Optimal Scheduling for Charging and Discharging of Electric Vehicles

Yifeng He, *Member, IEEE,* Bala Venkatesh, *Senior Member, IEEE,* and Ling Guan, *Fellow, IEEE*

***Abstract*—The vehicle electrification will have a significant impact on the power grid due to the increase in electricity consumption. It is important to perform intelligent scheduling for charging and discharging of Electric Vehicles (EVs). However, there are two major challenges in the scheduling problem. First, it is challenging to find the globally optimal scheduling solution which can minimize the total cost. Second, it is difficult to find a distributed scheduling scheme which can handle a large population and the random arrivals of the EVs. In this paper, we propose a globally optimal scheduling scheme and a locally optimal scheduling scheme for EV charging and discharging. We first formulate a global scheduling optimization problem, in which the charging powers are optimized to minimize the total cost of all EVs which perform charging and discharging during the day. The globally optimal solution provides the globally minimal total cost. However, the globally optimal scheduling scheme is impractical since it requires the information on the future base loads and the arrival times and the charging periods of the EVs that will arrive in the future time of the day. To develop a practical scheduling scheme, we then formulate a local scheduling optimization problem, which aims to minimize the total cost of the EVs in the current ongoing EV set in the local group. The locally optimal scheduling scheme is not only scalable to a large EV population but also resilient to the dynamic EV arrivals. Through simulations, we demonstrate that the locally optimal scheduling scheme can achieve a close performance compared to the globally optimal scheduling scheme.**

***Index Terms*—Optimal scheduling, electric vehicle, charging and discharging, Vehicle-to-Grid (V2G), convex optimization, distributed solution, smart grid**

NOMENCLATURE

$\mathbf{N}$	Interval set
$\mathbf{M}$	Set of Electric Vehicles (EVs)
$\mathbf{M}^{CHG}$	Charging-only EV set
$\mathbf{M}^{V2G}$	Vehicle-to-Grid (V2G) EV set
x_{mi}	Charging power of EV m in interval i
T_m	Charging period of EV m
τ	Length of an interval
E_m^{ini}	Initial energy of EV m
E_m^{cap}	Battery capacity of EV m
E_m^{fin}	Final energy of EV m
P^{max}	Maximum charging power
γ_m	Final energy ratio of EV m
$\mathbf{F}$	Charging-interval matrix
z_i	Total load in interval i
L_i^b	Real base load in interval i
L_i^{bF}	Forecasted base load in interval i
y_i	Charging load in interval i
k_0	Intercept in the real-time pricing model
k_1	Slope in the real-time pricing model
C_i	Cost for EV charging in interval i
$\mathbf{Q}^{(i)}$	Previous-interval set of interval i
$\mathbf{B}$	Group set
$\mathbf{H}_k^{(i)}$	Ongoing EV set at the beginning of interval i in group k
$\mathbf{W}_k^{(i)}$	Sliding window at the beginning of interval i in group k
$\mathbf{H}_k^{(i)CHG}$	Charging-only EV set at the beginning of interval i in group k
$\mathbf{H}_k^{(i)V2G}$	V2G EV set at the beginning of interval i in group k
t_m^{arr}	Arrival time of EV m
t_m^{dep}	Departure time of EV m
t_m^{C-s}	Start time of the charging period of EV m
t_m^{C-e}	End time of the charging period of EV m

Yifeng He, Bala Venkatesh, and Ling Guan are with the Department of Electrical and Computer Engineering, Ryerson University, Toronto, Ontario, M5B2K3, Canada. E-mail: yhe@ee.ryerson.ca, bala@ryerson.ca, and lguan@ee.ryerson.ca

I. INTRODUCTION

The automotive industry is heavily investing in Plug-in Hybrid Electric Vehicles (PHEVs) and fully Electric Vehicles (EVs) mainly in order to reduce the CO2 emissions and oil dependency of current automotive technology. The vehicle electrification will have significant impacts on the power grid due to the increase in electricity consumption.

The overall load profile of electric system will be changed due to the introduction of EV charging and discharging. The charging of a large population of EVs has a significant impact on the power grid. It have been estimated that the total charging load of the EVs in US can reach 18% of the US summer peak at the EV penetration level of 30% [1]. On the other hand, an EV can also provide energy to the power grid by discharging the battery, which is known as Vehicle-to-Grid (V2G) [2]. An intelligent scheduling scheme can optimally schedule the EV charging patterns such that the load profile of the electric system can be effectively flattened. This will reduce potential capital costs and minimize operational costs. Intelligent scheduling for EV charging and discharging has become a vital step towards smart grid implementation [3][4]. The essential principle in intelligent scheduling is to reshape the load profile by charging the EV battery from the grid at the time when the demand is low and discharging the EV battery to the grid when the demand is high. However, it is challenging to schedule the patterns of EV charging and discharging in an optimal way. First, it is difficult to find the globally optimal scheduling solution which can minimize the

overall charging cost, especially in the presence of a large EV population. Second, the scheduling scheme is required to have the capacity to efficiently handle the random arrivals of the EVs.

In the recent literature, a number of scheduling schemes for EV charging and discharging have been proposed [5][6][7][8]. However, the scheduling schemes in [5][6] only dealt with battery charging without V2G function. Though the existing work on V2G scheduling [7][8] tried to optimize the charging and discharging powers to minimize the cost, their methods are essentially centralized algorithms, which may not be suitable for the EV charging and discharging systems with a large population and dynamic arrivals.

In this paper, we propose a globally optimal scheduling scheme and a locally optimal scheduling scheme for EV charging and discharging. Our contributions are summarized as follows.

- We formulate a global scheduling optimization problem, which aims to minimize the total cost for charging all EVs within the day. The optimization problem is a convex optimization problem, which can be solved efficiently. The globally optimal scheduling scheme determines the optimal charging powers for all EVs for all intervals by solving a single *global scheduling optimization problem*, thus obtaining the globally minimal total cost.
- We formulate a local scheduling optimization problem for the EVs in the local group. Based on the *local scheduling optimization problem*, we develop a locally optimal scheduling scheme, which is performed in an independent and distributed way. The locally optimal scheduling scheme is very appropriate for the EV charging and discharging systems with a large population and dynamic arrivals. The performance of the locally optimal scheduling scheme is lower than but very close to that of the globally optimal scheduling scheme.

The globally optimal scheduling scheme provides the globally minimal total cost. However, the globally optimal scheduling scheme is impractical since it requires the information on the future base loads and the arrival times and the charging periods of the EVs that will arrive in the future time of the day. Though the locally optimal scheduling scheme performs a little worse than the globally optimal scheduling scheme, it it is a practical scheme which can efficiently handle a large EV population and dynamic EV arrivals. Therefore, the locally optimal scheduling scheme is the final solution suggested in the paper. With the globally minimal total cost provided by the globally optimal scheduling scheme, we can find out the optimality gap between the two schemes.

The remainder of the paper is organized as follows. Section II discusses the related work. In Section III, we formulate and solve the global scheduling optimization problem. In Section IV, we formulate and solve the local scheduling optimization problem. The simulation results are presented in Section V, and the conclusions are drawn in Section VI.

II. Related Work

Depending on the direction of energy flow, existing work on EV charging scheduling can be classified into two classes: 1) scheduling for charging only, and 2) scheduling for both charging and discharging.

In charging-only scheduling, the scheduler tries to optimize the energy flow from the grid to the battery of the EV. In [5], Shrestha *et al.* optimized the EV battery charging during the low-cost off-peak period to minimize the charging cost in the context of Singapore. The paper in [9] examined the problem of optimizing the charge trajectory of a PHEV, defined as the time and the rate with which the PHEV obtains electricity from the power grid. In [1], a decentralized charging control algorithm was proposed to schedule charging for large populations of EVs. The paper in [10] optimized EV battery charging behavior to minimize charging costs, achieving satisfactory state-of-energy levels, and optimal power balancing. Mets *et al.* in [6] presented smart energy control strategies for charging residential PHEVs, aiming to minimize the peak load and flatten the overall load profile. The impact of different battery charging rates of EVs on the power quality of smart grid distribution systems was studied in [11]. In [12], Clement *et al.* proposed coordinated charging with stochastic programming, which was introduced to represent the error in the load forecasting.

In charging and discharging scheduling, the scheduler tries to optimize the bidirectional energy flows: from the grid to the EV battery and from the EV battery to the grid. Binary particle swarm methods were employed to optimize the V2G scheduling in a parking lot to maximize the profit [7][8][13]. Sortomme *et al.* proposed an unidirectional regulation at the aggregator, in which several smart charging algorithms were examined to set the point about which the rate of charge varies while performing regulation [14]. The paper in [16] developed an aggregator for V2G frequency regulation with the optimal control strategy, which aims to maximize the revenue. Jang *et al.* proposed a method for an analytic estimation of the probability distribution of the Procured Power Capacity (PPC), based on which the optimal contract size was decided [17]. The paper in [18] presented a real-time model of a fleet of plug-in vehicles performing V2G power transactions. In [19], Singh *et al.* demonstrated that the coordinated charging and discharging of EVs can improve the voltage profile and reduce the power transmission loss. The paper in [15] discussed the vehicle to grid integration and described the vehicle-to-grid communication interface.

III. Global Scheduling Optimization

In this section, we formulate a global scheduling optimization for EV charging and discharging based on a real-time pricing model. The solution to the optimization problem provides a globally optimal scheduling scheme which minimizes the total cost.

A. System Models

We study the battery charging and discharging of EVs during a day, which is evenly divided into a set of intervals. The interval set is denoted by $\mathbf{N}$. The length of an interval is denoted by τ. We assume that the charging or discharging power in an interval is kept unchanged. In this paper, we divide

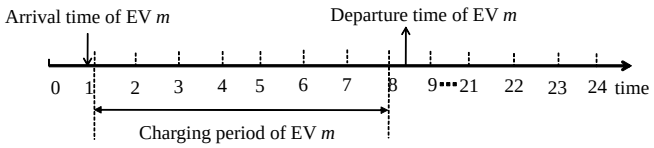


Fig. 1. Charging period of EV m

the day into 24 intervals such that the interval length is given by $\tau = 1$ hour.

The set of the EVs, which perform charging and discharging during the day, is denoted by $\mathbf{M}$. The EV set $\mathbf{M}$ consists of two sets: 1) the charging-only EV set $\mathbf{M}^{CHG}$, which includes the EVs that only charge their battery and do not provide the battery energy to the grid, and 2) the V2G EV set $\mathbf{M}^{V2G}$, which includes the EVs that perform both battery charging and battery discharging. We have $\mathbf{M} = \mathbf{M}^{CHG} + \mathbf{M}^{V2G}$. The charging or discharging power of EV m in interval i is denoted by x_{mi} ($\forall m \in \mathbf{M}, \forall i \in \mathbf{N}$). In order to unify the notation, we just call x_{mi} the charging power of EV m in interval i. If $x_{mi} > 0$, it means that EV m charges its battery in interval i. If $x_{mi} < 0$, it means that EV m discharges its battery in interval i. The EVs in the charging-only set $\mathbf{M}^{CHG}$ always satisfy $x_{mi} \geq 0$ since they do not discharge their battery at any time. On the other hand, the EVs in the V2G set $\mathbf{M}^{V2G}$ may have a positive, zero, or negative charging power x_{mi} in interval i ($\forall i \in \mathbf{N}$) since they have bidirectional energy flows between the battery and the power grid.

The arrival time of EV m, denoted by t_m^{arr}, is the time when EV m is plugged into the charging station. The departure time of EV m, denoted by t_m^{dep}, is the time when EV m is plugged out of the charging station. The charging period of EV m, denoted by T_m, is the period in which EV m charges or discharges its battery. Since we divide the time into multiple intervals, we define the charging period T_m of EV m as the set of continuous intervals that fall between the arrival time t_m^{arr} and the departure time t_m^{dep} of EV m, as illustrated in Fig. 1. The initial energy of EV m, denoted by E_m^{ini}, is defined as the battery energy at the arrival time t_m^{arr}. The battery capacity of EV m is denoted by E_m^{cap}. The final energy of EV m, denoted by E_m^{fin}, is defined as the battery energy at the departure time t_m^{arr}. The final energy E_m^{fin} is no larger than the battery capacity E_m^{cap}. We define a *final energy ratio* of EV m as $\gamma_m = E_m^{fin}/E_m^{cap}$ where $0 \leq \gamma_m \leq 1$. The charging station can automatically detect the arrival time, the initial energy and the battery capacity of EV m when the EV is connected to the charging station. The departure time and the final energy ratio of EV m are provided to the charging station by the user of EV m before charging is started. The charging station can determine the charging period T_m of EV m from the parameters t_m^{arr} and t_m^{dep}. EV m performs charging and discharging activities during the charging period T_m. To represent the relationship between the charging/discharging activities and the intervals, we define a *charging-interval matrix* $\mathbf{F} \subset \{0,1\}^{|\mathbf{M}|\times|\mathbf{N}|}$ where $|\mathbf{M}|$ and $|\mathbf{N}|$ denote the number of elements in the set $\mathbf{M}$ and the set $\mathbf{N}$, respectively. The elements of $\mathbf{F}$ are defined as

$$f_{mi} = \begin{cases} 1, & \text{if interval } i \text{ falls within the charging period } T_m \\ & \text{of EV } m, \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

In this paper, we consider the scheduling of EV charging and discharging in a small geographic area. In our real-time pricing model, we make two assumptions: 1) the losses between nodes are small and thus neglectable, and 2) there is no congestion in transmission. The two assumptions allow us to neglect the spatial variation of the electricity prices. The electricity price at a time instant is the same regardless of the charging location. The optimizations of EV charging based on only temporal variation but not spatial variation of the price have be seen in [1][6]. The electricity price is modeled as a linear function of the instant load [1], which is given as follows.

$$g(z_t) = k_0 + k_1 z_t, \tag{2}$$

where k_0 is the intercept and k_1 is the slope, which are both non-negative real number, and z_i is the total load at time t.

The total load in interval i consists of two parts: 1) the base load L_i^b, which represents the load of all electricity consumptions in interval i except EV charging, and 2) the charging load y_i, which represents the load of EV charging in interval i. We assume that the base load L_i^b remains constant in interval i. The charging load in interval i is given by $y_i = \sum_{m\in\mathbf{M}} x_{mi} f_{mi}$. If the load from the grid to the batteries of the EVs is greater than that from the batteries of the EVs to the grid in interval i, the charging load y_i is positive. Otherwise, it is negative. The total load in interval i is given by $z_i = L_i^b + y_i = L_i^b + \sum_{m\in\mathbf{M}} x_{mi} f_{mi}$. Since both the base load L_i^b and the charging power x_{mi} ($\forall m \in \mathbf{M}, \forall i \in \mathbf{N}$) remain constant in interval i, the total load z_i is constant in interval i.

In this paper, we define the charging cost in interval i, denoted as C_i, as the total amount of the money that the customers pay for charging and discharging of their EVs in interval i. Based on the pricing model, the charging cost in interval i ($\forall i \in \mathbf{N}$) is given by

$$\begin{aligned} C_i &= \int_{L_i^b}^{z_i} (k_0 + k_1 z_t) dz_t \\ &= (k_0 z_i + \tfrac{k_1}{2} z_i^2) - (k_0 L_i^b + \tfrac{k_1}{2} (L_i^b)^2). \end{aligned} \tag{3}$$

As shown in Equation (3), the charging cost C_i can be positive or negative. If the charging load y_i, given by $y_i = z_i - L_i^b$, in interval i is positive, the charging cost C_i is positive. Otherwise, it is negative.

B. Problem Formulation and Solution

In order to find a globally optimal scheduling scheme for the EVs that perform charging and discharging during the day, we make the following assumptions: 1) the arrival time and the departure time of each EV in the EV set $\mathbf{M}$ are known (this is realistic in the case where each EV user signs the charging contract and bring in the EV at a designated time); 2) the initial energy and the final energy of the battery for each EV in the EV set $\mathbf{M}$ are known; 3) the base load in each interval

of the day is known; and 4) a central controller collects all the information and then performs the scheduling optimization.

The total cost is defined as the sum of the charging costs over the interval set $\mathbf{N}$. The total cost is then given by

$$\begin{aligned} C_{tot} &= \sum_{i\in\mathbf{N}} C_i \\ &= \sum_{i\in\mathbf{N}}((k_0 z_i + \tfrac{k_1}{2} z_i^2) - (k_0 L_i^b + \tfrac{k_1}{2}(L_i^b)^2)). \end{aligned} \quad (4)$$

The *global scheduling optimization problem* can be stated as to minimize the total cost of the EVs which perform charging and discharging during the day, by optimizing the total load z_i in interval i ($\forall i \in \mathbf{N}$) and the charging power x_{mi} ($\forall m \in \mathbf{M}, \forall i \in \mathbf{N}$), subject to the relationship between the total load in an interval and the charging power of an individual EV, the instant energy constraints, the final energy constraints, and the lower bound and the upper bound of the charging power. Mathematically, the optimization problem can be formulated as follows.

$$\underset{\mathbf{x},\mathbf{z}}{\text{Minimize}} \quad \sum_{i\in\mathbf{N}}((k_0 z_i + \frac{k_1}{2} z_i^2) - (k_0 L_i^b + \frac{k_1}{2}(L_i^b)^2)) \quad (5a)$$

subject to

$$z_i = L_i^b + \sum_{m\in\mathbf{M}} x_{mi} f_{mi}, \forall i \in \mathbf{N}, \quad (5b)$$

$$0 \le E_m^{ini} + \sum_{k\in\mathbf{Q}^{(i)}} \tau x_{mk} f_{mk} \le E_m^{cap}, \forall m \in \mathbf{M}, \forall i \in \mathbf{N}, \quad (5c)$$

$$E_m^{ini} + \sum_{i\in\mathbf{N}} \tau x_{mi} f_{mi} \ge \gamma_m E_m^{cap}, \forall m \in \mathbf{M}, \quad (5d)$$

$$0 \le x_{mi} \le P^{max}, \forall m \in \mathbf{M}^{CHG}, \forall i \in \mathbf{N}, \quad (5e)$$

$$-P^{max} \le x_{mi} \le P^{max}, \forall m \in \mathbf{M}^{V2G}, \forall i \in \mathbf{N}. \quad (5f)$$

In the optimization problem (5), the objective function (5a) to be minimized is the total cost of the EVs which perform charging and discharging during the day. Constraints (5b) represent the relationship between the total load in an interval and the charging power of an individual EV. Constraints (5c) are the instant energy constraints, which require the energy of EV m ($\forall m \in \mathbf{M}$) at the end of interval i ($\forall i \in \mathbf{N}$), given by $E_m^i = E_m^{ini} + \sum_{k\in\mathbf{Q}^{(i)}} \tau x_{mk} f_{mk}$, to be no less than 0 and no larger than the battery capacity E_m^{cap} of EV m. Constraints (5d) are the final energy constraints, which require the final energy of EV m ($\forall m \in \mathbf{M}$), given by $E_m^{fin} = E_m^{ini} + \sum_{i\in\mathbf{N}} \tau x_{mi} f_{mi}$, to be no less than the specified energy level, which is given by $\gamma_m E_m^{cap}$. Constraints (5e) specify the lower bound 0 and the upper bound P^{max} of the charging power x_{mi} for the EVs in the charging-only set $\mathbf{M}^{CHG}$. Constraints (5f) specify the lower bound $(-P^{max})$ and the upper bound P^{max} of the charging power x_{mi} for the EVs in the V2G set $\mathbf{M}^{V2G}$.

In the optimization problem (5), the objective function (5a) is convex, and all the constraint functions are linear. Therefore the optimization problem (5) is a convex optimization problem, which can be solved efficiently with the interior point methods [20]. The solution to the optimization problem (5) provides the globally optimal scheduling scheme for EV charging and discharging during the day.

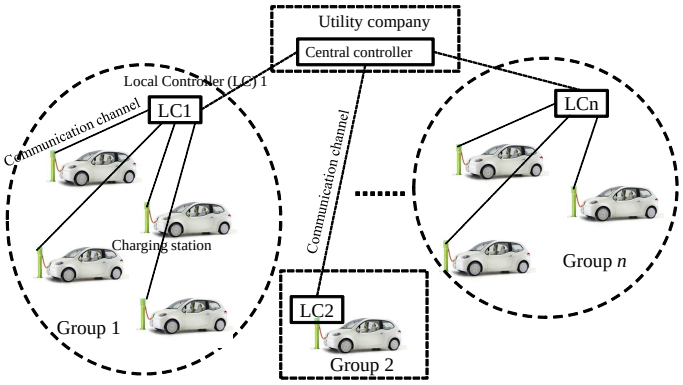


Fig. 2. Illustration of communications and controls in the locally optimal scheduling scheme

IV. Local Scheduling Optimization

The globally optimal scheduling scheme gives the globally minimal total cost. However, the globally optimal scheduling scheme is impractical due to the following reasons. First, the EVs that will arrive in the future time of the day are unknown at the current moment. Second, the base load in the future time of the day is unknown at the current moment. Third, it is not scalable for a centralized scheduling scheme in which the central controller may be overrun by a large number of EVs.

In this section, we formulate a local scheduling optimization problem, which relaxes the assumptions used in the global scheduling optimization problem (5). The solution to the local scheduling optimization problem is a locally optimal scheduling scheme, which can achieve the performance close to that in the globally optimal scheduling scheme. Compared to the globally optimal scheduling scheme, the locally optimal scheduling scheme is practical and scalable.

A. Problem Formulation and Solution

In the globally optimal scheduling scheme, since we assume that we have the global knowledge of the information about the EVs and the base load within the day, we can find the optimal charging powers at each interval by solving the global scheduling optimization problem (5) only once. In the locally optimal scheduling scheme, we do not know the information of the future load and the future EVs. We propose a locally optimal scheduling scheme to find the optimal charging powers in the next interval for the local EVs by using a sliding window mechanism.

In the locally optimal scheduling scheme, we perform the scheduling optimization based on groups. A group of EVs includes the EVs in one location or multiple nearby locations. For example, the EVs which perform charging and discharging in a parking lot can be classified into a group, and the EVs in a residential garage can be classified into another group. There is a Local Controller (LC) for each group. The communications and controls in the locally optimal scheduling scheme are illustrated in Fig 2. The local controller establishes communication connections with the central controller located in the utility company and the charging stations at the local site. The local controller receives the forecasted loads for the day from the

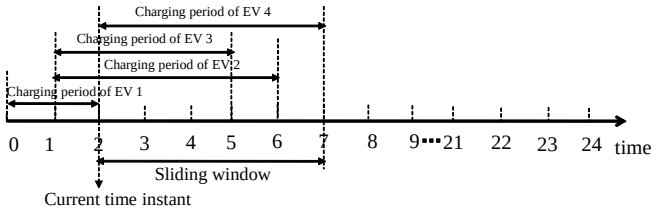


Fig. 3. Illustration of the ongoing EV set and the sliding window in the locally optimal scheduling scheme

central controller. The local controller communicates with each charging station in real time to collect the EV information, based on which it performs scheduling optimization and then instructs each local EV to charge or discharge the battery with the optimal charging powers.

We denote the group set by $\mathbf{B}$. Since each local controller performs scheduling independently, we will just study the scheduling optimization in group k ($\forall k \in \mathbf{B}$). The local controller does not know the future arrivals of the EVs in the group. Therefore, we propose to update the charging powers at the beginning of each interval by using a sliding window. At the beginning of interval i ($\forall i \in \mathbf{N}$), we need to first determine the current ongoing EV set $\mathbf{H}_k^{(i)}$ and the current sliding window $\mathbf{W}_k^{(i)}$. Let the current time t^{cur} be the beginning of interval i ($\forall i \in \mathbf{N}$). Each EV has a charging period. The start time and the end time of the charging period of EV m is denoted by t_m^{C-s} and t_m^{C-e}, respectively. If EV m satisfies $t_m^{C-s} \leq t^{cur}$ and $t_m^{C-e} > t^{cur}$, we say that EV m belongs to the current ongoing EV set $\mathbf{H}_k^{(i)}$. The current sliding window $\mathbf{W}_k^{(i)}$ at the beginning of interval i is defined as the set of the consecutive intervals between the start time t_i^{W-s} and the end time t_i^{W-e} of the sliding window. The start time of the sliding window is always given by $t_i^{W-s} = t^{cur}$, and the end time of the sliding window is defined by $t_i^{W-e} = \max\{t_m^{C-e} | m \in \mathbf{H}_k^{(i)}\}$. Fig. 3 illustrates the ongoing EV set and the sliding window at the beginning of interval 2. As shown in Fig. 3, EV 1 has completed charging since $t_1^{C-s} \leq t^{cur}$ and $t_1^{C-e} \leq t^{cur}$. EVs 2, 3, and 4 satisfy $t_m^{C-s} \leq t^{cur}$ and $t_m^{C-e} > t^{cur}$. Therefore the current ongoing EV set is given by $\mathbf{H}_k^{(2)} = \{\text{EVs } 2, 3, 4\}$, and the current sliding window is given by $\mathbf{W}_k^{(2)} = \{\text{intervals } 2, 3, 4, 5, 6\}$.

EV m ($\forall m \in \mathbf{H}_k^{(i)}$) performs charging and discharging activities during its charging period. At the beginning of interval i ($\forall i \in \mathbf{N}$), we define a charging-interval matrix $\mathbf{F}^{(i)} \subset \{0,1\}^{|\mathbf{H}_k^{(i)}| \times |\mathbf{W}_k^{(i)}|}$ whose elements are given by

$$f_{mj}^{(i)} = \begin{cases} 1, & \text{if interval } j \text{ falls within } \mathbf{W}_k^{(i)} \text{ and within} \\ & \text{the charging period of EV } m, \\ 0, & \text{otherwise.} \end{cases} \tag{6}$$

In order to determine the charging powers in the current sliding window, we need to know the base loads in the sliding window $\mathbf{W}_k^{(i)}$, which can be forecasted using similar-day approach, regression methods or time-series methods [21]. In this paper, we adopt the similar-day approach [21], in which the base load in each interval of the sliding window is estimated by averaging the base loads of the same interval of the recent days with similar weather conditions. The forecasted base load is denoted by L_j^{bF} for $j \in \mathbf{W}_k^{(i)}$.

Based on the current ongoing EV set $\mathbf{H}_k^{(i)}$ and the current sliding window $\mathbf{W}_k^{(i)}$, we formulate the *local scheduling optimization problem* for the current moment in group k. The optimization problem can be stated as to minimize the total cost of the EVs in the current ongoing EV set $\mathbf{H}_k^{(i)}$ during the current sliding window $\mathbf{W}_k^{(i)}$, by optimizing the total load z_j in interval j ($\forall i \in \mathbf{W}_k^{(i)}$) and the charging power x_{mj} ($\forall m \in \mathbf{H}_k^{(i)}, \forall j \in \mathbf{W}_k^{(i)}$), subject to the relationship between the total load in an interval and the charging power of an individual EV, the instant energy constraints, the final energy constraints, and the lower bound and the upper bound of the charging power. Mathematically, the optimization problem can be formulated as follows.

$$\underset{\mathbf{x},\mathbf{z}}{\text{Minimize}} \quad \sum_{j \in \mathbf{W}_k^{(i)}} ((k_0 z_j + \frac{k_1}{2} z_j^2) - (k_0 L_j^{bF} + \frac{k_1}{2} (L_j^{bF})^2)) \tag{7a}$$

subject to

$$z_j = L_j^{bF} + \sum_{m \in \mathbf{H}_k^{(i)}} x_{mj} f_{mj}^{(i)}, j \in \mathbf{W}_k^{(i)}, \tag{7b}$$

$$0 \leq E_m^{(i)ini} + \sum_{s \in \mathbf{Q}_k^{(j)}} \tau x_{ms} f_{ms}^{(i)} \leq E_m^{cap}, m \in \mathbf{H}_k^{(i)}, j \in \mathbf{W}_k^{(i)}, \tag{7c}$$

$$E_m^{(i)ini} + \sum_{j \in \mathbf{W}_k^{(i)}} \tau x_{mj} f_{mj}^{(i)} \geq \gamma_m E_m^{cap}, \forall m \in \mathbf{H}_k^{(i)}, \tag{7d}$$

$$0 \leq x_{mj} \leq P^{max}, \forall m \in \mathbf{H}_k^{(i)CHG}, j \in \mathbf{W}_k^{(i)}, \tag{7e}$$

$$-P^{max} \leq x_{mj} \leq P^{max}, \forall m \in \mathbf{H}_k^{(i)V2G}, j \in \mathbf{W}_k^{(i)}. \tag{7f}$$

In the local scheduling optimization problem (7), the objective function (7a) to be minimized is the total cost of the EVs in the current ongoing EV set $\mathbf{H}_k^{(i)}$ during the current sliding window $\mathbf{W}_k^{(i)}$. Constraints (7b) represent the relationship between the total load and the charging power of an individual EV in an interval of the current sliding window $\mathbf{W}_k^{(i)}$. Constraints (7c) are the instant energy constraints, which require the energy of EV m ($\forall m \in \mathbf{H}_k^{(i)}$) at the end of interval j ($\forall j \in \mathbf{W}_k^{(i)}$), given by $E_m^{(i)j} = E_m^{(i)ini} + \sum_{s \in \mathbf{Q}_k^{(j)}} \tau x_{hs} f_{hs}^{(i)}$, to be no less than 0 and no larger than the battery capacity E_m^{cap} of EV m. In Constraints (7c), $E_m^{(i)ini}$ denotes the energy at the beginning of interval i, $\mathbf{Q}_k^{(j)}$ denotes the current previous-interval set, defined as the set of intervals that belong to the current sliding window $\mathbf{W}_k^{(i)}$ but are no later than interval j. Constraints (7d) are the final energy constraints, which require the final energy of EV m ($\forall m \in \mathbf{H}_k^{(i)}$) to be no less than $\gamma_m E_m^{cap}$. Constraints (7e) specify the lower bound 0 and the upper bound P^{max} of the charging power x_{mj} for the EVs in the current charging-only EV set $\mathbf{H}_k^{(i)CHG}$. Constraints (7f) specify the lower bound $(-P^{max})$ and the upper bound P^{max} of the charging power x_{mj} for the EVs in the current V2G EV set $\mathbf{H}_k^{(i)V2G}$.

The local scheduling optimization problem (7) at the beginning of interval i is a convex optimization problem, which can be solved efficiently with the interior point methods [20]. By solving the optimization problem (7), we obtain optimal charging powers x^*_{mj} ($\forall m \in \mathbf{H}^{(i)}_k, j \in \mathbf{W}^{(i)}_k$), among which we only accept and execute the optimal charging powers x^*_{mi} ($\forall m \in \mathbf{H}^{(i)}_k$) for interval i, and discard the other charging powers x^*_{mj} ($\forall m \in \mathbf{H}^{(i)}_k, j \in \mathbf{W}^{(i)}_k, j > i$) which will be finally updated at the beginning of interval j ($j > i$).

B. Distributed Scheduling Protocol

Based on the local scheduling optimization problem (7), we develop a distributed scheduling protocol to implement the locally optimal scheduling scheme. The protocol for locally optimal scheduling executed at the local controller k is shown in Table I. The functionalities of the central controller are to perform load forecasting and collect the actual charging load for each EV. At the beginning of the day, the central controller forecasts the base loads of the day using similar-day approach, and then broadcasts the forecasted base loads to all the local controllers. After receiving the forecasted base loads of the day, the local controller k for group k ($\forall k \in \mathbf{B}$) performs the scheduling optimization at the beginning of each interval, starting from the first interval until the last interval in the interval set $\mathbf{N}$ in sequence. In group k at the beginning of interval i ($\forall i \in \mathbf{N}$), the local controller k determines the current ongoing EV set $\mathbf{H}^{(i)}_k$ and the current sliding window $\mathbf{W}^{(i)}_k$. Since the local controller k does not know the real base loads in the future intervals, the price in interval j ($\forall j \in \mathbf{W}^{(i)}_k$) is determined based on the forecasted base load and the charging load of the local EVs in the interval. The price in interval j varies from $(k_0+k_1L^{bF}_j)$ at the beginning of interval j to $(k_0+k_1(L^{bF}_j+\sum_{m\in\mathbf{H}^{(i)}_k} x_{mj}f^{(i)}_{mj}))$ at the end of interval j. At the end of each interval, each local controller reports the actual charging load of each local EV in this interval to the central controller.

There are two major advantages for the locally optimal scheduling scheme. First, it is scalable. Even when the number of the total EVs is large, each local controller only needs to take care of the scheduling optimization for the local EVs. Second, it is resilient to the dynamics of EV arrivals. The locally optimal scheduling scheme collects the EV information and then updates the charging powers at the beginning of each interval, thus responding quickly to the dynamic arrivals of the EVs.

C. Considering the Cost of Battery Lifetime Reduction

The lifetime of the battery of an EV will be reduced due to frequent charging and discharging. In this section, we consider the the *cost of battery lifetime reduction* caused by EV charging and discharging. We model the *cost of battery lifetime reduction* for EV m, denoted by ψ_m, as the sum of two cost components: the cost component ψ^A_m caused by the amount of charging and discharging power in each interval, and the cost component ψ^F_m caused by the fluctuation of charging and discharging power between any two consecutive intervals.

The cost component ψ^A_m of EV m depends on the amount of charging and discharging power of EV m in each interval of the day, and it is given by

$$\psi^A_m = \textstyle\sum_{i\in\mathbf{N}} \beta x^2_{mi}, \tag{8}$$

where β is a model parameter, and x_{mi} is the charging power of EV m in interval i. As shown in Equation (8), the cost component ψ^A_m of EV m is proportional to the sum of the squares of charging powers over the interval set $\mathbf{N}$. Given the same initial energy and the same final energy, the EV which discharges more energy during the day will have a higher cost compared to the one which discharges less energy.

The cost component ψ^F_m of EV m depends on the fluctuations of charging and discharging powers of EV m during the day, and it is given by

$$\psi^F_m = \textstyle\sum_{i=2}^{|\mathbf{N}|} \eta(x_{mi} - x_{m(i-1)})^2, \tag{9}$$

where $|\mathbf{N}|$ represents the number of the intervals in the interval set $\mathbf{N}$, and η is a model parameter. As shown in Equation (9), the cost component ψ^F_m of EV m is proportional to the sum of the squared differences of the charging powers between two consecutive intervals over the interval set $\mathbf{N}$. If the charging powers in the two consecutive intervals, intervals i and $(i-1)$ for $i = 2, ..., |\mathbf{N}|$, have opposite signs, a higher value will be added to the cost component ψ^F_m of EV m, compared to the case in which the charging powers in the two consecutive intervals have the same sign. In other words, a change of charging direction of EV m in interval i ($i = 2, ..., |\mathbf{N}|$) adds a higher value to the cost component ψ^F_m. Given the same initial energy and the same final energy, the EV which frequently switches between charging and discharging during the day will have a higher cost compared to the one which does not frequently switch between charging and discharging.

The *cost of battery lifetime reduction* for all EVs in the EV set $\mathbf{M}$ during the day is given by

$$\begin{aligned}\psi &= \textstyle\sum_{m\in\mathbf{M}} \psi_m \\ &= \textstyle\sum_{m\in\mathbf{M}} (\psi^A_m + \psi^F_m) \\ &= \textstyle\sum_{m\in\mathbf{M}} (\sum_{i\in\mathbf{N}} \beta x^2_{mi} + \sum_{i=2}^{|\mathbf{N}|} \eta(x_{mi} - x_{m(i-1)})^2).\end{aligned} \tag{10}$$

By adding the *cost of battery lifetime reduction* to the total cost, the objective function of the global scheduling optimization problem is changed to

$$\begin{aligned}f_{gso} = \; & \textstyle\sum_{i\in\mathbf{N}} ((k_0 z_i + \frac{k_1}{2} z^2_i) - (k_0 L^b_i + \frac{k_1}{2}(L^b_i)^2)) \\ & + \textstyle\sum_{m\in\mathbf{M}} \sum_{i\in\mathbf{N}} \beta x^2_{mi} \\ & + \textstyle\sum_{m\in\mathbf{M}} \sum_{i=2}^{|\mathbf{N}|} \eta(x_{mi} - x_{m(i-1)})^2.\end{aligned} \tag{11}$$

The global scheduling optimization problem considering the the cost of battery lifetime reduction is the same as the optimization problem (5) except a different objective function given by f_{gso} in Equation (11).

The local scheduling optimization problem considering the *cost of battery lifetime reduction*, at the beginning of interval i ($\forall i \in \mathbf{N}$) in group k ($\forall k \in \mathbf{B}$), is the same as the optimization problem (7) except a different

TABLE I
PROTOCOL FOR LOCALLY OPTIMAL SCHEDULING EXECUTED AT THE LOCAL CONTROLLER k

Initialize: At the beginning of the day, the central controller forecasts the base loads of the day using similar-day approach, and then broadcasts the forecasted base loads to all the local controllers.
At the beginning of interval i, $i = 1, 2, ..., |\mathbf{N}|$ where $|\mathbf{N}|$ represents the number of intervals in the set $\mathbf{N}$, the local controller k does the following:
1. Communicate with each charging station respectively to collect the EV information,
2. Determine the current ongoing EV set $\mathbf{H}_k^{(i)}$ and the current sliding window $\mathbf{W}_k^{(i)}$,
3. Determine the current charging-interval matrix $\mathbf{F}^{(i)}$,
4. Find the optimal charging powers x_{mi}^* ($\forall m \in \mathbf{H}_k^{(i)}$) by solving the local scheduling optimization problem (7),
5. Instruct EV m ($\forall m \in \mathbf{H}_k^{(i)}$) to perform charging with the optimal charging power x_{mi}^* in interval i.

objective function given by $f_{lso}^{(i)} = \sum_{j\in\mathbf{W}_k^{(i)}}((k_0 z_j + \frac{k_1}{2} z_j^2) - (k_0 L_j^{bF} + \frac{k_1}{2}(L_j^{bF})^2)) + \sum_{m\in\mathbf{H}_k^{(i)}} \sum_{j\in\mathbf{W}_k^{(i)}} \beta x_{mj}^2 + \sum_{m\in\mathbf{H}_k^{(i)}} \sum_{j=2}^{|\mathbf{W}_k^{(i)}|} \eta(x_{mj} - x_{m(j-1)})^2$, where $\mathbf{H}_k^{(i)}$ is the ongoing EV set and $\mathbf{W}_k^{(i)}$ is the sliding window at the beginning of interval i.

The global scheduling optimization problem and the local scheduling optimization problem, which consider the *cost of battery lifetime reduction*, are both convex optimization problems. Therefore, they can be solved efficiently with the interior point methods [20].

V. SIMULATIONS

We perform extensive simulations to evaluate the proposed scheduling schemes for EV charging and discharging.

A. Simulation Setting

We consider the electric load in a microgrid. We examine EV charging and discharging during a day (24 hours) starting from 12:00 AM in midnight. The day is evenly divided into 24 intervals. Each interval has a length of 1 hour. The base load at each interval is simulated by scaling the real load in Toronto on August 21, 2009 (Friday) by a factor of 1/1500 [22]. The unit of the electricity price is Canadian dollar (C$)/kWh, and the unit of the cost is C$. In the pricing model shown in Equation (2), we set $k_0 = 10^{-4}$ C$/kWh and $k_1 = 1.2 \times 10^{-4}$ C$/kWh/kW. The battery parameters of the EVs are based on the specifications of the Chrevolet Volt [23]. The battery capacity is 16 kWh with electric range up to 64.0 kM [23]. We assume the same specifications for every EV. The battery energy is required to reach at least 90% of the battery capacity at the end of the charging period. The maximum charging power for all EVs is set to $P^{max} = 5.0$ kW.

The arrival times, the charging periods, and the initial energy of the EVs are modeled as follows. The total number of the EVs is set to 200 by default. The arrival times of the EVs are uniformly distributed across the day, and the percentage of arriving vehicles in any hour of the day is less than 15%. The charging periods of the EVs are uniformly distributed between 4 and 12 hours. The initial energy of the EVs is uniformly distributed between 0 and 80% of the battery capacity.

To solve the optimization problems (5) and (7), we use CVX, a package for specifying and solving convex programs [24][25].

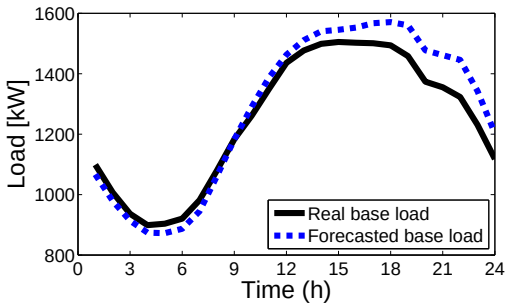
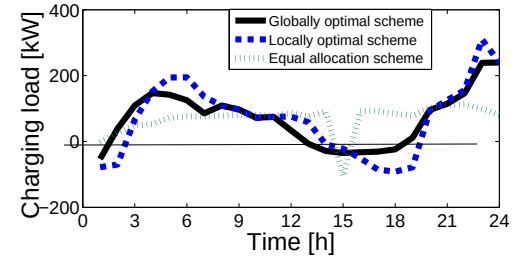


Fig. 4. Comparison of the real base load and the forecasted base load

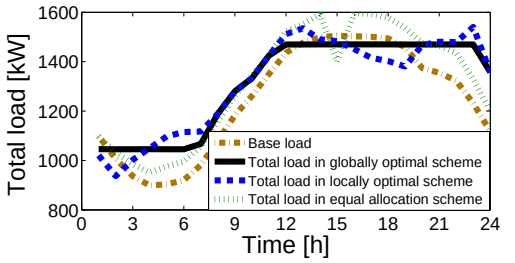
B. Simulation Results

The globally optimal scheduling scheme is a globally optimal solution which requires the perfect information. Therefore, the real base loads are used in the global scheduling optimization problem. However, in practical systems, the real base loads in the future intervals are unavailable. The locally optimal scheduling scheme is a practical solution. Therefore, the forecasted base loads are used in the local scheduling optimization problem. The comparison of the real and forecasted base loads is shown in Fig. 4. The real base load is obtained by scaling the load in Toronto on August 21, 2009 (Friday) by a factor of 1/1500. The forecasted base load is obtained with a similar-day approach, in which we average the loads of 8 weekdays in Toronto from August 11, 2009 to August 20, 2009 [22]. The *mean relative error* between the forecasted and real base loads, defined as $\epsilon = (1/|\mathbf{N}|) \sum_{i\in\mathbf{N}} |L_i^{bF} - L_i^b|/L_i^b$, is 0.041, which is quite small.

We compare three scheduling schemes: 1) the globally optimal scheduling scheme, which is the optimal solution to the global scheduling optimization problem (5), 2) the locally optimal scheduling scheme, which is the optimal solution to the local scheduling optimization problem (7), and 3) the equal allocation scheme, in which the charging power of an EV in an interval is allocated based on the following criteria: a) charging or discharging of an EV in an interval is determined based on the electricity price on the previous day, and b) the absolute value of the charging power of the EV is equal in each interval. The comparison is performed under the following simulation setting. The number of the total EVs is 200, and all EVs can perform both charging and discharging. The total EVs are divided into two groups, and each group consists of 100 EVs. In order for fair comparison, the total costs in the three schemes are all calculated based on

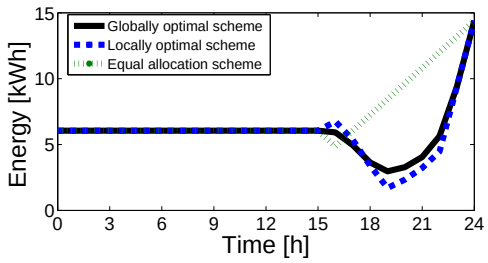


(a)

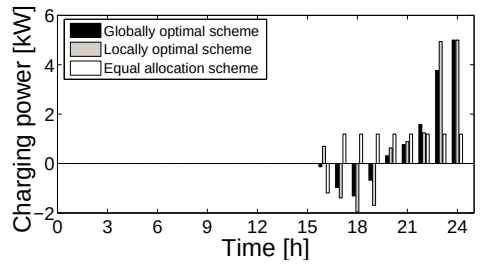


(b)

Fig. 5. Variation of charging load and total load in each interval: (a) the charging load, and (b) the total load



(a)



(b)

Fig. 6. Variation of energy and charging power of EV 5 in each interval: (a) the energy, and (b) the charging power

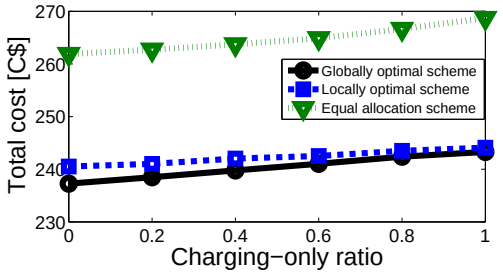


Fig. 7. Variation of total cost with different charging-only ratio

the real base loads. The total costs in the globally optimal scheduling scheme, the locally optimal scheduling scheme and the equal allocation scheme are 237.26 C$, 240.52 C$, and 261.88 C$, respectively. The globally optimal scheduling scheme and the locally optimal scheduling scheme reduce the total cost by 9.40% and 8.16%, respectively, compared to the equal allocation scheme. The variation of the charging load and the total load in each interval in the three schemes is shown in Fig. 5. We can see from Fig. 5(a) that the globally optimal scheduling scheme and the locally optimal scheduling scheme charge the battery from the grid in the intervals with a lower demand and discharge the battery to the grid in the intervals with a higher demand to achieve a low total cost. The globally optimal scheduling scheme and the locally optimal scheduling scheme can reshape the total load profile, as shown in Fig. 5(b). The globally optimal scheduling scheme flattens the total load profile in intervals 1-7 and intervals 12-23 to minimize the total cost. The globally optimal scheduling scheme determines the optimal charging powers for all EVs for all intervals by solving a single *global scheduling optimization problem*, thus obtaining the globally minimal total cost. The locally optimal scheduling scheme determines the optimal charging powers for a group of EVs for interval i ($i \in \mathbf{N}$) by solving the *local scheduling optimization problem* for interval i, respectively. The local scheduling optimization problem is formulated based on the local knowledge, while the global scheduling optimization problem is formulated based on the global knowledge. Therefore, the total cost obtained in the locally optimal scheduling scheme is close to but always larger than that in the globally optimal scheduling scheme.

We next examine the scheduling of charging power for a randomly chosen EV (e.g., EV 19) in Fig. 6. The charging period of EV 19 is from interval 16 to interval 24. As shown in Fig. 6(b), the equal allocation scheme discharges the battery in interval 16, and then charges the battery in intervals 17-24, with a constant charging or discharging power. The globally optimal scheduling scheme and the locally optimal scheduling scheme determine the charging powers by solving the optimization problems (5) and (7), respectively. All the three schemes enable EV 19 to reach the same final energy, as shown in Fig. 6(a).

Each EV decides whether it is willing to discharge the battery to the grid before starting charging. Therefore, each EV is classified into either the charging-only set $\mathbf{M}^{CHG}$ or the V2G set $\mathbf{M}^{V2G}$. We define a *charging-only ratio* as the ratio between the number of EVs in the charging-only set $\mathbf{M}^{CHG}$ and the number of the total EVs. Fig. 7 shows the impact of the charging-only ratio to the total cost. The increase of the charging-only ratio means more EVs in the charging-only set $\mathbf{M}^{CHG}$ and less EVs in the V2G set $\mathbf{M}^{V2G}$, thus causing a higher total cost in all three schemes, as shown in Fig. 7.

In the locally optimal scheduling scheme, the local controller schedules the EVs in the local group in an independent and distributed way. We define the *group size* as the number of the EVs in the group, and evaluate the performance under different average group size in Fig. 8. The total number of EVs is fixed at 200. Therefore, a larger average group size indicates

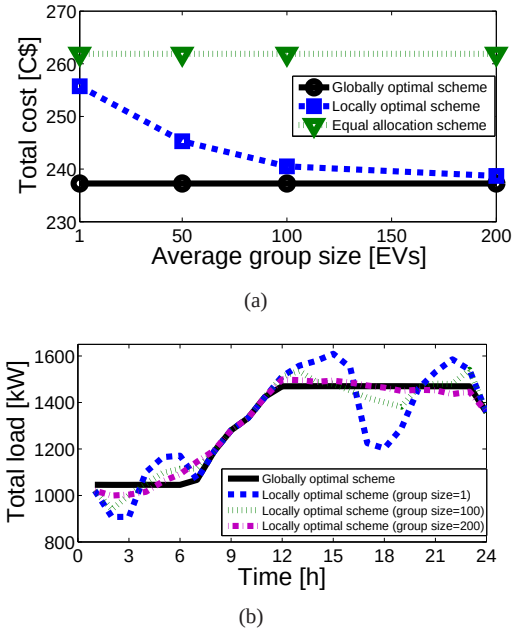


Fig. 8. Performance evaluation under different group size: (a) the total cost, and (b) the total load

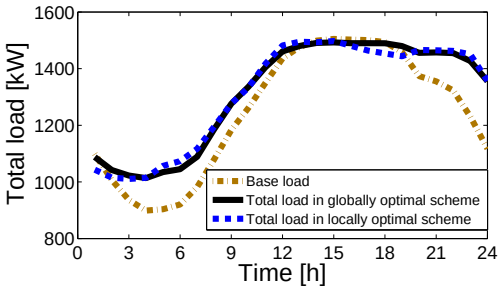


Fig. 9. Comparison of total load when considering the cost of battery lifetime reduction

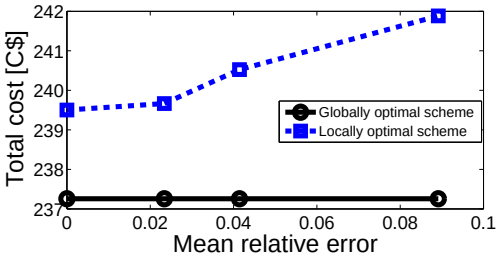


Fig. 10. Variation of total cost with different load forecasting error

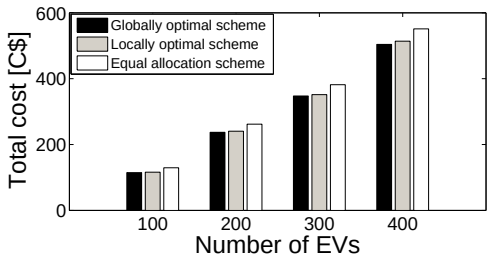
a smaller number of groups. The locally optimal scheduling scheme determines the optimal charging powers for a group of EVs for interval i ($i \in \mathbf{N}$) based on the local knowledge. A larger group size means more local knowledge available at the local controller, thus leading to a lower total cost, as shown in Fig. 8(a). The highest total cost is obtained in the case of group size of 1 EV, in which each local controller has the least local knowledge (e.g., only the information of one EV) and optimizes the charging power of one EV. The lowest total cost is obtained in the case of group size of 200 EVs, in which there is only one central controller, which has the information of all EVs. If the installation cost of the local controllers is considered, the case with a larger number of groups (or equivalently a smaller average group size) will have a higher installation cost. However, in the case that there are a smaller number of groups, each local controller needs to control more EVs in a larger area, thus introducing a higher cost in data communications between the local controller and the EVs in the group. In Fig. 8(b), we can see that the total load profile in the locally optimal scheduling scheme is changed closer to that in the globally optimal scheduling scheme as the average group size is increased from 1 to 200 EVs.

Fig. 9 shows the total load profiles in both the globally optimal scheduling scheme and the locally optimal scheduling scheme considering the cost of battery lifetime reduction. The model parameters are set as: $\beta = 5 \times 10^{-4}$ C\$/kWh^2 and $\eta = 10^{-3}$ C\$/kWh^2. The charging powers in each interval are obtained by solving the global scheduling optimization problem or the local scheduling optimization problem with the revised objective function, as described in Section IV-C. The total costs in the globally optimal scheduling scheme and the locally optimal scheduling scheme considering the *cost of battery lifetime reduction* are 244.58 C\$ and 246.14 C\$, respectively.

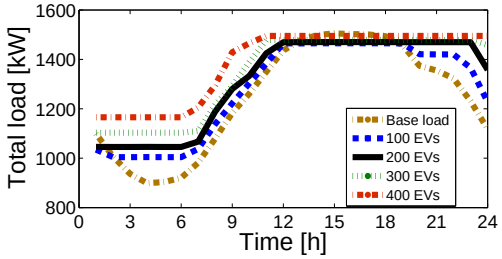
We show the impact of load forecasting error in the locally optimal scheduling scheme in Fig. 10. In similar-day approach for load forecasting, the forecasting error depends on the loads of the chosen similar days. In the simulation, we choose three sets of similar days. The future load in an interval is estimated by averaging the loads in this interval over the set of the similar days. The three sets lead to three *mean relative errors*, which are 0.023, 0.041, and 0.089, respectively. We also find the total cost in the locally optimal scheduling scheme in the simulation when the forecasted loads are assumed to be exactly equal to the real loads (e.g., the mean relative error is 0). As shown in Fig. 10, a lower forecasting error leads to a lower total cost in the locally optimal scheduling scheme.

We can see from Fig. 10 that the total cost in the locally optimal scheduling scheme approaches closer to that in the globally optimal scheduling scheme when the forecasting error approaches 0. We can also see from Fig. 8(a) that the total cost in the locally optimal scheduling scheme approaches closer to that in the globally optimal scheduling scheme when the average group size approaches the maximum (e.g., 200 EVs). In an extreme case where the forecasting error is 0 and the group size is 200, the total cost obtained in the locally optimal scheduling scheme is 238.28 C\$, which is higher than globally optimal result (237.256 C\$) by 0.43%. The reason is that the globally optimal scheduling scheme determines the optimal charging powers for all EVs for all intervals by solving a single *global scheduling optimization problem*, while the extreme case of the locally optimal scheduling scheme determines the optimal charging powers for all EVs for interval i ($i \in \mathbf{N}$) by solving the *local scheduling optimization problem* for interval i, respectively.

In the default simulation setting, the number of the total EVs is set to 200. In Fig. 11, we vary the number of the EVs



(a)



(b)

Fig. 11. Performance evaluation under different number of EVs: (a) the total cost, and (b) the total load

from 100 to 400, and then compare the total cost and the total load. All EVs are required to reach 90% of the battery capacity at the end of the charging period. A higher number of EVs means that a higher amount of energy is required to fill the battery, thus leading to a higher total cost. The globally optimal scheduling scheme provides the lowest cost under different number of EVs. As shown in Fig. 11(a), the locally optimal scheduling scheme outperforms the equal allocation scheme, and performs very close to the globally optimal scheduling scheme, under different number of EVs. Fig. 11(b) shows the comparison between the base loads without EV charging and the total loads with different number of EVs using the globally optimal scheduling scheme. As shown in Fig. 11(b), a higher number of EVs can reshape the total load profile to be flatter.

VI. Conclusions

In this paper, we study the scheduling optimization problem for EV charging and discharging. We first formulate a global scheduling optimization problem, in which the charging powers are optimized to minimize the total cost of all EVs which perform charging and discharging during the day. The globally optimal solution provides the globally minimal total cost. However, the globally optimal scheduling scheme is impractical since it assumes that the arrivals of all EV and the base loads during the day are known in advance. To develop a practical scheduling scheme, we formulate a local scheduling optimization problem, which aims to minimize the total cost of the EVs in the current ongoing EV set in the local group. The locally optimal scheduling scheme is performed in an independent and distributed way, which is not only scalable to a large EV population but also resilient to the dynamic EV arrivals. The simulation results demonstrated that the locally optimal scheduling scheme can achieve a close performance compared to the globally optimal scheduling scheme.

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Yifeng He (M'09) received his Ph.D. degree in Electrical Engineering from Ryerson University, Canada in 2008. He is currently an Assistant Professor in the department of Electrical and Computer Engineering at Ryerson University, Canada. His research interests include network resource optimization, smart grid communications, wireless sensor networks, and Peer-to-Peer (P2P) overlay networks. He is the recipient of 2008 Governor General's Gold Medal in Canada, and 2007 Pacific-rim Conference on Multimedia (PCM) best paper award.

Bala Venkatesh (SM '08) is an Associate Professor in the Department of Electrical and Computer Engineering and Academic Director of the Centre for Urban Energy at Ryerson University. His interests are in Power Systems Analysis and Optimization. He is a registered Professional Engineer in the provinces of Ontario and New Brunswick, Canada.

Ling Guan (S'88-M'90-SM'96-F'08) is a Tier I Canada Research Chair in Multimedia and Computer Technology, and a Professor of Electrical and Computer Engineering at Ryerson University, Toronto, Canada. He received his Bachelor's Degree from Tianjin University, China, Master's Degree from University of Waterloo and Ph.D. Degree from University of British Columbia. Dr. Guan has been working on image, video and multimedia signal processing and published extensively in the field. He chaired the 2006 IEEE International Conference on Multimedia and Expo in Toronto, co-chaired the 2008 ACM International Conference on Image and Video Retrieval in Niagara Falls, and served as the Founding General Chair of IEEE Pacific-Rim Conference on Multimedia in 2000. Dr. Guan is a Fellow of the IEEE, a Fellow of the Engineering Institute of Canada and a Fellow (Elected) of the Canadian Academy of Engineering. He is an IEEE Circuits and System Society Distinguished Lecturer (2010-2011) and a recipient of the 2005 IEEE Transactions on Circuits and Systems for Video Technology Best Paper Award.