Error Performance of Maximal-Ratio Combining with Transmit Antenna Selection in Flat Nakagami-m Fading Channels

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***Abstract*—In this paper, the performance of an uncoded multiple-input-multiple-output (MIMO) scheme combining single transmit antenna selection and receiver maximal-ratio combining (the TAS/MRC scheme) is investigated for independent flat Nakagami-m fading channels with arbitrary real-valued m. The outage probability is first derived. Then the error rate expressions are attained from two different approaches. First, based on the observation of the instantaneous channel gain, the binary phase-shift keying (BPSK) asymptotic bit error rate (BER) expression is derived, and the exact BER expression is obtained as an infinite series, which converges for reasonably large signal-to-noise ratios (SNRs). Then the exact symbol error rate (SER) expressions are attained as a multiple infinite sum based on the moment generating function (MGF) method for M-ary phase-shift keying (M-PSK) and quadrature amplitude modulation (M-QAM). The asymptotic SER expressions reveal a diversity order equal to the product of the m parameter, the number of transmit antennas and the number of receive antennas. Theoretical analysis is verified by simulation.**

***Index Terms*—Antenna selection, diversity, maximal-ratio combining (MRC), Nakagami-m fading.**

I. Introduction

THE error performance of an uncoded multiple-input-multiple-output (MIMO) scheme combining single transmit antenna selection (TAS) and receiver maximal-ratio combining (MRC), referred to as the TAS/MRC scheme[1], has been analyzed in several papers [1]–[5]. In this scheme, a single transmit antenna, which maximizes the total received signal power at the receiver, is selected for uncoded transmission at any time, and all the other transmit antennas are inactive. It was shown in [1] that a diversity order equal to the product of the number of the transmit antennas and the number of the receive antennas can be achieved for independent flat Rayleigh fading channels at high signal-to-noise ratios (SNRs), as if all the transmit antennas were used.

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It is well known that Nakagami-m fading [6] covers a wide range of fading scenarios via the m ($m \geq 1/2$) parameter, which includes the Rayleigh fading ($m = 1$) as a special case. Several recent papers [7]–[10] have extended the analysis of TAS/MRC systems to Nakagami-m fading scenarios. In [7], the error performance was analyzed under the assumption of integer m for multicarrier direct-sequence (DS) code-division multiple access (CDMA). The exact binary phase-shift keying (BPSK) bit error rate (BER) expression was derived in [9] with generalized selection criterion for integer m. Similar efforts were carried out in [8] and [10] for integer m. The impact of fading severity m on the system performance was not explicitly revealed in [7]–[10]. To the best knowledge of the authors, the *exact* analytical error performance of TAS/MRC systems in Nakagami-m fading channels for *arbitrary* m has not been thoroughly investigated.

This paper presents a comprehensive performance analysis of TAS/MRC systems in independent flat Nakagami-m fading channels for *arbitrary* m. The outage probability, which is indicative of error performance, is first developed. Then both asymptotic and exact error performances are investigated with two different methods. First the asymptotic BER is derived for arbitrary real-valued m and binary phase-shift keying (BPSK) based on the behavior of the probability density function (pdf) of the instantaneous channel power gain. Moreover, the exact BER of such a system with two antennas at the transmitter is obtained as an infinite series. Then the exact symbol error rate (SER) expressions are obtained for M-ary phase-shift keying (M-PSK) and M-ary quadrature amplitude modulation (M-QAM) by utilizing the moment generating function (MGF) method [11]. The asymptotic SER expressions are also derived. An interesting conclusion is reached that the diversity order is equal to the product of three parameters: the Nakagami parameter m, the number of transmit antennas, and the number of receive antennas. Simulation results are also provided to verify the analysis.

Compared with [7]–[10], this paper not only investigates the exact symbol error performance of a TAS/MRC system in a more general fading scenario for *arbitrary* m from two different approaches, but also explicitly reveals the impact of the fading severity on error performance.

II. System and Channel Model

We consider an $(L_t, 1; L_r)$ TAS/MRC system equipped with L_t transmit and L_r receive antennas in independent flat Nakagami-m fading channels. At any time, a single transmit antenna is selected for uncoded transmission, and all the L_r

receive antennas are used for MRC. Assume that the fading coefficients between the i-th transmit and j-th receive antennas are independent and denoted by $h_{j,i}$, where $1 \le i \le L_t$ and $1 \le j \le L_r$. Then the amplitude $|h_{j,i}|$ follows a Nakagami distribution with fading parameter m. It is assumed that $E\{|h_{j,i}|^2\} = 1$, where $E\{\cdot\}$ denotes the expected value. The single selected transmit antenna, denoted by I, is determined by

$$I = \underset{1\le i\le L_t}{\text{argmax}} \left\{ C_i = \sum_{j=1}^{L_r} |h_{j,i}|^2 \right\}. \quad (1)$$

Let γ denote the instantaneous SNR of the maximal-ratio combiner output, or instantaneous post-processing SNR. For a $(1, L_r)$ MRC system with a single transmit antenna and L_r receive antennas in Nakagam-m fading channels, the pdf of γ is given by [12]

$$p_\gamma(\gamma) = \frac{\gamma^{mL_r-1}}{\left(\frac{\bar{\gamma}}{m}\right)^{mL_r}\Gamma(mL_r)} \exp\left(-m\frac{\gamma}{\bar{\gamma}}\right), \quad (2)$$

where the gamma function is defined as $\gamma(z) = \int_0^\infty x^{z-1}e^{-x}dx$, and the average SNR $\bar{\gamma} = E_s/N_0$, in which E_s is the average energy per symobl at the transmitter and N_0 is the power spectral density of the additive white Gaussian noise (AWGN) at each receive antenna.

For arbitrary real-valued m, the cumulative distribution function (cdf) of γ is calculated as

$$P_\gamma(\gamma) = \frac{\gamma\left(mL_r, m\frac{\gamma}{\bar{\gamma}}\right)}{\Gamma(mL_r)}, \quad (3)$$

where $\gamma(\alpha, x) = \int_0^x e^{-t}t^{\alpha-1}dt$, $\text{Re}\{\alpha\} > 0$, is the lower incomplete gamma function.

We rearrange C_i in ascending order of magnitude, and denote them by $C_{(l)}$, where $1 \le l \le L_t$, and $C_{(1)} \le \cdots \le C_{(L_t)}$. The instantaneous post-processing SNR of the $(L_t, 1; L_r)$ TAS/MRC system, denoted by $\gamma_{(L_t)}$, can be written as $\gamma_{(L_t)} = C_{(L_t)}\bar{\gamma}$. The pdf of $\gamma_{(L_t)}$ is given by [13]

$$\begin{aligned} p_{\gamma_{(L_t)}}(\gamma) &= L_t\left[P_\gamma(\gamma)\right]^{L_t-1}p_\gamma(\gamma) \\ &= L_t\left(\frac{m}{\bar{\gamma}}\right)^{mL_r}\frac{\gamma^{mL_r-1}}{\Gamma(mL_r)}\exp\left(-m\frac{\gamma}{\bar{\gamma}}\right)\left[\frac{\gamma\left(mL_r, m\frac{\gamma}{\bar{\gamma}}\right)}{\Gamma(mL_r)}\right]^{L_t-1}. \end{aligned} \quad (4)$$

III. Outage Probability

The outage probability is defined as the probability that the instantaneous capacity is less than a given capacity R [14]. For an $(L_t, 1; L_r)$ MIMO system, the outage probability is given by [1, (7)]

$$\begin{aligned} P_{out}(R, \bar{\gamma}) &= \Pr\left\{\log_2(1 + C_{(L_t)}\bar{\gamma}) < R\right\} \\ &= \Pr\left\{C_{(L_t)} < \frac{2^R - 1}{\bar{\gamma}}\right\}. \end{aligned} \quad (5)$$

Let $y = \frac{2^R-1}{\bar{\gamma}}$, $P_{out}(R, \bar{\gamma})$ in (5) can be written as

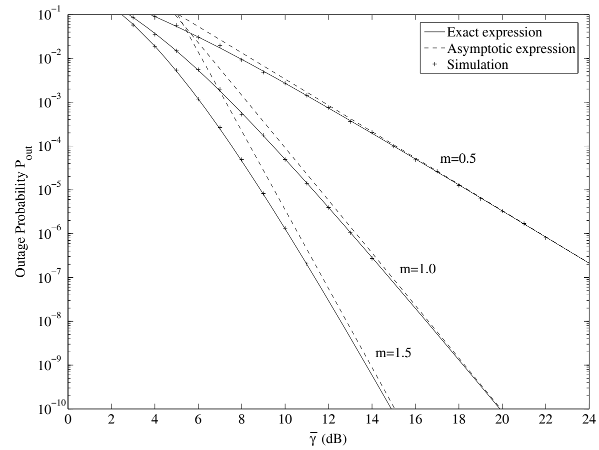

Fig. 1. Outage probability of a $(3, 1; 2)$ MIMO system with $m = 0.5$, 1.0, and 1.5, bandwidth efficiency $R = 2$ bits/s/Hz.

$$\begin{aligned} P_{out}(R, \bar{\gamma}) &= \int_0^y p_{C_{(L_t)}}(x)dx \\ &= \frac{L_t m^{mL_r}}{\left[\Gamma(mL_r)\right]^{L_t}} \int_0^y x^{mL_r-1}\exp(-mx)\left[\gamma(mL_r, mx)\right]^{L_t-1}dx \\ &= \frac{L_t m^{mL_tL_r}}{\left[\Gamma(mL_r)\right]^{L_t}} \sum_{n_1=0}^{+\infty}\sum_{n_2=0}^{+\infty}\cdots\sum_{n_{L_t-1}=0}^{+\infty} m^{\sum_{i=0}^{L_t-1}n_i} \\ &\quad \times \prod_{i=1}^{L_t-1}\frac{(-1)^{n_i}}{n_i!\,(mL_r+n_i)} \\ &\quad\quad \times \int_0^y x^{mL_tL_r+\sum_{i=0}^{L_t-1}n_i-1}\exp(-mx)\,dx, \end{aligned} \quad (6)$$

in which the identity [15, (8.354-1)]

$$\gamma(\alpha, x) = \sum_{n=0}^{+\infty}\frac{(-1)^n x^{\alpha+n}}{n!(\alpha+n)} \quad (7)$$

is employed. By utilizing [15, (3.381-1)], the outage probability in (6) can be simplified as

$$\begin{aligned} P_{out}(R, \bar{\gamma}) &= \frac{L_t}{\left[\Gamma(mL_r)\right]^{L_t}}\sum_{n_1=0}^{+\infty}\sum_{n_2=0}^{+\infty}\cdots\sum_{n_{L_t-1}=0}^{+\infty}\prod_{i=1}^{L_t-1}\frac{(-1)^{n_i}}{n_i!\,(mL_r+n_i)} \\ &\quad \times \sum_{k=0}^{+\infty}\frac{(-1)^k\left[\frac{m(2^R-1)}{\bar{\gamma}}\right]^{mL_tL_r+\sum_{i=0}^{L_t-1}n_i+k}}{k!\left(mL_tL_r+\sum_{i=0}^{L_t-1}n_i+k\right)}. \end{aligned} \quad (8)$$

We let $n_i = 0$, $1 \le i \le L_t - 1$, and $k = 0$ in (8), and can obtain $P_{out}(R, \bar{\gamma})$ at high SNRs ($\bar{\gamma} \to \infty$) as

$$P_{out}(R, \bar{\gamma}) = \frac{\left[m\left(2^R - 1\right)\right]^{mL_tL_r}}{\left[\Gamma(mL_r + 1)\right]^{L_t}}\bar{\gamma}^{-mL_tL_r} + o\left(\bar{\gamma}^{-mL_tL_r}\right), \quad (9)$$

where we write $f(x) = o(g(x))$, $x \to x_0$, if $\lim_{x\to x_0}\frac{f(x)}{g(x)} = 0$. This indicates that a diversity order of mL_tL_r can be potentially achieved in an Nakagami-m fading channel for arbitrary m.

For illustration purpose, the outage probability of a (3,1;2) MIMO system was depicted in Fig. 1 for Nakagami-m fading with $m = 0.5$, 1.0, and 1.5, and $R = 2$ bits/s/Hz. The exact expression (8), the asymptotic expression (9), and simulation results are presented. Note that the result corresponding to the evaluation of (8) was obtained by truncating the infinite series to 10 terms. It is observed that simulation results match the exact expression (8), and that (9) is a tight upper bound of (8) at high SNRs, which confirms the diversity order of mL_tL_r.

IV. Asymptotic and Exact Error Performance for BPSK

It is shown in [16] that the asymptotic error performance of wireless transmissions in fading channels depends on the behavior of the pdf of the instantaneous channel power gain β, which is defined as $\beta = \frac{\gamma}{\bar{\gamma}}$. Assume that β-dependent instantaneous SER is given by $P_E(\beta) = Q\left(\sqrt{k\beta\bar{\gamma}}\right)$, where k is a positive fixed constant ($k = 2$ for BPSK). If the pdf $p(\beta)$ can be approximated by a single polynomial term for $\beta \to 0^+$ as

$$p(\beta) = a\beta^t + o(\beta^t), \quad a > 0, \tag{10}$$

then at high SNRs, the average SER is given by [16]

$$P_E = \frac{2^t\, a\, \Gamma(t + 3/2)}{\sqrt{\pi}(t + 1)}(k\bar{\gamma})^{-(t+1)} + o(\bar{\gamma}^{-(t+1)}). \tag{11}$$

If $p(\beta)$ can be expanded in a series form at the origin as

$$p(\beta) = \sum_{i=0}^{+\infty} a_i \beta^{t+i}, \tag{12}$$

then the average SER can be written as [16]

$$P_E = \sum_{i=0}^{+\infty} \frac{2^{t+i}\, a_i\, \Gamma(t + i + 3/2)}{\sqrt{\pi}(t + i + 1)}(k\bar{\gamma})^{-(t+i+1)} \tag{13}$$

for a $\bar{\gamma}$ range in which the series converges.

Considering $\beta = \frac{\gamma}{\bar{\gamma}}$, we can write the pdf of β for the $(L_t, 1; L_r)$ TAS/MRC system as

$$\begin{aligned} p(\beta) &= \bar{\gamma} p_{\gamma_{(L_t)}}(\beta\bar{\gamma}) \\ &= \frac{L_t m^{mL_r}}{\Gamma(mL_r)} \left[\frac{\gamma\left(mL_r, m\beta\right)}{\Gamma(mL_r)}\right]^{L_t - 1} \beta^{mL_r - 1} e^{-m\beta}. \end{aligned} \tag{14}$$

Using the identity [15, (8.354-1)], we have

$$\begin{aligned} p(\beta) &= \frac{L_t m^{mL_r}}{[\Gamma(mL_r)]^{L_t}} \left[\sum_{n=0}^{+\infty} \frac{(-1)^n (m\beta)^{mL_r+n}}{n!(mL_r + n)}\right]^{L_t - 1} \times \beta^{mL_r - 1} e^{-m\beta} \\ &= \frac{L_t m^{mL_r}}{[\Gamma(mL_r)]^{L_t}} \left[\frac{(m\beta)^{mL_r}}{mL_r} + o\left(\beta^{mL_r}\right)\right]^{L_t - 1} \times \beta^{mL_r - 1} e^{-m\beta} \\ &= \frac{m^{mL_tL_r+1} L_t L_r}{[\Gamma(mL_r + 1)]^{L_t}} \beta^{mL_tL_r - 1} + o\left(\beta^{mL_tL_r - 1}\right), \quad \beta \to 0. \end{aligned} \tag{15}$$

In the last equation of (15), $e^{-m\beta}$ is dropped because it can be expressed as a power series and only the first term (being 1) is taken with the remaining terms being absorbed into $o\left(\beta^{mL_tL_r-1}\right)$.

The comparison between (15) and (10) reveals that

$$a = \frac{m^{mL_tL_r+1} L_t L_r}{[\Gamma(mL_r + 1)]^{L_t}} \tag{16}$$

and

$$t = mL_tL_r - 1. \tag{17}$$

Substituting (16) and (17) into (11) and letting $k = 2$, we obtain the BER for a BPSK $(L_t, 1; L_r)$ TAS/MRC system at high SNRs for an arbitrary value of m as

$$P_2 = \frac{m^{mL_tL_r}\Gamma(mL_tL_r + \frac{1}{2})}{2\sqrt{\pi}\,[\Gamma(mL_r + 1)]^{L_t}} \bar{\gamma}^{-mL_tL_r} + o\left(\bar{\gamma}^{-mL_tL_r}\right). \tag{18}$$

Equation (18) clearly indicates that an asymptotic diversity order of mL_tL_r is achieved.

If mL_r is an integer, (18) can be further written as

$$P_2 = \frac{m^{mL_tL_r}(2mL_tL_r - 1)!}{2^{2mL_tL_r}\left[(mL_r)!\right]^{L_t}(mL_tL_r - 1)!} \bar{\gamma}^{-mL_tL_r} + o\left(\bar{\gamma}^{-mL_tL_r}\right). \tag{19}$$

For Rayleigh fading ($m = 1$), (19) simply reduces to (34) in [1].

Next we will illustrate that the exact BER expression could be attained following the similar method by considering a special case where $L_t = 2$ ($(2, 1; L_r)$ TAS/MRC) and mL_r is an integer.

For the $(2, 1; L_r)$ TAS/MRC with integer mL_r, based on [15, (8.352-1)], the pdf of β in (14) can be written as

$$\begin{aligned} p(\beta) &= \frac{2m^{mL_r}}{(mL_r - 1)!} \left[1 - e^{-m\beta} \sum_{i=0}^{mL_r - 1} \frac{(m\beta)^i}{i!}\right] \beta^{mL_r - 1} e^{-m\beta} \\ &= \sum_{i=mL_r-1}^{+\infty} \frac{2m^{mL_r+i+1}}{(mL_r - 1)!} \left[\frac{(-1)^i(2^{i+1} - 1)}{(i + 1)!} - \sum_{j=1}^{mL_r-1} \frac{(-2)^{i-j+1}}{j!(i - j + 1)!}\right] \beta^{i+mL_r}. \end{aligned} \tag{20}$$

The comparison between (20) and (12) readily reveals the values of a_i and t in (12) as

$$a_i = \frac{2m^{mL_r+i+1}}{(mL_r - 1)!} \left[\frac{(-1)^i(2^{i+1} - 1)}{(i + 1)!} - \sum_{j=1}^{mL_r-1} \frac{(-2)^{i-j+1}}{j!(i - j + 1)!}\right] \tag{21}$$

and

$$t = mL_r. \tag{22}$$

Substituting these values into (13), we obtain the exact BER expression for BPSK as

$$P_2 = \sum_{i=mL_r-1}^{+\infty} \frac{m^{mL_r+i+1}(2mL_r + 2i + 1)!}{2^{2mL_r+2i+1}(mL_r - 1)!(mL_r + i + 1)!} \times \left[\frac{(-1)^i(2^{i+1} - 1)}{(i + 1)!} - \sum_{j=1}^{mL_r-1} \frac{(-2)^{i-j+1}}{j!(i - j + 1)!}\right] \bar{\gamma}^{-(mL_r+i+1)}. \tag{23}$$

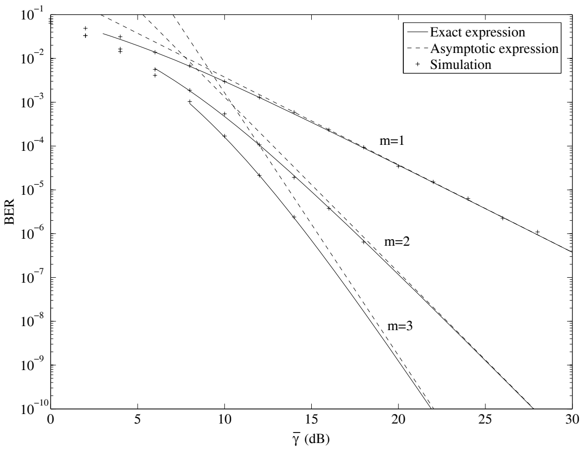

Fig. 2. Comparison of the exact expression, asymptotic expression, and simulation for the BPSK $(2,1;1)$ TAS/MRC scheme with $m = 1, 2$, and 3.

Writing (23) in the form of $P_2 = \sum_{i=mL_r-1}^{+\infty} u_i$, we have the convergence condition as

$$\lim_{i\to\infty}\left|\frac{u_{i+1}}{u_i}\right| = \frac{2m}{\bar{\gamma}} < 1. \quad (24)$$

Therefore, the equality in (23) holds for $\bar{\gamma} > 2m$.

If we only calculate the term associated with $i = mL_r - 1$ in (23), and treat all the other terms associated with $i > mL_r - 1$ as $o\left(\bar{\gamma}^{-2mL_r}\right)$ at high SNRs, based on

$$\frac{(-1)^{mL_r-1}\left(2^{mL_r}-1\right)}{(mL_r)!} - \sum_{j=1}^{mL_r-1}\frac{(-2)^{mL_r-j}}{j!\,(mL_r-j)!} = \frac{1}{(mL_r)!}, \quad (25)$$

the exact BER expression (23) can be written as

$$P_2 = \frac{m^{2mL_r}\,(4mL_r-1)!}{2^{4mL_r}\left[(mL_r)!\right]^2(2mL_r-1)!}\bar{\gamma}^{-2mL_r} + o\left(\bar{\gamma}^{-2mL_r}\right). \quad (26)$$

It is interesting to note that (26) is exactly the same as (19) with $L_t = 2$, which shows that the first term in (23), associated with $i = mL_r - 1$, is actually the asymptotic BER expression. This illustrates the favorable property of (13) in evaluating the average error rate in wireless channels.

Fig. 2 presents the comparison of the exact expression in (23), the asymptotic expression in (19) and the simulation for the BPSK $(2,1;1)$ TAS/MRC scheme for different integer values of m. It clearly indicates that the simulation result matches the analytical one in (23). It also shows that (23) asymptotically approaches (19) at high SNRs. The curve corresponding to the analytical evaluation of (23) was obtained by truncating the infinite series to 70 terms. Also note that the convergence areas, given by $\bar{\gamma} > 2m$, for $m = 1$, 2, and 3 are $\bar{\gamma} > 3.01$ dB, $\bar{\gamma} > 6.02$ dB, and $\bar{\gamma} > 7.78$ dB, respectively. It is shown in Fig. 2 that the convergence areas cover the SNR range of practical interest.

V. Exact and Asymptotic Symbol Error Rate for M-PSK and M-QAM

In this section, with MGF-based method [11], we will derive the SER expressions for a TAS/MRC scheme over Nakagami-m fading channels for both M-PSK and M-QAM modulations. Arbitrary real-valued m ($m \geq \frac{1}{2}$) is assumed.

The MGF associated with γ is given by

$$\begin{aligned}
M_\gamma(s) &= \int_0^\infty p_{\gamma_{(L_t)}}(\gamma)e^{s\gamma}d\gamma \\
&\overset{(a)}{=} \int_0^\infty \frac{L_t}{\Gamma(mL_r)}\left(\frac{m}{\bar{\gamma}}\right)^{mL_r}\gamma^{mL_r-1} \\
&\quad\times\left[\frac{\gamma\left(mL_r, m\frac{\gamma}{\bar{\gamma}}\right)}{\Gamma(mL_r)}\right]^{L_t-1}\exp\left[-\left(\frac{m}{\bar{\gamma}}-s\right)\gamma\right]d\gamma \\
&\overset{(b)}{=} \frac{L_t}{\left[\Gamma(mL_r)\right]^{L_t}}\left(\frac{m}{\bar{\gamma}}\right)^{mL_tL_r} \\
&\quad\times\sum_{n_1=0}^{+\infty}\sum_{n_2=0}^{+\infty}\cdots\sum_{n_{L_t-1}=0}^{+\infty}\left(\frac{m}{\bar{\gamma}}\right)^{\sum_{k=1}^{L_t-1}n_k} \\
&\quad\times\prod_{i=1}^{L_t-1}\frac{(-1)^{n_i}}{n_i!\,(mL_r+n_i)} \\
&\quad\times\int_0^{+\infty}\gamma^{mL_tL_r+\sum_{k=1}^{L_t-1}n_k-1} \\
&\quad\times\exp\left[-\left(\frac{m}{\bar{\gamma}}-s\right)\gamma\right]d\gamma. \quad (27)
\end{aligned}$$

Note that identity [15, (8.354-1)] is employed in the attainment of (b) from (a). Using the identity [15, (3.381-4)]

$$\int_0^\infty x^{\nu-1}e^{-\mu x}dx = \frac{1}{\mu^\nu}\Gamma(\nu), \qquad \mathrm{Re}\{\mu\} > 0, \mathrm{Re}\{\nu\} > 0, \quad (28)$$

the MGF in (27) can be further written as

$$\begin{aligned}
M_\gamma(s) =& \frac{L_t}{\left[\Gamma(mL_r)\right]^{L_t}}\left(\frac{m}{\bar{\gamma}}\right)^{mL_tL_r} \\
&\times\sum_{n_1=0}^{+\infty}\sum_{n_2=0}^{+\infty}\cdots\sum_{n_{L_t-1}=0}^{+\infty}\left(\frac{m}{\bar{\gamma}}\right)^{\sum_{k=1}^{L_t-1}n_k} \\
&\times\prod_{i=1}^{L_t-1}\frac{(-1)^{n_i}}{n_i!\,(mL_r+n_i)}\Gamma\left(mL_tL_r+\sum_{k=1}^{L_t-1}n_k\right) \\
&\times\left(\frac{m}{\bar{\gamma}}-s\right)^{mL_tL_r+\sum_{k=1}^{L_t-1}n_k}. \quad (29)
\end{aligned}$$

Based on the MGF of γ given in (29), we are ready to evaluate the symbol error performance of the TAS/MRC scheme in Nakagami-m fading channels for different modulation schemes.

A. Symbol Error Rate for M-PSK

Following [11], the SER expression for a M-PSK $(L_t, 1; L_r)$ TAS/MRC scheme can be written as

$$\begin{aligned}
P_s(E) =& \frac{1}{\pi}\int_0^{\pi-\frac{\pi}{M}} M_\gamma\left(-\frac{\sin^2\frac{\pi}{M}}{\sin^2\theta}\right)d\theta \\
=& \frac{L_t}{\left[\Gamma(mL_r)\right]^{L_t}}\left(\frac{m}{\bar{\gamma}}\right)^{mL_tL_r} \\
&\times\sum_{n_1=0}^{+\infty}\sum_{n_2=0}^{+\infty}\cdots\sum_{n_{L_t-1}=0}^{+\infty}\left(\frac{m}{\bar{\gamma}}\right)^{\sum_{k=1}^{L_t-1}n_k} \\
&\times\prod_{i=1}^{L_t-1}\frac{(-1)^{n_i}}{n_i!\,(mL_r+n_i)}\Gamma\left(mL_tL_r+\sum_{k=1}^{L_t-1}n_k\right)
\end{aligned}$$

$$\times \underbrace{\frac{1}{\pi}\int_0^{\pi-\frac{\pi}{M}} \left(\frac{m}{\bar{\gamma}}+\frac{\sin^2\frac{\pi}{M}}{\sin^2\theta}\right)^{-\left(mL_tL_r+\sum_{k=1}^{L_t-1} n_k\right)} d\theta}_{I_1}. \tag{30}$$

The integral I_1 can be written as

$$I_1 = \left(\frac{m}{\bar{\gamma}}\right)^{-\left(mL_tL_r+\sum_{k=1}^{L_t-1} n_k\right)} \times \underbrace{\frac{1}{\pi}\int_0^{\pi-\frac{\pi}{M}} \left(\frac{\sin^2\theta}{\sin^2\theta+\frac{\bar{\gamma}}{m}\sin^2\frac{\pi}{M}}\right)^{-\left(mL_tL_r+\sum_{k=1}^{L_t-1} n_k\right)} d\theta}_{I_2}. \tag{31}$$

Based on [17, (43)] [1], the integral I_2 can be expressed as in (32), where ${}_2F_1(a,b;c;x)$ is the Gauss hypergeometric function [15, (9.111)], and $F_1(a,b,b';c;x,y)$ is the Appell hypergeometric function [15, (9.180-1)].

Therefore, the SER expression for M-PSK modulation can be expressed as in (33).

Letting $n_i=0$, $1\le i\le L_t-1$, in (33), and utilizing

$$\sum_{n=0}^{+\infty}\frac{\left(\frac{1}{2}-mL_tL_r\right)_n}{(2n+1)n!}\cos^{2n}\frac{\pi}{M} = {}_2F_1\left(\frac{1}{2},\frac{1}{2}-mL_tL_r;\frac{3}{2};\cos^2\frac{\pi}{M}\right), \tag{34}$$

where $(a)_n$ denotes the Pochhammer symbol, we have the asymptotic expression for $\bar{\gamma}\to\infty$ as

$$\begin{aligned} P_s(E) = {} & \frac{\Gamma(mL_tL_r+1)}{[\Gamma(mL_r+1)]^{L_t}}\left(\frac{m}{\sin^2\frac{\pi}{M}}\right)^{mL_tL_r} \\ & \times\left\{\frac{1}{2\sqrt{\pi}}\frac{\Gamma\left(mL_tL_r+\frac{1}{2}\right)}{\Gamma(mL_tL_r+1)} + \frac{\cos\frac{\pi}{M}}{\pi}{}_2F_1\left(\frac{1}{2},\frac{1}{2}-mL_tL_r;\frac{3}{2};\cos^2\frac{\pi}{M}\right)\right\} \\ & \times\bar{\gamma}^{-mL_tL_r}+o\left(\bar{\gamma}^{-mL_tL_r}\right) \end{aligned} \tag{35}$$

after some mathematical manipulation based on the definition of Gauss and Appell hypergeometric functions. Expression (35) clearly indicates an asymptotic diversity order of mL_tL_r.

B. Symbol Error Rate for M-QAM

For M-QAM, the SER expression for a $(L_t,1;L_r)$ TAS/MRC scheme can be written as [11]

$$P_s(E) = \frac{4}{\pi}\left(1-\frac{1}{\sqrt{M}}\right)\underbrace{\int_0^{\frac{\pi}{2}} M_\gamma\left(-\frac{3}{2(M-1)\sin^2\theta}\right)d\theta}_{I_3} - \frac{4}{\pi}\left(1-\frac{1}{\sqrt{M}}\right)^2\underbrace{\int_0^{\frac{\pi}{4}} M_\gamma\left(-\frac{3}{2(M-1)\sin^2\theta}\right)d\theta}_{I_4}. \tag{36}$$

[1] Note that there is a misprint in [17, (43)]. For the second Appell hypergeometric function, it should be $F_1\left(\frac{1}{2},n,\frac{1}{2}-n;\frac{3}{2};\frac{\cos^2\psi}{1+\omega},\cos^2\psi\right)$, instead of $F_1\left(\frac{1}{2},n,\frac{1}{2}-n;\frac{3}{2};\frac{\cos^2\omega}{1+\omega},\cos^2\omega\right)$.

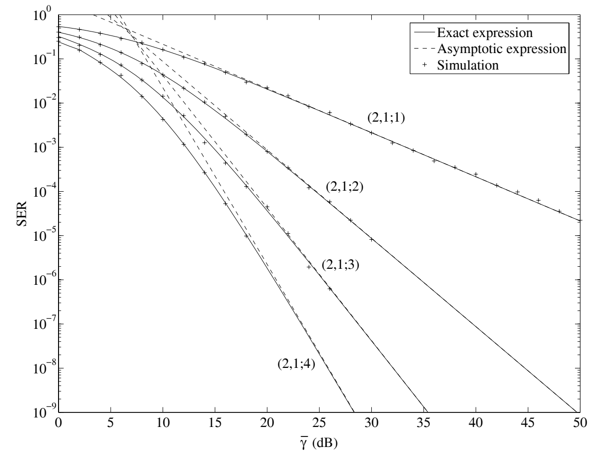

Fig. 3. Comparison of the exact expression, asymptotic expression, and simulation for the 8-PSK $(2,1;L_r)$ TAS/MRC scheme with $m=0.5$ and different number of receive antennas, $1\le L_r\le 4$.

Based on [17, (46)] and [17, (48)], the integral I_3 can be derived as in (37) and I_4 can be expressed as in (38).

Substituting (37) and (38) into (36), we obtain the SER expression for M-QAM as in (39).

Let $n_1=n_2=\cdots n_{L_t-1}=0$ in (39), after some simple mathematical manipulation, we obtain the asymptotic SER expression as

$$\begin{aligned} P_s(E) = {} & 2\left(1-\frac{1}{\sqrt{M}}\right)\frac{\Gamma(mL_tL_r+1)}{[\Gamma(mL_r+1)]^{L_t}}\left[\frac{3}{m(M-1)}\right]^{-mL_tL_r} \\ & \times\left\{\frac{2^{mL_tL_r}}{\sqrt{\pi}}\frac{\Gamma\left(mL_tL_r+\frac{1}{2}\right)}{\Gamma(mL_tL_r+1)} - \frac{\left(1-\frac{1}{\sqrt{M}}\right)F_1\left(1,mL_tL_r,1;mL_tL_r+\frac{3}{2};\frac{1}{2},\frac{1}{2}\right)}{(2mL_tL_r+1)\pi}\right\} \\ & \times\bar{\gamma}^{-mL_tL_r}+o\left(\bar{\gamma}^{-mL_tL_r}\right), \end{aligned} \tag{40}$$

which indicates an asymptotic diversity order of mL_tL_r.

It is worthwhile to point out that, although being a special case of M-PSK investigated in Section V-A, Section IV adopted a different approach based on the behavior of the pdf of the instantaneous channel power gain, which was demonstrated to be a simple and effective alternative to MGF method in Section V-A. In addition, the exact BER expression developed in Section IV takes a different form from that in Section V-A although they are equivalent.

C. Numerical Results

Fig. 3 presents the performance comparison between the exact expression (33), the asymptotic expression (35) and simulation for the $(2,1;L_r)$ TAS/MRC scheme, where $2\le L_r\le 4$, with 8-PSK modulation and $m=0.5$. It is shown that the simulation matches the theoretical analysis, and (35) is a tight upper bound of (33). Similar conclusion could be drawn in Fig. 4 for 16-QAM.

Fig. 5 compares the symbol error performance by simulation of the (2,1;2) TAS/MRC scheme and the (2,2) Alamouti space-time block code (STBC) [18] for different m. For Alamouti

$$I_2 = \left(1+\frac{\bar{\gamma}}{m}\sin^2\frac{\pi}{M}\right)^{-\left(mL_tL_r+\sum_{k=1}^{L_t-1}n_k\right)} \left\{ \frac{1}{2\sqrt{\pi}} \frac{\Gamma\left(mL_tL_r+\sum_{k=1}^{L_t-1}n_k+\frac{1}{2}\right)}{\Gamma\left(mL_tL_r+\sum_{k=1}^{L_t-1}n_k+1\right)} \right.$$
$$\times {}_2F_1\left(mL_tL_r+\sum_{k=1}^{L_t-1}n_k, \frac{1}{2}; mL_tL_r+\sum_{k=1}^{L_t-1}n_k+1; \frac{1}{1+\frac{\bar{\gamma}}{m}\sin^2\frac{\pi}{M}}\right)$$
$$\left. +\frac{\cos\frac{\pi}{M}}{\pi}F_1\left(\frac{1}{2}, mL_tL_r+\sum_{k=1}^{L_t-1}n_k, \frac{1}{2}-mL_tL_r-\sum_{k=1}^{L_t-1}n_k; \frac{3}{2}; \frac{\cos^2\frac{\pi}{M}}{1+\frac{\bar{\gamma}}{m}\sin^2\frac{\pi}{M}}, \cos^2\frac{\pi}{M}\right)\right\}. \quad (32)$$

$$P_s(E) = \frac{L_t}{\left[\Gamma(mL_r)\right]^{L_t}} \sum_{n_1=0}^{+\infty}\sum_{n_2=0}^{+\infty}\cdots\sum_{n_{L_t-1}=0}^{+\infty} \left(\prod_{i=1}^{L_t-1}\frac{(-1)^{n_i}}{n_i!\,(mL_r+n_i)}\right)\Gamma\left(mL_tL_r+\sum_{k=1}^{L_t-1}n_k\right)$$
$$\times \left(1+\frac{\bar{\gamma}}{m}\sin^2\frac{\pi}{M}\right)^{-\left(mL_tL_r+\sum_{k=1}^{L_t-1}n_k\right)} \left\{ \frac{1}{2\sqrt{\pi}} \frac{\Gamma\left(mL_tL_r+\sum_{k=1}^{L_t-1}n_k+\frac{1}{2}\right)}{\Gamma\left(mL_tL_r+\sum_{k=1}^{L_t-1}n_k+1\right)} \right.$$
$$\times {}_2F_1\left(mL_tL_r+\sum_{k=1}^{L_t-1}n_k, \frac{1}{2}; mL_tL_r+\sum_{k=1}^{L_t-1}n_k+1; \frac{1}{1+\frac{\bar{\gamma}}{m}\sin^2\frac{\pi}{M}}\right)$$
$$\left. +\frac{\cos\frac{\pi}{M}}{\pi}F_1\left(\frac{1}{2}, mL_tL_r+\sum_{k=1}^{L_t-1}n_k, \frac{1}{2}-mL_tL_r-\sum_{k=1}^{L_t-1}n_k; \frac{3}{2}; \frac{\cos^2\frac{\pi}{M}}{1+\frac{\bar{\gamma}}{m}\sin^2\frac{\pi}{M}}, \cos^2\frac{\pi}{M}\right)\right\}. \quad (33)$$

$$I_3 = \frac{L_t}{\left[\Gamma(mL_r)\right]^{L_t}} \sum_{n_1=0}^{+\infty}\sum_{n_2=0}^{+\infty}\cdots\sum_{n_{L_t-1}=0}^{+\infty} \prod_{i=1}^{L_t-1}\frac{(-1)^{n_i}}{n_i!\,(mL_r+n_i)}\Gamma\left(mL_tL_r+\sum_{k=1}^{L_t-1}n_k\right)$$
$$\times \frac{\left[1+\frac{3\bar{\gamma}}{2m(M-1)}\right]^{-\left(mL_tL_r+\sum_{k=1}^{L_t-1}n_k\right)}}{2\sqrt{\pi}} \frac{\Gamma\left(mL_tL_r+\sum_{k=1}^{L_t-1}n_k+\frac{1}{2}\right)}{\Gamma\left(mL_tL_r+\sum_{k=1}^{L_t-1}n_k+1\right)}$$
$$\times {}_2F_1\left(mL_tL_r+\sum_{k=1}^{L_t-1}n_k, \frac{1}{2}; mL_tL_r+\sum_{k=1}^{L_t-1}n_k+1; \frac{1}{1+\frac{3\bar{\gamma}}{2m(M-1)}}\right), \quad (37)$$

$$I_4 = \frac{L_t}{\left[\Gamma(mL_r)\right]^{L_t}} \sum_{n_1=0}^{+\infty}\sum_{n_2=0}^{+\infty}\cdots\sum_{n_{L_t-1}=0}^{+\infty} \prod_{i=1}^{L_t-1}\frac{(-1)^{n_i}}{n_i!\,(mL_r+n_i)}\Gamma\left(mL_tL_r+\sum_{k=1}^{L_t-1}n_k\right)$$
$$\times \frac{\left[1+\frac{3\bar{\gamma}}{m(M-1)}\right]^{-\left(mL_tL_r+\sum_{k=1}^{L_t-1}n_k\right)}}{2\pi\left[2\left(mL_tL_r+\sum_{k=1}^{L_t-1}n_k\right)+1\right]}$$
$$\times F_1\left(1, mL_tL_r+\sum_{k=1}^{L_t-1}n_k, 1; mL_tL_r+\sum_{k=1}^{L_t-1}n_k+\frac{3}{2}; \frac{1+\frac{3\bar{\gamma}}{2m(M-1)}}{1+\frac{3\bar{\gamma}}{m(M-1)}}, \frac{1}{2}\right). \quad (38)$$

$$P_s(E) = 4\left(1-\frac{1}{\sqrt{M}}\right)\frac{L_t}{\left[\Gamma\left(mL_r\right)\right]^{L_t}}\sum_{n_1=0}^{+\infty}\sum_{n_2=0}^{+\infty}\cdots\sum_{n_{L_t-1}=0}^{+\infty}\prod_{i=1}^{L_t-1}\frac{(-1)^{n_i}}{n_i!\,(mL_r+n_i)}\Gamma\left(mL_tL_r+\sum_{k=1}^{L_t-1}n_k\right)$$
$$\times\left\{\frac{\left[1+\frac{3\bar{\gamma}}{2m(M-1)}\right]^{-\left(mL_tL_r+\sum_{k=1}^{L_t-1}n_k\right)}}{2\sqrt{\pi}}\frac{\Gamma\left(mL_tL_r+\sum_{k=1}^{L_t-1}n_k+\frac{1}{2}\right)}{\Gamma\left(mL_tL_r+\sum_{k=1}^{L_t-1}n_k+1\right)}\right.$$
$$\times\,{}_2F_1\left(mL_tL_r+\sum_{k=1}^{L_t-1}n_k,\frac{1}{2};mL_tL_r+\sum_{k=1}^{L_t-1}n_k+1;\frac{1}{1+\frac{3\bar{\gamma}}{2m(M-1)}}\right)$$
$$-\left(1-\frac{1}{\sqrt{M}}\right)\frac{\left[1+\frac{3\bar{\gamma}}{m(M-1)}\right]^{-\left(mL_tL_r+\sum_{k=1}^{L_t-1}n_k\right)}}{2\pi\left[2\left(mL_tL_r+\sum_{k=1}^{L_t-1}n_k\right)+1\right]}$$
$$\left.\times F_1\left(1,mL_tL_r+\sum_{k=1}^{L_t-1}n_k,1;mL_tL_r+\sum_{k=1}^{L_t-1}n_k+\frac{3}{2};\frac{1+\frac{3\bar{\gamma}}{2m(M-1)}}{1+\frac{3\bar{\gamma}}{m(M-1)}},\frac{1}{2}\right)\right\}. \quad (39)$$

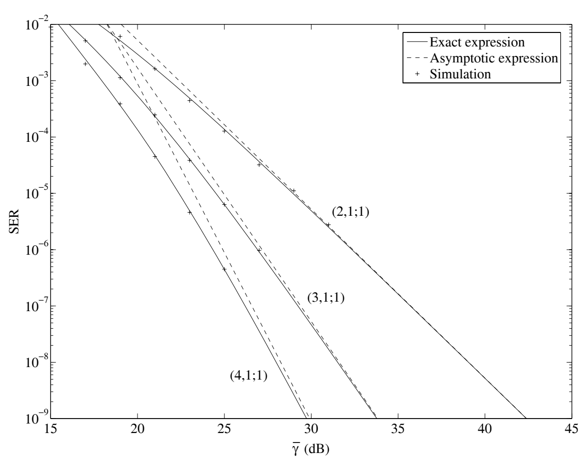

Fig. 4. Comparison of the exact expression, asymptotic expression, and simulation for the 16-QAM $(L_t, 1; 1)$ TAS/MRC scheme with $m = 1.5$ and different number of transmit antennas, $2 \leq L_t \leq 4$.

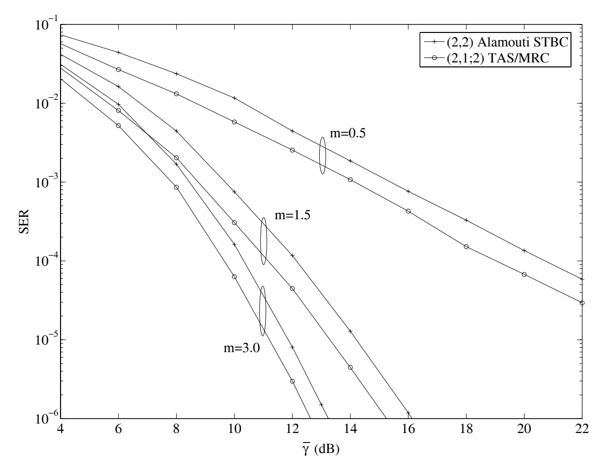

Fig. 5. Error performance comparison between the (2,1;2) TAS/MRC scheme and (2,2) Alamouti STBC, 4-PSK, $m = 0.5$, 1.5, and 3.0.

STBC, we have $\bar{\gamma} = E_s/N_0$, where E_s is the total energy per symbol across the two transmit antennas. It is shown that both schemes have the same diversity order for the same m. The TAS/MRC scheme has an SNR advantage of 1.7, 0.9, and 0.6 dB for $m = 0.5$, 1.5, and 3, respectively. This clearly demonstrates the superiority of the TAS/MRC scheme over the Alamouti STBC.

VI. Conclusion

The outage probability and symbol error performance of the TAS/MRC scheme in flat Nakagam-m fading channels were investigated. Utilizing the behavior of the pdf of the instantaneous channel gain, the asymptotic BER expression was derived for BPSK modulation, and the exact BER expression was also derived for the special case with two available transmit antennas in the form of an infinite series. The exact SER expressions were obtained by MGF method for M-PSK and M-QAM modulations, based on which the asymptotic SER property was revealed. It was shown that the asymptotic diversity order is equal to the product of the m parameter, the number of transmit antennas, and the number of receive antennas. Simulation results were provided to substantiate the analysis.

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