Channel Estimation for OFDM over Fast Rayleigh Fading Channels

Jin-Goog Kim, Tae-Joon Kim, Jae-Seang Lee, and Jong-Tae Lim

***Abstract*—**This paper presents a channel estimation scheme for orthogonal frequency division multiplexing (OFDM) in fast varying Rayleigh channels. To estimate the current fading channel, we use not only the known pilot symbols but also the previously estimated channel coefficients. With the previously obtained channel coefficients, we estimate the data symbols and compensate ICI terms. In simulation results, the effectiveness of the proposed method is shown as compared with the polynomial..

***Keywords*—**Channel estimation, ICI, OFDM.

I. INTRODUCTION

IN recent years, orthogonal frequency division multiplexing (OFDM) transmission have been studied widely in wireless communications because they have high transmission capability and they are robust to multi-path delay. Channel parameters are needed to coherently decode the transmit signal and to combine the diversity. Channel estimation has been studied extensively for single antenna systems [2]-[3], [6]-[7]. Most of the channel estimation scheme have focused on pilot-assisted approaches [2], [7] based on a quasi-static fading model that allows the channel to be constant for a block of OFDM symbols. However, in fast fading channels, the time variation of a fading channel over an OFDM symbol period results in a loss of subchannel orthogonality which leads to inter-carrier interference (ICI) [8]-[10]. In order to support a high-speed mobilities, the time variation of a fading channel over an OFDM block must be considered. In fast fading channels, there are more channel parameters compared with these in quasi-static fading channels. Hence, if we use a channel estimation method that estimates only a few coefficients corresponding to different multipath delays, then we can improve the estimation performance. In [1], [3] a channel estimation algorithm based on a D -th order polynomial function has been proposed, but the D -th order polynomial approximation does not reflect the channel characteristic. Generally, we know the relationship between the channel coefficients for each path at different time instants.

In this paper, we propose a channel estimation technique based on ICI compensation. First, we estimate the transmitted data symbol based on the previously obtained the channel coefficients. Then, we compensate the ICI terms for pilot tones. We approximate the channel as polynomials and propose the OFDM structure with equispaced pilots.

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Authors are with the Dept. of Electrical Engineering and Computer Science, Korea Advanced Institute of Science and Technology, 373-1 Guseong-dong Yuseong-gu Daejeon, 305-701, Korea (e-mail: genuinekjg@kaist.ac.kr, jtlim@stcon.kaist.ac.kr).

II. SYSTEM MODEL

Consider an OFDM system with N subcarriers which employs the quadrature phase shift keying (QPSK) modulation. The transmitted symbols are denoted by $\boldsymbol{X} = [X(0), X(1), \cdots, X(N-1)]^T$ and the received symbols are denoted by $Y = [Y(0), Y(1), \cdots, Y(N-1)]^T$. The transmitted symbols are fed to an inverse discrete fourier transform (IDFT) to produce the OFDM signal, and a guard interval is inserted, which is a cyclic extension of the IDFT output sequence, in order to eliminate the inter-symbol interference (ISI). The guard interval is chosen to be longer than the maximum delay spread of the channel. This signal is transmitted over the multipath channel and the received samples are demodulated by taking the DFT after removal of the guard interval.

As shown in [6]-[10], it is well known that the general form of OFDM over slowly fading channels and time-invariant channels over several OFDM symbol periods can be expressed as

$$Y(k) = \Lambda(k)X(k) + Z(k) \tag{1}$$

where $X(k)$ and $Y(k)$ represent, the transmitted and received signals on subcarrier k respectively, $\Lambda(k)$ is an channel frequency response between transmitted symbol and received symbol and Z is the zero-mean AWGN with covariance σ_z^2 for subcarrier k .

However, in fast fading channels, the channel within an OFDM block is time-varying and the overall system can be expressed in a matrix notation as

$$\boldsymbol{Y} = \boldsymbol{GX} + \boldsymbol{Z} \tag{2}$$

where

$$\boldsymbol{G} = \begin{bmatrix} g(0,0) & \cdots & g(0,N-1) \\ \vdots & \ddots & \vdots \\ g(N-1,0) & \cdots & g(N-1,N-1) \end{bmatrix} \tag{3}$$

g of matrix $\boldsymbol{G}$ is as

$$g(p,q) = \frac{1}{N}\sum_{n=0}^{N-1}\sum_{l=0}^{L-1} h(n,l)e^{-j2\pi n(p-q)/N}e^{-j2\pi ql/N} \tag{4}$$

It is observed that the channel matrix $\boldsymbol{G}$ in (2) is no longer a diagonal matrix as in the case of slowly fading channels. This shows that time-selective fading causes ICI, which is represented by the off-diagonal blocks of $\boldsymbol{G}$. Note that (1) can be re-written as

$$Y(p) = g(p,p)X(p) + \sum_{q=0,q\neq p}^{N-1} g(p,q)X(q) + Z(p) \qquad (5)$$

In a time-selective channel, the first term in the right-hand side of (5) is the desired term without ICI in the frequency domain, which denotes the contribution from the same symbol, and the second term in the right-hand side of (5) is the ICI term in the frequency domain, which denotes the contributions from other symbols.

The time-varying fading channel introduces a random amplitude and phase shift to the transmitted signal. The fading channel process $h(t)$ is modeled as a normalized, zero-mean and equal variance wide-sense stationary Gaussian process with a correlation function [5] $r_h(\Delta t) = E\{h(t)h^*(t+\Delta t)\}$, where $E\{\cdot\}$ denotes statistical mean and $(\cdot)^*$ represents complex conjugate of $(\cdot)$. As shown in [4], the spaced-time correlation function of the channel in a typical mobile communication environment can be modeled as

$$r_h(\Delta t) = J_0(2\pi f_d \Delta t) \qquad (6)$$

where f_d is the maximum Doppler shift and $J_0(\cdot)$ is the zero-th order Bessel function of the first kind.

III. Channel Estimation

In this section, we consider the problem of channel estimation in an system with the time- and Frequency-selective channel. To estimate parameters, we rewrite the equation $\boldsymbol{Y} = \boldsymbol{GX} + \boldsymbol{Z}$ as the form of $\boldsymbol{Y} = \boldsymbol{Wh} + \boldsymbol{Z}$. Let $\boldsymbol{h} = [h(0) \cdots h(N-1)]^T$.

The p-th received symbol at the received antenna $Y(p)$ is expressed as follows:

$$\begin{aligned} Y(p) &= \frac{1}{N}\sum_{n=0}^{N-1} h(n)x(n)e^{-j\frac{2\pi}{N}np} \\ &= \sum_{n=0}^{N-1} W(p,n)h(n) \end{aligned}$$

where $W(p,n) = \frac{1}{N}\sum_{q=0}^{N-1} D(p,q,n)X(q)$ and $D(p,q,n) = e^{-j\frac{2\pi}{N}(p-q)n}$.

Assuming that there are N_P pilot tones, which are placed at subcarriers $\mathrm{S} = \{P(1),\cdots,P(N_P)\}$, we know only $\boldsymbol{X} = \{X(p) \mid p \in \mathrm{S}\}$.

Thus, $W(p,n)$ is split in two matrices as follows:

$$\begin{aligned} W(p,n) &= \frac{1}{N}\sum_{k\in \mathrm{S}} D(p,k,n)X(k) + \frac{1}{N}\sum_{k\notin \mathrm{S}} D(p,k,n)X(k) \\ &=: W^p(p,n) + W^n(p,n) \end{aligned} \qquad (7)$$

where S is the set of pilot tones. Thus, we obtain

$$Y(p) = W^p(p)\boldsymbol{h} + W^n(p)\boldsymbol{h} + Z(p) \qquad (8)$$

with $W^p[p] = [W^p(p,0),\cdots,W^p(p,N-1)]$ and $W^n[p] = [W^n(p,0),\cdots,W^n(p,N-1)]$. Since we do not know $X(p)$ for $p \notin \mathrm{S}$, $W^n(p)$ has unknown elements.

A. Channel Estimation Scheme

Let $\check{\boldsymbol{Y}} = [Y(P(1)),\cdots,Y(P(N_P))]^T$ and $\check{\boldsymbol{W}} = [W^p[P(1)],\cdots,W^p[P(N_P)]]^T$. Then we can form the N_P linear equations

$$\begin{pmatrix} Y(P(1)) \\ \vdots \\ Y(P(N_P)) \end{pmatrix} = \begin{pmatrix} W^p(P(1),0) & \cdots & W^p(P(1),N-1) \\ \vdots & \ddots & \vdots \\ W^p(P(N_P),0) & \cdots & W^p(P(N_P),N-1) \end{pmatrix}\boldsymbol{h} + \begin{pmatrix} Z(P(1)) \\ \vdots \\ Z(P(N_P)) \end{pmatrix}$$

or

$$\check{\boldsymbol{Y}} = \check{\boldsymbol{W}}\boldsymbol{h} + \check{\boldsymbol{Z}} \qquad (9)$$

However, $\check{\boldsymbol{W}}$ is an $N_P \times N$ matrix and $N_P \le N$. To obtain $\boldsymbol{h}$ as the least square solution, the number of rows of $\check{\boldsymbol{W}}$ is larger than the number of columns of $\check{\boldsymbol{W}}$. Thus, in order to satisfy the condition $N_P \ge N$, we reduce the number of channel parameters.

To reduce the number of parameters needed for channel estimation, we can approximate the time varying channel to the polynomials. The channel model is shown as follows:

$$h(n) \approx \sum_{q=0}^{Q} \bar{h}_q \cdot n^q \qquad (10)$$

where $\bar{h}_q$ is the polynomial coefficients and Q is the polynomial order. Let $\bar{\boldsymbol{h}} = [\bar{h}_0,\cdots,\bar{h}_Q]^T$. Consider the $N \times (Q+1)$ matrix $\boldsymbol{A}$ such that $\boldsymbol{h} = \boldsymbol{A}\bar{\boldsymbol{h}}$. Then, we rewrite the system model as follows:

$$\check{\boldsymbol{Y}} = \check{\boldsymbol{W}}\boldsymbol{A}\bar{\boldsymbol{h}} + \check{\boldsymbol{Z}} \qquad (11)$$

where $\check{\boldsymbol{W}}\boldsymbol{A}$ has a full column rank. The estimated channel vector is $\tilde{\boldsymbol{h}} = \boldsymbol{A}\bar{\boldsymbol{h}} = \boldsymbol{A}(\check{\boldsymbol{W}}\boldsymbol{A})^{\dagger}\check{\boldsymbol{Y}}$ where $(\cdot)^{\dagger}$ denotes the pseudoinverse of $(\cdot)$.

B. Improved Channel Estimation Scheme

Now, we consider a ICI compensation method to reduce the channel estimation error. In [11], two step channel estimation algorithm is proposed for reducing estimation error.

$$\begin{pmatrix} Y(0)) \\ \vdots \\ Y(N-1) \end{pmatrix} = \begin{pmatrix} W(0,0) & \cdots & W(0,N-1) \\ \vdots & \ddots & \vdots \\ W(N-1,0) & \cdots & W^{p}(P(N_P),N-1) \end{pmatrix} \boldsymbol{h} + \begin{pmatrix} Z(0) \\ \vdots \\ Z(N-1) \end{pmatrix}$$

or

$$\boldsymbol{Y} = \boldsymbol{W}\boldsymbol{h} + \boldsymbol{Z} \tag{12}$$

where $\boldsymbol{W}$ is consisted of the pilot-symbols and the data symbols obtained by using two step channel estimation algorithm in [11]. However, the method has high computational complexity, because the two step algorithm calculates the transmitted symbols based on the currently received symbols. However, to compensate ICI, we use a spaced-time correlation between channel coefficients in time domain. Assuming that there are no significant changes in channel statistics, we obtain the channel coefficients $\tilde{\boldsymbol{h}}$ using the sliding window approach based on the (i-1)th OFDM block as follows:

$$\tilde{h}(n) \;=\; a_1 h(n-M) + \cdots + a_M h(n-1) \tag{13}$$

where M is the window size and tap-weight a_m is obtained by [14] and using a relationship in (6). Thus, $\tilde{\boldsymbol{h}}$ is used to obtain data symbols in time domain [13].

Now, we can compensate ICI terms by obtaining $\breve{\boldsymbol{W}}^{n}(p,n)$

$$\breve{\boldsymbol{W}}^{n}(p,n) = \frac{1}{N}\sum_{k=0,k\notin S}^{N-1} D(p,k,n)\breve{X}(k) \tag{14}$$

and

$$\begin{aligned} Y(p) \;&=\; \boldsymbol{W}^{p}[p]\boldsymbol{h} + \breve{\boldsymbol{W}}^{n}[p]\boldsymbol{h} + Z(p) \\ &=: \breve{\boldsymbol{W}}[p]\boldsymbol{h} + Z(p) \end{aligned} \tag{15}$$

The proposed algorithm as follows:

Step 1: Using the previously obtained channel coefficients, we can estimate the transmitted data symbols $\boldsymbol{X}$.

Step 2: We use the obtained $\breve{\boldsymbol{X}}$ to estimate $\breve{\boldsymbol{W}}$ which includes the ICI terms.

Then, the estimated channel is obtained through (15) and denoted as

$$\hat{\boldsymbol{h}} = \boldsymbol{A}\hat{\boldsymbol{h}} = \boldsymbol{A}(\breve{\boldsymbol{W}}\boldsymbol{A})^{\dagger}\boldsymbol{Y} \tag{16}$$

IV. SIMULATION

In this section, we demonstrate the performance of the proposed channel estimation scheme. To construct an OFDM signal, we assume that the entire channel band is divided into 128 tones. The 12 tones are used as pilot tones. Moreover, the QPSK modulation with coherent demodulation is used. The equispaced pilot symbols are inserted in the system. In our simulation, we consider time-varying flat fading channel with normalized frequency offset, with $\varepsilon = 0.1$, $\varepsilon = 0.4$ and $\varepsilon = 0.6$ [12].

Figs. 1, 2 and 3 show the bit error rate (BER) performance of channel estimation schemes. For comparison, the error rate curve with the polynomial approximation channel is shown in Figs. In Figs., method 1 is channel estimation scheme for $\boldsymbol{W}^{n} = 0$, which is associated with unknown data symbols in (8), method 2 is channel estimation scheme with two step algorithm in (15) and method 3 consider only neighboring ICI terms for pilot tones.

In the proposed scheme, the error performance improves significantly. We know that the proposed scheme has better BER performance over the entire E_b / N_o . In Fig. 4, we show the intercarrier interference with the various frequency offset errors.

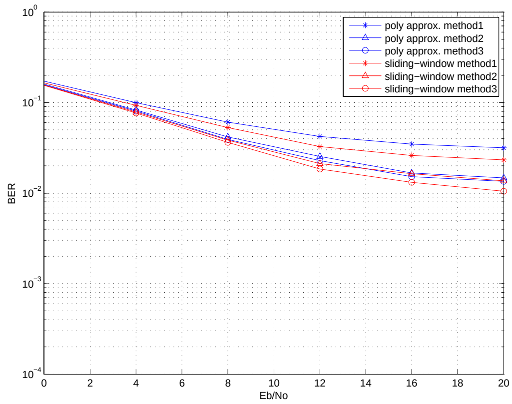

Fig. 1 BER performance of channel estimation schemes (normalized frequency offset $\varepsilon = 0.6$)

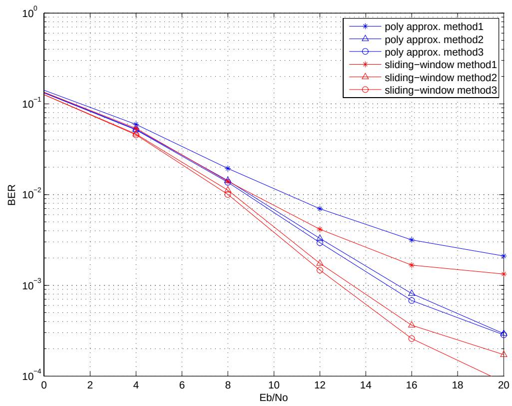

Fig. 2 BER performance of channel estimation schemes (normalized frequency offset $\varepsilon = 0.4$)

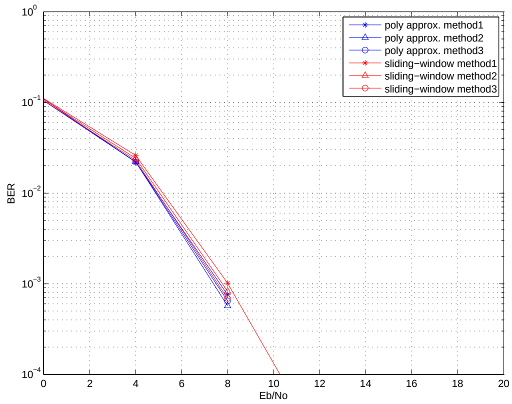

Fig. 3 BER performance of channel estimation schemes (normalized frequency offset $\varepsilon = 0.1$)

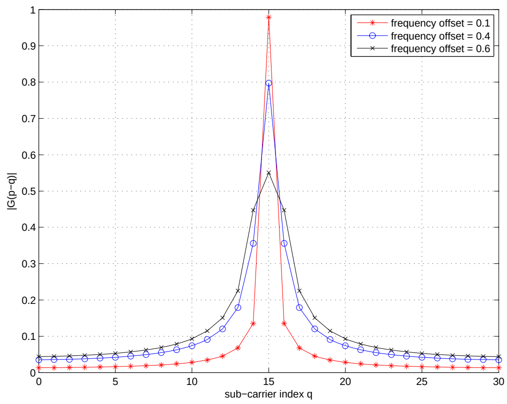

Fig. 4 Amplitude of G(p-q) where frequency offset is normalized by the subcarrier separation

V. Conclusion

In this paper, we proposed the channel estimation method for OFDM over fast Rayleigh fading channel in OFDM systems. We use the sliding window approach to compensate the ICI terms for pilot tones. The proposed estimation scheme has better performance than the conventional channel estimation scheme because the proposed estimation scheme considers ICI terms for pilot tones.

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