a Bernoulli process executed during nodes' idle state. Considering that the IEEE 802.15.4 protocol provides a maximum capacity of 250 Kbps, we have considered three scenarios, in which a nodes generate packets with a mean frequency of 10, 20, and 40 packets per second. In terms of data generation rate, this setting corresponds to a raw data generation ranging from 4.8 Kbps up to 38.4 Kbps per node, which defines an extensive spectrum of input traffic scenarios that are realistic for a wireless sensor network. Generation rates and packet sizes are network homogeneous, all communications occur as local broadcast.

In terms of traffic loads, we have a total of $2 \times 3 = 6$ different combinations of packet sizes and generation rates.

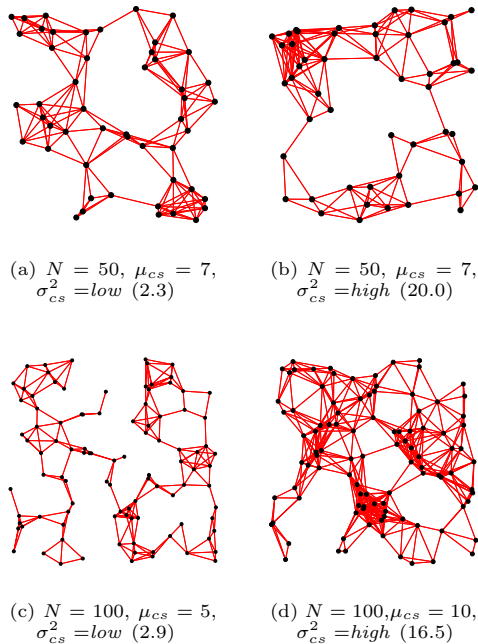


Figure 4: Examples of the generated network topologies.

N	50, 100
μ_{cs}	5, 7, 10
Packet size	60 bytes, 120 bytes
Generation rates	10, 20, 40 packets/sec

Table 1: Parameter values used in the experiments.

5.3 Simulation environment

Using the above setup for networks and traffic loads, we ran extensive simulations with the *TOSSIM* [17] network simulator, in order to compare the P_{Fail} values obtained from our analytical model with the values of channel access failures empirically measured through simulation. *TOSSIM* is an event-driven simulation tool explicitly developed for wireless sensor networks. *TOSSIM* simulates applications written for the TinyOS operating system [18]. The main reason behind the choice of *TOSSIM* as simulation environment for our evaluation is because it can provide results enough close to reality as a consequence of its quite accurate wireless channel model and hardware emulation. *TOSSIM* profits from the component-based architecture of TinyOS, and transparently defines a hardware abstraction layer that simulates the TinyOS network stack at the processor level. *TOSSIM* also provides a generic implementation of a CSMA/CA protocol. We set the MAC parameters following the specifications of the 802.15.4 standard: in all simulations we use the default values for the 802.15.4 as defined in [1]. We use a perfect capture model, and the carrier sense range is equal to the transmission range, both set to 10 m. There are no routing nor packet forwarding policies.

5.4 Experimental evaluation

In this section, we compare the estimates provided by our model against the results obtained by the network simulations according to the experimental setup introduced above.

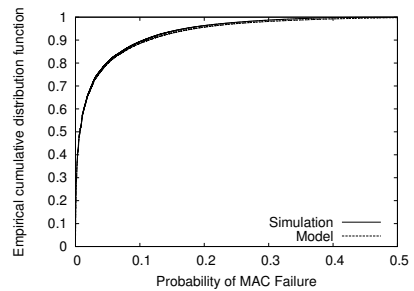


Figure 5: Comparison between simulation and model estimates of probabilities of MAC failures.

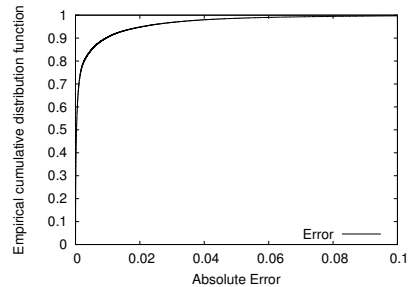


Figure 6: Paired, per node, analysis of the results: empirical cumulative distribution of absolute errors.

For each one of the selected 1,800 topologies, we consider all the possible traffic loads (i.e., all the 6 possible combinations of packet size and generation rate), and for each pair (topology, traffic) we performed 25 simulation runs, corresponding to 10 minutes of data traffic. This amounts to a total of $1,800 \times 6 = 10,800$ different instances, and $10,800 \times 25 = 270,000$ simulation runs.

For each node in the network, we count the number of MAC failures experienced by the node during a simulation run, and derive a sample estimate of the median of the probability of a MAC failure. For each instance, the corresponding input configuration for topology and traffic is fed to the analytical model, to calculate the values of the vector P_{Fail} .

The cumulative results for all the experiments are reported in Figure (5), where we perform a per node analysis, showing the *empirical cumulative distribution function of the probability of MAC failures* measured from simulations and calculated using the model. The two empirical distributions are practically overlapping all over the probability range. This means that the model predictions of the per node probability of MAC failures are fully confirmed with high accuracy by the empirical evidence gathered through the very extensive and realistic set of simulations.

Figure (6) reports a more in-depth view of the results. It shows the empirical cumulative distribution function of the *absolute error*, that is, the absolute value of the difference between the sample median obtained through simulation and the estimates of the numerical model, calculated separately for *each node* in each experiment. Remarkably, for the 95 % of the values, this error is less than 0.022, and for the 99 % of the data is less than 0.05. This observation confirms that our model is very accurate estimating the per node probability of MAC failures, with the prediction errors at the nodes being always very low, if not negligible at all.

So far, the model evaluation has been done considering the cumulative results from all the experiments. As a further step, we analyze separately the impact of topology and traffic parameters on the accuracy of the model estimates. Table (2) shows, for all possible combinations of settings of topology parameters, the maximum absolute error corresponding to the 95% of the cases. In practice, each row in the table refer to all set of experiments associated to one specific assignment of topology-related parameters and to all possible combinations of traffic load parameters. Table (3) shows an equivalent information, but in this case, each row of the table refers to the case in which are the traffic parameters that are maintained fixed, and all topology-related parameters are varied. From Table (2) we can observe that, still remaining very low, the increase in the degree of a node, expressed by μ_{cs} , is associated with a relative increase of the error (about one order of magnitude passing from $\mu_{cs} = 5$ to $\mu_{cs} = 10$. on the other hand, increasing the spread of the value of μ_{cs} or the total number of nodes seems to have quantitatively a very little effect or no effect at all. From Table (3) we can see the increasing the data rate degrades the accuracy of the model predictions. Also the increase in packet size seems to have a tangible effect only for the higher data rates. Overall, larger values of μ_{cs} and elevated traffic loads seems to have some negative impact on the accuracy of the model, but it could have been expected since node degree and data production are the most critical aspects governing MAC behavior. However, even in these cases, the error is still within more than acceptable boundaries. Moreover, for larger values of μ_{cs} and elevated traffic loads the probability of MAC failures is itself higher because of the higher number of collisions occurring, therefore a greater variability of the estimation error is expected to be observed.

In summary, we ran a wide test of experiments covering multiple scenarios to validate accuracy and robustness of the proposed analytical model. We observed that the error remains significantly low for all parameter settings and shows a little increase only in correspondence of extreme conditions in terms of node degree and traffic loads.

N	σ_{cs}^2	μ_{cs}	error
50	low	5	0.002
50	low	7	0.009
50	low	10	0.033
50	medium	5	0.003
50	medium	7	0.015
50	medium	10	0.037
50	high	5	0.005
50	high	7	0.022
50	high	10	0.041
100	low	5	0.002
100	low	7	0.010
100	low	10	0.032
100	medium	5	0.004
100	medium	7	0.014
100	medium	10	0.036
100	high	5	0.006
100	high	7	0.022
100	high	10	0.037

Table 2: Effect of topology characteristics: Maximum absolute error for the 95% of the cases.

6. CONCLUSIONS AND FUTURE WORK

In this work, we have proposed a novel and accurate an-

packet size (bytes)	data rate(pkts/s)	error
60	10	0.001
60	20	0.003
60	40	0.018
120	10	0.007
120	20	0.027
120	40	0.063

Table 3: Effect of traffic load: Maximum absolute error for the 95% of the cases.

alytical model for the IEEE 802.15.4 non beacon enabled MAC in multi-hop networks. First, we modeled the CSMA/CA mechanism for a single node by means of a discrete time Markov chain. We identify as an important aspect the relationship between the busy assessment probability and the backoff stage. This dependency is not considered in most of previous works. Our model tackles this issue using a novel approximation strategy. As a second stage of the modeling process, we considered the multihop aspect. Contrary to single hop networks, in multihop topologies different nodes have different views of the wireless channel, which represent a challenge for modeling. In our approach we used the notion of independent set in a graph to determine the possible simultaneous transmissions from neighbor nodes. Coupling all together the models for the individual nodes and their local environments, we obtained a multivariate non-linear system which we can solve numerically, given a description of the traffic load and the specification of the topological relations among the nodes in the network. From the solution of the system we obtain estimates of the probability of a MAC failure for each single node in the network.

In order to asses the accuracy of the model, we performed an extensive set of evaluation experiments, considering a large number of network scenarios in terms of number of nodes, topological connectivity, and traffic loads. We built a data set of 10,800 different network instances and used the model to calculate the estimates of the probabilities of MAC failure for each network node. These values were compared to those obtained by simulation, that is, by executing the same instances using TOSSIM, a state-of-the-art network simulator for WSNs. The results have shown a near perfect agreement between the values predicted by the model and those empirically measured by simulation. On the same data, we also performed a sensitivity analysis of the model, investigating which topological and traffic aspects are more critical in terms of degrading the accuracy of the predictions. We have shown that increasing the average node degree and/or the rate of packet generation, the model seems to slightly decrease its accuracy, still showing an excellent average prediction error of order 10^{-2} .

As future work, we will extend our approach relaxing the assumption that packet generation can only happen during the idle states of the MAC protocol, in order to allow nodes to generate packets, more realistically, at any time. Further work will also consider the inclusion of a radio propagation model at the level of the CCA, instead of the assumption of perfect channel. Finally, we plan to check the accuracy of the model using a real sensor network testbed.

7. REFERENCES

- [1] IEEE Standard for Information Technology, Specific Requirements Part 15.4: Specifications for Wireless Medium Access Control and Physical Layer, 2006.

- [2] G. Bianchi. Performance analysis of the IEEE 802.11 distributed coordination function. *IEEE J. on Selected Areas in Communications*, 18(3):535–547, 2000.
- [3] R. Boorstyn, a. Kershenbaum, B. Maglaris, and V. Sahin. Throughput Analysis in Multihop CSMA Packet Radio Networks. *IEEE Transactions on Communications*, 35(3):267–274, 1987.
- [4] C. Buratti and R. Verdone. A mathematical model for performance analysis of IEEE 802.15.4 non-beacon enabled mode. In *Proceedings of the 14th European Wireless Conference (EW 2008)*, pages 1–7, 2008.
- [5] C. Buratti and R. Verdone. Performance analysis of IEEE 802.15.4 non beacon-enabled mode. *IEEE Transactions on Vehicular Technology*, 58(7):3480–3493, 2009.
- [6] M. Carvalho and J. Garcia-Luna-Aceves. A scalable model for channel access protocols in multihop ad hoc networks. In *Proceedings of the 10th Annual International Conference on Mobile Computing and Networking (MobiCom 2004)*, pages 330–344, 2004.
- [7] Z. Chen, C. Lin, H. Wen, and H. Yin. An analytical model for evaluating IEEE 802.15.4 CSMA/CA protocol in low-rate wireless application. In *Proceedings of the 21st International Conference on Advanced Information Networking and Applications Workshops (AINAW 2007)*, pages 899–904, 2007.
- [8] P. Di Marco, P. Park, C. Fischione, and K. Johansson. Analytical modelling of IEEE 802.15.4 for multi-hop networks with heterogeneous traffic and hidden terminals. In *Proc. of the IEEE Global Telecomm. Conf. (GLOBECOM 2010)*, pages 1–6, 2010.
- [9] M. Garetto, T. Salonidis, and E. Knightly. Modeling per-flow throughput and capturing starvation in CSMA multi-hop wireless networks. *IEEE/ACM Transactions on Networking*, 16(4):864–877, 2008.
- [10] F. Gebali. *Analysis of computer and communication networks*. Springer, 2008.
- [11] M. Goyal, D. Rohm, W. Xie, S. Hosseini, K. Trivedi, Y. Bashir, and A. Divjak. A stochastic model for beaconless IEEE 802.15.4 MAC operation. *Computer Communications*, pages 199–207, 2010.
- [12] T. Kim, H. Kim, J. Lee, J. Park, and B. Choi. Performance analysis of IEEE 802.15.4 with non-beacon-enabled CSMA/CA in non-saturated condition. *Embedded and Ubiquitous Computing*, pages 884–893, 2006.
- [13] T. O. Kim, J. S. Park, H. J. Chong, K. J. Kim, and B. D. Choi. Performance analysis of IEEE 802.15.4 non-beacon mode with the unslotted CSMA/CA. *IEEE Communications Letters*, 12(4):238–240, 2008.
- [14] L. Kleinrock and F. Tobagi. Packet switching in radio channels: Part I – Carrier sense multiple-access modes and their throughput-delay characteristics. *IEEE Trans. on Communications*, 23(12):1400–1416, 1975.
- [15] B. Latré, P. D. Mil, I. Moerman, B. Dhoedt, P. Demeester, and N. V. Dierdonck. Throughput and delay analysis of unslotted IEEE 802.15.4. *Journal of Networks*, 1(1):20–28, 2006.
- [16] B. Lauwens, B. Scheers, and A. Capelle. Performance analysis of unslotted CSMA/CA in wireless networks. *Telecommunication Systems*, 44(1–2):109–123, 2009.
- [17] P. Levis, N. Lee, M. Welsh, and D. Culler. TOSSIM: Accurate and scalable simulation of entire TinyOS applications. In *Proceedings of the 1st International conference on Embedded networked sensor systems (SenSys 2003)*, pages 126–137, 2003.
- [18] P. Levis, S. Madden, J. Polastre, R. Szewczyk, K. Whitehouse, A. Woo, D. Gay, J. Hill, M. Welsh, E. Brewer, and C. D. TinyOS: An operating system for sensor networks. In W. Weber, J. Rabaey, and E. Aarts, editors, *Ambient Intelligence*. Springer-Verlag, 2004.
- [19] M. Martalò, S. Busanelli, and G. Ferrari. Markov Chain-based performance analysis of multihop IEEE 802.15.4 wireless networks. *Performance Evaluation*, 66(12):722–741, 2009.
- [20] J. Misic, V. Misic, and S. Shafi. Performance of a beacon enabled IEEE 802.15.4 cluster with downlink and uplink traffic. *IEEE Transactions on Parallel and Distributed Systems*, 17(4):361–376, 2006.
- [21] P. Park, P. Di Marco, P. Soldati, C. Fischione, and K. H. Johansson. A generalized Markov chain model for effective analysis of slotted IEEE 802.15.4. In *Proc. of the 6th IEEE Int. Conf. on Mobile Adhoc and Sensor Systems (MASS’09)*, pages 130–139, 2009.
- [22] T. Park, T. Kim, J. Choi, S. Choi, and W. Kwon. Throughput and energy consumption analysis of IEEE 802.15.4 slotted CSMA/CA. *Electronics Letters*, 41(18):1017–1019, 2005.
- [23] S. Pollin, M. Ergen, S. Ergen, B. Bougard, L. Der Perre, I. Moerman, A. Bahai, P. Varaiya, and F. Cathoor. Performance analysis of slotted carrier sense IEEE 802.15.4 medium access layer. *IEEE Transactions on Wireless Communications*, 7(9):3359–3371, 2008.
- [24] I. Ramachandran, A. Das, and S. Roy. Analysis of the contention access period of IEEE 802.15.4 MAC. *ACM Transactions on Sensor Networks*, 3(1), 2007.
- [25] F. Shu, T. Sakurai, M. Zukerman, and H. Vu. Packet loss analysis of the IEEE 802.15.4 MAC without acknowledgements. *IEEE Communications Letters*, 11(1):79–81, 2007.
- [26] F. Tobagi and L. Kleinrock. Packet switching in radio channels: Part II – The hidden terminal problem in carrier sense multiple-access and the busy-tone solution. *IEEE Transactions on Communications*, 23(12):1417–1433, 1975.
- [27] X. Wang and K. Kar. Throughput modelling and fairness issues in CSMA/CA based ad-hoc networks. In *Proc. of the 24th INFOCOM*, pages 23–34, 2005.
- [28] Y. Wang and J. Garcia-Luna-Aceves. Performance of collision avoidance protocols in single-channel ad hoc networks. In *Proceedings of the 10th IEEE International Conference on Network Protocols (ICNP 2002)*, pages 68–77, 2002.
- [29] L. Wu and P. Varshney. Performance analysis of CSMA and BTMA protocols in multihop networks. Single channel case. *Information Sciences*, 120(1–4):159–177, 1999.
- [30] Y. Yang, J. C. Hou, and L.-C. Kung. Modeling the effect of transmit power and physical carrier sense in multi-hop wireless networks. In *Proc. of the 26th IEEE Int. Conference on Computer Communications (INFOCOM’07)*, pages 2331–2335, 2007.