Synchronisation of Cycles *

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November 24, 2003

Abstract

Many interesting issues are posed by synchronisation of cycles. In this paper we define synchronisation and show how the degree of synchronisation can be measured. We propose tests of the hypotheses that cycles are either unsynchronised or perfectly synchronized. Unlike previous tests of synchronization in the literature our procedures are robust to heteroscedasticity and serial correlation in the random variables making up the test statistic.

Tests of synchronization are performed using data on industrial production, on monthly stock indices and on series that are used to construct the reference cycle for the United States and Australia.

Where synchronisation is found interest focuses on extracting the common cycle. We discuss the relationship between various definitions of common cycles that have been proposed based on parametric models. Then an algorithm is detailed which utilizes NBER dating procedures for identifying the common cycle that they identify as the reference cycle. This algorithm is used to extract a the reference cycle for the United States and Australia.

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*Earlier versions of this paper were presented at the 5th Australian Macroeconomics Workshop, University of Queensland, Brisbane; the 20th International Symposium on Forecasting, Lisbon June 2000; the Conference on Growth and Business Cycles in Practice, University of Manchester, July 2000, the Monash Workshop in Business and Economic Forecasting, September 2001 and the Common Features in Rio Conference, July 2002. We would like to thank the many people who made comments on it but particularly two anonymous referees and the editor J. Issler.

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JEL Classification: C12, C14, C22, C33, E32.

Keywords: Business Cycles, Common Cycles, Synchronization, Turning Points, Factor Models.

1. Introduction

A viewing of the graphs of many specific series have often suggested to researchers that the cycles seen in them are synchronized, in the sense that their turning points occur at either roughly the same points in time or differ by intervals that are roughly constant i.e. the turning points "cluster together". Such clustering of turning points was a major theme in the work of Burns and Mitchell (1946). In particular it underpinned their idea of a "reference cycle".[1] The question of synchronization is of interest since many actions that are contemplated often require an answer as to whether it is present e.g. when countries are considering forming a monetary union the question of whether their business cycles are coordinated arises. Apart from economic activity, there are also many other series which exhibit cycles and which encourage questions regarding synchronization e.g. do 'bull' and 'bear' markets align either in different stock market indices in a single country (e.g. the NASDAQ vs the Dow) or across countries?

Basic to any investigation of the question of synchronization of cycles is a description of how one is to recognize a cycle. Broadly speaking we can find three suggestions in the literature. Each involves the construction of a set of indicators of a cycle from the information available on a continuous random variable y_t. In the oldest tradition the indicators are *turning points* in y_t, with the output being formally described as a binary random variable S_t which shows when the economy is in the different phases that are separated by the turning points e.g. an expansion in the business cycle can be associated with $S_t = 1$ while a contraction is indicated by $S_t = 0$.

The other two suggestions proceed in a different way. Common to both is the prior transformation of y_t so as to remove a permanent component, leaving only a transitory one, z_t. It is the cycle in z_t rather than y_t that is then examined, with the requisite indicators being derived from observations on z_t. With the z_t in hand, the first of these two traditions then defines the cycle indicator as the presence or absence of complex roots in an AR fitted to z_t or, more generally, a peak in the spectral density of z_t. Such an indicator is often mentioned in undergraduate

[1] It is important not to overstate the extent to which Burns and Mitchell focused on synchronization. Burns and Mitchell (1946 p 70), for example, observe that at any point in time 'some activities [are] in an expanding phase, some beginning to recede from their peaks, some contracting, and some beginning to revive from their troughs'. Nevertheless, they observe from their studies 'that at any one time one phase is dominant'.

textbooks. In contrast, the second tradition adopts the proposal set out in Blinder and Fischer (1981, p 277), who say that a cycle is indicated by "serially correlated deviations of output from trend" i.e. for a cycle to exist in z_t there should be serial correlation in z_t.[2] It is important to note that it is the *existence* of a cycle which is checked for by these measures. In all cases the resulting indicators of the existence of a cycle are quite distinct from the underlying series, whether it is y_t or z_t; the latter *are not the cycle*, although their nature will determine the characteristics of the cycle.[3]

The turning points view of a cycle is widespread in media and policy analysis and is implicitly invoked whenever lectures and textbooks either show graphs of y_t or quote the dates of recessions such as those established by the NBER. Cycle characteristics established via the turning points in y_t are determined by the nature of the process Δy_t. Of course one might also be interested in the cycle in z_t found by examining the turning points in that series. If the emphasis is in fact shifted to cycles in z_t, then one might compare the three definitions using a common base. Doing so reveals that there is little relation between the first and second views, since the duration of time between the turning points in z_t has no close relation to the length of any periodic cycle indicated by the position of the peak of the spectral density of z_t. Moreover, one does not need any complex roots in an AR process in order to generate a turning point cycle in z_t. Turning to the relation between the first and third views, it was shown in Harding and Pagan (2002) that it was not necessary to have serial correlation in the z_t process in order to produce a cycle. Moreover, that paper also showed that the nature of the second order moments of Δz_t influenced the type of cycle that would be seen in z_t - since the probability of encountering a turning point in z_t could be expressed in terms of the second-order moments of the Δz_t process. Because the second order moments of Δz_t are just transformations of those for z_t it is clear that the two views work with the same inputs but focus on different outputs.

As the diversity of viewpoints would indicate there is probably no right or wrong way of defining a cycle. But even a cursory reading of the financial press would point to the fact that the turning point view seems to be what is meant by a cycle when one reads economic and policy commentary. So it seems natural

[2] They define a "detrending" operation as removal of a permanent component.

[3] If y_t is a vector then one can provide equivalent concepts. Thus the extension of the turning point view is the "reference cycle" which we will explain later, while a complex root in an AR becomes a complex eigenvalue in a VAR etc.

to adopt such a definition. At the very least one should subject it to academic analysis rather than simply moving on to work with different views. If it turned out that the analysis with a turning point view of cycles was intractable, then there would be a good case for moving to some other framework, but in our mind this has never been established.

Given our orientation towards turning points, the issues we deal with in this paper are how to define and measure synchronization of cycles, when these are defined through their turning points; how to test hypotheses about the extent of any synchronization; and how to extract and talk about the "common cycle" that arises when synchronization is found. Formally, synchronization will be viewed as the phenomenon whereby turning points in *specific cycles* cluster at particular dates. Section 2 briefly outlines how the specific cycles associated with n variables $y_{1t}, ..., y_{nt}$ will be represented by binary time series S_{it}.

We show, in section 3, that for the bivariate case synchronization can be defined in terms of the joint density of (S_{xt}, S_{yt}) and establish links between this definition and definitions based on the correlation between (S_{xt}, S_{yt}) and the proportion of time S_{xt} and S_{yt} are in the same state. Then, for the multivariate case, we propose to measure synchronization via the correlation matrix of the vector $(S_{1t}, ..S_{nt})$. We then show how our concept of synchronization is linked to the notion of "co-movement" that is widely used in the literature.

Section 4 concentrates upon defining a "common cycle" in a set of variables. There are two ways one might do this. One is the parametric approach, which constructs parametric models of the series and then extracts a continuous factor whose cycle will be taken to represent the common cycle. Again, the factor itself is not the cycle. There is a literature on such a parametric approach, which varies according to the nature of the underlying factor and the way it is extracted. Examples would be Vahid and Engle's (1993) common cycles and the common factor approach of Stock and Watson (1991), Chauvet (1998) and Forni et al (1999). This methodology essentially constructs an aggregate and then locates the turning points in it. An alternative approach works in a non-parametric way and aggregates the specific cycle turning points into a single set of turning points. For this latter task we develop an algorithm which yields a common cycle that is closely related to the reference cycle produced by NBER business cycle dating techniques. In this sense the algorithm formalizes the methods of Burns and Mitchell and their followers such as Boehm and Moore (1984).

Section 5 turns to testing for the presence and degree of synchronization. Here, attention is focused on three facets of testing for synchronization. The first of these is the necessity to correct for serial correlation and heteroscedasticity in the S_{jt} in order to make valid inference. In two applications we show that correcting for these features of the data can modify the conclusions drawn about the extent of synchronization. The second issue is that testing for perfect synchronization involves testing on the boundary of the parameter space. We develop tests that are appropriate for this case. The section concludes by providing an illustration of how the testing procedures and our analysis of synchronization can be used to guide practitioners in constructing NBER-like reference cycles. In this regard we bring the construction of NBER-like reference cycles into the econometric mainstream.

Section 6 develops the non parametric approach to extracting common cycles. The method is calibrated against the Australian reference cycle which is known to be determined using the NBER methodology. The non parametric approach is then evaluated in terms of its capacity to match the NBER reference cycle (an out of sample test) and it is then applied to extract the common cycle in industrial production and stock market cycles across countries. Conclusions are presented in section 7.

2. Measuring specific cycles

Specific cycles refer to the cycles in individual series Y_t as expressed though their turning points; the latter being local maxima and minima in the sample path of the time series. It is convenient to work with $y_t = \ln(Y_t)$ rather than Y_t, mainly because many empirical models fitted to series use such a transformation. Turning points in y_t and Y_t are identical so that the transformation loses no information.

A standard "turning points" definition of a cycle in quarterly data is provided by the following rules that are the basis of the NBER procedures summarized in the Bry and Boschan (1971) program:[4]

$$peak\ at\ t \quad = \quad \{y_{t-2} < y_t, y_{t-1} < y_t; y_t > y_{t+1}, y_t > y_{t+2}\}$$

[4]For monthly data y_t must exceed $y_{t\pm 5}$ for there to be a peak and must be less than these for a trough. We will retain the quarterly emphasis. Other rules could be adopted.

$$trough \quad at \ t \quad = \quad \{y_{t-2} > y_t, y_{t-1} > y_t; y_t < y_{t+1}, y_t < y_{t+2}\}. \tag{2.1}$$

This rule is supplemented by censoring procedures used in NBER dating methods which ensure that phases of the cycle have a required minimum duration of six months and that completed cycles have a minimum duration of fifteen months. Further details on the algorithms that are used to find turning points in this manner can be found in Harding and Pagan (2002).

The discussion above has focussed on cycles in the levels of a series y_t. These are commonly referred to as *classical cycles*. Classical cycle peaks are points at which a series moves from positive growth rates to negative growth rates and classical cycle troughs are points at which a series moves from negative growth rates back to positive growth rates. As mentioned in the introduction the classical cycle is not the only cycle that has been investigated. It may sometimes be desirable to study cycles in series from which a permanent component has been removed. We designate such a series as z_t.[5] The cycles established through turning points in z_t are often referred to as *growth cycles* but this name is potentially misleading; *deviation cycle* might be a better description.

Once we have identified the phases of the cycle we can associate them with a binary random variable S_t that takes the values unity and zero. We will refer to S_t as being the *specific cycle* in a designated variable. It might be asked why one wants to focus upon the binary variable S_t rather than y_t itself? A simple justification is that the binary classification underpins a great deal of the discussion over developments in the level of economic activity. One simply needs to follow the concerns in the past few years over whether economies were likely to go into recession or to have a "double-dip" recession to see that great emphasis is placed upon events summarized by the binary indicator. As the definition of a recession implied from the rules above involves a sustained *reduction* in the level of activity, and it is a well known fact from the psychological literature that agents are loss averse, it may well be that this accounts for the marked concentration upon the binary outcomes. In passing it might be noted that other

[5] We do not like referring to z_t as a "de-trended" series since the permanent component in y_t is an important factor in determining the nature of the cycle in y_t, and so it is not possible to produce a meaningful "trend/cycle" decomposition. Generally it is better to refer to what has been removed from the series in typical "de-trending" operations such as Hodrick-Prescott and Band-Width filtering as the "permanent" rather than "trend" component.

research topics e.g. those looking at the predictability of "crises", also convert continuous random variables into binary ones before analysis. Finally, there are some pragmatic reasons for needing a framework for the analysis of binary random variables. One of these is simply that the data often comes only in this form e.g. the NBER business cycle dates are presented with only general indications of the behavior of the specific continuous series used to determine them.

What are the properties of S_t i.e. what is its DGP? It is clear that the DGP of S_t depends on the nature of the rule to identify a cycle and the nature of the series Δy_t that enters into the dating rules. In general S_t is a high order stationary and ergodic Markov Chain. To illustrate this, if Δy_t is a mean-zero stationary Gaussian process, and phases are identified with the rule that $S_t = 1(\Delta y_t > 0)$, Kedem(1980, p34) sets out the relation between the autocorrelations of the Δy_t and $S(t)$ processes. Letting $r_{\Delta y}(k) = corr\,(\Delta y_t, \Delta y_{t-k})$ and $r_S(k) = corr\,(S_t, S_{t-k})$, he determines that

$$r_S(k) = \frac{2}{\pi} \arcsin\left(r_{\Delta y}(k)\right). \tag{2.2}$$

Thus, given an estimate of $r_{\Delta y}(k)$, we can immediately find an estimate of $r_S(k)$ and *vice versa*. It is clear from the nature of these autocorrelations that (say) an $AR(1)$ process for Δy_t will imply a much more complex DGP for S_t than an $AR(1)$.

As the dating rule differs the nature of the DGP for S_t will also change. In Harding and Pagan (2001) we work through the case where the dating rule is that a recession involves two successive quarters of negative growth and y_t is a random walk with drift. We show that there is substantial serial correlation in the S_t even when there is none in Δy_t. Thus, in general there will be extensive serial correlation in S_t, and this must be allowed for when S_t appears in any test statistic.

3. Defining and Measuring Synchronization

3.1. Density Measures for Bivariate Cycles

It is useful to start a discussion of synchronization by concentrating upon the relations between the unconditional densities of two cycles S_{xt} and S_{yt}. It seems natural to define *strong perfect positive synchronization (SPPS)* as the case when the two random variables S_{xt} and S_{yt} are identical. Because of the binary nature of the random variables, necessary and sufficient conditions for this type of synchronization are

$$\begin{aligned} (a)\ \Pr(S_{yt} &= 1, S_{xt} = 0) = 0 && (3.1) \\ (b)\ \Pr(S_{yt} &= 0, S_{xt} = 1) = 0. && (3.2) \end{aligned}$$

In the same vein *strong perfect negative synchronization (SPNS)* will obtain when

$$\begin{aligned} \Pr(S_{yt} &= 0, S_{xt} = 1) = 1 && (3.3) \\ \Pr(S_{yt} &= 1, S_{xt} = 0) = 1. && (3.4) \end{aligned}$$

We will couch our discussion in terms of positive synchronization since it is easy to translate the requisite tests to the other case and, in most instances, it is positive synchronization that is of most interest. Cycles that are *strongly non-synchronized (SNS)* might then be regarded as the case when S_{xt} and S_{yt} are independent i.e. the joint probability function for S_{xt} and S_{yt} factorizes into the product of the marginal probability functions.

Because S_{yt} and S_{xt} are binary indicators it is easily seen that the probabilities in (3.1) and (3.2) can be expressed as expectations and doing so yields the following moment conditions that need to hold under the two null hypotheses relating to synchronization mentioned above.

$$\begin{aligned} SPPS\ (a) &: E(S_{yt}(1 - S_{xt})) = E(S_{yt}) - E(S_{xt}S_{yt}) = 0 && (3.5) \\ SPSS\ (b) &: E(S_{xt}(1 - S_{yt})) = E(S_{xt}) - E(S_{xt}S_{yt}) = 0 && (3.6) \\ SNS &: E(S_{xt}S_{yt}) - E(S_{xt})E(S_{yt}) = 0 && (3.7) \end{aligned}$$

By subtracting the two conditions in $(SPPS)$ from each other one could get equivalent moment conditions

$$\begin{aligned} SPPS(i) &: E(S_{yt}) - E(S_{xt}) = 0 && (3.8) \\ SPPS(ii) &: E(S_{xt}) - E(S_{xt}S_{yt}) = 0 && (3.9) \end{aligned}$$

These are useful since the first implies that the unconditional densities of S_{xt} and S_{yt} are identical while the second is a property of the conditional density. Indeed we can express $SPPS(ii)$ as

$$\mu_{S_x} - \sigma_{S_x}\sigma_{S_y}\rho_S + \mu_{S_x}\mu_{S_y} = 0, \qquad (3.10)$$

where $\mu_{S_x} = E(S_{xt}), \mu_{S_y} = E(S_{yt})$ and ρ_S is the correlation coefficient between S_{xt} and S_{yt}. When $SPPS(i)$ holds $E(S_{yt}) = E(S_{xt}) = \mu_S$ and $\sigma^2_{S_x} = E(S_{xt})(1 - E(S_{xt})) = \sigma^2_{S_y}$, so that (3.10) becomes

$$(1-\rho_S)\mu_S(1-\mu_S) = 0, \tag{3.11}$$

which implies that $\rho_S = 1$. Thus when testing perfect synchronization we can test if $\mu_{S_x} = \mu_{S_y}$ and $\rho_S = 1$. Although it is clear that, when $\rho_S = 1$ it has to be the case that $\mu_{S_x} = \mu_{S_y}$, our examples later show the value in performing the tests sequentially, since this is more informative about the reasons for any failure of perfect synchronization. When testing (SNS) we have $\sigma_{S_x}\sigma_{S_y}\rho_S = 0$ and so $\rho_S = 0$ is required. By concentrating upon $\hat{\rho}_S$ we are therefore able to provide a natural measure of the *degree* of synchronization.

The discussion above also leads to the following quantities which might be the basis of test statistics,

$$\begin{aligned} SPPS(i) &: \quad \hat{\mu}_{S_x} - \hat{\mu}_{S_y} \\ SPSS(ii) &: \quad \hat{\rho}_S - 1 \\ SNS &: \quad \hat{\rho}_S \end{aligned}$$

For later reference it should be noted that perfect synchronization between S_{yt} and S_{xt} only occurs when S_{yt} is identical to S_{xt}, and so one could have derived the moment conditions in (3.8) and (3.9) directly from that equality. This alternative interpretation is useful when looking at multivariate issues.

3.2. Measures Based Upon Phase States for Binary Cycles

Rather than focus directly upon turning points a different way of measuring the degree of synchronization of cycles is to ask what fraction of time the cycles are in the same phase. This *concordance index,* which is the sample analog of $Pr\left(S_{xt} = S_{yt}\right)$, was advocated in Harding and Pagan (2002) and has the form (for two series y_t and x_t and a sample size of T)

$$\hat{I} = \frac{1}{T}\{\sum_{t=1}^{T} S_{xt}S_{yt} + \sum_{t=1}^{T}(1-S_{xt})(1-S_{yt})\}. \tag{3.12}$$

There are close connections between this index and those advanced in the meteorological literature to assess forecast accuracy, see Granger and Pesaran (2000).

Artis et al (1997) and Artis et al. (1999) use a modified version of $\hat{I}$ that is transformed to lie between zero and 100.

It useful to re-write and re-parameterize this index in a different way

$$\hat{I} = 1 + \frac{2}{T}\sum_{t=1}^{T} S_{xt}S_{yt} - \hat{\mu}_{S_x} - \hat{\mu}_{S_y} \quad (3.13)$$

$$= 1 + 2\hat{\sigma}_{S_xS_y} + 2\hat{\mu}_{S_x}\hat{\mu}_{S_y} - \hat{\mu}_{S_x} - \hat{\mu}_{S_y} \quad (3.14)$$

where $\hat{\sigma}_{S_xS_y}$ is the estimated covariance between S_{xt} and S_{yt}. For the discussion that follows it will be convenient to write (3.14) as

$$\hat{I} = 1 + 2\hat{\rho}_S(\hat{\mu}_{S_s}(1-\hat{\mu}_{S_x}))^{1/2}(\hat{\mu}_{S_y}(1-\hat{\mu}_{S_y}))^{1/2} + 2\hat{\mu}_{S_x}\hat{\mu}_{S_y} - \hat{\mu}_{S_x} - \hat{\mu}_{S_y} \quad (3.15)$$

where $\hat{\rho}_S$ is the estimated correlation coefficient between S_{xt} and S_{yt}. Because of the binary nature of S_{xt} and S_{yt} the estimated standard deviations have the form $\sqrt{(\hat{\mu}_{S_x} - \hat{\mu}_{S_x}^2)}$. Now the concordance index has a maximum value of unity when $S_{xt} = S_{yt}$ and zero when $S_{xt} = (1 - S_{yt})$. Consequently, it is easily shown that, when either of these holds, $\hat{\sigma}_{S_x}\hat{\sigma}_{S_y} = \hat{\sigma}_{S_x}^2$, and so the value of $\hat{\rho}_S = 1$ corresponds to a concordance index of one and $\hat{\rho}_S = -1$ to a concordance index of zero. Since the concordance index is also monotonic in ρ_S, it is natural to shift attention away from the former to the latter i.e. to focus upon the correlation between the two states S_{xt} and S_{yt}. Consequently, the tests based on $\hat{\rho}_S$ laid out in the previous section will be those employed in the paper, although it can sometimes be useful to reinterpret the value of $\hat{\rho}_S$ as a value for $\hat{I}$. [6]

[6] A problem with looking at the value of $\hat{I}$ can be seen when when $\rho_S = 0$. Then $E(\hat{I}) = 1 + 2\mu_{S_x}\mu_{S_y} - \mu_{S_x} - \mu_{S_y}$ so that $E(\hat{I}) = .5$ only if $\mu_{S_x} = .5, \mu_{S_y} = .5$. Since μ_{S_x} is the probability of x_t being in an expansion, for the business cycle it is likely that it will be closer to .9 than .5. In that case $E(\hat{I}) \simeq .82$ and so one could easily think that the cycles are synchronized even though there is no relation between them. Of course a policy maker may not be too concerned with that fact, as they may only be interested in the fraction of time that (say) two economies are in the same phase and not the reason for it. But the example points to how what might appear to be a high degree of association between cycles can be quite misleading, as it is simply an artifact of expansions lasting for long periods of time relative to the sample. If one is to use $\hat{I}$ as a test statistic it is necessary to mean correct it, and that is essentially what happens when one uses $\hat{\rho}_S$.

3.3. Multivariate Synchronization

Turning to the general case where there are n series $x_{1t}, ..., x_{nt}$ which will have associated specific cycles $S_{jt}, j = 1, ..., n$, we will refer to the hypothesis where all pairs (S_{jt}, S_{kt}) $j \neq k$ are strongly non- synchronized as strong multivariate non-synchronization (SMNS) We can test for whether there is SMNS by asking if the correlation matrix of the S_{jt} is diagonal i.e. all the pairwise correlations ρ_S^{ij} are tested for whether they are zero. For perfect synchronization we observe that the cycles to which this pertains must have $\mu_{S_i} = \mu_{S_j}$ $\forall i.j = 1...n$ and that all pairwise correlations ρ_S^{ij} are unity. In many instances there will be an obvious choice of numeraire e.g. the US would often be that for business cycle analysis. In such an instance, let it be the first series, in which case we would then test $H_0 : \mu_j - \mu_1 = 0, j = 2, ..., n$. Notice that the numeraire does not matter for this test as test statistics will be invariant to it, since the vector of mean differences with (say) the second series as a numeraire is a non-singular transformation of that with the first. The situation is less clear for the test that all ρ_S^{ij} are unity. However, if the null hypothesis $H_0 : \rho^{1j} = 1 \forall j$ is accepted (rejected) it implies that S_{it} and S_{jt} are identical (non-identical) so that $\rho^{ij} = 1 (\neq 1)$ must hold for all (some) i.

3.4. Co-Movement of Cycles

Loosely speaking, if variables have cycles which are synchronized we would like to say that they possess a common cycle. To be more precise about this concept we need to examine the determinants of ρ_S in the two series case. From the definition of ρ_S,

$$\begin{aligned}
\rho_S &= \frac{E(S_{xt}S_{yt}) - [E(S_{xt})E(S_{yt})]}{\sqrt{E(S_{xt})(1 - E(S_{xt}))}\sqrt{E(S_{yt})(1 - E(S_{yt}))}} \\
&= \frac{\Pr(S_{xt} = 1, S_{yt} = 1) - [\Pr(S_{xt} = 1)\Pr(S_{yt} = 1)}{\sqrt{\Pr(S_{xt} = 1)\Pr(S_{xt} = 0)}\sqrt{\Pr(S_{yt} = 1)\Pr(S_{yt} = 0)}} \qquad (3.16)
\end{aligned}$$

and we see that the degree of synchronization of cycles depends upon two items: the characteristics of the specific cycles which are determined by $\Pr(S_{xt} = 1)$ and $\Pr(S_{yt} = 1)$ and the probability of the event $\{S_{xt} = 1, S_{yt} = 1\}$. The latter event is more likely to occur when the turning points in both cycles are located at the same point in time i.e. turning points *cluster* around a given date. From the expression for ρ_S it is clear that, given individual cycle characteristics summarized by $\Pr(S_{xt} = 1)$ and $\Pr(S_{yt} = 1)$, the higher is ρ_S the greater will be the probability that turning points will occur together, and so the greater will be the chance of observing synchronized cycles.

Now the $\Pr(S_{xt} = 1)$ and $\Pr(S_{yt} = 1)$ are characteristics of the marginal densities of S_{xt} and S_{yt} respectively and these derive from the marginal densities of x_t and y_t. The joint density of x_t and y_t will be involved in determining $\Pr(S_{xt} = 1, S_{yt} = 1)$. If we keep $\Pr(S_{xt} = 1)$ and $\Pr(S_{yt} = 1)$ unchanged then, as $\Pr(S_{xt} = 1, S_{yt} = 1)$ changes, so will ρ_S. Consider then the case when Δx_t and Δy_t are jointly normal with expected values $\mu_x = E(\Delta x_t)$, $\mu_y = E(\Delta y_t)$,variances σ_x^2 and σ_y^2 and correlation ρ. If the marginal density parameters are held constant it is clear that $\Pr(S_{xt} = 1, S_{yt} = 1)$ varies directly with ρ and so ρ_S and ρ are related.

This connection is useful since it shows how our concept of synchronization relates to that used in most of the literature on cyclical "co-movement", as it is the latter which has been the focus of attention of RBC researchers. That group studies the correlation among series from which the permanent component has been removed through some form of filtering, and so it is effectively studying the growth cycle. Examples of this methodology applied to a single economy include Cooley and Prescott (1995) and Cooley and Hansen (1995) and involves the correlation between variables such as GDP, consumption, investment, employment, unemployment, hours worked and prices from which permanent components have been removed. There are also several papers that study correlation between the z_t from different countries, including Backus et al (1992), Canova and Dellas (1992), Canova (1993), Engle and Kozicki (1993) and Artis and Zhang (1997). The autocorrelations and cross correlations between the z_t of different countries can be used to reconstruct the equivalent quantities for Δz_t, and it is the latter which will be important to the nature and existence of growth cycles. However it is the correlation of Δx_t and Δy_t that is the appropriate quantity to study synchronization of classical cycles. The latter cycle depends upon all the second order moments of Δx_t and Δy_t, although in a very complex way, since ρ_S also depends on what determines the marginal probabilities like $\Pr(S_{xt} = 1)$ as well as the joint probability.[7] Consequently the moments of the series Δx_t and Δy_t, as well as their covariance, will determine ρ_S. Studying any individual moment, such as the covariance, will not be very informative about synchronization.

[7] Of course it is really the joint density of Δy_t and Δx_t which determines the cycle characterictics rather than the second moments *per se*.

4. Defining and extracting a Common Cycle

4.1. Through Parametric Models

Now what is a common cycle? Firstly, we might define a common cycle as occurring when there is perfect positive synchronization i.e. when $\rho_S = 1$. This turns out to be the definition used by Vahid and Engle (1993), at least when the cycles being examined are growth cycles. To see this, note that they propose a test statistic for a common cycle by writing

$$y_t = a_y T_{BN,y,t} + z_{yt} \tag{4.1}$$

$$x_t = a_x T_{BN,xt} + z_{xt}, \tag{4.2}$$

where $T_{BN,y,t}$ is the Beveridge-Nelson "trend" decomposition, and then test for a linear relation between z_{xt} and z_{yt} (dropping the BN identifiers). A "common cycle" is said to exist if $z_{yt} = dz_{xt}$. Since the emphasis is upon z_t it is clear that it is a common growth cycle that is being tested for. As shown in those papers, when a "common cycle" exists among the z_t cycles, one can write each z_t as a multiple of a factor f_t (see Vahid and Engle (1993,p344)). Consequently, provided the factors of proportionality have the same sign, the turning points in each of the series z_{yt} and z_{xt} are identical, simply being those of f_t. Because the growth cycles in z_{yt} and z_{xt} are identical there is therefore perfect synchronization between them.

More precisely, the test supplied by Vahid and Engle is a test of whether what might be termed "Beveridge-Nelson growth cycles" are perfectly synchronized. There are no implications from this test for whether there is synchronization of the classical cycles in y_t and x_t. To see this, we note that, if the test statistic indicates that the null hypothesis of a BN common cycle is accepted between two series y_t and x_t, then $z_{yt} = g_y f_t$ and $z_{xt} = g_x f_t$ and, after differencing (4.1) and (4.2), one can write

$$\Delta y_t = a_y \varepsilon_{yt} + g_y \Delta f_t \tag{4.3}$$

$$\Delta x_t = a_x \varepsilon_{xt} + g_x \Delta f_t \tag{4.4}$$

where ε_{yt} and ε_{xt} are the innovations into the permanent component of each series (the assumption in the BN framework being that all series are $I(1)$). Thus the series will generally have different classical cycles since the nature of the autocorrelation in Δy_t and Δx_t will depend upon the relative variances of the two elements in each equation, and there is no reason for these to be the same, unless $a_y = a_x$, $g_y = g_x$, and $var(\varepsilon_y) = var(\varepsilon_x)$. Even if the series are co-integrated, as in Vahid and Engle's analyses, so that $\varepsilon_{xt} = \varepsilon_{yt}$ are the innovations into a common trend, there will likely be a disparity in the specific cycle lengths.[8] It should be noted that, in the co-integration case, there will be error correction terms entering into Δy_t and Δx_t, and these may assist in producing common factors in the levels of the series.

Now the Engle-Vahid common cycle model implies that the series Δy_t and Δx_t are driven by common factors and, in fact, since the underlying framework for their analysis is that y_t and x_t follow a VAR, the factor must also have such a structure. More generally one might have non-linear models for the series and, hence, the factors. Thus one might have Δy_t (or Δf_t) following a Markov Switching process as in Hamilton (1989) or Chauvet (1998), or even some other non-linear structure.[9] But all of these are just different models for Δy_t and Δx_t and they do not determine the existence or non-existence of a common cycle, which is our focus. However, they will certainly be important for the nature of the cycle, as we will now illustrate.

Consider defining a common cycle from the viewpoint of a *lack* of synchronization. The simplest instance of this would be when Δx_t and Δy_t are independent, so that S_{xt} and S_{yt} are independent, and $\rho_S = 0$. From this perspective a common cycle would exist whenever $\rho_S \neq 0$. Now to make this more precise let us follow the common factor literature and assume that there is a factor driving Δy_t and Δx_t. Then

$$\Delta y_t = a_y f_t + \varepsilon_{yt} \tag{4.5}$$
$$\Delta x_t = a_x f_t + \varepsilon_{xt} \tag{4.6}$$

[8] There is also an issue of the drift in each series i.e. μ_y and μ_x. Unless these are the same the classical cycles will have to be of different length i.e. the series need to co-trend as well as co-integrate.

[9] A referee argued that the difference between our approach and Hamilton's was that the latter made Δy_t depend on S_t while we had S_t being dependent on Δy_t. This claim is misleading for two reasons. First, the S_t we construct are not the latent states ξ_t that the Markov switching model contains. Indeed our S_t are the equivalent of the cycle dates ζ_t that Hamilton produces by using the dating rule $\zeta_t = 1(\Pr(\xi_t = 1|\Delta y_t, \Delta y_{\pm t-1}, ...) - .5)$. Secondly, there is nothing in our approach which says that Δy_t cannot be a function of $S_{t-j}(j > 1)$. If this was true one would just have a model for Δy_t that featured non-linear dependence.

where ε_{yt}, ε_{xt} are independently distributed (of one another) and the correlation between Δy_t and Δx_t is ρ. This correlation clearly depends upon the effect of the factor f_t upon the Δy_t and Δx_t, i.e. the magnitude of the loadings. Provided these are non-zero it natural to say that a common cycle exists within this model and that it can be extracted from f_t. The degree of synchronization will of course depend upon ρ_S, which will be a function of many things - the relative magnitudes of the factor loadings, the relative variances of f_t to ε_{yt} and ε_{xt}, the type of serial correlation in the idiosyncratic shocks, whether there is drift etc. Unless one has a completely specified parametric model it is hard to be precise about the degree of synchronization in the series and how important the common cycle is.

4.2. A Non-parametric Approach

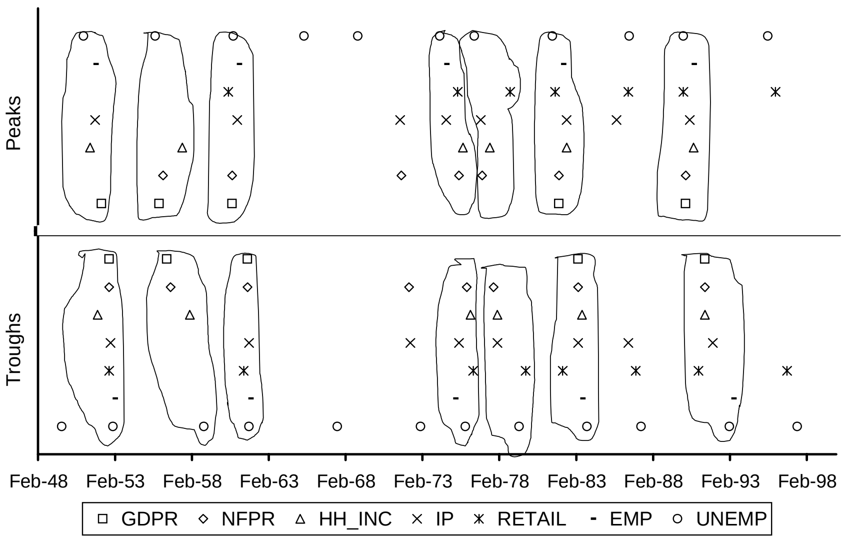
A final approach is to focus directly upon the turning points in the specific series when considering the construction of a common cycle. This leads to a non-parametric method for common cycle extraction or, as it would be known in the NBER typology, the *reference cycle*. Burns and Mitchell (1946, p13) provide a starting point for a definition of what constitutes a set of synchronized cycles with the observation that:

> At an early stage of the investigation we thought it prudent to compare the specific cycles in numerous series. Rough tabulations of specific cycle turns suggested that they clustered around certain months, which usually came in years when business annals reported a recession or revival.

Figure 4.1 is a stylized version of the worksheet used by Ernst Boehm to date the Australian Business cycle: it shows the location of turning points in seven Australian series.[10] Clusters of turning points are marked on the chart, making the phenomenon very apparent. The median date of each cluster determined the Australian reference cycle chronology. The visual evidence for the seven Australian series being synchronized is that almost all of the turning points fall within seven clusters. Later, we will formalize this eye-ball test, thereby providing a way of measuring synchronized cycles.

[10] The series are real GDP (GDPR), real non-farm product (NFPR), real household income (HH_INC), industrial production (IP), employment (EMP) and minus the unemployment rate (UNEMP).

Figure 4.1: Clustering of turning points in the Australian classical business cycle



The notion of clusters of turning points is visually appealing but requires careful definition in order to precisely quantify the phenomenon that the eye identifies. Burns and Mitchell had, and their followers at the NBER business cycle dating committee have, a long history of interpreting such visual information. Harding (2003) shows that the implicit rules used to construct the NBER business cycle chronology have changed over time and in particular the implicit dating rule used to construct the pre-WWII chronology differs from that used to construct the post WWII chronology. He also shows that starting from about 1959.1 the NBER seems to have used approximately the same rule to locate turning points. We have therefore sought to extract and codify the rules implicit in the NBER procedures used to construct the post 1959.1 chronology.

The NBER has increased the amount of information that it provides about its dating procedures (see http://www.nber.org/cycles/main.html) but it remains the case that there is insufficient information provided by the NBER to allow their dating procedures to be replicated from the information they provide. However, the late Geoffrey Moore (1983), a colleague of Burns and Mitchell and long time member of the NBER Dating Committee, and Boehm and Moore (1984), who use these procedures to obtain an NBER-like reference cycle for Australia, provide a description of the NBER procedures that is sufficient to enable us to write down an algorithm. Their procedures can be summarized in the following steps:

- Find a set of series that are believed to be roughly coincident.
- Adjust those series so that they are all pro-cyclical i.e. positively synchronized.
- Identify the turning points in each of those series via peak and trough dating.
- Visually identify clusters of turning points by seeking to minimize the distance between the turning points in each cluster.
- Construct a coincident index as the weighted sum of the set of coincident series and then find the turning points of the coincident index.
- Obtain the candidate reference cycle as the consensus of the turning points in each cluster.

These steps contain the essence of what Boehm and Moore, and by extension the NBER dating committee, consider to be the defining features of synchronization and the associated common cycle. Inspection of the Boehm and Moore article suggests that the last two steps are of minor significance. The second step is a normalization that is avoided by assuming, for the moment, that all series are pro-cyclical. The third step can be achieved via some dating algorithm, such as Harding and Pagan's (2002) quarterly adaptation of the Bry and Boschan (1971) procedures. This suggests that the main unresolved issue is to codify step four above. This asserts that the defining feature of synchronization is a formal description of the minimum distance between the nearest turning points of the same type in a vector of specific cycles. We will later describe an algorithm that can be used to implement these steps and will use an updated version of the Boehm and Moore (1984) Australian reference cycle data to calibrate the algorithm. This means that, when we apply the algorithm to US data later in the paper, and compare the chronology with that of the NBER, we are effectively performing an out-of-sample test - something that is not true of other procedures that are calibrated directly on the NBER data.

5. Testing Synchronization

5.1. Test Statistics

5.1.1. Bivariate Tests

In the case of bivariate cycles we have proposed that $SPPS(i)$ be tested by considering whether

$$E(S_{yt} - S_{xt}) = 0.$$

This involves testing if two sample means are equal and is easily done. GMM methods can be employed to produce a robust standard error.

For testing non-synchronization we recommended the correlation between S_{xt} and S_{yt}, ρ_S. To estimate ρ_S we have the moment condition

$$E[\sigma_{S_x}^{-1}(S_{xt} - \mu_{Sx})\sigma_{S_y}^{-1}(S_{yt} - \mu_{Sy}) - \rho_S] = 0 \tag{5.1}$$

and the estimator generating equation is just

$$\frac{1}{T}\sum_{t=1}^{T} \hat{\sigma}_{S_x}^{-1}(S_{xt} - \hat{\mu}_{Sx})\hat{\sigma}_{S_y}^{-1}(S_{yt} - \hat{\mu}_{Sy}) - \hat{\rho}_S = 0. \tag{5.2}$$

Since we need to find estimates of the means and variances of S_{xt} and S_{yt} in order to compute $\hat{\rho}_S$, the estimated correlation coefficient is a sequential method of moments estimator, to use Newey's (1984) term. The moment condition can be written as

$$E[m_t(\theta, S_{xt}, S_{yt}) - \rho_S] = 0, \tag{5.3}$$

where $\theta' = [\mu_{Sx}, \sigma_{S_x}, \mu_{Sy}, \sigma_{S_y}]$. Now, because $E\{\frac{\partial m_t}{\partial \theta}\} = 0$ under the null hypothesis that $\rho_S = 0$, the fact that θ has been estimated from the data does not impact upon the asymptotic distribution of $T^{1/2}(\hat{\rho}_S - \rho_S)$.

Testing for the second criterion used in perfect synchronization ($SPPS(ii)$) is a little more complex. When testing (SNS) it would be expected that $T^{1/2}\hat{\rho}_S$ would be asymptotically $N(0, v)$, and so $T^{1/2}\hat{v}^{-1/2}\hat{\rho}_S$ would be $N(0, 1)$ asymptotically. One cannot be entirely precise about the stationarity properties of the states S_{xt}, S_{yt} , since they depend upon the dating rule employed, but, for standard ones, like the NBER rule, these states follow stationary Markov Chains. It is conceivable that there do exist some dating rules for which this might not be true. However, the proposed $SPSS(ii)$ test involves testing on the boundary of the parameter space since $|\rho_S| \leq 1$. There is a literature on the distribution of $T^{1/2}\hat{v}^{-1/2}(\hat{\rho}_S - 1)$ in that case. As Chant (1974) and Andrews (2001) point out it is asymptotically a half normal. Since the series S_{xt} and S_{yt} are serially correlated the value of v will not be unity and will need to be estimated by using a robust covariance estimator. In this scalar case it is simply a matter of doing a one tail rather than two tail test. One could also generate p values numerically from the empirical density of $T^{1/2}\hat{v}^{-1/2}(1(\tilde{\rho}_S < 0)\tilde{\rho}_S - 1)$, where $\tilde{\rho}_S$ are drawn from an $N(1, T^{-1}\hat{v})$ density.

Although method of moments is an obvious way to perform estimation and inference about ρ_S it is often useful to recognize that $\hat{\rho}_S$ can be found from the regression

$$\hat{\sigma}_{S_y}^{-1} S_{yt} = a_1 + \rho_S \hat{\sigma}_{S_y}^{-1} S_{xt} + u_t, \tag{5.4}$$

since this makes clear difficulties that can arise with some procedures advocated in the existing literature. In particular, the critical role played by their implicit assumption that u_t is *i.i.d.* Thus both the market timing test of Pesaran and Timmermann (1992) and its close relative, Pearson's test of independence in a contingency table (see Artis et al (1997)), effectively make this assumption. Artis et.al. (1997) and Artis et.al. (1999), who work with transformations of the concordance index, derive statistics for independence of cycles that effectively assume the state S_{yt} to be *i.i.d.*

As one can see from the regression, when the null $\rho_S = 0$ holds the error term inherits the serial correlation properties of S_{yt}. We have seen that S_{yt} is strongly positively serially correlated and, as is well known, positive serial correlation sharply increases the chance of rejecting the null that $\rho_S = 0$, unless inferences are made robust to the serial correlation as well as to any heteroskedasticity in the errors, as can be easily done within the method of moments framework. Thus in applications below we report the t ratios for testing if $\rho_S = 0$ using the method of moments estimator and with inferences that do and do not make an allowance for serial correlation and heteroskedasticity. Notice that an advantage of the method of moments approach over the regression model is that we are making no assumptions about which of S_{yt} and S_{xt} are "exogenous".

The regression interpretation is also useful for looking at questions about whether the degree of synchronization has changed over time. It is possible to compute ρ_S recursively and to study its evolution over time. For formal testing of parameter stability one can utilize the methods in Sowell (1996).

5.1.2. Tests of multivariate synchronization

To test for multivariate non-synchronization ($SMNS$) we can also use the GMM estimator based on the following $n(n+1)/2$ moment conditions

$$ES_{jt} = \mu_{S_j} \qquad\qquad j = 1, .., n$$

$$E\left[\frac{(S_{jt} - \mu_{S_j})(S_{it} - \mu_{S_i})}{\sqrt{\mu_{S_j}\left(1 - \mu_{S_j}\right)\mu_{S_i}\left(1 - \mu_{S_i}\right)}} - \rho_S^{ji}\right] = 0 \qquad j = 1, .., n, \quad i > j$$

Let $\theta' = \left[\mu_{S_1}, .., \mu_{S_n}, \rho_S^{12}, .., \rho_S^{n(n-1)}\right]$ be a vector of parameters and S_t be the $1 \times n$ matrix with typical element S_{jt}. Then we can write the stacked moment conditions as $h_t(\theta, S_t)$ as follows

$$h_t(\theta, S_t) = \begin{bmatrix} S_{1t} - \mu_{S_1} \\ \vdots \\ S_{nt} - \mu_{S_n} \\ \frac{(S_{1t} - \mu_{S_1})(S_{2t} - \mu_{S_2})}{\sqrt{\mu_{S_1}\left(1 - \mu_{S_1}\right)\mu_{S_2}\left(1 - \mu_{S_2}\right)}} - \rho_S^{12} \\ \vdots \\ \frac{(S_{(n-1)t} - \mu_{S_{n-1}})(S_{nt} - \mu_{S_n})}{\sqrt{\mu_{S_{(n-1)}}\left(1 - \mu_{S_{(n-1)}}\right)\mu_{S_n}\left(1 - \mu_{S_n}\right)}} - \rho_S^{(n-1)n} \end{bmatrix} \quad \textit{and} \quad g\left(\theta, \{S\}_{t=1}^{T}\right) = \frac{1}{T}\sum_{t=1}^{T} h_t(\theta, S_t)$$

Let $\widehat{\theta}' = \left[\widehat{\mu}_{S_1}, .., \widehat{\mu}_{S_n}, \widehat{\rho}_S^{12}, .., \widehat{\rho}_S^{n(n-1)}\right]$ be the vector of sample means and sample pair wise correlations for the S_{jt}. Then

$$\widehat{V} = \widehat{\Gamma}_0 + \sum_{k=1}^{m}\left[1 - \frac{k}{m+1}\right]\left[\widehat{\Gamma}_k + \widehat{\Gamma}_k'\right],$$

Where,

$$\widehat{\Gamma}_k = \frac{1}{T}\sum_{t=k+1}^{T} h_t(\widehat{\theta}, S_t) h_{t-v}(\widehat{\theta}, S_{t-v})',$$

is a consistent estimate of the covariance matrix for $\sqrt{T}g\left(\theta,\{S\}_{t=1}^{T}\right)$. Letting $\theta_0' = \left[\mu_{S_1},..,\mu_{S_n},0,..,0\right]$ be the restricted parameter vector for the SMNS case (ie $\rho_S^{ji}=0$), under the SMNS null the statistic,

$$W_{SMNS} = \sqrt{T}g\left(\theta_0,\{S\}_{t=1}^{T}\right)'\widehat{V}^{-1}\sqrt{T}g\left(\theta_0,\{S\}_{t=1}^{T}\right)$$

has an asymptotic $\chi^2_{n(n-1)/2}$ distribution. In applying this test we need to choose a value for m the window width. Unlike the regression case we don't have an automatic procedure for doing this and have chosen to set m equal to the integer part of $(T-n(n-1)/2)^{\frac{1}{3}}$.

Testing the necessary condition for perfect synchronization among a number of series can be done by testing if $\mu_{S_1}=\mu_{S_2}=...=\mu_{S_K}$. As we observed early we can convert this into an $(n-1)\times 1$ vector of differences by relating all the μ_{S_j} to μ_{S_1} and the choice of the series to normalize upon is irrelevant. The GMM approach just described then provides a standard way of effecting such a joint test. It also motivates a test of the second criterion $SPPS(ii)$. Again we have a boundary value problem and now the distribution of the joint test for a number of correlation coefficients being unity is complex. The standard test statistic of $H_0:\rho_S=\rho_{S0}$ would be to form

$$T(\hat{\rho}_S-\rho_{S0})'V^{-1}(\hat{\rho}_S-\rho_{S0}),$$

where ρ_S would be the $\frac{n\times(n-1)}{2}$ vector of correlations and V would be the asymptotic variance of $T^{1/2}(\hat{\rho}_S-\rho_{S0})$. Now ρ_{S0} is a vector of ones and it is known that the true density in these circumstances would be a weighted average of χ^2 densities, see Gourieroux et al (1982), but getting the weights is complex, and it seems simplest to generate it by simulation methods viz. by drawing realizations of $\hat{\rho}^{1j}$ from an $N(i_n,V)$ density, where i_n is an $n\times 1$ vector of ones, and then forming the standard test statistic, but discarding draws of $\hat{\rho}_{ij}^{S}$ that exceed unity when computing the empirical p value. This is the analogue of what would be done in the scalar case.

5.2. Some Applications

Our first two investigations of synchronization of cycles are with industrial production and stock prices. In this investigation our focus is on the extent to which serial correlation and heteroscedasticity distort inferences about synchronization.

We then turn to the data used by Boehm and Moore (1984) to construct the Australian reference cycle and by the NBER to construct the reference cycle for the United States. Investigation of synchronization in these data sets serves two purposes. First, it illustrates how the methods developed in this paper can be used by practitioners seeking to construct NBER-like reference cycles. Second, it is a precursor to later sections where we use this data to calibrate and test the non-parametric procedure that is employed in extracting a common cycle.

5.2.1. Industrial Production

Our first investigation of synchronization of cycles is with the data on industrial production for the twelve countries in Artis et al (1997, Table 2). We first focus on the statistics $\left\{\widehat{I}, \widehat{\rho}_S, \widehat{\mu}_S\right\}$ that are presented in Table 5.1, where the countries are ranked according to the magnitude of $\widehat{\mu}_S$; the concordance statistic $\widehat{I}$ is above the diagonal while $\widehat{\rho}_S$ is below the diagonal and $\widehat{\mu}_S$ is in the bottom two rows of the table. Reported values of $\widehat{I}$ are large suggesting that industrial production in these 12 countries spends much of the time in the same state of the classical cycle. However, the pair wise correlations $\widehat{\rho}_S$ are typically small which, together with (3.15), suggests that it is the high values for $\widehat{\mu}_S$ rather than a strong correlation between industrial production in different countries that lies behind the high degree of concordance. This effect is most evident for the UK, which shows concordance with other countries in the range of 0.58 to 0.71; yet it only shows correlations in the range of -0.04 to 0.31.

There is extensive serial correlation in the states. For example, the first order serial correlation coefficient in $S_{GER,t}$ is .95, so that there will need to be a serial correlation correction performed to get the correct t ratio for $\hat{\rho}_S$. Consequently, we use a heteroskedastic and autocorrelation consistent (HACC) standard error with Bartlett weights to account for the serial correlation. We set the number of lags to be the integer part of $\widehat{\gamma}T^{\frac{1}{3}}$, where $\widehat{\gamma}$ is estimated using the procedures in Newey and West (1994).[11] Other estimators might be used to improve the power of the test e.g. the method outlined in Kiefer and Vogelsang (2002) and Phillips et. al (2003). Results are in Table 5.2, where the uncorrected t-statistics are above the diagonal, while those based on HACC standard errors are below the diagonal. The robust t-ratio shows that the evidence for the null hypothesis of

[11] Estimated values of γ for each pair of countries are available from the authors on request.

Table 5.1: Concordance indexes and correlations of cycles in industrial production for selected countries

	UK	CAN	LUX	ITA	NET	GER	BEL	US	JAP	FRA	SPA	IRE
UK	$\ddots$	0.61	0.62	0.58	0.62	0.66	0.68	0.65	0.58	0.63	0.67	0.71
CAN	0.11	$\ddots$	0.56	0.56	0.68	0.62	0.64	0.83	0.70	0.66	0.66	0.72
LUX	0.12	-0.04	$\ddots$	0.70	0.64	0.76	0.74	0.62	0.72	0.75	0.74	0.74
ITA	0.02	-0.05	0.27	$\ddots$	0.84	0.83	0.82	0.67	0.79	0.84	0.77	0.73
NET	0.12	0.23	0.11	0.59	$\ddots$	0.83	0.85	0.77	0.80	0.81	0.84	0.75
GER	0.20	0.08	0.40	0.57	0.57	$\ddots$	0.81	0.74	0.84	0.83	0.80	0.80
BEL	0.23	0.07	0.30	0.53	0.61	0.47	$\ddots$	0.75	0.81	0.88	0.91	0.85
US	0.14	0.60	-0.04	0.09	0.36	0.26	0.20	$\ddots$	0.76	0.75	0.79	0.83
JAP	-0.04	0.22	0.23	0.42	0.46	0.55	0.39	0.20	$\ddots$	0.84	0.86	0.81
FRA	0.05	0.11	0.32	0.59	0.49	0.50	0.61	0.13	0.43	$\ddots$	0.86	0.88
SPA	0.16	0.09	0.27	0.39	0.56	0.40	0.69	0.22	0.46	0.42	$\ddots$	0.84
IRE	0.31	0.27	0.25	0.19	0.24	0.41	0.44	0.29	0.20	0.46	0.12	$\ddots$
$\widehat{\mu}_S$	0.66	0.68	0.71	0.72	0.72	0.74	0.80	0.82	0.82	0.84	0.87	0.92

no association is quite strong; something that was not true of the test performed with the uncorrected t ratios.

Table 5.2: Standard and robust t-statistics for the null hypothesis of no correlation of classical cycle states in industrial production for selected countries

	UK	CAN	LUX	ITA	NET	GER	BEL	US	JAP	FRA	SPA	IRE
UK	$\ddots$	4.0	4.3	0.9	4.6	8.1	10.6	6.6	-2.0	2.7	9.2	24.1
CAN	0.6	$\ddots$	-1.6	-2.0	8.8	3.3	3.1	34.9	10.6	5.3	5.0	20.4
LUX	0.5	-0.2	$\ddots$	10.6	4.3	16.9	14.0	-1.9	11.1	16.9	15.3	19.1
ITA	0.1	-0.3	1.2	$\ddots$	27.9	26.9	27.7	4.3	21.9	36.0	23.0	14.2
NET	0.53	1.0	0.5	3.5	$\ddots$	27.1	34.5	18.1	24.6	28.0	37.5	18.1
GER	0.9	0.4	1.6	2.9	3.1	$\ddots$	23.8	12.3	31.1	28.9	24.2	33.1
BEL	1.2	0.4	1.5	3.5	6.3	2.0	$\ddots$	9.6	20.1	38.6	51.7	36.2
US	0.6	6.9	-0.2	0.4	1.7	1.1	1.2	$\ddots$	9.7	6.6	12.4	22.4
JAP	-0.2	0.9	1.0	1.9	2.2	3.3	1.7	0.6	$\ddots$	23.5	28.6	14.8
FRA	0.2	0.4	1.8	4.4	3.4	2.8	4.4	0.7	1.6	$\ddots$	25.7	38.2
SPA	0.7	0.4	1.0	1.3	5.8	2.2	10.4	0.8	1.8	1.6	$\ddots$	8.9
IRE	2.9	2.1	1.4	0.8	1.5	3.9	4.7	1.8	0.9	3.3	0.5	$\ddots$

It is worth testing for the necessary condition for perfect synchronization. To do this we define the vector of moment conditions $\widetilde{h}_t(S_t)$ implied by that condition as

$$\widetilde{h}_t(S_t) = \begin{bmatrix} -i_{n-1} & I_{n-1} \end{bmatrix}' \begin{bmatrix} S_{UK,t} \\ \vdots \\ S_{IRE,t} \end{bmatrix} \quad and \quad \widetilde{g}\left(\{S\}_{t=1}^{T}\right) = \frac{1}{T}\sum_{t=1}^{T}\widetilde{h}_t(S_t)$$

wherei_{n-1} is an $(n-1\times 1)$ vector of ones and I_{n-1} is an $(n-1\times n-1)$ identity matrix. Under the null of $SPPS(i)$ the statistic

$$W_{PS} = T\widetilde{g}\left(\{S\}_{t=1}^{T}\right)' V_{T,m}^{-1}\widetilde{g}\left(\{S\}_{t=1}^{T}\right),$$

where $V_{T,m}$ is a HACC estimate of the covariance matrix estimated with Bartlett weights and lag length m, is asymptotically distributed as $\chi^2(n-1)$. Using the information on specific cycles in industrial production, $W_{PS} = 34.4$ with p-value 0.0003, leading to a rejection of perfect synchronization.

5.2.2. Stock prices

Another example of cycles that are possibly synchronized relates to international stock market movements. We examine data on monthly stock price indices for three countries - Australia, the United Kingdom and the U.S. The data sets were used in Pagan (1998) and the rules to determine the phases of the cycles are described there (with a short description for the US data being available in Pagan and Sossonouv (2002)). Two sample periods are provided; from 1875/1- 1997/5 and the "post-WW2" period of 1945/1-1997/5. A striking feature of these data, seen in Table 5.3, is that, while the means of the stock states ($\widehat{\mu}_S$) were quite different in the pre-WWII era, they became close in the post-WWII era, and we cannot reject the null hypothesis that they satisfy the necessary condition for perfect positive synchronization in that era. We can however reject the second of the $SPPS$ conditions since the robust t ratio for testing if $\rho_S^{Aus/UK}$ was unity would be 4.2 which, when referred to the half normal density, would provide a strong rejection. Nevertheless, even though not perfectly synchronized, there is strong evidence that the cycles are highly correlated, although the robust t ratios do dampen the strength of this evidence.

Table 5.3: Evidence on the necessary conditions for perfect synchronization across three stock markets

	$\widehat{\mu}_S$				
	Australia	United Kingdom	United States	W_{PS}	p-value
1875/1-1997/5	0.68	0.56	0.61	9.5	0.009
1945/1-1997/5	0.67	0.64	0.64	0.9	0.65

5.2.3. Components of the Australian reference cycle

Perhaps the best known example of synchronization relates to the NBER reference cycle. Previously investigation of that phenomena has largely been considered to be outside of the scope of econometrics. As an example of how our methods can be used to redress this situation we apply the techniques developed in this paper

Table 5.4: Concordance Indices and Correlations of Cycles in Equity Prices

	UK/US	Aust/US	Aust/UK
1875/1-1997/5			
$\hat{I}$	0.66	0.61	.70
$\hat{\rho}_S$	0.29	0.16	.39
t	18.8	10.2	24.5
robust t	3.9	2.0	4.9
$\widehat{\gamma}$	1.6	0.4	1.7
1945/1-1997/5			
$\hat{I}$	0.67	0.69	0.79
$\hat{\rho}_S$	0.28	0.33	0.54
t	12.3	14.3	27.0
robust t	2.4	2.7	5.0
$\widehat{\gamma}$	1.1	0.3	2.0

to investigate the synchronization between the component series that Boehm and Moore (1984) used to define the Australian reference cycle. The dates of specific cycle peaks in the component series and the Boehm and Moore reference cycle peaks are in Table 5.5, while the information on troughs is in Table 5.6.

We first comment on the statistics $\left\{\widehat{\rho}_S, \widehat{t}_{HACC}, \widehat{\mu}_S\right\}$ that are presented in Table 5.7. Here $\widehat{\rho}_S$ is above the diagonal, $\widehat{t}_{HACC}$ is the heteroscedasticity and autocorrelation robust t-statistic for the SNS hypothesis reported below the diagonal and $\widehat{\mu}_S$ is in the bottom row of the table. Reported values of the pair-wise correlations $\widehat{\rho}_S$ are often small, which might be of some concern to those who use this data to construct the Australian reference cycle. The generally high values of $\widehat{\mu}_S$, together with (3.15), suggests that it is the high values for $\widehat{\mu}_S$, rather than a strong correlation between the components of the Australian reference cycle, that creates strong concordance between the specific cycles for those variables. Inspection of the final column of Table 5.7 points to the fact that inclusion of the unemployment rate in the set of variables used to construct the reference cycle may not be justified, since its specific cycle is weakly correlated with the other components. The test statistic for the null hypothesis of the necessary condition for perfect synchronization takes the value 37.8 (p-value $1.3e^{-6}$), and so we can emphatically reject it for the Australian data. Inspection of the last row in Table 5.7 shows that the unemployment rate is the culprit as it spends too little time in expansions ($\widehat{\mu}_S = 0.48$) to be perfectly synchronized with the other series. This suggests

Table 5.5: Peaks in the components of the Australian reference cycle

Real GDP (GDPR)	Real non farm product (NFPR)	Real household income (HH_INC)	Real industrial production (IP)	Real retail sales (Retail)	Employment (Emp)	Unemployment rate (Unemp)	Boehm and Moore Reference Cycle (Ref)
52.02	57.02	51.05	51.09	60.05	50.08	50.12	51.04
55.11	60.08	57.05	60.12	75.04	60.02	55.08	56.12
60.08	71.08	75.08	71.07	78.09	74.06	60.09	60.09
81.01	75.05	77.05	74.07	81.08	82.01	65.04	74.07
90.02	76.11	82.05	76.01	86.05	90.07	68.10	76.08
	81.01	90.08	82.05	89.12		74.02	81.09
	90.02		85.08	95.12		76.05	89.12
			90.05			81.06	
						86.06	
						89.12	
						95.06	

Table 5.6: Troughs in the components of the Australian reference cycle

Real GDP (GDPR)	Real non farm product (NFPR)	Real household income (HH_INC)	Real industrial production (IP)	Real retail sales (Retail)	Employment (Emp)	Unemployment rate (Unemp)	Boehm and Moore Reference Cycle (Ref)
52.02	52.08	51.11	52.09	52.08	52.11	52.11	52.09
56.05	56.08	57.11	61.09	61.05	61.09	58.10	57.12
61.08	61.08	76.02	72.03	76.04	75.01	61.09	61.09
83.02	72.02	77.11	75.05	79.09	83.04	67.06	75.10
91.05	75.11	83.05	77.11	82.02	93.02	72.11	77.10
	77.08	91.05	83.02	86.11		75.10	83.05
	83.02		86.05	90.12		79.04	92.12
	91.05		91.11	96.09		83.09	
						87.03	
						92.12	
						97.05	

that the unemployment rate is unsuitable for inclusion in the construction of a reference cycle for Australia.

Table 5.7: Bivariate correlations and robust t-statistics for null hypothesis of strong non synchronization between components of the United States reference cycle

	Unemp	NFPR	Retail	IP	Emp	GDPR	HH_INC
Unemp	$\ddots$	0.33	0.22	0.31	0.35	0.29	0.22
NFPR	2.5	$\ddots$	0.59	0.43	0.33	0.57	0.32
Retail	1.7	4.5	$\ddots$	0.07	0.09	0.24	0.11
IP	2.0	3.6	0.5	$\ddots$	0.57	0.45	0.35
Emp	2.0	2.2	0.7	4.5	$\ddots$	0.52	0.32
GDPR	2.0	2.2	1.5	2.1	2.8	$\ddots$	0.31
HH_INC	2.3	3.0	0.8	1.6	1.5	1.4	$\ddots$
$\widehat{\mu}_S$	0.48	0.80	0.80	0.85	0.87	0.91	0.93

In this paper our interest is in using the Australian reference cycle data to calibrate our algorithm for extracting a reference cycle against the procedures used by Boehm and Moore. Thus we continue to include the unemployment rate in the set of series from which a common cycle is extracted. Given this focus we need to check whether a common cycle exists. The χ^2_{21} test statistic for the SMNS null hypothesis takes the value 37 with p-value 0.02. Consequently, there is reasonably strong evidence for synchronization among the components of the Australian reference cycle when taken as a whole, although Table 5.7 hints that the specific cycle in household income is only weakly correlated with the other specific cycles. Most importantly, the discussion above suggests that practitioners should select the specific cycles that are to be used in constructing a reference cycle by first selecting only the subset of variables where $\mu_{S_i} = \mu_{S_j}$ and then selecting from the subset of specific cycles that are highly correlated.

5.2.4. Components of the United States reference cycle

The NBER business cycle dating committee pays particular attention to four series viz, Total Nonfarm Employment; Industrial production; Manufacturing and trade sales; and Personal income less transfer payments.[12] Specific cycle turning points for these four series are shown in Table 5.8 for the period 1959.1 to 2002.4. The specific cycle turning points were found using a version of the Bry Boschan algorithm and agree closely with those on Robert Hall's NBER spreadsheets.

Table 5.8: Specific cycle turningpoints for industrial production, employment, sales and income, United States, 1959.1 to 2002.4

Industrial Production		Employment		Sales		Income	
Trough	Peak	Trough	Peak	Trough	Peak	Trough	Peak
	1960.1		1960.4		1960.1		np
1961.2	1967.1	1961.2	np	1961.1	np	nt	np
1967.7	1969.10	nt	1970.3	nt	1969.10	nt	np
1970.11	1973.11	1970.11	1974.10	1970.11	1973.11	nt	1973.11
1975.3	1979.6	1975.3	np	1975.3	1979.3	1975.4	1979.12
1980.7	1981.7	nt	1981.7	1981.1	nt	1980.7	1981.8
1982.12	1984.7	1982.11	np	np	nt	nt	np
1985.12	np	nt	np	np	nt	nt	np
nt	1990.9	nt	1990.6	nt	1990.8	nt	1990.7
1991.3	2000.6	1992.2	2001.3	1991.1	2001.9	1991.2	np

Here our investigation of this data is to meet a referee's request to evaluate the algorithm developed in section 6 in terms of its capacity to generate the NBER reference cycle. Thus, the information in table 5.8 is presented to ensure our work is replicable.[13] Given, our focus of interest we will not investigate whether the component series used to construct the reference cycle are well chosen. However, we do need to check whether there is a common cycle in the four series used by the NBER to construct the reference cycle. The value of the χ^2_6 test statistic for SMNS in the components of the NBER reference cycle is 20 with p-value 0.003. Thus, there is strong evidence for the existence of a common cycle in these four series. In a later section we extract that common cycle and compare it to the NBER reference cycle.

[12]The data was obtained from the spreadsheet constructed by Robert Hall that is available from the NBER home page http:www.nber.org/cycles/hall.xlw.

[13]A more extensive analyis of this data is given in Harding (2003).

6. A Non-parametric Method for Extracting the Common Cycle

6.1. The Algorithm

There is an extensive literature on the extraction of dynamic common factors from time series and, as mentioned earlier, the factors are normally used to construct series whose turning points can be dated with specific cycle dating techniques. Because of this literature on the construction of common cycles using parametric models we will focus upon the relatively neglected non-parametric approach.

Implementing the non-parametric method requires some definitions of the key concepts appearing in it. The first of these is the definition of a function $\tau_i^P(t)$ that measures the distance from t to the nearest peak in the i^{th} specific cycle.[14]

Definition 1. ***Distance to nearest turning points.*** *Let t_i^P and t_i^T be vectors containing the dates to peaks and troughs respectively in the i^{th} specific cycle,$i = 1, .., n$. Let $d(.)$ be a measure of distance and $\tau_i^P(t) = \min d\left(t_i^P - t\right)$ be the distance to the nearest peak in the i^{th} specific cycle.*

The next step is to define a function $\tau^P(t)$ that measures the "average" distance from t to the set of nearest peaks. Local minima in $\tau^P(t)$ define the *central dates of clusters of peaks;* these comprise dates at which the distance in time to the set of nearest peaks is minimized.

Definition 2. ***Centres of clusters of turning points.*** *Let $g()$ be a function that measures the centre of tendency of the distances to the nearest turning point for a collection of specific cycles.*[15] *Define $\tau^P(t) = g\left(\tau_1^P(t), .., \tau_n^P(t)\right)$ to be the centre of tendency of the distances to the peaks nearest to date t.. Define M^P as the vector of dates of local minima in $\tau^P(t)$. Formally,*

$$M^P = \left\{t \in 1, .., T | \tau^P(t + \Delta t) \geq \tau^P(t) \quad for\, all\, \Delta t\, such\, that\ |\Delta t| \leq \delta\right\}, \quad (6.1)$$

[14] One proceeds in the same way for troughs.

[15] Typically, $g(.)$ will be selected from either the family of generalized means or from the median.

where δ is some positive constant used to define "local" and M^P is the vector containing the central date of the clusters of peaks. The vector containing the central dates of the cluster of troughs, M^T, can be defined in a similar fashion.

Once the centre of each cluster is located, attention turns to determining, for each specific cycle, whether or not the peak nearest to the centre of that cluster is in that cluster. The rule used in the definition below is that, for each specific cycle, the nearest peak to the centre of a cluster is in that cluster if two conditions are met

1. It is not nearer to the centre of another cluster; and
2. It is less than $\overline{d}$ from the centre of the cluster.

Definition 3. ***Cluster of turning points.*** *Let m_j^P be the j^{th} element of M^P and $C\left(m_j^P\right)$ represent the cluster of peaks centered on m_j^P. $C\left(m_j^P\right)$ is defined as follows*

$$C\left(m_j^P\right) = \left\{t_{ji} \in \left(t_1^P, .., t_n^P\right) | d\left(m_j^P, t_{ji}\right) < d\left(m_k^P, t_{ji}\right) \quad for\, all\, k \neq j;\, and\, d\left(m_j^P, t_{ji}\right) \leq \overline{d}\right\},$$

where $\overline{d}$ is a constant. Clusters of troughs can be defined in a similar way.

Thus to implement the algorithm one needs to make choices about:

1. A function $d(.)$ to measure the distance between turning points.
2. A function $g(.)$ used to measure the centre of tendency of turning points in a cluster.
3. A constant $\overline{d}$ that determines the maximum width of a cluster.

We have adopted the choices made by Boehm and Moore (1984); specifically we use $d(t_1, t_2) = |t_1 - t_2|$ and choose the median as the measure of the centre of tendency $(g(.))$. Boehm and Moore do not make a recommendation for the choice of $\overline{d}$, but, inspection of Ernst Boehm's worksheets suggests that $\overline{d}$ was never chosen to be greater than 24 months and, in several instances, clusters were chosen with the distance from the median date to the furthest date in the cluster being 20, 21 and 22 months respectively. This suggests that a choice of $\overline{d} = 24$ for monthly data (8 for quarterly data) would provide a good approximation to their procedures.

Described in words the algorithm proceeds in the following three stages

1. At date t find the number of months to the nearest peak (trough) for each series. This gives a vector of dimension n. The median of the elements in this vector is then found. The interpretation of this median is that, at time t, it is the median distance to the nearest peak. Designate this item at time t by m_t.

2. Step 1 is done for each t, producing $m_t(t = 1, ..., T)$. The series m_t is then examined and, wherever a local minimum is encountered, this is taken to be a candidate for a turning point in the reference cycle.

3. The candidate turning points are then modified in two ways. First, owing to the fact that m_t is discrete, it may be necessary to break ties e.g. m_{J+1} and m_J may be equal, and one has to decide whether it is J or $J+1$ that is the turning point. In this situation the algorithm looks at higher percentiles than the median until a unique local minimum is found. We feel this appeal to clustering in higher order percentiles is a natural way to resolve any non-uniqueness of the local median. Second, turning points may need to be censored so that peaks and troughs alternate and to maintain the NBER criteria regarding minimum completed phase and cycle durations. Here we use the censoring procedure described in Harding and Pagan (2002).

6.2. Calibration

The algorithm described above was applied to the Australian data used by Ernst Boehm to identify the Australian reference cycle. Our objective is to check whether the choice of $\overline{d} = 24$ to calibrate the algorithm yielded a reference cycle for Australia that was a good approximation to that obtained by Boehm and Moore using the NBER procedures. The results are presented in Table 6.1 The first four columns of the table relate to peaks and the second four columns relate to troughs. The columns headed "B&M" give the dates of the centre point (median) of clusters of specific cycle peaks and troughs as identified in Ernst Boehm's spreadsheets. These represent the patterns of reference cycle turning points data that the algorithm is seeking to match. The columns headed "ALG" give the centre point (median) of clusters of specific cycle peaks and troughs as identified by the algorithm. The column labeled "Difference" contains the difference in months between the turning point date identified by Boehm and that identified by the algorithm — this comparison is made feasible because the algorithm identifies the same number of turning points as does Boehm. It is evident that the algorithm does quite a good job at matching turning points, with the largest difference being 7 months and the median difference being zero for peaks and one month for troughs. But here we remark that it is the finding just cited regarding the median distance between B&M's turning points and those located by the algorithm which validate our calibration of $\overline{d} = 24$. We will use this calibrated value later when we apply the algorithm to the NBER reference cycle. The column headed "Cluster tightness" measures the median distance between specific cycle turning points in the cluster around the reference cycle turning point. Overall, the clusters are tight with a median distance between specific cycle turning points of 5 months at peaks and 3 months at troughs.

In summary, while the algorithm does not perfectly replicate the Australian reference cycle constructed by Boehm and Moore, we feel that it provides an extremely good approximation, given that it is an automated selection method.

6.3. Application to the NBER reference cycle

It is of interest to investigate how well the algorithm developed earlier and calibrated on Australian data in the preceding section can replicate the decisions of the NBER Business Cycle Dating Committee. The algorithm aggregates the specific cycle turning points in Industrial Production, Employment, Sales and Income that are reported in Table 5.8 to yield reference cycle peaks and troughs that are reported in Table 6.2. This table has the same structure as Table 6.1, the only difference being that the columns in Table 6.2 headed "NBER" reports the

Table 6.1: Comparison of chronologies for two methods of dating the Australian reference cycle

Peaks				Troughs			
B&M	ALG	Difference (ALG-B&M)	Cluster tightness	B&M	ALG	Difference (ALG-B&M)	Cluster tightness
51.08	51.10	2	5	52.08	52.08	0	1
55.12	56.07	7	11	57.04	57.08	4	15
60.08	60.07	-1	2	61.09	61.08	-1	1
74.12	74.12	0	6	75.10	76.01	3	3
76.11	76.11	0	6	77.11	78.06	7	10
81.11	81.08	-3	3	83.02	83.03	1	1
90.02	90.01	-1	1	91.05	91.02	-3	3
Sum		4	34	Sum		11	34
Average		0.6	4.9	Average		1.6	4.9
Median		0	5	Median		1	3

reference cycle dates as determined by the NBER. Looking at the four columns related to troughs it is evident that the algorithm does very well in providing exact matches for four out of the six troughs, differing by one month either way in the date of the two remaining troughs, and yielding an average (and median) difference between the algorithm and the NBER of one half of one month. The clusters of specific cycles at troughs are very tight, with median distance between specific cycle troughs being one-half of one month.

On average, the algorithm locates peaks 2.7 months before the NBER dating committee, with a median distance of 2 months. The clusters of specific cycle peaks are relatively tight with an average distance between specific cycle peaks and the reference cycle peak of 2 months and a median distance of 1.5 months. It is important to observe that the capacity of the algorithm to match the NBER dating committee is not a result of over-fitting. Indeed, no parameters to calibrate the algorithm were chosen by reference to US data. Rather, as described earlier the parameters of the algorithm were selected to replicate an NBER-like reference cycle for Australia. As such it provides very strong evidence in support of the hypothesis that the algorithm effectively summarizes the main aspects of the decisions of the NBER dating committee.

Table 6.2: Comparison of chronologies for two methods of dating the United States reference cycle

Peaks				Troughs			
NBER	ALG	Difference (NBER-ALG)	Cluster tight-ness	NBER	ALG	Difference (NBER-ALG)	Cluster tight-ness
1960.04	1960.01	-3	1.5	1961.02	61.02	0	0.5
1969.12	1969.10	-2	2.5	1970.11	1970.11	0	0.0
1973.11	1973.11	0	0.0	1975.03	1975.03	0	0.5
1980.01	1979.05	-8	4.5	1980.07	1980.07	0	0.5
1981.07	1981.07	0	0.5	1982.11	1982.12	1	0.5
1990.07	1990.08	1	1.0	1991.03	1991.02	-1	1.0
2001.03	2000.08	-7	4.5				
Sum		-19	14.5	Sum		0	3.0
Average		-2.7	2.0	Average		0	0.5
Median		-2	1.5	Median		0	0.5

Of course, one would not expect the algorithm to exactly replicate the decisions of that committee. One reason for this is that the composition of the committee has changed over time. The most recent change resulted from the death of Dr Geoffrey Moore and it may be that the procedures of that committee have changed since his death. Such changes to the composition of the dating committee provide a rationale for using algorithms of the type developed in this paper to provide a consistent method of combining turning points to construct a reference cycle.

7. Conclusion

In this paper we have defined synchronization of cycles, related that definition to the existing literature on common cycles, and shown how to test for synchronization, allowing for the complications caused by serial correlation and heteroscedasticity in cycle states. Applying this test we found weak evidence of synchronization in industrial production and strong evidence in stock prices. We have also developed and applied an algorithm to extract the common (reference) cycle. The attractiveness of the algorithm lies in the fact that it yields an automatic method for identifying the reference cycle from a given set of specific cycles and therefore formalizes the informal procedures used by the NBER.

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