Cooperation as a Transmitted Cultural Trait

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Abstract

This paper studies the survival over time of cooperative behavior in the context of strategic interactions such as those modeled in the Prisoner's Dilemma. In our model, agents are characterized by a 'cooperative' or a 'non-cooperative' preference trait, and rationally take costly actions to socialize their children to their own trait. We then study the implications of this cultural transmission mechanism for the population dynamics of preference traits. In particular, we ask whether 'cooperative' agents survive in the long run, when they interact strategically with agents who do not cooperate.

In contrast to the evolutionary game theory literature, we find that our cultural selection mechanism generally leads to long run polymorphic populations, in which cooperation can be sustained. This is because parents of the minority cultural trait have higher incentives to spend resources to socialize their offspring, than parents in majority groups. This result holds both under complete and incomplete information in the matching process.

Keywords: Cultural transmission, cooperation, endogenous preferences, evolutionary selection mechanisms.

In various real economic and social environments in which interactions are such that collectively beneficial actions are not supported by the non-cooperative equilibrium behavior of rational individual agents, agents in fact often do nonetheless cooperate to sustain collective benefits. Examples include the adherence to collective norms of behavior in families, firms, and most social groups at large. Cooperative behavior in e.g., the Prisoner's Dilemma has been also frequently reproduced in experimental exercises.[1]

While this observation is consistent with the functionalist tradition in sociology, it is not consistent with methodological individualism. Consequently, explaining the emergence of norms of behavior in general, and cooperative behavior in particular, in terms of rational agent theory has become one of the most fundamental problems addressed in economics and in social sciences.

A large literature exists on the issue.[2] Since the early Folk Theorem results (Friedman (1971)), it is well known that cooperative behavior can be enforced by rational agents engaging in infinitely repeated relationships.[3] A complementary literature looks at the problem of emergence of cooperative behavior from the different perspective of evolutionary game theory. Instead of asking, Why is it that cooperative behavior occurs when rational agents play non-cooperative games ? this literature postulates that some agents play cooperatively even if not rational for them to do so, and asks instead the question, How can agents who play cooperatively survive evolutionary selection when they interact strategically with agents who do not cooperate ?[4]

This paper attempts to contribute to the literature on the emergence of cooperative behavior by analyzing it from a perspective which is related but distinct from that of evolutionary game theory. In our context, agents might play cooperatively (e.g., in the Prisoner's Dilemma) because, we postulate, they receive a psychological payoff from doing so.[5] We then ask, how can these agents survive a *cultural* rather than evolutionary selection mechanism when they interact strategically with agents who do not cooperate?

In other words, we study the implications of a model of inter-generational transmission of preferences for the population dynamics of preferences which favor cooperation *per se*. In our set-up in particular, agents are randomly matched to interact once

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[1]A recent survey of this literature, centered around the original contribution by Axelrod (1984), is contained in Axelrod-D'ambrosio (1996).

[2]See Hechter-Opp (2001) for recent surveys within a multi-disciplinary perspective.

[3]These results have been extended in many directions; see Fudenberg-Tirole (1991).

[4]Evolutionary arguments in the study of cooperative behavior and altruism have been put forward by economists, e.g., Becker (1970), Kockesen-Ok-Sethi (2000), Stark (1999), sociologists, e.g., Axelrod (1984), Cohen-Riolo-Axelrod (2000), Bendor-Swistak (2001), biologists, e.g., Maynard Smith (1982), and anthropologists, e.g., Rogers (1988).

[5]In a similar spirit Becker-Madrigal (1995) study cooperation which is induced by habitual behavior: agents gain psychological payoffs by playing cooperatively repeatedly.

in a life-time, and hence no Folk theorem repeated-games considerations apply. We build on population dynamics models of cultural transmission used in biology and cultural anthropology (Cavalli-Sforza-Feldman (1981), Boyd-Richerson (1983)), and on the work on socialization by Coleman (1994). With respect to this literature, though, the cultural selection mechanism we adopt has the added fundamental property that parents rationally take costly actions to socialize their children to some preference specification (Bisin-Verdier (2000)). For instance parents spend time with their children and invest resources (e.g., in the form of private school's tuition) to socialize them to their preferred social norms. Some parents may favor cooperative norms, prescribing caring socially conscious behavior, while others may favor more competitive individualistic norms. We assume that parents who socialize their children do so motivated by altruism, but we assume altruism is not perfect in the sense that parents evaluate their children payoffs with their own preferences (i.e., parents which gain subjective psychological utility when cooperating, also gain subjective psychological utility out of their children cooperation, even though their children might not).

A first contribution of the paper is therefore to provide a cultural selection mechanism of preferences to investigate how such a mechanism affects the evolution of cooperation. We can therefore compare cultural transmission with the common evolutionary selection mechanisms selection, like the replicator dynamics (see Weibull (1995)), adopted by most of the evolutionary game theory literature.[6]

Indeed the implications of cultural and evolutionary selection for the evolution of cooperation are at odds. In our approach in fact, cultural transmission generally leads to long run polymorphic populations of preferences in which cooperation can be sustained (a positive fraction of the steady state population of agents will play cooperatively in equilibrium); while, on the contrary, only non-cooperative behavior is evolutionary stable under random matching and replicator dynamics (see Weibull (1995), example 2.1, pg. 39). Two crucial aspects of cultural transmission processes account for such a difference. First, the selection criterion is not based on objective (or purely material) payoffs but on payoffs as perceived by cultural parents according to their own preferences and values: hence an 'imperfect empathy bias' in the cultural transmission process. Though sometimes counterbalanced by the logic of material payoffs, this imperfect empathy bias induces cultural parents to take actions to transmit preferably their own preferences to the next generation. In particular, it induces cooperative parents to value *per se* the transmission of the cooperative trait in their children. Second, cultural parents are rational in their socialization decisions and therefore react optimally to changes in the cultural environment in which their children are immersed and may be socialized. The implications is the fact that parents in minority cultural groups

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[6]Such comparison is meaningful because material constraints are not stringent for biological survival, the concept of fitness in the replicator dynamics is generally understood as metaphor for some social selection mechanism (learning, imitation, socialization, economic competition).

have, everything else being equal, higher incentives to spend resources to socialize their children to their own preferences than parents in majority cultural groups. These two features introduce therefore a persistence effect which allows for the stability of minority cultural groups and the existence of polymorphic populations. In the present context, it allows cooperative preferences to survive culturally, even though they would be associated generally with lower purely material payoffs.

Such different implications for the dynamics of cooperation of cultural and evolutionary mechanisms in our analysis emphasize the importance of taking explicitly into account the nature of the social selection mechanism at work in the evolution of behaviors and preferences.

The Prisoner's Dilemma has become the paradigm to study cooperation in abstract settings of social interaction, and therefore we follow the literature and restrict our analysis to this game. The other main component of the social interaction is the information structure in matchings. We distinguish two cases, complete and incomplete information. In the first case preferences and payoffs of each player in a match are common knowledge. This setting therefore captures in an abstract manner social relationships which are well established, like relationships with friends, peers and family members (but, recall, we do not consider infinitely repeated relationships). When information is incomplete, instead, players do not know their matches' relevant traits, like preferences and payoffs, and must infer those from the aggregate population distribution of such traits. This setting is meant to capture, again abstractly, more casual relationships across agents in society.

We characterize the conditions under which cooperation is favored both under complete information, and under incomplete information. Interestingly, we show that matching under incomplete information may in some circumstances promote more cooperation in the long run than matching under complete perfect information.

1 Cultural transmission and evolution of cooperation.

Consider overlapping generations of two period lived (young and adult) agents. Reproduction is asexual and fertility is exogenous: each adult at time t has a child who is young at time $t+1$. Adult agents are randomly matched to play the standard Prisoner's Dilemma with 'objective' payoffs represented by the following symmetric matrix:

[**Prisoner's Dilemma table for NC players** table here]

with $T, R, V > 0$.

Since the payoffs when both players do not cooperate, play nc, is normalized to

zero, the parameter T represents the gains from cooperation. R represents instead the cost associated to cooperating, playing c when the other player does not; while V represents the gains associated to not cooperating when the other player in the match does. We restrict ourselves to games where cooperation (c, c) is efficient from a social surplus point of view namely, $T > V - R$.

Players with 'non-cooperative' preferences (NC players for short) have payoffs as in the payoff matrix above.[7] On the other hand, players with 'cooperative' preferences (C players) receive $d > 0$ extra units of subjective psychological payoff any time they play cooperatively (choose action c) independently of the other player's action.[8] Let q_t^C denote the fraction of C players in the population at time t. Also, let q_t^{NC} denote the fraction of NC Players. Of course $q_t^C + q_t^{NC} = 1$.

Young agents are born with no well-defined preferences. Parents with preferences of type $i \in \{C, NC\}$ choose effort τ^i to socialize their children to their own preferences, at cost $H(\tau^i)$. [9]

The socialization mechanism works as follows. Consider a parent with i preferences. His child is first directly exposed to the parent's preferences (and is socialized with probability τ^i chosen by the parent); if this direct socialization is not successful, with probability $1 - \tau^i$, he is socialized to the preferences of a role model picked at random in the population, that is to preferences i with probability q^i and to preferences $j \neq i$ with probability $q^j = 1 - q^i$.

Formally, parents with preferences of type i at time t will then have children with their own same preferences with probability

$$\pi_t^{ii} = \tau^i + (1 - \tau^i)q_t^i,$$

while they will have children with different preferences with probability

$$\pi_t^{ij} = (1 - \tau^i)(1 - q_t^i)$$

As a consequence the dynamics of the fraction of C players in the population is governed by the difference equation:

$$q_{t+1}^C - q_t^C = q_t^C(1 - q_t^C)(\tau^C - \tau^{NC}) \tag{1}$$

The probability of direct socialization of each population group, τ^i, $i \in \{C, NC\}$ is determined as the optimal choice of each parent with trait i. We assume in fact that

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[7]We use the wordings NC player, NC agent, agent with NC preferences, interchangeably; the same for C player, etc.

[8]Our analysis is unchanged if we model C preferences with a psychological cost to play non-cooperatively.

[9]Parents have no technology to socialize their children to preferences different to their own, even if they might want to do so. The assumption can be relaxed at some cost without changing qualitative results.

parents are altruistic, and altruism motivates the transmission of culture. But we assume parents evaluate their children payoff from playing the game with their own (the parents') preferences. For instance a parent who is a C player derives utility from his child playing c, even if the child does not.[10]

The parents' socialization problem is

$$\max_{\tau^i} \pi_t^{ii} V^{ii}([q_{t+1}^C]^e) + \pi_t^{ij} V^{ij}([q_{t+1}^C]^e) - H(\tau^i)$$

where $V^{ij}([q_{t+1}^C]^e)$ represents the type i parent's evaluation of his child's payoff from playing the prisoner's dilemma in period $t+1$, if the child has preferences of type $j \in \{C, NC\}$ and the expected fraction of agents in the population with preferences of type C is $[q_{t+1}^C]^e$.

We assume socialization costs are quadratic, $H(\tau^i) = \frac{1}{2}(\tau^i)^2$. This is just for simplicity, as it allows closed form solutions, but our qualitative results are preserved for more general convex costs. The solution of the socialization problem is then simply given by:

$$\tau^i(q_t^i, [q_{t+1}^C]^e) = (1 - q_t^i)\Delta V^i([q_{t+1}^C]^e) \tag{2}$$

where $\Delta V^i([q_{t+1}^C]^e) = V^{ii}([q_{t+1}^C]^e) - V^{ij}([q_{t+1}^C]^e)$.[11]

We are looking for dynamic cultural processes such that the time path $\{q_t^C\}_{t \geq 0}$ is a *rational expectation* path with $[q_{t+1}^C]^e = q_{t+1}^C$, so that our analysis does not depend on any systematic mistake in the agents prediction of the dynamics of the distribution of the population with respect to preferences. The dynamics of the fraction of the population with trait C, in equation (1), can then be written as

$$q_{t+1}^C - q_t^C = q_t^C(1 - q_t^C)\left(\tau^C(q_t^C, q_{t+1}^C) - \tau^{NC}(1 - q_t^C, q_{t+1}^C)\right) \tag{3}$$

A first informal discussion of some of the general properties of the dynamics of our model of cultural transmission may be useful to interpret our results. Suppose, for the sake of illustration, that the gains from socialization, ΔV^i are constant (independent of q_{t+1}^C) and positive, for both traits i. Then, from equation (2), τ^i only depends on q_t^i, and it is decreasing. Moreover in this case τ^i is 0 if $q_t^i = 1$. In other words, perfect majorities do not socialize their children, because they are freely socialized to the majority trait by society at large. Minorities on the other hand do socialize their children. In fact the direct probability of socialization of group i, τ^i, is decreasing in the fraction of the population with trait i. By equation (3), then, homomorphic steady

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[10]Some evidence for this form of 'paternalistic' altruism (or 'imperfect empathy') is discussed in Bisin-Verdier (2000), while a justification in terms of natural selection in sufficiently rapidly changing environments is provided by Bisin-Verdier (2001).

[11]We implicitly normalize the parameters of the model, i.e., the payoffs R, V, d, so that $\tau^i \leq 1$, for any q_t^C, $[q_t^C]^e$. This is necessary, since τ^i is a probability.

states, in which q^i is either 1 or 0, are not stable, and a unique polymorphic steady state, $q^{i*} \in (0,1)$ is dynamically stable (with the whole $(0,1)$ as basin of attraction).[12]

In the strategic environment at study, in which agents are randomly matched to play the Prisoner's Dilemma, the gains from socialization, ΔV^i, are not constant, though: an agent with a preference for cooperation will generally gain more from transmitting this preference to his children when the fraction of cooperative agents in society is large, since in this case his children will more often avoid the costly interactions with non-cooperative agents. Socialization rates will therefore depend in general on the distribution of the population in terms of preferences.

Moreover, how costly is the interaction with a non-cooperative agent for a cooperative agent will depend on the information structure of the game: if a non-cooperative agent is known to be such in the match, then cooperative agents might adopt conditional strategies, e.g., play c with C agents and nc with NC agents.

In general, homomorphic steady states are unstable as long as

$$\left(\tau^i(q_t^i, q_{t+1}^i) - \tau^j(1 - q_t^i, q_{t+1}^i) \right)_{q_t^i = 0}, \quad \text{for } i \neq j,$$

Therefore, the crucial aspect of the analysis, which will determine whether homomorphic or polymorphic population will survive in the long run, will consist of characterizing the endogenous gains from socialization for very small minorities, that is populations of agents with common preferences i where q^i is close to 0. In particular it will be important to understand how such endogenous gains from socialization are a function of the payoff, the information structure of the game, and the properties of the matching mechanism. This will allow us to derive implications for the dynamics of the distribution of traits in the population.

1.1 Matching with complete information

In this section we study the case in which players can observe the preference type of the opponent after having been matched to play the game.

A characterization of the Nash equilibria of the one-shot prisoner's dilemma, for various configuration of the parameters, is straightforward and is reported in the following table.

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[12]We will look for simplicity at the continuous time limit of the dynamics of q_t^C, by assuming that socialization is instantaneous. In other words, we will consider a discrete time model with periods of length h letting then $h \to 0$. The dynamics reduces to the following form (see the proofs of the Propositions):

$$\dot{q}_t^C = q_t^C (1 - q_t^C) \left(\tau^C(q_t^C, q_{t+h}^C) - \tau^{NC}(1 - q_t^C, q_{t+h}^C) \right) \tag{4}$$

and we can derive explicit closed form solutions in terms of parameters R, d, V.

[**Nash Equilibria (complete information)** table here]

Playing nc is a dominant strategy for an NC player, independently of which type of player he is matched with. The equilibrium action of C players depends instead from the parameters of the game.

An environment in which the psychological gains from cooperation are small enough, $d < \min\{V, R\}$, is one where nc is a dominant strategy for both NC and C players, in any match. In this case then NC and C players are indistinguishable in terms of equilibrium actions. We exclude this trivial uninteresting case from the analysis and restrict ourselves to the case in which $d > \min\{V, R\}$. We distinguish the three remaining region of the parameters:

the environment in which $d > \max\{R, V\}$ is one where the preferences for cooperation of C agents, their psychological gains from cooperating, are very intense;

the environment in which $V < d < R$ is one where non cooperating in a match in which one agent plays c and the other nc is associated to small gains, but possibly imposes large costs on a cooperating player; while

the environment in which instead $R < d < V$ is one where non cooperating in a match in which one agent plays c and the other nc is associated to large gains but imposes small costs.

If $d > \max\{V, R\}$, c is a dominant strategy for C players. If $\min\{R, V\} < d < \max\{R, V\}$, on the other hand, each type C agent's equilibrium action may be contingent on the type of the agent he is matched with. In this case multiple equilibria exists. Fortunately a natural selection of equilibrium exist and we study the dynamics of the population distribution of traits, represented by q_t^C for such a selection:

agents of type C play action c when matched with a C agent in the parameter region $V < d < R$; otherwise, trivially, this parameters' region is undistinguishable from the region in which $d > \max\{V, R\}$;

each agent of type C plays action c one-half of the times he is matched with a C agent in the parameter region $R < d < V$, for symmetry.

Proposition 1 *There exists a unique stable stationary state of the population dynamics; this state is polymorphic for $d > \max\{V, R)$ and $R < d < V$; it is monomorphic at $q^{C*} = 1$ for $V < d < R$, .*

Consider first the environment in which non cooperating imposes big costs and has small gains, $V < d < R$. In this case our cultural transmission process gives rise to a stable monomorphism with full cooperation: in the limit the whole population is composed of agents with preferences for cooperation. How is this possible ? When non cooperating imposes big costs and has small gains players with a preference for cooperation in equilibrium adopt a strategy of conditional cooperation: they cooperate when they match agents with their same preferences and they do not cooperate otherwise. We could also call this strategy *reciprocation*. This strategy is obviously quite effective, as it allows agents with cooperative preferences to avoid the cost R of cooperation with non cooperators.[13] But the reason why such a strategy is so successful in terms of our cultural selection mechanism is that parents with non cooperative preferences have no incentives to socialize their kids to their own preferences in this case. While parents with non cooperative preferences, in fact, only care about the 'objective' payoffs of the game, conditional cooperation fares very well in terms of objective payoffs, actually better than the strategy of non cooperation that their children would adopt if socialized. Parents with preferences for cooperation, on the other hand, always socialize their children because they gain psychologically if they cooperate. Consequently, in this environment, parents with preferences for cooperation have higher incentives to socialize their children to their own preferences than parents with non cooperative preferences, and hence cultural transmission will select the 'cooperative ' preference trait.

The pure cultural transmission mechanism which, as we have seen, favors the socialization of minorities, is responsible instead for the polymorphic stationary state when $d > max\{V, R\}$ and $R < d < V$, and players with preferences for cooperation do in fact cooperate in equilibrium when matched with agents with non cooperative preferences also.

1.2 Matching with incomplete information

In this section we study the case in which players cannot observe the preference type of the opponent after having been matched to play the game.

The appropriate equilibrium concept in this case is Bayesian Nash equilibrium (see Myerson (1997)). A characterization of the Bayesian Nash equilibria of the one-shot prisoner's dilemma, for various configuration of the parameters, is reported in the following table.

[**Bayesian Nash Equilibria (incomplete information)** table here]

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[13]Evolutionary biologists have noted early on such selective advantages of these class of strategies; see Trivers (1971).

Playing nc is a dominant strategy for an NC player, for any distribution of preference types in the population. If the psychological gains from cooperation are small enough, $d < \min\{V, R\}$, nc is also a dominant strategy for C players, for any distribution of types. As in the complete information case, then, for $d < \min\{V, R\}$, NC and C players are indistinguishable in terms of play; and again we restrict the analysis to the interesting case in which $d > \min\{V, R\}$. If $d > \max\{V, R\}$, C agents always play c, as in the case of complete information. In this region of the parameters also, then, the dynamics of q_t^C are as in the case of complete information.

The most interesting cases are in the region of the parameters in which $\min\{R, V\} < d < \max\{V, R\}$. In this case in fact, with complete information, C agents choose actions contingent on the type of the match, whereas this is not possible with incomplete information. With incomplete information C agents can only condition their action in equilibrium on the fraction of agents of type C in the population, q_t^{C}.[14]

The interesting question we address in this environment is whether incomplete information helps or hinders the evolution of cooperation in the population. Consider the two cases in turn: $V < d < R$ first, and then $R < d < V$.

Proposition 2 *If $V < d < R$, there exists a cutoff population distribution $\overline{q}^C = \frac{R-d}{R-V}$ such that:*
   - any initial distribution $q_0^C < \overline{q}^C$ is a stable stationary state;
   - $\overline{q}^C$ is a dynamically stable stationary state, with basin of attraction $[\overline{q}^C, 1]$.

With complete information, in this case, agents with preferences for cooperation adopt the conditional strategy of playing c only when facing a C agent in equilibrium; as a consequence, in the limit all of the population is composed of agents with a preference of cooperation, $q_t^c \rightarrow 1$. With incomplete information, instead, agents of type C can only condition their action in equilibrium on the fraction of agents of type C in the population, q_t^C. In equilibrium they in fact play c only when the fraction of agents with cooperative preferences in the population is large enough (above the cutoff). As a consequence, when C agents are a majority in the population, and they play c in all matches, agents with non-cooperative preferences have some incentive to socialize their children to their own preferences, to avoid them bearing cost R in any match with NC agents. This contrast with the complete information case in which non-cooperative parents have no incentive to socialize their children when C parents are a majority. The dynamics of the population reflects this reduced advantage of C agents with incomplete information when cooperation is associated with big costs and small gains, that is when $V < d < R$. We conclude that in this case incomplete information unambiguously hinders the evolution of cooperation.

Consider now instead the case in which $R < d < V$.

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[14]As in the previous analysis of complete information, multiple equilibria arise and equilibrium selection is necessary; we adopt the same selection mechanism with incomplete information.

Proposition 3 *If $R < d < V$, there exists a unique stable stationary state of the population dynamics, $q^C = \frac{d-R}{d-R+V}$ whose basin of attraction is $(0,1)$.*

With complete information, in this case, there also exists a unique stable stationary state of the population dynamics whose basin of attraction is $(0,1)$. The composition of the population at the stationary state is different in the complete and incomplete information cases: with complete information $q^c = \frac{d-R}{\frac{1}{2}(T+d-R+V)}$ while with incomplete information $q^C = \frac{d-R}{d-R+V}$.

In this region of the parameters it is therefore possible that incomplete information favors rather than hinders cooperation. If in fact $T > d-R+V$, incomplete information is associated with a higher fraction of agents with preferences for cooperation at the stationary state. The intuition of this result is rather subtle. The disadvantage of agents with a preference for cooperation when information is complete follows from the fact that in this case NC agents have strong socialization incentives even if they represent a relatively large share of the population. This because in this case C agents actually play the conditional strategy of cooperating in matches with NC agents, and randomize in matches with other cooperative agents. When instead information is incomplete, such conditional strategies are not possible, and C agents randomize when they represent a large enough fraction of the population. This strategy is not as costly from the point of view of NC agents relying on objective payoffs only.

Appendix

Proposition 1: Proof. We compute the parent's expected evaluation of the future payoff of his child and analyze the dynamics of q_t^C for any parameter configuration of the game.

Case 1: $d > max\{V, R\}$. In this case, the subjective flow gains per unit of time can be computed as:

$$\begin{array}{rcl}
V^{C,C}(q_{t+h}^C) & = & q_{t+h}^C(T+d) + (1 - q_{t+h}^C)(-R+d) \\
V^{C,NC}(q_{t+h}^C) & = & q_{t+h}^C(T+V) \\
V^{NC,NC}(q_{t+h}^C) & = & q_{t+h}^C(T+V) \\
V^{NC,C}(q_{t+h}^C) & = & q_{t+h}^C T + (1 - q_{t+h}^C)(-R)
\end{array}$$

Let $\Delta V^C(q_{t+h}^C) = V^{C,C}(q_{t+h}^C) - V^{C,NC}(q_{t+h}^C)$, and $\Delta V^{NC}(q_{t+h}^C) = V^{NC,NC}(q_{t+h}^C) - V^{NC,C}(q_{t+h}^C)$. Then, we have

$$\begin{array}{rcl}
\Delta V^C(q_{t+h}^C) & = & q_{t+h}^C(d-V) + (1 - q_{t+h}^C)(d-R) > 0 \\
\Delta V^{NC}(q_{t+h}^C) & = & q_{t+h}^C V + (1 - q_{t+h}^C)R > 0
\end{array}$$

From this and 2, it follows that τ^C and τ^{NC} are always strictly positive for $(q_t^C; q_{t+h}^C) \in (0,1) \times [0,1]$. Substituting 2 into 1 the difference equation becomes

$$q_{t+h}^C - q_t^C = hq_t^C(1 - q_t^C)\frac{[d - R - q_t^C(d+V-R)]}{1 - hq_t^C(1 - q_t^C)[R-V]}$$

Taking the limit $h \to 0$, one gets the differential equation for q_t^C as

$$\dot{q}_t^C = q_t^C(1 - q_t^C)[d - R - q_t^C(d+V-R)]$$

As a consequence, the homomorphic steady states $q^C = 0$ and $q^C = 1$ are dynamically unstable and the unique polymorphic steady state $q^{\bar{C}*} = \frac{d-R}{d-R+V}$ is dynamically stable (its basin of attraction is $(0,1)$).

Case 2: $V < d < R$. As type C agents play C when faced with a type C agent (and NC when faced with a NC agent). The 'subjective' flow gains per unit of time are:

$$\begin{array}{rcl}
V^{C,C}(q_{t+h}^C) & = & q_{t+h}^C(T+d) \\
V^{C,NC}(q_{t+h}^C) & = & 0 \\
V^{NC,NC}(q_{t+h}^C) & = & 0 \\
V^{NC,C}(q_{t+h}^C) & = & q_{t+h}^C T
\end{array}$$

and

$$\begin{array}{rcl}
\Delta V^C(q_{t+h}^C) & = & q_{t+h}^C(T+d) > 0 \\
\Delta V^{NC}(q_{t+h}^C) & = & -q_{t+h}^C T < 0
\end{array}$$

From this and 2, it follows that τ^C is positive and that $\tau^{NC} = 0$ for $(q_t^C; q_{t+h}^C) \in (0, 1) \times [0, 1]$. Substituting 2 into1 and reorganizing, one gets

$$q_{t+h}^C - q_t^C = h \frac{(q_t^C)^2(1 - q_t^C)^2(T + d)}{1 - hq_t^C(1 - q_t^C)^2[T + d]}$$

and taking the continuous time limit gives:

$$\dot{q}_t^C = (q_t^C)^2(1 - q_t^C)^2(T + d) > 0$$

As a consequence, the homomorphic steady states $q^C = 1$ is dynamically stable with a basin of attraction $(0, 1))$.

Case 3: $R < d < V$. We compute the 'subjective' flow gains per unit of time as:

$$
\begin{aligned}
V^{C,C}(q_{t+h}^C) &= q_{t+h}^C \cdot \tfrac{1}{2}(T + V + d - R) + (1 - q_{t+h}^C)(d - R) \\
V^{C,NC}(q_{t+h}^C) &= q_{t+h}^C(T + V) \\
V^{NC,NC}(q_{t+h}^C) &= q_{t+h}^C(T + V) \\
V^{NC,C}(q_{t+h}^C) &= q_{t+h}^C : \tfrac{1}{2}(T + V - R) - (1 - q_{t+h}^C)R
\end{aligned}
$$

and

$$\Delta V^C(q_{t+h}^C) = -\frac{1}{2}q_{t+h}^C(T + V + d - R) + (d - R)$$

$$\Delta V^{NC}(q_{t+h}^C) = q_{t+h}^C\frac{1}{2}(T + V + R) + (1 - q_{t+h}^C)R > 0$$

Note that, for q_{t+h}^C close to 1, $\Delta V^C(q_{t+h}^C) < 0$. Agents of type C do not want to socialize children as C players.[15] Formally then

$$\tau^C = (1 - q_t^C) \max\{0, \Delta V^C(q_{t+h}^C)\}$$

and $\tau^C = 0$ for $q_{t+h}^C > \frac{d-R}{\frac{1}{2}(T+V+d-R)}$.

Let us note for convenience,

$$g_0(q, h) = \frac{q + hq(1 - q)[d - R - dq]}{1 + \frac{1}{2}hq(1 - q)[T + V + d - R - dq]}$$

$$g_1(q, h) = \frac{q - hq^2(1 - q)R}{1 + \frac{1}{2}h[q^2(1 - q)](T + V - R)}$$

$g_0(q)$ (resp. $g_1(q)$) characterizes the evolution of the time path of q_t^C when $\tau^C > 0$ (resp. $\tau^C = 0$). More precisely, denoting $q_\gamma = \frac{d-R}{\frac{1}{2}(T+V+d-R)}$:

$$
\begin{aligned}
q_{t+h}^C &= g_0(q_t^C, h) \text{ when } q_{t+h}^C \leq q_\gamma \\
q_{t+h}^C &= g_1(q_t^C, h) \text{ when } q_{t+h}^C > q_\gamma
\end{aligned}
$$

---

[15]If we allowed agents of type C to socialize their children to trait NC, they would, for q_{t+h}^C close to 1. Qualitative results are though unchanged.

Lemma 1 *There exists a $\overline{q}^C(h) \in [q_\gamma, 1)$ such that: i) the dynamics of the system is described by: $q_{t+h}^C = g_0(q_t^C, h)$ when $q_t^C \leq \overline{q}^C(h)$ and $q_{t+h}^C = g_1(q_t^C, h)$ when $q_t^C > \overline{q}^C(h)$ with $\lim_{h \to 0} \overline{q}^C(h) = q_\gamma$.*

Proof. i) Consider first that we have $q_{t+h}^C > q_\gamma = \frac{d-R}{\frac{1}{2}(T+V+d-R)}$, then the rational expectation path is given by $q_{t+h}^C = g_1(q_t^C, h)$ and the domain of validity of this dynamic equation should satisfy $g_1(q_t^C, h) > q_\gamma$. Substitution of the expression of $g_1(q_t^C, h)$ provides the following inequality:

$$\Psi(q,h) = -hd(T+V)\frac{q^2(1-q)}{2} + \frac{1}{2}(T+V+d-R)q - (d-R) \geq 0$$

One can see that $\Psi'(q,h) = -dh(T+V)[q(1-q) - \frac{q^2}{2}] + \frac{1}{2}(T+V+d-R)$ and $\Psi''(q,h) = -dh(T+V)[1-3q]$. Hence $\Psi'(q,h)$ has a minimum at $q = \frac{1}{3}$ and $\Psi'(\frac{1}{3}, h) = \frac{1}{2}[(d-R) + (T+V)(\frac{1}{3} - dh)] > 0$ when $dh < \frac{1}{3}$. Hence for h small enough $\Psi'(q,h)$ is positive for all $q \in [0,1]$ and $\Psi(q,h)$ is increasing with $\Psi(0,h) = -(d-R) < 0$ and $\Psi(1,h) = \frac{1}{2}(T+V++R-d) > 0$ (as $V > d$). Hence there exists a unique $\overline{q}^C(h)$ such that $\Psi(\overline{q}^C(h), h) = 0$ and that $\Psi(q,h) \geq 0$ if and only if $q \geq \overline{q}^C(h)$. Moreover as $\Psi(q_\gamma) = -\left[dh(T+V)\frac{q^2(1-q)}{2}\right]_{q=q_\gamma} \leq 0$ it follows that $\overline{q}^C(h) \geq q_\gamma = \frac{d-R}{\frac{1}{2}(T+V+d-R)}$ and that $\lim_{h \to 0} \overline{q}^C(h) = q_\gamma$

Consider now that $q_{t+h}^C < q_\gamma$, then the rational expectation path is given by $q_{t+h}^C = g_0(q_t^C, h)$ and the domain of validity of this dynamic equation should satisfy $g_0(q_t^C, h) > q_\gamma$. Substitution of the expression of $g_0(q_t^C, h)$ provides the inequality:

$$\Psi(q,h) = -dhK(T+V)\frac{q^2(1-q)}{2} + \frac{1}{2}(T+V+d-R)q - (d-R) < 0$$

Hence $q < \overline{q}^C(h)$. Thus the characterization of the dynamics $q_{t+h}^C = g_0(q_t^C, h)$ when $q_t^C \leq \overline{q}^C(h)$ and $q_{t+h}^C = g_1(q_t^C, h)$ when $q_t^C > \overline{q}^C(h)$.

When $q_t^C \leq \overline{q}^C(h)$, this can be rewritten as:

$$q_{t+h}^C - q_t^C = \frac{hq_t^C(1-q_t^C)[d-R-q_t^C d - \frac{1}{2}q_t^C(T+V+d-R) + \frac{1}{2}\left(q_t^C\right)^2]}{1 + \frac{1}{2}hq_t^C(1-q_t^C)[T+V-R+d-dq_t^C]}$$

which gives at the limit $h \to 0$:

$$\dot{q}_t^C = q_t^C(1-q_t^C)\left[d\frac{(q_t^C)^2}{2} - dq_t^C - q_t^C\frac{T+V+d-R}{2} + (d-R)\right] \text{ when } q_t^C \leq \lim_{h \to 0} \overline{q}^C(h) = q_\gamma$$

In order to characterize the dynamics in this case we need to study the sign of the second order polynomial $P(q_t^C, q_t^C)$ given by:

$$P(q_t^C, q_t^C) = [d-R-dq_t^C - \frac{1}{2}q_t^C(T+V+d-R) + \frac{1}{2}d\left(q_t^C\right)^2]$$

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With some algebra we re-write

$$P(q_t^C, q_t^C) = \left[q_t^C \left[\frac{1}{2} q_t^C d - \frac{1}{2}(T + V + d - R) \right] + (d - R - q_t^C d) \right]$$

Note that $P(q_t^C, q_t^C) > 0$ (resp. < 0) iff

$$q_t^C d \left(\frac{1}{2} q_t^C - 1 \right) > \frac{1}{2} q_t^C (T + V + d - R) - (d - R) \quad (\text{ resp. } < 0) \tag{5}$$

Consider the function $\Theta(q) = qd \left(\frac{1}{2} q - 1 \right) - \frac{1}{2} q(T + V + d - R) + (d - R)$. This function is decreasing in q for $q \in [0, 1]$ and $\Theta(0) = d - R > 0$ and $\Theta(1) = -\frac{T+V-R}{2} - R < 0$. Hence there is unique polymorphic solution $q^{C*} \in (0, 1)$ such that $\Theta(q^{C*}) = 0$. Moreover $\Theta(\frac{d-R}{\frac{1}{2}(T+V+d-R)}) = -\frac{2d(d-R)(T+V)}{(T+V+d-R)^2} < 0$. Hence $q^{C*} < q_\gamma$. It is also easy to see that $\dot{q}_t^C > 0$ (resp. < 0) when $q_t^C < q^{C*}$ (resp. $q^{C*} < q_t^C < q_\gamma$).

One can finally check that, at the limit $h \to 0$, when $q_t^C > \overline{q}^C(h)$,

$$\dot{q}_t^C = -q_t^C(1 - q_t^C) \left[q_t^C R + \frac{1}{2}(q_t^C)^2 (T + V - R) \right] < 0 \text{ for } q_t^C > \lim_{h \to 0} \overline{q}^C(h) = q_\gamma$$

Therefore the qualitative dynamics has unstable corner solutions $q^C = 0$ and $q^C = 1$ and that $q^{C*} < q_\gamma$. is a stable interior stationary state with a basin of attraction $(0, 1)$.
∎

Taking the limit $h \to 0$, the dynamic system becomes

$$
\begin{aligned}
\dot{q}_t^C &= q_t^C(1 - q_t^C) \left[d \frac{(q_t^C)^2}{2} - d q_t^C - q_t^C \frac{T + V + d - R}{2} + (d - R) \right] \text{ when } q_t^C \le q_\gamma \\
&= -q_t^C(1 - q_t^C) \left[q_t^C R + \frac{1}{2}(q_t^C)^2 (T + V - R) \right] < 0 \text{ when } q_t^C > q_\gamma
\end{aligned}
$$

There is then a unique dynamically stable polymorphic steady state $q^{C*} \in (0, q_\gamma)$ (whose basin of attraction is $(0, 1)$ and the homomorphic states $q = 0$ and $q = 1$ are dynamically unstable.

Case 4: $d < \ min\{V, R\}$ In this case trivially

$$\Delta V^C(q_{t+h}^C) = \tau^C(q_t^C, q_{t+h}^C) = \Delta V^{NC}(q_{t+h}^C) = \tau^{NC}(1 - q_t^C, q_{t+h}^C) = 0$$

As a consequence any initial condition for q_t^C is maintained over time (as a stable stationary point). ∎

Proposition 2: Proof. Agents of type C play c iff

$$q_{t+h}^C(T + d) + (1 - q_{t+h}^C)(d - R) > q_{t+h}^C(T + V)$$

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i.e., iff

$$q_{t+h}^C > \frac{R-d}{R-V} < 1 \qquad (6)$$

If (6) is satisfied then

$$\tau^C(.) = (1 - q_t^C)\left(d - R + q_{t+h}^C(R - V)\right)$$

which is positive for $q_{t+h}^C > \frac{R-d}{R-V}$, is $= 0$ for $q_{t+h}^C = \frac{R-d}{R-V}$ and $q_t^C = 1$.
    Also, for $q_{t+h}^C > \frac{R-d}{R-V}$,

$$\tau^{NC}(.) = q_t^C\left(R - q_{t+h}^C(R - V)\right),$$

and hence is strictly positive and decreasing in q_{t+h}^C, for $q_{t+h}^C > \frac{R-d}{R-V}$. For $q_{t+h}^C \leq \frac{R-d}{R-V}$, instead, $\tau^{NC}(.) = 0$. From this, the dynamics of q_t^C can be described as :

$$
\begin{aligned}
q_{t+h}^C &= \frac{q_t^C + hq_t^C(1 - q_t^C)\left[-R + (1 - q_t^C)d\right]}{1 - hq_t^C(1 - q_t^C)(R - V)} && \text{for } q_{t+h}^C > \frac{R-d}{R-V} \\
&= q_t^C && \text{otherwise}
\end{aligned}
$$

which gives

$$
\begin{aligned}
q_{t+h}^C - q_t^C &= \frac{hq_t^C(1 - q_t^C)\left[-(R - d) + q_t^C(R - V - d)\right]}{1 - hq_t^C(1 - q_t^C)(R - V)} && \text{for } q_{t+h}^C > \frac{R-d}{R-V} \\
&= 0 && \text{otherwise}
\end{aligned}
$$

As $-(R - d) + q_t^C(R - V - d) \leq 0$ for all $q_t^C \in [0, 1]$ and taking the limit $h \to 0$, the characterization of the dynamics then follows immediately. $\blacksquare$

Proposition 3: Proof. Agents of type C play c iff

$$q_{t+h}^C(T + d) + (1 - q_{t+h}^C)(d - R) > q_{t+h}^C(T + V)$$

i.e., iff

$$q_{t+h}^C < \frac{d-R}{V-R} < 1 \qquad (7)$$

If (7) is satisfied then

$$\tau^C(.) = (1 - q_t^C)\left(d - R - q_{t+h}^C(V - R)\right)$$

which is decreasing in q_t^C and q_{t+h}^C and is $= 0$ at the cutoff $q_{t+h}^C = \overline{q}^C = \frac{d-R}{V-R}$. If (7) is not satisfied, then $\tau^C(.) = 0$.

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Also, for any $(q_t^C, q_{t+h}^C) \in (0,1) \times [0,1]$, $\tau^{NC} > 0$. Also,

$$\tau^{NC}(.) = q_t^C \left(q_{t+h}^C(V - R) + R \right)$$

if (7) is satisfied, and hence is increasing in q_t^C and q_{t+h}^C, is > 0 for $q_t^C > 0$, and is $= 0$ for $q_t^C = 0$. From this, we get the following dynamics

$$
\begin{aligned}
q_{t+h}^C - q_t^C &= \frac{hq_t^C(1 - q_t^C)\left[d - R - q_t^C(d - R + V)\right]}{1 - hq_t^C(1 - q_t^C)(R - V)} \quad \text{for } q_{t+h}^C < \frac{d - R}{V - R} \\
&< 0 \qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad \text{otherwise}
\end{aligned}
$$

Taking the limit $h \to 0$, the characterization of the dynamics is then

$$
\begin{aligned}
\dot{q}_t^C &= q_t^C(1 - q_t^C)\left[d - R - q_t^C(d - R + V)\right] \quad \text{for } q_t^C < \frac{d - R}{V - R} \\
&< 0 \qquad\qquad\qquad\qquad\qquad\qquad\quad \text{otherwise}
\end{aligned}
$$

From this, it is straightforward to get the characterization of the dynamics as given in the proposition. ∎

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Prisoner's Dilemma:
NC Player's Payoffs

	c	nc
c	T, T	-R, T+V
nc	T+V, -R	0, 0

Pl. 2

Pl. 1

Nash Equilibria (complete information)

Match	(NC,NC)	(NC,C)	(C,C)
Parameters:			
d>max{V,R}	(nc,nc)	(nc,c)	(c,c)
V<d<R	(nc,nc)	(nc,nc)	(c,c) and (nc,nc)
R<d<V	(nc,nc)	(nc,c)	(nc,c) and (c,nc)
d<min{V,R}	(nc,nc)	(nc,nc)	(nc,nc)

Bayesian Nash Equilibria (incomplete information)

Pop. of Coop. Agents	Below Cutoff	Above Cutoff
Parameters:		
d>max{V,R}	(c,nc)	(c,nc)
V<d<R	(nc,nc)	(c,nc) and (nc,nc)
R<d<V	(c,nc)	(mix,nc)
d<min{V,R}	(nc,nc)	(nc,nc)

Dynamics   (complete information)

Stability of	Heterog. Pop.	Coop. Pop.	Non-Coop. Pop.
Parameters:			
d>max{V,R}	x		
V<d<R	x		
R<d<V	x		
d<min{V,R}	x	x	x