Wideband Power Spectrum Sensing Using Sub-Nyquist Sampling

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***Abstract*—Compressive sampling (CS) is famous for its ability to perfectly reconstruct a sparse signal based on a limited number of measurements. In some applications, such as in spectrum sensing for cognitive radio, perfect signal reconstruction is not really needed. Instead, only statistical measures such as the power spectrum or equivalently the auto-correlation sequence are required. In this paper, we introduce a new approach for reconstructing the power spectrum based on samples produced by sub-Nyquist rate sampling. Depending on the compression rate, the entire problem can be presented as either under-determined or over-determined. In this paper, we mainly focus on the over-determined case, which allows us to employ a simple least-squares (LS) reconstruction method. We show under which conditions this LS reconstruction method yields a unique solution, without including any sparsity constraints.**

I. Introduction

In the last few years, new wireless applications have placed great demands on premium radio resources such as bandwidth and radio spectrum. In addition, current trends in wireless technologies have put an additional burden on the receiver hardware, especially the analog to digital converter (ADC) due to the wideband nature of the signals produced by specific applications, such as ultra wideband (UWB) communications. According to the Whittaker-Kotelnikov-Shannon-Nyquist theorem, a signal with a frequency support between $-f_0$ and f_0 can be perfectly recovered from its samples if the sampling rate is more than or equal to the Nyquist rate of $2f_0$. Sampling a very large bandwidth signal at the Nyquist rate will yield a high power consumption for the ADC [1].

Apart from its significance in perfect signal reconstruction, Nyquist rate sampling is an important topic in the field of cognitive radio networks, where licensed frequency bands can be used by rental users when the primary users who own the licensed bands are inactive. To facilitate such networks, the rental users are required to perform spectrum sensing, which plays a key role to gauge the wireless environment over a broad frequency band and to identify both the occupied and unoccupied bands. The output of spectrum sensing is used by the rental users to decide whether to enter or leave the observed spectrum (spectrum allocation).

In order to mitigate the high sampling rate problems, several researchers have been looking into sub-Nyquist rate sampling. In [2], sub-Nyquist sampling based on non-uniform (also known as multi-coset) sampling is investigated for the case of multiband signals having a frequency support on a union of finite intervals. This multi-coset approach has been shown to reach the Landau's lower bound in [3] suggesting that the minimal required sampling rate to perfectly reconstruct the multiband signal is equal to the frequency occupancy, and thus smaller than the Nyquist rate. Appropriate sampling patterns for the multi-coset approach have been investigated in [2]. Similarly, [4] discusses sub-Nyquist sampling for sparse multiband analog signals by means of a so-called modulated wideband converter (MWC), which consists of multiple channels where each channel employs a different mixing function followed by low-pass filtering and low-rate uniform sampling. The conditions for perfect reconstruction of an analog signal from the output of the MWC can be found in [4].

In recent years, compressive sampling based on [5] has been a new emerging field. Also compressive sampling provides the possibility to reconstruct the original signal from a limited number of measurements with no or little information loss as long as the signal has a sparse representation in a particular basis. The signal is randomly projected onto a second basis by the so-called measurement matrix or compressive sampling matrix thereby reducing the number of samples compared to the Nyquist rate. Several works have exploited this concept for different applications. In [6], wideband spectrum sensing based on compressive sampling has been proposed by exploiting the inherent sparsity feature of the edge spectrum. However, the approach in [6] actually still samples the received wideband signal at the Nyquist rate since compressive sampling is applied to the auto-correlation sequence of the Nyquist rate samples. Therefore, [7] proposes to directly conduct compressive sampling on the received signal and tries to exploit the relationship between the auto-correlation sequence of the measurements and that of the Nyquist rate samples. Unfortunately, [7] assumes that the measurements are still wide-sense stationary, which is not true for most compressive sampling matrices.

Although [6] and [7] focus on the auto-correlation sequence, most of the approaches listed in the previous paragraphs intend to obtain perfect reconstruction of the original signal itself, which puts high demands on the type of signals that can be reconstructed. On the other hand, several applications, such as spectrum sensing, only need perfect reconstruction of some statistical measures of the signal. In this paper, we propose a new approach based on sub-Nyquist sampling for reconstructing the power spectrum of the original signal. This

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approach exploits the cyclo-stationarity of the measurements (or stationarity of the measurement vector) and might not even need the sparsity assumption that is generally required for signal reconstruction. This article is organized as follows. The original problem formulation is given in Section II, which discusses the relationship between the correlation matrix of the measurement vector and the auto-correlation sequence of the Nyquist rate samples as well as how this relationship can be exploited. In Section III, we try to reformulate the problem by viewing the elements of the measurement vector as the outputs of parallel filters whose coefficients are given by the rows of the sampling matrix. Section IV discusses the reconstruction of the auto-correlation sequence or equivalently the power spectrum, for under- and over-determined systems even though we put more focus on the latter case due to its simplicity. In Section V, the compressive sampling matrix choices for the over-determined case are discussed. We consider two possible matrices, namely a random matrix and a multi-coset matrix. Section VI elaborates on some simulation studies and finally, Section VII provides conclusions.

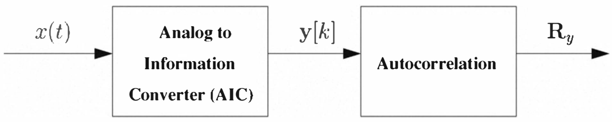

Figure 1. Representation of the proposed compressive sampling based spectrum sensing approach

II. Problem Formulation

We consider compressive sampling of the received wide-sense stationary signal $x(t)$, which is sampled using an analog to information converter (AIC) leading to a sequence of measurement vectors $\mathbf{y}[k]$, as shown in Fig. 1. As argued in [6] and [7], the AIC can theoretically be perceived as a block containing a basic ADC operating at Nyquist rate followed by a multiplexing operation collecting N consecutive Nyquist rate samples, and concluded by a multiplication with a compressive sampling matrix reducing the number of samples from N to M. Note that this AIC block can also model the multi-coset sampler introduced in [2]. Based on the above description, we denote the output of the ADC operating at Nyquist rate by $x[n]$ and the output of the multiplexing operation by the $N \times 1$ vector sequence $\mathbf{x}[k]$ defined as:

$$\mathbf{x}[k] = \left[x[kN], x[kN+1], \ldots, x[kN+N-1]\right]^T \quad (1)$$

Every $N \times 1$ vector $\mathbf{x}[k]$ is then compressed from N samples to M samples by the $M \times N$ compressive sampling matrix $\mathbf{\Phi}$ leading to the $M \times 1$ vector sequence $\mathbf{y}[k]$:

$$\mathbf{y}[k] = \mathbf{\Phi}\mathbf{x}[k] \quad (2)$$

We describe the auto-correlation sequence of the Nyquist rate samples $x[n]$ as $r_x[l] = E\{x[n]x^*[n-l]\}$, where $(.)^*$ denotes the complex conjugate operation. We can then construct the $N \times N$ auto-correlation matrix of $\mathbf{x}[k]$ in (1) as $\mathbf{R}_x = E\left\{\mathbf{x}[k]\mathbf{x}^H[k]\right\}$, where the elements of $\mathbf{R}_x$ are given by:

$$[\mathbf{R}_x]_{ij} = r_x[i-j] = r_x^*[j-i] \quad (3)$$

Based on (2), the $M \times M$ auto-correlation matrix of $\mathbf{y}[k]$ in (2) can be expressed as:

$$\mathbf{R}_y = E\left\{\mathbf{y}[k]\mathbf{y}^H[k]\right\} = \mathbf{\Phi}\mathbf{R}_x\mathbf{\Phi}^H \quad (4)$$

Observe that the elements of the measurement vectors $\mathbf{y}[k]$ are generally not wide-sense stationary due to the nature of the compressive sampling matrix $\mathbf{\Phi}$. As a result, the elements of $\mathbf{R}_y$ can generally not be expressed in a similar form as (3).

While all columns of $\mathbf{R}_x$ basically contain the same information, every column of $\mathbf{R}_y$ has a different content. As a result, it is theoretically possible to exploit all columns of $\mathbf{R}_y$ to estimate one of the columns of $\mathbf{R}_x$. First of all, we stack all columns of $\mathbf{R}_y$ into the $M^2 \times 1$ vector $\text{vec}(\mathbf{R}_y)$, where $\text{vec}(.)$ is the operator that stacks all columns of a matrix in a large column vector. From (4), it is then clear that $\text{vec}(\mathbf{R}_y)$ can be expressed as:

$$\text{vec}(\mathbf{R}_y) = (\mathbf{\Phi}^* \otimes \mathbf{\Phi})\text{vec}(\mathbf{R}_x) \quad (5)$$

where $\otimes$ denotes the Kronecker product operation. As mentioned before, all columns of $\mathbf{R}_x$ contain the same information, which can be collected into the $2N-1$ vector $\mathbf{r}_x$ defined as:

$$\mathbf{r}_x = [r_x(0), r_x(1), \ldots, r_x(N-1), r_x(1-N), \ldots, r_x(-1)]^T$$

The relationship between $\text{vec}(\mathbf{R}_x)$ and $\mathbf{r}_x$ can then be written as:

$$\text{vec}(\mathbf{R}_x) = \mathbf{T}\mathbf{r}_x \quad (6)$$

where $\mathbf{T}$ is a special $N^2 \times (2N-1)$ repetition matrix. As a result, from (5) and (6), we obtain:

$$\text{vec}(\mathbf{R}_y) = (\mathbf{\Phi}^* \otimes \mathbf{\Phi})\mathbf{T}\mathbf{r}_x \quad (7)$$

In order to simplify the analysis, we introduce the $M^2 \times (2N-1)$ matrix $\mathbf{\Theta} = (\mathbf{\Phi}^* \otimes \mathbf{\Phi})\mathbf{T}$ and rewrite (7) as:

$$\text{vec}(\mathbf{R}_y) = \mathbf{\Theta}\mathbf{r}_x \quad (8)$$

Given (8), our intention is to reconstruct the auto-correlation sequence $\mathbf{r}_x$ from $\text{vec}(\mathbf{R}_y)$ and to use it to compute the $(2N-1) \times 1$ power spectrum vector $\mathbf{p}_x$ based on the following relationship:

$$\mathbf{p}_x = \mathbf{F}\mathbf{r}_x \quad (9)$$

where $\mathbf{F}$ is the $(2N-1) \times (2N-1)$ discrete Fourier transform matrix.

III. Problem Reformulation

We can also view every row of $\mathbf{\Phi}$ in (7) as a unique discrete waveform or filter and rewrite $\mathbf{\Phi}$ in terms of its row vectors:

$$\mathbf{\Phi} = \left[\boldsymbol{\varphi}_0, \boldsymbol{\varphi}_1, \boldsymbol{\varphi}_2, \ldots, \boldsymbol{\varphi}_{M-1}\right]^T \quad (10)$$

with $\boldsymbol{\varphi}_i = [\varphi_i[0], \varphi_i[-1], \ldots, \varphi_i[1-N]]^T$. If we define the vector sequence $\mathbf{y}[k]$ in (2) as a collection of M parallel scalar sequences $y_i[k]$, i.e., $\mathbf{y}[k] = [y_0[k], y_1[k], \ldots, y_{M-1}[k]]^T$, we can view $y_i[k]$ as the N-fold downsampled version of the sequence obtained by filtering $x[n]$ with $\varphi_i[n]$:

$$y_i[k] = \sum_{n=1-N}^{0} \varphi_i[n]x[kN-n] \tag{11}$$

In that case, the auto- and cross-correlations between the elements of the measurement vector $\mathbf{y}[k]$ in $\text{vec}(\mathbf{R}_y)$ can be described in terms of the auto- and cross-correlation sequences between the different sequences $y_i[k]$ but at lag zero. Denoting the correlation sequence between $y_i[k]$ and $y_j[k]$ at lag l as $r_{y_i,y_j}[l]$ and taking the deterministic nature of the filter coefficients into account, we can write:

$$\begin{aligned} r_{y_i,y_j}[l] &= E\left\{y_i[k]y_j^*[k-l]\right\} \\ &= \sum_{n=1-N}^{0} \varphi_i[n] \sum_{p=1-N}^{0} \varphi_j^*[p] r_x[lN+p-n] \end{aligned} \tag{12}$$

By using variable substitution, we can rewrite (12) as:

$$r_{y_i,y_j}[l] = \sum_{s=1-N}^{N-1} r_{\varphi_i,\varphi_j}[s] r_x[lN-s] \tag{13}$$

where $r_{\varphi_i,\varphi_j}[l] = \sum_{n=1-N}^{0} \varphi_i[n]\varphi_j^*[n-l]$ is the deterministic correlation sequence between $\varphi_i[n]$ and $\varphi_j[n]$. By taking (13) into account, $\text{vec}(\mathbf{R}_y)$ in (8) can be written as:

$$\text{vec}(\mathbf{R}_y) = \left[\mathbf{r}_{y_0}^T[0], \mathbf{r}_{y_1}^T[0], \ldots, \mathbf{r}_{y_{M-1}}^T[0]\right]^T \tag{14}$$

where $\mathbf{r}_{y_i}[0] = \left[r_{y_0,y_i}[0], r_{y_1,y_i}[0], \ldots, r_{y_{M-1},y_i}[0]\right]^T$. From (13) as well as from some elementary mathematical calculations on (7) and (8), we can now see that $\boldsymbol{\Theta}$ in (8) is composed of the auto- and cross-correlations between the rows of $\boldsymbol{\Phi}$. If we specify $\mathbf{r}_{\varphi_i,\varphi_j}$ as $\mathbf{r}_{\varphi_i,\varphi_j} = \left[r_{\varphi_i,\varphi_j}[0], \ldots, r_{\varphi_i,\varphi_j}[1-N], r_{\varphi_i,\varphi_j}[N-1], \ldots, r_{\varphi_i,\varphi_j}[1]\right]^T$ we can express $\boldsymbol{\Theta}$ as:

$$\boldsymbol{\Theta} = \left[\mathbf{r}_{\varphi_0,\varphi_0}, \ldots, \mathbf{r}_{\varphi_{M-1},\varphi_0}, \ldots, \mathbf{r}_{\varphi_0,\varphi_{M-1}}, \ldots, \mathbf{r}_{\varphi_{M-1},\varphi_{M-1}}\right]^T \tag{15}$$

From (14) and (15), we can observe that $\text{vec}(\mathbf{R}_y)$ is obtained by simply multiplying the auto-correlation sequence $\mathbf{r}_x$ with $\boldsymbol{\Theta}$, whose elements are given by the deterministic auto- and cross-correlations between the rows of $\boldsymbol{\Phi}$.

IV. Reconstruction

In this section, we attempt to reconstruct the power spectrum by first recovering the auto-correlation sequence $\mathbf{r}_x$ from (8) for $\boldsymbol{\Theta}$ given by (15). In general, the reconstruction problem can be divided into two different cases, the under-determined and over-determined cases (we view the determined case as part of the over-determined case). In the under-determined case, additional constraints (such as sparsity considerations, as discussed in [5]) are clearly needed. However, this is not the case for the over-determined case which will therefore be an interesting case to study. It is quite important to point out that even with compression (i.e., $M \ll N$), our approach might result in an over-determined system, while common compressive sampling problems generally boil down to an under-determined system. The reason for this is the fact that we focus on reconstructing statistical measures (namely auto- and cross-corelation sequences) which allows us to gather much more system equations.

A. Case 1: Under-Determined System

When $M^2 < 2N-1$, our reconstruction problem becomes under-determined. Then, we can exploit the fact that the signal is sparse in some basis and apply one of the many recently proposed CS-based reconstruction methods. One approach is to assume that the power spectrum is sparse, which is reasonable in a cognitive radio scenario. In this case, we simply exploit the relationship between the $(2N-1)\times 1$ power spectrum vector $\mathbf{p}_x$ and the $(2N-1)\times 1$ auto-correlation vector $\mathbf{r}_x$ given in (9) and thus we can formulate the reconstruction of the power spectrum as an l_1-norm minimization problem:

$$\hat{\mathbf{p}}_x = \arg\min_{\mathbf{p}_x} \|\mathbf{p}_x\|_1 \text{ s.t. } \text{vec}(\mathbf{R}_y) = \boldsymbol{\Theta}\mathbf{F}^{-1}\mathbf{p}_x \tag{16}$$

When the power spectrum is not sparse, we can make use of the wavelet based edge detection introduced in [6], [7], which relates the $(2N-1)\times 1$ auto-correlation vector $\mathbf{r}_x$ in (8) with the $(2N-1)\times 1$ sparse edge spectrum vector $\mathbf{z}_p$ according to:

$$\mathbf{z}_p = \boldsymbol{\Gamma}\mathbf{F}\mathbf{W}\mathbf{r}_x \tag{17}$$

where $\mathbf{W}$ is a $(2N-1)\times(2N-1)$ smoothing matrix and $\boldsymbol{\Gamma}$ is the $(2N-1)\times(2N-1)$ first-order difference matrix (for more details see [6], [7]). The reconstruction of the edge spectrum can then be formulated as the following l_1-norm minimization problem:

$$\hat{\mathbf{z}}_p = \arg\min_{\mathbf{z}_p} \|\mathbf{z}_p\|_1 \text{ s.t. } \text{vec}(\mathbf{R}_y) = \boldsymbol{\Theta}(\boldsymbol{\Gamma}\mathbf{F}\mathbf{W})^{-1}\mathbf{z}_p \tag{18}$$

Based on the estimated edge spectrum $\hat{\mathbf{z}}_p$, we can easily recover $\hat{\mathbf{r}}_x$ from (17) and $\hat{\mathbf{p}}_x$ by performing a cumulative sum on $\hat{\mathbf{z}}_p$ as shown in [7].

B. Case 2: Over-Determined System

When $M^2 \geq 2N-1$ and $\boldsymbol{\Theta}$ has full column rank, it is possible to compute the auto-correlation vector $\mathbf{r}_x$ as the least-squares solution of (8):

$$\hat{\mathbf{r}}_x = (\boldsymbol{\Theta}^H\boldsymbol{\Theta})^{-1}\boldsymbol{\Theta}^H\text{vec}(\mathbf{R}_y) \tag{19}$$

The power spectrum estimate $\hat{\mathbf{p}}_x$ can then be computed from (9). The most interesting aspect about this case is that we can recover the statistics of the signal without requiring any sparsity assumptions. This is in contrast with the general compressive sampling framework where sparsity of the signal is needed to guarantee perfect reconstruction. In the next section, we focus our attention on this case as the under-determined case is similar to the approaches that have been intensively explored in [5], [6], [7].

V. Sampling Matrix Design

In this section, we discuss the design of the sampling matrix in order to ensure that the least-squares solution is unique ($\boldsymbol{\Theta}$ has full column rank).

A. Random Sampling Matrix

When each element of $\mathbf{\Phi}$ is randomly generated, there is a very high probability that $\mathbf{\Theta}$ will have full column rank once $M^2 \geq (2N-1)$. Note that this can occur for $M \ll N$. We thus propose the use of a random matrix such as a complex Gaussian matrix as one possible realization of the sampling matrix $\mathbf{\Phi}$ to guarantee the full column rank property of $\mathbf{\Theta}$.

B. Multi-Coset Sampling Matrix

It is also possible to adopt multi-coset sampling. As indicated in (10), we can construct a multi-coset sampling matrix by selecting M different rows from the identity matrix $\mathbf{I}_N$ leading to an $M \times N$ multi-coset sampling matrix $\mathbf{\Phi}$. Different from Section V-A, we cannot simply select the rows of $\mathbf{I}_N$ in a random way for a given M since some requirements have to be satisfied in order to ensure the full column rank property of $\mathbf{\Theta}$ in (15). When a multi-coset sampling matrix is employed, every row of $\mathbf{\Theta}$ will only contain a single one and will have zeros elsewhere. Hence, to achieve a full column rank $\mathbf{\Theta}$, we have to select an appropriate combination of rows of $\mathbf{I}_N$ that result in $\mathbf{\Theta}$ having at least a single one in each column. Furthermore, it is desirable that the number of rows we select is minimal since we want to minimize the compression rate M/N.

Assuming that $\varphi_j[n] = \delta[-n-n_j]$ for $j = 0, 1, 2, \ldots, N-1$, it is clear from (13) that the correlation $r_{\varphi_i,\varphi_j}[l]$ is given by:

$$r_{\varphi_i,\varphi_j}[l] = \delta[l - n_i + n_j] \tag{20}$$

Our task is now to construct $\mathbf{\Phi}$ by selecting M out of N possible rows of $\mathbf{I}_N$ subject to the constraints on $\mathbf{\Theta}$ mentioned before. Introducing S as a set of M indexes selected from $\{0, 1, \ldots, N-1\}$ representing the rows from $\mathbf{I}_N$ that we are going to select and Ω as a set given by:

$$\Omega = \{|n_i - n_j| \, |\forall n_i, n_j \in S\} \tag{21}$$

our multi-coset sampling matrix construction problem can be stated as:

$$\min_S |S| \text{ s.t. } \Omega = \{0, 1, \ldots, N-1\} \tag{22}$$

where $|S|$ denotes the cardinality of the set S. This problem actually corresponds to a so-called minimal length-$(N-1)$ sparse ruler problem. A sparse ruler with length $N-1$ can be regarded as a ruler that has $k < N$ distance marks $0 = n_0 < n_1 < \cdots < n_{k-1} = N-1$ but is still able to measure all integer distances from 0 to $N-1$. Note that Ω in (22) represents the set of integer distances that can be measured by the length-$(N-1)$ sparse ruler with all marks $n_i \in S$. The length-$(N-1)$ sparse ruler with k distance marks is called minimal if there is no length-$(N-1)$ sparse ruler with $k-1$ marks. If we solve this minimal sparse ruler problem, we basically minimize the compression rate M/N while maintaining uniqueness of the solution of the least-squares reconstruction problem. The minimal sparse ruler problem and how to solve it is discussed in [8].

VI. Simulation Study

In this section, we illustrate our approach with numerical results from a simulation study for both random complex Gaussian and multi-coset sampling matrices. We consider a complex baseband representation of an OFDM signal with 16 QAM data symbols, 8192 frequency tones that span a frequency band from $-\pi$ to π, and a cyclic prefix length of 1024. We only activate 3072 frequency tones in the bands $[-\pi, -0.75\pi]$, $[0, 0.25\pi]$ and $[0.5\pi, 0.75\pi]$. The transmitted signal $x(t)$ has 10dB power. We set N to $N = 128$ and vary the compression rate M/N from 0.1563 to 0.5 while ensuring that $\mathbf{\Theta}$ has full column rank.

The complex Gaussian sampling matrix is randomly generated with zero mean and variance $1/M$ and it is kept fixed over the different runs. In Fig. 2, the mean squared error (MSE) between the estimated power spectrum and the theoretical one is computed for the random complex Gaussian sampling matrix case. No noise is considered in this figure. The MSE is calculated according to:

$$MSE = \frac{E\left\{\|\hat{\mathbf{p}}_x - \mathbf{p}_x\|_2^2\right\}}{\|\mathbf{p}_x\|_2^2} \tag{23}$$

where $\mathbf{p}_x$ represents the theoretical power spectrum vector. The MSE is computed for different numbers of collected measurement vectors (MVs) $\mathbf{y}[k]$ in (2) as an attempt to represent different sensing times. It is clear from the figures that the quality of the estimation improves with M/N, although the performance seems to saturate at a particular point. We can also notice that the MSE improves as the sensing time increases, which is to be expected as our estimated auto-correlation value $r_{y_i,y_j}[0]$ in (14) approaches the actual value. In Fig. 3, the estimated power spectrum is depicted together with the theoretical one for $M/N = 0.5$ and different values of sensing times. Again a random complex Gaussian sampling matrix is assumed here. Obviously, the presence of the active bands can be better located for longer sensing times.

For multi-coset sampling, it turns out that the minimum number of distance marks for a length-127 sparse ruler is 20. Hence, we select the corresponding 20 rows from the 128 rows of the identity matrix $\mathbf{I}_{128}$ to form a 20×128 matrix $\mathbf{\Phi}$. The larger M/N cases are then realized by randomly adding additional rows of $\mathbf{I}_{128}$ into the already selected 20 rows. In general, the trends seem to be similar as before. However, multi-coset sampling seems to offer a better performance than complex Gaussian sampling. This is clear from Figs. 4 and 5, which show the same results as Figs. 2 and 3, respectively, but now for multi-coset sampling.

VII. Conclusion

In this paper, we have introduced a new approach for estimating the power spectrum based on samples obtained from a sub-Nyquist rate sampling device. We exploit the cyclo-stationarity of the measurements to gain more linear equations for our reconstruction problem. We have focused on the over-determined case and investigated the full column

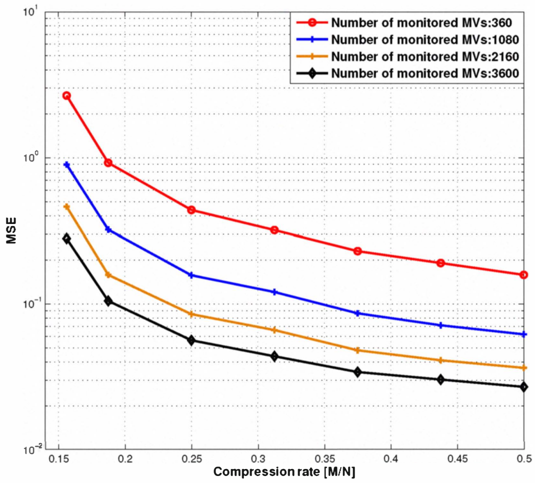

Figure 2. The MSE between the estimated power spectrum (random complex Gaussian sampling) and the theoretical one for a noiseless signal.

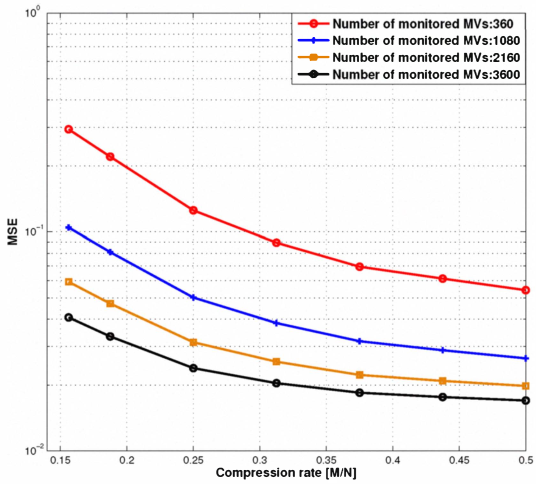

Figure 4. The MSE between the estimated power spectrum (multi-coset sampling) and the theoretical one for a noiseless signal.

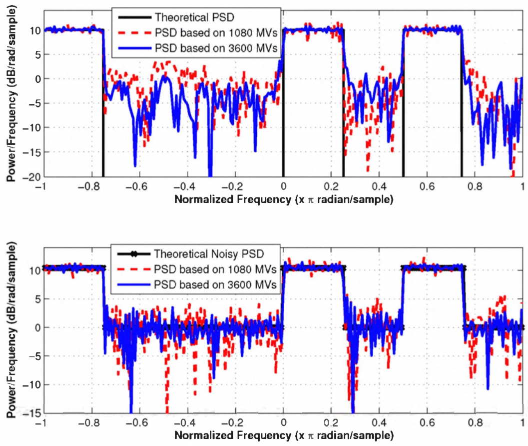

Figure 3. Estimated power spectrum for $M/N = 0.5$ (random complex Gaussian sampling) and the theoretical one; top: noise-free; bottom: noisy.

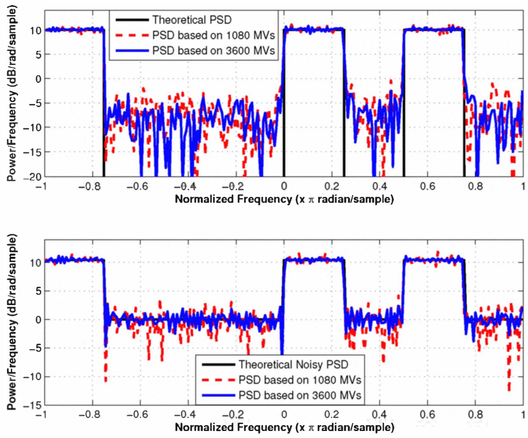

Figure 5. Estimated power spectrum for $M/N = 0.5$ (multi-coset sampling) and the theoretical one; top: noise-free; bottom: noisy.

rank property of the reconstruction matrix Θ such that a simple least-squares algorithm can be used to reconstruct the power spectrum. When this full column rank property can be guaranteed, our approach can be used to estimate the power spectrum of wide-sense stationary signals even without any sparsity assumption. Two possible realizations of the sampling matrix are discussed, namely a random complex Gaussian and a multi-coset sampling matrix. The simulation study for both sampling matrix realizations has clearly shown that our approach performs satisfactory and is able to locate the active bands, thereby making it a promising candidate for power spectrum sensing in a cognitive radio environment.

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