

# Partial Relay Selection in Underlay Cognitive Networks with Fixed Gain Relays

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**Abstract**—In a communication system with multiple cooperative relays, selecting the best relay utilizes the available spectrum more efficiently. However, selective relaying poses a different problem in underlay cognitive networks compared to the traditional cooperative networks due to interference thresholds to the primary users. In most cases, a best relay is the one which provides the maximum end-to-end signal to noise ratio (SNR). This approach needs plenty of instantaneous channel state information (CSI). The CSI burden could be reduced by partial relay selection. In this paper, a partial relay selection scheme is presented and analyzed for an underlay cognitive network with fixed gain relays operating in the vicinity of a primary user. The system model is adopted in a way that each node needs minimal CSI to perform its task. The best relay is chosen on the basis of maximum source to relay link SNR which then forwards the message to the destination. We derive closed form expressions for the received SNR distributions, system outage, probability of bit error and average channel capacity of the system. The derived results are confirmed through simulations.

**Index Terms**—cognitive radio, underlay networks, overlay networks, user cooperation, and relay selection etc.

## I. INTRODUCTION

The emerging wireless communications standards allow non-licensed or secondary users to access the licensed spectrum dedicated for the primary users. Various access methods have been proposed for smooth and transparent co-existence of primary and secondary users. One approach is called interweaved in which the secondary users detect and use the vacant primary spectrum [1] until the primary user remains inactive in that band. The secondary users can access the same band being used by the primary users in overlay and underlay approaches. The overlay cognition is based on interference avoidance [1], [2] whereas in underlay cognitive networks the secondary users must maintain a strict interference threshold toward the primary users [1], [2]. The interference constraint forces the secondary users to transmit at low powers which also reduces their operational area. In this situation, reaching remote users without increasing the transmission power is possible through cooperative relaying [3].

A cooperative relay provides an amplified copy of the source's message to the destination in addition to the directly received signal, if there exists one. Thus, it increases the diversity order of the received signal. Multiple relays can

further increase the diversity order; however, since they use orthogonal channels for transmission in most cases, they consume more bandwidth to convey the same message [3], [4]. Hence, the idea of best relay selection was introduced [5] and has been a topic of interest recently in non-cognitive networks [5]-[7]. Generally, a relay which could provide the maximum end-to-end signal to noise ratio (SNR) is selected as the best relay for forwarding the source's message to the destination. In underlay cognitive networks, a secondary relay with large end-to-end SNR means favourable channel conditions which may also result in more interference to the primary user. Hence, only the SNR may not be the optimum criterion for selective relaying in underlay settings.

Selective relaying in cognitive networks recently received some attention. A relay selection and channel allocation method is discussed in [8] for interweaved approach and can not be applied in underlay mode. A similar relay selection and power allocation scheme with limited interference to the primary users is proposed in [9]. A hop selection scheme is proposed in [10] for multi-hop networks which involves power control to meet the interference threshold at the primary user. A modified relay selection criterion for the underlay networks is proposed in [11] where the relays are assumed to be operating in decode-and-forward (DF) mode. Another relay selection scheme based on satisfying a certain outage probability of the primary network is proposed in [12] with DF relays. In [8]-[12], a common assumption is that the secondary nodes can adapt their transmission power in order to satisfy the interference constraint. This assumption may not be true in all cases and the secondary nodes may have fixed output power due to hardware constraints. Relay selection in an underlay cognitive network with fixed transmission power nodes is presented in [13]. However, the decision about the best relay is taken at the destination requiring the global CSI knowledge which may not be practical in some cases.

To reduce the amount of CSI required at different nodes, we consider a secondary cooperative network in which the relays operate in amplify-and-forward (AF) mode with fixed gains. Hence, without knowing the source to relay channel they amplify the received signal with a fixed gain. This causes each relay to transmit at different power depending the first hop channel gain. The relay selection is performed partially, i.e., the relay with the strongest first hop link is selected to forward source's message to the destination. Hence, the destination does not need any CSI, while the source needs to know the

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first hop channels only, greatly reducing the amount of CSI needed in the whole system. We derive probability density function (PDF) and cumulative distribution function (CDF) of the received SNR at the destination and use them to evaluate various performance parameters of the system in closed form.

## II. SYSTEM MODEL

We consider a system in which a secondary source  $S$  is transmitting its signal to a secondary destination  $D$  with the help of  $L$  secondary relays shown by  $R_i, i = 1, 2, \dots, L$ , in Fig. 1. This network is operating in underlay mode in the vicinity of a primary user  $P$ . A traditional two time slot communication procedure is followed in AF mode with fixed gain relays. The channel gain on the direct link is  $h_0$  whereas  $S \rightarrow R_i$  and  $R_i \rightarrow D$  channels are  $h_{1i}$  and  $h_{2i}$ , respectively. The interference channel from the source to the primary user is  $h_{SP}$  and from the  $i^{th}$  relay to the primary user is  $h_{iP}$ . We assume that all the channels are Rayleigh distributed; therefore, their squared amplitudes are exponentially distributed. Each link in the system, either communication or interference, is subjected to additive white Gaussian noise (AWGN) with zero mean and variance  $N_0$ .

In order to co-exist with the primary users, underlay cognitive networks must comply with stringent interference constraints and ensure that the primary transmission is not affected. This is realized through an interference threshold at the primary user. Let  $\lambda$  be the interference threshold. Since the relays are equipped with fixed gains and no transmit power adaptation is available, the interference channel  $h_{iP}$  may be strong enough for some relays that they may violate this threshold. This is also possible for the  $S \rightarrow P$  channel  $h_{SP}$ . However, if this is the case with the source, it will refrain from initiating the transmission. The interference to the primary user through the source  $\mathcal{I}_{SP}$ , its PDF and CDF can be given, respectively, as

$$\mathcal{I}_{SP} = E_s |h_{SP}|^2, \quad p_{\mathcal{I}_{SP}}(x) = \frac{1}{\rho} e^{-\frac{x}{\rho}} \quad \text{and} \quad P_{\mathcal{I}_{SP}}(x) = 1 - e^{-\frac{x}{\rho}}, \quad (1)$$

where  $E_s$  is the transmission power at the source and  $\rho$  is the average strength of the  $S \rightarrow P$  interference channel.

From (1), the probability of having  $\mathcal{I}_{SP} > \lambda$  or no transmission from the source is  $e^{-\frac{\lambda}{\rho}}$ . To analyze the whole scheme, we consider a situation when the source satisfies the interference constraint. Similarly, the relays must also satisfy  $\lambda$  to become a part of the transmission. Each relay can amplify whatever it receives from the source with a fixed gain  $g$ . However, the strengths of  $S \rightarrow R_i$  and  $R_i \rightarrow P$  channels determine if it could satisfy the interference constraint or not. The interference from the  $i^{th}$  relay to the primary user is

$$\mathcal{I}_{iP} = E_s g^2 |h_{1i}|^2 |h_{iP}|^2. \quad (2)$$

As mentioned earlier  $|h_{1i}|^2$  and  $|h_{iP}|^2$  are exponentially distributed; hence, the PDF and CDF of  $\mathcal{I}_{iP}$  can be given, respectively, as

$$p_{\mathcal{I}_{iP}}(y) = \frac{2}{\alpha\sigma} K_0\left(2\sqrt{\frac{y}{\alpha\sigma}}\right) \quad \text{and} \quad P_{\mathcal{I}_{iP}}(y) = 1 - 2\sqrt{\frac{y}{\alpha\sigma}} K_1\left(2\sqrt{\frac{y}{\alpha\sigma}}\right), \quad (3)$$

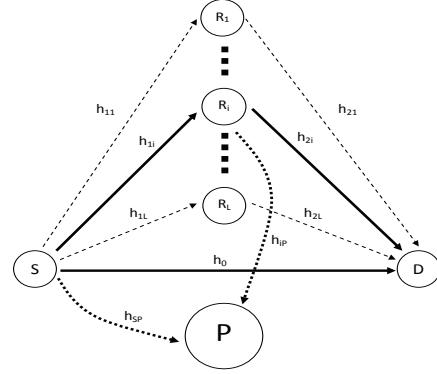


Fig. 1. System Model: A cognitive network operating near a primary user.

where  $K_0(\cdot)$  and  $K_1(\cdot)$  are the zero and first-order modified Bessel functions of the second kind,  $\alpha$  are the average SNRs of  $S \rightarrow R_i$  links and  $\sigma$  are the average strengths of  $R_i \rightarrow P$  links.

Using (3), we can find out the probability of satisfying the interference constraint by each relay. Let us call it  $P_\lambda = P_{\mathcal{I}_{iP}}(\lambda)$ . Hence, each relay out of the available  $L$  could become a candidate for the best relay selection with a probability  $P_\lambda$  or could be dropped from the selection pool with probability  $\bar{P}_\lambda = 1 - P_\lambda$ . In other words, the number of relays satisfying the interference constraint is a random variable (RV). We assume that  $\ell$  relays out of  $L$  satisfy the interference constraint such that  $\ell = 0, 1, \dots, L$ . We also assume that each relay is aware of its interference channel to the primary. This information could be gathered when the primary user is transmitting or acknowledging any received signal. Each relay shares this information with the source on a dedicated feedback channel. This enables the source, which already knows  $S \rightarrow R_i$  channels, to pick up  $\ell$  relays satisfying  $\lambda$  and generate a set  $\mathcal{A}$  containing their indexes. In the first time slot, the source picks up the best relay based on maximum first hop SNR, i.e.

$$\gamma_s = \max_{i \in \mathcal{A}} \gamma_{1i} = \max_{i \in \mathcal{A}} \frac{E_s}{N_0} |h_{1i}|^2, \quad (4)$$

and broadcasts its message to all the relays and the destination with the index of the selected relay on a feedback or a control channel. In case of centralized implementation, a dedicated node can collect the required CSI for this scheme and broadcast the decision to all involved nodes. However, in this paper, we are more focused on statistical and performance analysis of this scheme rather than its implementation detail.

Now, with the selected first hop link, the received SNR at the destination can be given as [14]

$$\hat{\gamma}_i = \frac{\gamma_s \gamma_{2i}}{C + \gamma_{2i}}, \quad (5)$$

where  $\hat{i}$  is the index of the selected relay,  $\gamma_{2i} = \frac{E_s |h_{2i}|^2}{N_0}$  is the SNR of the second hop and  $C = \frac{E_s}{g^2 N_0}$  is constant for a particular system.

The CDF and PDF of the selected link ( $S \rightarrow R_{\hat{i}}$ ) SNR  $\gamma_s$  among the  $\ell$  relays satisfying the interference constraint are

given, respectively, as

$$P_{\gamma_s}(\gamma|\ell) = \left(1 - e^{-\frac{\gamma}{\alpha}}\right)^\ell, \text{ and } p_{\gamma_s}(\gamma|\ell) = \frac{\ell}{\alpha} e^{-\frac{\gamma}{\alpha}} \left(1 - e^{-\frac{\gamma}{\alpha}}\right)^{\ell-1}. \quad (6)$$

As mentioned earlier, finding  $\ell$  relays meeting the interference threshold out of the total  $L$  depends upon  $P_\lambda$  and  $\bar{P}_\lambda = 1 - P_\lambda$  and thus follows a binomial distribution

$$p_\ell(\ell; L, P_\lambda) = \binom{L}{\ell} P_\lambda^\ell \bar{P}_\lambda^{L-\ell}, \quad (7)$$

where  $\binom{L}{\ell} = \frac{L!}{\ell!(L-\ell)!}$ .

Now, the PDF of the end-to-end SNR through the selected relay using the proposed partial selection scheme can be evaluated using the outage probability of the relay links, which can be defined as follows

$$P_{out|\ell}^i = Pr[\gamma_i < \gamma_{th}] = Pr\left[\frac{\gamma_s \gamma_{2i}}{C + \gamma_{2i}} < \gamma_{th}\right], \quad (8)$$

where  $P_{out|\ell}^i$  is the outage due to the relay links for a given  $\ell$  and  $\gamma_{th}$  is the outage threshold SNR.

For a given  $\gamma_{2i}$ , (8) can be evaluated over the PDF of  $\gamma_{2i}$  as in [14]

$$P_{out|\ell}^i = \int_0^\infty Pr\left[\gamma_s < \gamma_{th}\left(1 + \frac{C}{\gamma_{2i}}\right) | \gamma_{2i}\right] p_{\gamma_{2i}}(\gamma) d\gamma. \quad (9)$$

The probability term in the above can be evaluated using the CDF in (6). Since  $\gamma_{2i}$  is also exponentially distributed with parameter  $\beta$  representing the average SNR of the  $i^{th}$  second hop, the above integral reduces to

$$P_{out|\ell}^i = \frac{1}{\beta} \int_0^\infty \left[1 - e^{-\frac{\gamma_{th}}{\alpha}(1 + \frac{C}{\gamma})}\right]^\ell e^{-\frac{\gamma}{\beta}} d\gamma. \quad (10)$$

Using binomial expansion and solving through [15, Eq. (3.324.1)], we get

$$P_{out|\ell}^i = 1 + 2 \sum_{n=1}^{\ell} \binom{\ell}{n} (-1)^n e^{-\frac{n\gamma_{th}}{\alpha}} \sqrt{\frac{n\gamma_{th}C}{\alpha\beta}} K_1\left(2\sqrt{\frac{n\gamma_{th}C}{\alpha\beta}}\right). \quad (11)$$

Differentiating (11) with respect to  $\gamma_{th}$  using [15, Eq. (8.486.12)] gives us the PDF of the SNR through the partially selected relay link conditioned over  $\ell$  as follows

$$p_{\gamma_i}(\gamma|\ell) = \frac{2}{\alpha} \sum_{n=1}^{\ell} \binom{\ell}{n} (-1)^{n+1} n e^{-\frac{n\gamma}{\alpha}} \left[ \sqrt{\kappa_n \gamma} K_1(2\sqrt{\kappa_n \gamma}) + \frac{C}{\beta} K_0(2\sqrt{\kappa_n \gamma}) \right], \quad (12)$$

where  $\kappa_n = \frac{nC}{\alpha\beta}$ .

The unconditional PDF of  $\gamma_i$  can be obtained by averaging (12) over (7) for  $\ell = 1, 2, \dots, L$ .

$$p_{\gamma_i}(\gamma) = \frac{2}{\alpha} \sum_{\ell=1}^L \binom{L}{\ell} P_\lambda^\ell \bar{P}_\lambda^{L-\ell} \sum_{n=1}^{\ell} \binom{\ell}{n} (-1)^{n+1} n e^{-\frac{n\gamma}{\alpha}} \times \left[ \sqrt{\kappa_n \gamma} K_1(2\sqrt{\kappa_n \gamma}) + \frac{C}{\beta} K_0(2\sqrt{\kappa_n \gamma}) \right]. \quad (13)$$

Note that in the above PDF we do not consider  $\ell = 0$ , i.e., a situation when none of the relays satisfy the interference

constraint and the destination receives the direct signal only. Hence, (13) is an unconditional but truncated PDF. The probability of receiving the direct signal only is  $\bar{P}_\lambda^L$ . Since the direct link SNR follows an exponential distribution, different from the one given in (13), therefore the situation when  $\ell = 0$  is not considered here. It will be included in the later analysis.

The destination combines the signal received through the best relay with the directly received signal making the total SNR at the destination  $\gamma_T = \gamma_0 + \gamma_i$ , where  $\gamma_0 = \frac{E_s |h_0|^2}{N_0}$  is the exponentially distributed direct link SNR with average SNR parameter  $\bar{\gamma}_0$  having PDF and CDF, respectively, as follows

$$p_{\gamma_0}(\gamma) = \frac{1}{\bar{\gamma}_0} e^{-\frac{\gamma}{\bar{\gamma}_0}}, \quad \text{and} \quad P_{\gamma_0}(\gamma) = 1 - e^{-\frac{\gamma}{\bar{\gamma}_0}}. \quad (14)$$

Since  $\gamma_0$  and  $\gamma_i$  are mutually independent, the PDF of  $\gamma_T$  is simply the convolution between (13) and (14).

$$p_{\gamma_T}(\gamma) = \int_0^\infty p_{\gamma_0}(\gamma - x) p_{\gamma_i}(x) dx. \quad (15)$$

Solving (15) using [15, Eq. (6.643.3)] and simplifying through [16, Eqs. (13.1.33), (13.6.28), (13.6.30) and (6.5.19)], we get

$$p_{\gamma_T}(\gamma) = e^{-\frac{\gamma}{\bar{\gamma}_0}} \sum_{\ell=1}^L \binom{L}{\ell} P_\lambda^\ell \bar{P}_\lambda^{L-\ell} \sum_{n=1}^{\ell} \binom{\ell}{n} (-1)^{n+1} n \times \left[ \frac{1}{n\bar{\gamma}_0 - \alpha} - \frac{\alpha C e^{\frac{nC\bar{\gamma}_0}{\beta(n\bar{\gamma}_0 - \alpha)}}}{\beta(n\bar{\gamma}_0 - \alpha)^2} E_1\left(\frac{nC\bar{\gamma}_0}{\beta(n\bar{\gamma}_0 - \alpha)}\right) \right], \quad (16)$$

where  $E_1(\cdot)$  is the exponential integral function.

A simple integration of (16) gives us the CDF of the received SNR at the destination as follows

$$P_{\gamma_T}(\gamma) = \bar{\gamma}_0 (1 - e^{-\frac{\gamma}{\bar{\gamma}_0}}) \sum_{\ell=1}^L \binom{L}{\ell} P_\lambda^\ell \bar{P}_\lambda^{L-\ell} \sum_{n=1}^{\ell} \binom{\ell}{n} (-1)^{n+1} n \times \left[ \frac{1}{n\bar{\gamma}_0 - \alpha} - \frac{\alpha C e^{\frac{nC\bar{\gamma}_0}{\beta(n\bar{\gamma}_0 - \alpha)}}}{\beta(n\bar{\gamma}_0 - \alpha)^2} E_1\left(\frac{nC\bar{\gamma}_0}{\beta(n\bar{\gamma}_0 - \alpha)}\right) \right]. \quad (17)$$

### III. PERFORMANCE ANALYSIS

In this section, using the statistics of the received SNR, we derive some important system performance parameters such as outage and bit error probabilities and channel capacity. It is worthy to note that from (13) to (17) it is assumed that at least one relay is satisfying the interference threshold ( $\ell = 1$ ). However, in the following we include the situation when  $\ell = 0$  and the system is operating on the direct link only.

#### A. Outage Probability

A communication system is supposed to be in outage if the received SNR falls below a certain threshold. Therefore, the outage probability can simply be evaluated through the CDF of the total SNR by using  $\gamma = \gamma_{th}$ , where  $\gamma_{th}$  is the outage threshold SNR. The probability that none of the relays satisfies the interference constraint and the destination receives the direct signal only is  $Pr[\ell = 0] = (1 - P_\lambda)^L = \bar{P}_\lambda^L$ . Additionally, the direct link SNR can fall below  $\gamma_{th}$  with a

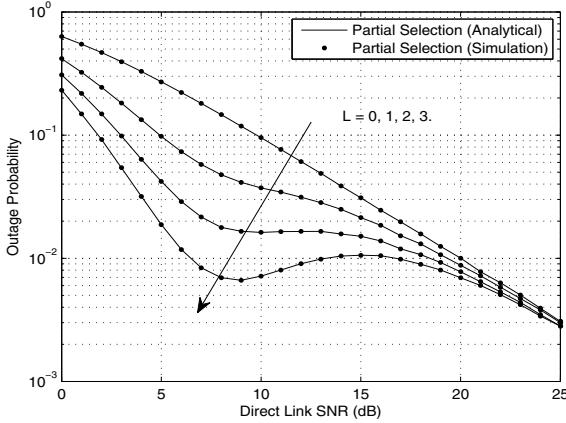


Fig. 2. Outage probability with  $\lambda = 10$  and  $\gamma_{th} = 1$  for  $L = 0, 1, 2, 3$ .

probability  $(1 - e^{\frac{\gamma_{th}}{\bar{\gamma}_0}})$ . Hence, the outage probability of the system becomes

$$P_{out} = P_{\gamma_T}(\gamma_{th}) + \bar{P}_{\lambda}^L(1 - e^{\frac{\gamma_{th}}{\bar{\gamma}_0}}). \quad (18)$$

#### B. Average Probability of Bit Error

Average bit error probability (BER) can be derived using either (16) or (17). For a given SNR, we can represent BER in AWGN for any linear modulation scheme in terms of standard  $Q$  function and then average it over (16). However, the final integral in this approach involves  $Q$  function, which may not be evaluated in closed form in some cases. Alternatively, we can use the approach in [17] to express the  $Q$  function integral in terms of CDF of the received SNR, making it possible to derive the BER using (17). Hence,

$$P_e = \underbrace{\frac{Pr[\ell = 0]}{\sqrt{2\pi}} \int_0^\infty P_{\gamma_0} \left( \frac{t^2}{\eta} \right) e^{-\frac{t^2}{2}} dt}_{\text{Direct link only}} + \underbrace{\frac{1}{\sqrt{2\pi}} \int_0^\infty P_{\gamma_T} \left( \frac{t^2}{\eta} \right) e^{-\frac{t^2}{2}} dt}_{\text{Direct and best relay links}}, \quad (19)$$

where  $\eta$  is a constant depending upon the modulation scheme. Replacing (14) and (17) in (19) and solving through [15, Eq. (3.321.3)], we get

$$P_e = \frac{1}{2} \left( 1 - \sqrt{\frac{\eta \bar{\gamma}_0}{2 + \eta \bar{\gamma}_0}} \right) \left[ \bar{P}_{\lambda}^L + \bar{\gamma}_0 \sum_{\ell=1}^L \binom{L}{\ell} P_{\lambda}^{\ell} \bar{P}_{\lambda}^{L-\ell} \sum_{n=1}^{\ell} \binom{\ell}{n} \right. \\ \times (-1)^{n+1} n \left[ \frac{1}{n \bar{\gamma}_0 - \alpha} - \frac{\alpha C e^{\frac{nC \bar{\gamma}_0}{\beta(n \bar{\gamma}_0 - \alpha)}}}{\beta(n \bar{\gamma}_0 - \alpha)^2} E_1 \left( \frac{nC \bar{\gamma}_0}{\beta(n \bar{\gamma}_0 - \alpha)} \right) \right] \right]. \quad (20)$$

#### C. Average Channel Capacity

The channel capacity of the considered system model becomes half of the classical Shannon's definition,  $\mathfrak{C} = \mathfrak{B} \log_2(1 + SNR)$ , where  $\mathfrak{B}$  is the signal bandwidth, because the information is conveyed in two time slots.

$$\mathfrak{C} = \frac{\mathfrak{B}}{2} \left[ \bar{P}_{\lambda}^L \int_0^\infty \log_2(1 + \gamma_0) p_{\gamma_0}(\gamma) d\gamma + \int_0^\infty \log_2(1 + \gamma_T) p_{\gamma_T}(\gamma) d\gamma \right]. \quad (21)$$

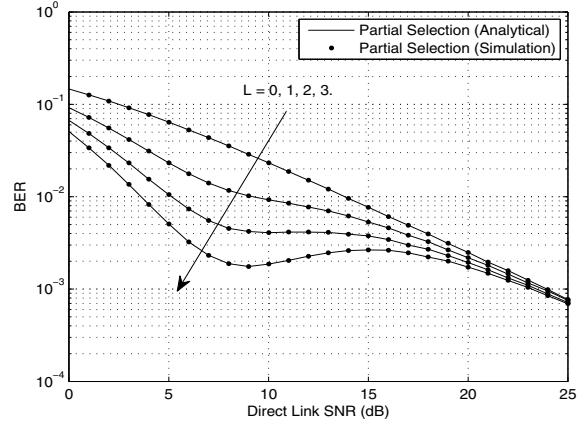


Fig. 3. BER with  $\lambda = 10$  for different number of relays.

Replacing (14) and (16) in (21) and solving using [15, Eqs. (4.337.1 or 2)] and [16, Eq. (5.1.7)], we get

$$\mathfrak{C} = \frac{\mathfrak{B}}{2 \ln 2} e^{\frac{1}{\bar{\gamma}_0}} E_1(1/\bar{\gamma}_0) \left[ \bar{P}_{\lambda}^L + \bar{\gamma}_0 \sum_{\ell=1}^L \binom{L}{\ell} P_{\lambda}^{\ell} \bar{P}_{\lambda}^{L-\ell} \sum_{n=1}^{\ell} \binom{\ell}{n} \right. \\ \times (-1)^{n+1} n \left[ \frac{1}{n \bar{\gamma}_0 - \alpha} - \frac{\alpha C e^{\frac{nC \bar{\gamma}_0}{\beta(n \bar{\gamma}_0 - \alpha)}}}{\beta(n \bar{\gamma}_0 - \alpha)^2} E_1 \left( \frac{nC \bar{\gamma}_0}{\beta(n \bar{\gamma}_0 - \alpha)} \right) \right] \right]. \quad (22)$$

#### IV. SIMULATION RESULTS

Simulations are performed by varying  $\bar{\gamma}_0$  while setting  $\alpha = 1.3\bar{\gamma}_0$  and  $\beta = 1.8\bar{\gamma}_0$  with zero mean and unit variance AWGN. The interference channels are simulated with parameter  $\sigma = 0.8\bar{\gamma}_0$ . The source is assumed to be transmitting at  $E_s = 1$  and each relay is equipped with  $g = 1$ . Binary phase shift keying (BPSK) with  $\eta = 2$  is used as the modulation technique. System configurations with different number of relays are compared with equal power conditions.

Fig. 2 depicts the outage probability of the system with  $\gamma_{th} = 1$ . In the upper curve, when  $\lambda \rightarrow 0$ , no relay could satisfy this constraint and the system operates on the direct link only ( $\ell = 0$ ). The other curves are for  $L = 1, 2$ , and  $3$  as marked with  $\lambda = 10$ . At low SNR, all the relays in the system could satisfy  $\lambda$  giving ( $\ell = L$ ) and diversity order of the system becomes  $L + 1$ . With fixed gains and no transmission power control, increasing SNR causes more interference to the primary user. Therefore, some relays do not satisfy  $\lambda$  resulting in  $\ell < L$  and reducing the diversity order of the system. At further high SNR, none of the relays could satisfy  $\lambda$  and the system operates on the direct link only. Hence, all the curves merge into the direct link curve at high SNR. Analytical and simulation results match perfectly.

Bit error probability (BER) of the system is shown in Fig. 3. Similar behaviour is evident because of the same reasons as above. An optimum SNR can be seen after which the performance starts degrading. Hence, a particular number of relays required to maintain a specific BER at a certain value of  $\lambda$  can be chosen. Also, normal operating range could be extended by using more number of relays. Analytical results closely follow the simulation results.

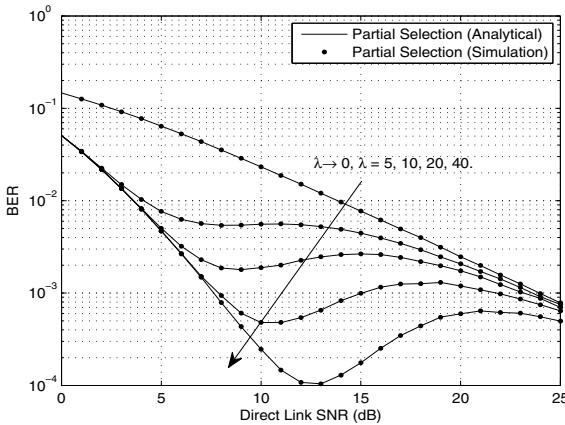


Fig. 4. BER with  $L = 3$  and different values of  $\lambda$ .

Fig. 4 shows the BER performance with  $L = 3$  for different values of  $\lambda$ . As the interference constraint is relaxed or increased, more relays could qualify for the selection process and the diversity order of the system remains  $L + 1$  at comparatively higher SNRs. Hence, the optimum operating SNR moves gradually forward with increasing  $\lambda$ .

Average channel capacity of the system is plotted in Fig. 5. Minimum capacity is resulted when  $\lambda \rightarrow 0$  or with direct link only. An increase in the number of relays gradually increases the channel capacity. However, a slight decrease can be seen after a certain SNR in each case, which again results from decreasing diversity order of the system due to interference constraint violation.

## V. CONCLUSION

We presented a partial relay selection scheme and analyzed its performance for a cognitive network operating near a primary user. The relays were equipped with fixed gains and no transmit power adaptation was possible. The partial relay selection was based on satisfying the primary interference threshold and on maximum first hop SNR. All these assumptions reduced the overall CSI requirement in the system. We derived closed form expressions for received SNR distributions, system outage, BER and channel capacity and verified through simulations. We concluded that with fixed transmit power and gain at the relays, selective relaying is only feasible in low SNR region.

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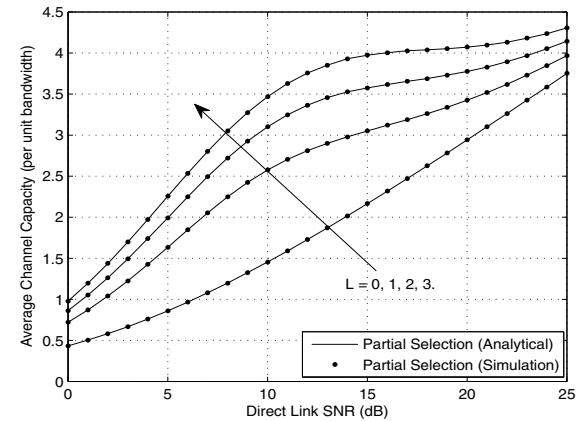


Fig. 5. Channel capacity with  $\lambda = 10$  for different number of relays.

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