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Measuring the Distribution of Human Development: Methodology and an Application to Mexico

By

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Abstract: The Human Development Index (HDI) improves upon per capita GDP as an indicator of development by incorporating information on health and education. However, like its predecessor, it fails to account for the inequality with which the benefits of development are distributed among the population. Subsequent work by Anand and Sen (1993) and Hicks (1997) has led to a useful distribution-sensitive measure of human development, but at the cost of a key property of the HDI that ensures consistency between regional and aggregate analyses. This paper presents a new parametric class of human development indices that includes the original HDI as well as a family of distribution sensitive indices that satisfy all the basic properties for an index of human development. An empirical application using the year 2000 Mexican Population Census data shows how the new measures can be applied to analyze the distribution of human development at the national level and for individual states.

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1 The Traditional HDI

Since its introduction in 1990, the Human Development Index (HDI) has become an established indicator of national and regional development and one of the few broadly used multidimensional welfare measures. The annual publication of the Human Development Report with its HDI country ranking is an eagerly awaited event that receives substantial media interest and public response. In addition, 135 countries around the world have produced national reports using the same methodology, and there are now several cases where the indicator is used for distributing resources among states and municipalities.[1]

What explains the popularity of the HDI? There is a general awareness among economists and policymakers alike that development is not simply income growth. Income is indeed an important intermediate product of development; however, as emphasized by Sen (1999), attention must be paid to other achievements like education and health that are closely linked to the life choices available to people. This is precisely what the HDI includes in its evaluation process.

Also important to the HDI's success is its simplicity. The HDI combines three intuitive components: health conditions (indicated by life expectancy at birth), educational attainment (measured by literacy and attendance rates), and income (represented by the log of per capita GDP). The three population averages are normalized to obtain values between zero and one, and then averaged again to obtain the overall level of human development. The index value, which lies between zero and one, allows for easy evaluation across time and place. In addition, the

[1] See for example the National Human Development Reports of Brazil and Egypt.

widespread availability of data on the three components has meant that it can be readily calculated at national and regional levels.[2]

The methodology of the HDI, like that of any practical index, entails many compromises and choices, and this in turn has led to queries at several levels. The first concerns the choice of "space" for evaluating human development. Why should the index be based on just three dimensions? Surely there are many other categories of achievements that are important for human development. And if only three are to be selected, why should the choices be income, education and health?[3] A second concern is the way that the variables are transformed and normalized to fall within a zero-one range. The income variable is transformed using a logarithmic function. Why is this transformation appropriate? The specific cutoffs used in the normalization procedure are bound to be arbitrary, and yet they implicitly determine the weights on the underlying variables.[4]

While these questions are indeed important, and worthy of further study, the present paper addresses a third problematic aspect of the HDI: the aggregation method it uses to combine the data into an overall index of human development. The current procedure of averaging within and then across dimensions can be criticized on several grounds,[5] and chief among them is the fact that the HDI ignores the distribution of human development across people. It simply does not distinguish whether the benefits of development are reaching all strata of society, or whether they are concentrated among a fortunate few. In countries with low ambient levels of inequality, this may not be such an important issue as the HDI itself will be

[2] Information on life expectancy is not always available, especially at the state or municipality level. Therefore, national reports commonly use infant mortality or survival rates as a proxy.

[3] For example, one could argue that the set of relevant dimensions should include information on human liberties (see Kelley (1991), and Anand and Sen, 1994). One recent empirical study that includes more dimensions is Brandolini and D´Alessio (1998).

[4] See Kelley (1991) and Srinivasan (1994) for discussions of this issue.

[5] See for example Kelley (1991), Srinivasan (1994), Anand and Sen (1994), Streeten (1994), Hicks (1997) and Ravallion (1997).

highly representative of the conditions of the population. However, in the presence of inequality, a given level of HDI may conceal wide variations in achievements across the population, with very high levels of income, schooling and health for some and low values of the same indicators for others. The HDI improves upon per capita income by including additional dimensions of development; but it is no more informative about distributional considerations than its predecessor[6].

Hicks (1997) proposed a distribution sensitive index that employs the Sen welfare standard to evaluate each dimension of development, and then averages across dimensions using the mean.[7] The Sen welfare standard, which is based on the well-known Gini coefficient of inequality, satisfies many of the key properties for welfare measures and has an intuitive link with the generalized Lorenz curve of Shorrocks (1983). However, it is not subgroup consistent, since it is possible for welfare to rise in one region and stay fixed in another while overall welfare falls[8]. The associated distribution sensitive development index inherits this characteristic, which makes it less than ideal for regional and disaggregated analyses of human development. At the current state of the literature, there appears to be a tradeoff between distribution sensitivity and subgroup consistency.

This paper presents a new class of distribution sensitive indices of human development based on distribution sensitive generalizations of the arithmetic mean called the general means.[9] The new human development indices satisfy all the basic properties including subgroup

[6] To be fair, distributional concerns have been discussed since the introduction of the HDI, especially in reference to the income component. See Anand and Sen (2000).
[7] The Sen welfare standard is the mean income discounted by the level of inequality as measured by the Gini coefficient. See section 3.2 below.
[8] A general discussion of subgroup consistency can be found in Foster and Sen (1997) on Section 2.2, below.
[9] See Hardy, Littlewood and Pólya (1952) or Atkinson (1970), for example.

consistency, and hence are well suited for regional analyses of human development. We illustrate the methodology using data from the 2000 Census of Population in Mexico.

Section 2 of the paper begins by setting up the framework for evaluating human development and defining a number of properties that an index of human development should satisfy. Section 3 reviews the relevant single and multidimensional work on inequality, and presents the distribution sensitive measure of Hicks. Section 4 proposes the new class of indices that satisfies all the desirable properties of an HDI and at the same time is distribution sensitive. Section 5 implements the methodology using data from Mexico. Section 6 offers some concluding remarks.

2 Axioms for Measuring Human Development

We now describe the multidimensional framework within which human development will be measured. There are three major steps for measuring the human development of a given population. The first is to identify the key dimensions or "spaces" of human development. The second is to find or construct variables from real world data to represent achievements in the various dimensions and to adjust and normalize them for comparability purposes. The third is to aggregate the normalized variables into an overall indicator of human development. As discussed above, there are significant challenges associated with the satisfactory completion of steps one and two; however, our focus here will be on the aggregation step and, more specifically, on how inequality might be included in the process.

2.1 The Framework

We assume that we have data on three dimensions of development -- income, education and health -- for a population of n units or persons. The data have been transformed and normalized according to the convention usually employed in this exercise, resulting in three distributions with positive entries, namely the distribution of income $x = (x_1,x_2,\ldots,x_n)$, the distribution of education $y = (y_1,y_2,\ldots,y_n)$, and the distribution of health $z = (z_1,z_2,\ldots,z_n)$. Normally, the entries are expected to lie between 0 and 1. However, depending on the level of disaggregation, it is possible that individual observations may exceed the upper limit set down for a variable, in which case the corresponding entry in the vector will exceed 1. We use the symbol $\mu(x)$ to denote the (arithmetic) mean, or per capita, income of a given income distribution x, which has the usual formula $\mu(x) = (x_1 + x_2 +\ldots+ x_n)/n$. Analogous definitions apply to y and z.

All three dimensions of development can be represented simultaneously in a 3xn data matrix D whose first row is the vector x, whose second row is y, and whose third row is z. For a fixed population $n \geq 1$, the domain under consideration is therefore the set of all positive 3xn arrays, which will be denoted here by D_n. Since countries differ in size, we must allow for arbitrary n, so that $\mathrm{D} = \mathrm{U}_n\mathrm{D}_n$ is the overall domain of the index. In what follows, we will sometimes combine two matrices, say D in D_n and D' in $\mathrm{D}_{n'}$, to obtain a new array in $\mathrm{D}_{n+n'}$. For ease of notation we will use the notation (D,D') to denote the matrix in which the first n columns make up D and the last n' columns make up D'.

A human development index is a function F: $\mathrm{D} \rightarrow \mathrm{R}$ from the set D of matrices to the real numbers R, where F(D) is interpreted to be the level of development associated with matrix D in D. The traditional human development index, which we denote by H, can be formally defined as the "mean of means"

$$H(D) = \mu[\mu(x),\mu(y),\mu(z)],$$

where x, y and z are the respective rows of D. In other words, to construct the usual human development index, find the mean achievement for each of the dimensions of development, and then average across the dimensions[10].

Equivalently, one could first aggregate across dimensions at the individual level, to obtain person i's level of development $h_i = \mu(x_i,y_i,z_i)$ and the corresponding personal distribution $h = (h_1,h_2,\ldots,h_n)$ of human development. Then the overall index of human development is simply $H(D) = \mu(h_1,h_2,\ldots,h_n)$, or the mean level of individual human development across all persons.[11] Finally, one can take a more direct view by extending the definition of the arithmetic mean to matrices, so that $\mu(D) = \Sigma_i(x_i+y_i+z_i)/(3n)$ for D in D_n. Then, $H(D) = \mu(D)$, the arithmetic mean of all entries in the matrix D.

2.2 Basic Properties

What properties should a general human development index satisfy? The traditional index H satisfies a collection of intuitive properties that may be seen as the basic set of properties for human development indices. These properties are now presented and discussed. In the ensuing definitions, A and B are taken to be matrices in D.

The first property places restrictions on the relative weight that the index places on the three dimensions of development. We say that B is obtained from A by a *dimensional permutation* if there is a 3x3 permutation matrix P such that B = PA. A permutation matrix has the effect of rearranging the rows of the matrix, such as would occur if the distribution of education were replaced by the distribution of income, and vice versa[12]. F is said to be

[10] In practice, the three variables may be drawn from different sample populations. See section 5.1 below.

[11] In other words, the same level of development is obtained irrespective of the order of aggregation. As will be seen below, a recently proposed generalization does not have this characteristic, and may be criticized for relying on an arbitrary order of aggregation.

[12] More formally, a permutation matrix is an nxn matrix such that each row and each column contains one "1" and the rest "0"s.

symmetric in dimensions if F(B) = F(A) whenever B is obtained from A by a dimensional permutation. Under this property, each dimension of human development is taken by F to be equally important; it rules out the possibility that F treats the three dimensions differentially with, say income receiving more emphasis than education.

The next property requires an analogous form of symmetry to hold for persons. We say that B is obtained from A by a *positional permutation* if there is an nxn permutation matrix Q such that B = AQ. The permutation matrix has the effect of reassigning development levels among people as would occur if, say, the income, education and health levels of the first person are replaced by the respective income, education and health levels of the second person, and vice versa. F is said to be *symmetric in people* if F(B) = F(A) whenever B is obtained from A by a positional permutation. This property requires the measure to treat people symmetrically in that the overall level of development is unchanged whenever two persons switch their respective levels of income, education and health.

In practice it is important to be able to compare development levels for countries or regions having different population sizes. The next property provides one way of ensuring coherent evaluations for different sized populations. We say that B is obtained from A by a *replication* if B = (A,A,…,A) (k times) for some $k \geq 2$. When A is replicated to obtain B, each person and hence each column in A is "cloned" k many times. F is said to be *replication-invariant* if F(B) = F(A) whenever B is obtained from A by a replication. This property essentially ensures that F adopts a per capita interpretation of development.

An aggregate index of human development should be sensitive to increases in individual achievements, and this is the intent of the next basic property. We say that B is obtained from A by *simple increment* if the matrix B – A is nonnegative with exactly one strictly positive entry. F is said to be *monotonic* if F(B) > F(A) whenever B is obtained from A by a simple increment. In other words, the measure of human development is monotonic if it is increasing in each component of A.

In addition, there are three properties satisfied by the usual human development index that are more technical in nature, but nonetheless add much to its ability to convey information. We say that F is *linearly homogeneous* if $F(B) = \alpha F(A)$ whenever $B = \alpha A$ for any given $\alpha > 0$. This property ties the level of development to the individual dimensions in such a way that if all entries of an array are cut in half, the overall level of development is cut in half. We say that F is *normalized* if F(A) = 1/2 whenever all entries in A are 1/2. When combined with linear homogeneity, normalization ensures that whenever everyone has the same level β in all dimensions, F must be β as well. Finally, we say that F is *continuous* if its restriction on D_n is a continuous function. This property ensures that small changes in the array are associated with small changes in the value of the function F.

The three properties of symmetry in population, replication invariance, and monotonicity are entirely analogous to the standard properties for welfare functions of a single dimension and share the same justifications.[13] Homogeneity and normalization are similar to properties characterizing living standards in income space, such the mean, median, and the well-known equally distributed equivalent (ede) or representative income function.[14] In the present, multidimensional context this ensures that aggregate development is measured within the same space as each of the dimensions of development. Symmetry in dimensions ensures that the aggregate index of human development gives each normalized development variable equal

[13] See Foster and Sen (1997) who discuss the unidemensional versions of the properties.
[14] The notion of a representative income or ede is due to Kolm (1969) and Atkinson (1970). The properties for such functions are discussed in Foster and Shneyerov (2000) and Foster and Szekely (2002).

weight. Note, though, that this interpretation relies heavily on the normalization step to ensure that the three variables are indeed commensurate.[15]

It is easy to show that the traditional human development index H satisfies these basic properties. To begin with, either form of permutation leaves the arithmetic mean of the entries of D, and hence H, unchanged. Consequently, H satisfies symmetry in dimension and symmetry in people. Replication invariance follows from the per capita nature of H. An increase in a single entry of D raises the average, implying that H is monotonic. The properties of linear homogeneity, normalization, and continuity follow immediately from linearity and the other basic properties of the arithmetic mean. Thus the standard human development H index satisfies each of the aforementioned basic properties.

2.2 Subgroup Consistency

Although it is conceptual in nature, the final property we consider here has its roots in very practical considerations. Suppose that the measured level of human development changes for one population subgroup and stays the same for its complement (with both subgroup population sizes remaining unchanged). It is quite natural to expect the direction of change in the overall level of human development to be consistent with the direction of change for the subgroup. If this were not the case, then a potential conflict could occur between, say, local and national efforts to augment human development, which in turn would raise questions about the policy relevance of the index.

A human development index F is said to be *subgroup consistent* if, for every A and A' in D_n and every B and B' in $D_{n'}$, we have F(B,B') > F(A,A') whenever F(B) = F(A) and F(B') > F(A'). In other words, a subgroup consistent index is one for which a ceteris paribus change in

[15] For example, expanding the upper limit in constructing the normalized version of the variable diminishes the effective weight on that variable.

development within a subgroup of the population is associated with the corresponding change for the population as a whole. This property is helpful in generating consistent "profiles" of human development and in formulating targeting strategies.[16]

To see that the index H satisfies subgroup consistency, recall that it is simply the mean of the entries in the development matrix. Now if the mean development level for one subgroup rises while the mean for the remaining subgroup is unchanged (with fixed subgroup population sizes), then since the overall mean development level is a weighted average of subgroup means (with population share weights), the overall level must rise. The standard human development index is subgroup consistent.

3 Inequality and Human Development

The case for including considerations of inequality in the evaluation of human development has been well made in the relevant literature, and mirrors the well-known critique of per capita GDP as an indicator of social welfare.[17] But exactly how should a measure of human development take into account inequality and among whom? Are there tools in the existing literature on inequality that might help in constructing a distribution-sensitive index of human development? To help in answering these questions, we will take a short digression to explore traditional measures of inequality and welfare in the context of a single variable.

3.1 Inequality in One Dimension

We begin with some basic definitions from the measurement of inequality in a single variable, say income. As before, $x = (x_1, x_2,\ldots,x_n)$ will denote the income distribution under consideration. An index of inequality I is a function from the set of all income distributions (of

[16] The property can be easily extended to arbitrary numbers of subgroups by repeated application. Anand and Sen (1994) briefly discuss the suitability of subgroup consistency in the context of the HDI and note that when the individual level data are not independent of the grouping of people, this may prevent the property from being applied in practice.
[17] See Anand and Sen (1994) Hicks (1997), and Sen (1997).

arbitrary population size) to the real numbers, where I(x) is the level of inequality associated with the income distribution x. Note that by its very definition, an inequality index provides a complete ranking of income distributions; in other words, it can always compare distributions, irrespective of how non-intuitive or ambiguous the particular comparison appears to be.

Inequality indices are expected to satisfy a collection of basic properties not unlike those described above for human development indices. *Symmetry* and *replication invariance*, appropriately redefined, are two of the fundamental requirements. A key difference, though, is that inequality indices are not monotonic, but instead satisfy an invariance property that links up distributions of different total amounts of income. The usual property of this sort is *scale invariance*: I is unchanged by a proportional scaling up or down of all incomes. This property ensures that the resulting measure evaluates the relative inequality in the distribution, independent of the total amount of income.

To this collection of three properties is added the defining characteristic of inequality measures, the transfer principle. Suppose that x and x' have the same population size and the same mean income. We say that x' is obtained from x by a progressive transfer if there exists persons i and j with $x_i < x_j$ such that $x_i < x'_i$ and $x'_j < x_j$, with $x_k = y_k$ for all k=i, j. In other words, a richer person (j) transfers income to a poorer person (i) in such a way that the incomes are closer together after the transfer than before. I satisfies the *transfer principle* if I(x') < I(x) whenever x' is obtained from x by progressive transfer.[18] The four basic properties are

[18] The transfer principle can also be defined using a type of matrix called a bistochastic matrix, which is a nonnegative square matrix having the property that each row and column sums to one. We say that x' is obtained from x by a *smoothing of incomes* if x' = Mx for some bistochastic matrix and x' is not a permutation of x. I satisfies the transfer principle if I(x') < I(x) whenever x is obtained from y by a smoothing of incomes.

equivalent to *Lorenz consistency*, which requires the inequality measure to follow the well-known Lorenz criterion when it applies.[19]

A wide range of inequality measures satisfies these basic properties. One well-known example is the Gini coefficient G(x) which can be interpreted as the expected (absolute value of the) difference between two incomes drawn at random from the distribution, normalized by twice the mean.[20] The Gini coefficient is perhaps the most commonly used inequality measure in empirical studies. In any case, it is certainly one of the main candidates for measuring inequality.

A second major form of inequality measure can defined with the help of a generalization of the mean to a family of income standards. The class of *general means* is given by $\mu_q(x) = [(x_1^q + \ldots + x_n^q)/n]^{1/q}$ for all $q \neq 0$ and by $\mu_q(x) = (x_1 \cdots x_n)^{1/n}$ for q = 0. When q = 1, the formula reduces to that of the arithmetic mean. The geometric mean is obtained when q = 0; it invariably places more emphasis on lower incomes than does the arithmetic mean.[21] The value q = -1 yields the harmonic mean, which by inverting all incomes, taking the average, and then inverting the result, places even greater emphasis on the lower incomes in x. Indeed, as q approaches infinity, μ_q tends to the "Rawlsian" standard of the minimum income in x. In the other direction, μ_q progressively places greater weight on higher incomes and tends to the maximum entry of x as q rises towards infinity. Parameter q thus indicates the extent to which μ_q emphasizes the upper end of the income distribution.

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[19] See Foster (1985).
[20] There are many other interpretations of G. A notable example links it to (twice) the area between the well-known Lorenz curve and the 45-degree line of complete equality. See Foster and Sen (1997) for the full range of definitions and interpretations of G.
[21] This is the well-known inequality between the geometric and arithmetic mean, e.g., see Hardy, Littlewood and Pólya (1952).

The graph of $\mu_q(x)$ as a function of q reveals useful information about the underlying income distribution x. For example, if $\mu_q(x)$ is constant in q, then the distribution x must be completely equal. Otherwise, $\mu_q(x)$ must be strictly increasing in q, indicating that at least two incomes are unequal in x. If the graph rises especially rapidly, so that the ratio $\mu_q(x)/\mu_{q'}(x)$ departs significantly from 1 for some q and q', then x can be expected to have a high level of income inequality. Indeed, as noted by Foster and Shneyerov (1999), virtually every income inequality measure is either a function of the ratio of two general means, or the limit of such functions. This includes Atkinson's parametric family of inequality measures, the generalized entropy class (and hence Theil's two measures and the coefficient of variation), and even the variance of logarithms that is commonly used in evaluating the distribution of earnings.[22]

Atkinson's (1970) family of inequality measures is defined for each distribution x by:

$$I_\varepsilon(x) = 1 - [\mu_{1-\varepsilon}(x)/\mu(x)] \qquad \text{for } \varepsilon > 0.$$

An Atkinson measure compares a "bottom-sensitive" general mean (with $q = 1- \varepsilon < 1$) and the "neutral" arithmetic mean (with $q = 1$). General means with parameters below 1 are always smaller than the arithmetic mean, and so the ratio $\mu_{1-\varepsilon}(x)/\mu(x)$ is less than one but greater than zero. Greater inequality is reflected in a larger relative gap between $\mu_{1-\varepsilon}(x)$ and $\mu(x)$, and hence a higher value for the measure. The parameter ε can be interpreted as an "inequality aversion" parameter, with a higher value reflecting a greater sensitivity to inequality at the lowest part of the distribution. All of the Atkinson measures satisfy the basic properties for inequality measures.

3.2 Welfare in One Dimension

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[22] See Foster and Sen (1997) for a discussion of these inequality indices; Foster and Ok (2000) provide a critique of the variance of logarithms.

A welfare measure W is a function from the set of all income distributions (of arbitrary population size) to the real numbers, where W(x) is the level of welfare or "well being" associated with the income distribution x. While an inequality measure evaluates the relative dispersion of a distribution independent of the total income (as required by the scale invariance axiom), a welfare measure offers a view of the income distribution that is sensitive to inequality, but rises as incomes rise. The set of properties usually required for welfare measures includes *symmetry*, *replication invariance*, and the *transfer principle* (where a progressive transfer leads to a higher level of welfare). However, scale invariance is dropped in favor of *monotonicity*: If one income rises and the rest are unchanged, then the level of welfare must rise. In addition, we assume here that the welfare measure W(x) is *linearly homogenous*, *normalized*, and *continuous* resulting in what we call here a *welfare standard*. Notice that by the transfer principle we have $W(x) \leq W(\mu(x),\ldots, \mu(x))$, while homogeneity and normalization ensure that $W(\mu(x),\ldots, \mu(x)) = \mu(x)$. Hence, $W(x) \leq \mu(x)$, with strict inequality whenever x is not completely equal.

There is a natural way of going between inequality measures and welfare standards (so long as the inequality measure takes values between zero and one):[23]

$$I(x) = [1 - W(x)/\mu(x)]$$

or equivalently

$$W(x) = \mu(x)[1 - I(x)].$$

The associated inequality level I(x) can be interpreted as the loss in welfare from inequality expressed as a percentage of the maximum achievable welfare.[24] Conversely, the welfare

[23] See Atkinson (1970), Kolm (1969), or Sen (1997).

[24] From the above, we know that completely equal distribution $(\mu(x),\ldots,\mu(x))$ maximizes the welfare standard W(x) among the set of all distributions having the same mean $\mu(x)$ as x, while the welfare level of this distribution is $W(\mu(x),\ldots,\mu(x)) = \mu(x)$. Therefore $I(x) = [\mu(x) - W(x)]/\mu(x)$ is the percentage loss in welfare due to inequality.

standard W(x) is the mean income discounted by the level of inequality in x. Notice that the formula for Atkinson's parametric family of inequality measures $I_\varepsilon(x)$ mirrors I(x) but with $\mu_{1-\varepsilon}(x)$ in place of W(x). This is no coincidence, since $\mu_{1-\varepsilon}(x)$ for $\varepsilon > 0$ is a welfare standard that satisfies each of the properties listed above.

The formula for constructing welfare standards from inequality measures can be applied to the Gini coefficient, yielding the Sen welfare standard $S(x) = \mu(x)(1 - G(x))$.[25] While S(x) satisfies the basic properties of symmetry, replication invariance, monotonicity, the transfer principle, linear homogeneity, normalization, and continuity, it does *not* satisfy *subgroup consistency*. Indeed, it is possible for the Sen welfare standard to register an increase in one region, remain unchanged for the rest of the population, and yet the Sen welfare standard for the combined population falls. Such examples, though, are impossible to find if $\mu_{1-\varepsilon}(x)$ is the welfare standard employed. Indeed, it can be proved that the general means are the *only* welfare standards satisfying subgroup consistency.[26] This provides a strong rationale for using Atkinson's class of welfare functions in constructing a distribution-sensitive human development index.

3.3 The Kolm Transfer Principle

Before we can evaluate whether a given human development index is distribution sensitive, we need to generalize the transfer principle to multidimensional environments. Recall that a bistochastic matrix M is a nonnegative square matrix whose columns and rows sum to one. Applying M to a distribution has the effect of smoothing the distribution (or perhaps permuting the entries). We say that B is obtained from A by a *common smoothing* if there is a bistochastic matrix M such that B = AM, and B is not simply a positional permutation of A. Notice that the

[25] S(x) is also linked to the generalized Lorenz curve of Shorrocks (1983), since it is twice the area below the curve.
[26] See Foster and Szekely (2002) for this characterization result.

rows xM, yM, and zM of development matrix B are obtained from the rows x, y and z of A by the same smoothing process M.

A human development index satisfies the *Kolm transfer principle* if H(A) < H(B) whenever B is obtained from A by a common smoothing.[27] A *distribution sensitive* human development index is one that satisfies the Kolm transfer principle, and rises in response to a common smoothing of entries. Since a transformation of this type necessarily leaves the mean of each dimension unchanged, the standard HDI just violates the Kolm transfer principle and is clearly not distribution sensitive.

3.4 The Hicks Index

Now let us return to the main question under consideration: how to construct an index of human development that is appropriately sensitive to the distribution of human development. Following a suggestion in Anand and Sen (1994), Hicks (1997) proposed the following index

$$H_G(D) = \mu[\ \mu(x)(1-G(x),\mu(y)(1-G(y)),\mu(z)(1-G(z)]$$

$$= \mu[\ S(x),S(y),S(z)],$$

where x, y and z are the rows of D. The index H_G discounts the mean of each variable by its Gini level of inequality, and then averages across the dimensional welfare levels using the standard mean. In other words, it is the mean of the Sen welfare levels across the three dimensions of income, education and health.

This is undoubtedly a natural way of introducing inequality across persons into the index, and the resulting measure is easy to comprehend and employs elements that are well-understood. Many of the basic properties for a human development index are satisfied by H_G, including symmetry in people (since S is symmetric), symmetry in dimensions (since μ is symmetric),

[27] See Kolm (1977).

replication invariance (since S is replication invariant), and monotonicity (as S and μ are both monotonic). The three properties of linear homogeneity, normalization and continuity follow directly from the analogous properties of S and μ. In addition, the fact that S satisfies the single dimensional transfer principle ensures that H_G satisfies the Kolm multidimensional transfer principle. Consequently, this approach is successful in incorporating distributional sensitivity into the human development index while maintaining many of the properties satisfied by the original index H.

However, it be shown that H_G violates subgroup consistency, so that local judgments about changes in human development can be reversed at the national level. It follows that H_G is not particularly well-suited for analyses of human development by population subgroup. This arises because of its use of S and G, which are well-known to violate subgroup consistency; they in turn transfer this violation to H_G, dimension by dimension. Given the key importance of regional analyses in targeting and other practical policy approaches, it would appear that this index of human development is not entirely satisfactory.

In addition, there is a second conceptual criticism that can be leveled against H_G – that it depends importantly on the order of aggregation across people and dimensions. Recall that the definition of H_G first applies S to each distribution and then applies μ across dimensions, yielding the formula $H_G(D) = \mu[\ S(x),S(y),S(z)]$. An alternative approach would be to apply μ first across dimensions for each person, and then to apply S to the resulting distribution of human development, yielding the alternative formula $H_G'(D) = S[\mu(x_1,y_1,z_1),\ldots,\mu(x_n,y_n,z_n)]$. Each is an equally defensible method of applying S across people and μ across dimensions to determine an overall level human development – yet the two typically lead to different values and even contradictory rankings of distributions. In contrast, the original index of human development H is independent of the order of aggregation since, as noted above, $\mu[\mu(x),\mu(y),\mu(z)] = \mu[\mu(x_1,y_1,z_1),\ldots,\mu(x_n,y_n,z_n)]$. Can distribution sensitivity be gained without sacrificing useful properties such as subgroup consistency or introducing arbitrariness into the definition of the index?

4 A New Class of Human Development Indices

The standard HDI finds the arithmetic means of the three dimensions of development, namely, $\mu(x)$, $\mu(y)$, and $\mu(z)$, and applies the arithmetic mean again to obtain $H(D) = \mu[\mu(x),\mu(y),\mu(z)]$. Our first departure from this approach will be to use a distribution sensitive general mean to summarize the dimension-specific level of human development, namely $\mu_{1-\varepsilon}(x)$, $\mu_{1-\varepsilon}(y)$, and $\mu_{1-\varepsilon}(z)$, where $\varepsilon > 0$. As noted above, this lowers the means of the three distributions in accordance with the level of inequality they exhibit. For example, the level for the income dimension is $\mu_{1-\varepsilon}(x) = \mu(x)[1 - I_\varepsilon(x)]$, or the arithmetic mean discounted by the level of inequality as given by the Atkinson inequality measure with parameter ε. This mirrors Hicks' use of a distribution sensitive indicator for each dimension, but unlike Hicks, we start with an indicator – the general mean – that is subgroup consistent, thus making it possible for the overall index of human development to exhibit this important property.

Now how are we to aggregate across dimensions? One natural possibility is to use the arithmetic mean, which yields the formula $\mu[\mu_{1-\varepsilon}(x),\mu_{1-\varepsilon}(y),\mu_{1-\varepsilon}(z)]$, or the "mean of general means". This expression has the advantage of being easily understood as the simple average of the dimension-specific achievements. However, even though each within-dimension indicator is subgroup consistent, it turns out that the resulting overall indicator is *not*. For example, set $\varepsilon = 2$ and suppose that initially both regions have the same development matrix $A = B$ in which the income distribution is $x = (0.70, 0.90)$, the education distribution is $y = (0.70, 0.70)$, and the health distribution is z = (0.20, 0.70). Applying the general mean to each of these distributions and taking the arithmetic mean yields a development level of 0.60 in each region and overall. Now suppose that the distribution matrix for region 2 becomes B' with income distribution x' = (0.40, 0.80), education distribution y' = (0.70, 0.70), and health distribution z' = (0.40, 0.80). Then applying the above formula yields a *higher* level of 0.62 for region 2 (there is no change in region 1), while the overall level has *fallen* to 0.59. Clearly, the proposed formula violates subgroup consistency.

The above violation of subgroup consistency arises because there is a mismatch in the aggregation method used within each dimension and the method used to aggregate across dimensions. If on the other hand, the *same* general mean were used within and across dimensions, might the resulting index satisfy subgroup consistency? We now present this class of measures and explore the properties that its members satisfy.

4.1 The H_ε Indices

Consider the following family of human development indices

$$H_\varepsilon(D) = \mu_{1-\varepsilon}[\mu_{1-\varepsilon}(x),\mu_{1-\varepsilon}(y),\mu_{1-\varepsilon}(z)] \quad \text{for } \varepsilon \geq 0,$$

where x, y and z are the rows of D. Each member of this family evaluates the level of human development with the help of a given distribution sensitive general mean – first, by summarizing the achievements *within* each dimension of development and, second, by aggregating *across* dimensions. In other words, the index is a “general mean of general means”. It can be shown that the same value is obtained when the general mean is first applied across dimensions to obtain person i’s level of development $h_i = \mu_{1-\varepsilon}(x_i,y_i,z_i)$ and then to the distribution of individual levels to obtain $H_\varepsilon(D) = \mu_{1-\varepsilon}(h_1,h_2,\ldots,h_n)$. Indeed, one can extend the definition of the general mean to apply directly to development matrices D in D, analogous to the arithmetic mean definition above,[28] in which case our class of human development indices is simply

$H_\varepsilon(D) = \mu_{1-\varepsilon}(D) \qquad \text{for } \varepsilon \geq 0,$

or, equivalently, the general mean of the entries of D.

The initial member of this class is $H_0(D) = \mu_1(D)$, or the usual human development index H. It represents the degenerate case where there is no concern at all for inequality – aggregation is done using the arithmetic mean. When $\varepsilon = 1/2$, the resulting distribution sensitive index $H_{1/2}(D) = \mu_{1/2}(D)$ transforms the entries in D by the square root before averaging and transforming back. The concave transformation ensures that the smaller entries in D receive greater relative weight, and hence additional inequality lowers the level of this index. With $\varepsilon = 1$, even greater weight is place on low entries and inequality, since the index $H_1(D) = \mu_0(D)$ aggregates the entries in D based on the geometric mean. Our final example of $\varepsilon = 2$ yields $H_2(D) = \mu_{-1}(D)$, a distribution-sensitive human development index which judges aggregate achievements according to the harmonic mean, and hence is even more sensitive to inequality. In general, $H_\varepsilon(D)$ has greater aversion to inequality as ε rises.

4.2 Properties and Interpretations

It is immediate from the properties of the general means that $H_\varepsilon(D)$ satisfies the basic properties for human development indices, including symmetry in dimensions, symmetry in people, replication invariance, monotonicity, linear homogeneity, normalization and continuity. In addition, $H_\varepsilon(D)$ inherits from the general means the property of subgroup consistency. Indeed, it is possible to derive a simple expression linking subgroup and aggregate human development

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[28] Specifically, the definitions are given by $\mu_{1-\varepsilon}(D) = [\Sigma_i(x_i^{1-\varepsilon}+y_i^{1-\varepsilon}+z_i^{1-\varepsilon})/(3n)]^{1/(1-\varepsilon)}$ for $\varepsilon \neq 1$, and $\mu_{1-\varepsilon}(D) = [\Pi_i(x_i y_i z_i)]^{1/(3n)}$ for $\varepsilon = 1$. The general mean is obtained by transforming all the entries of D, averaging them using the arithmetic mean, and then applying the reverse transformation.

levels. For simplicity, let us suppose that A and B have the same population size. Then the overall level of human development $H_\varepsilon(A,B)$ is related to the subgroup levels $H_\varepsilon(A)$ and $H_\varepsilon(B)$ as follows:

$$H_\varepsilon(A,B) = \mu_{1-\varepsilon}(H_\varepsilon(A),H_\varepsilon(B)).$$

The overall human development level can be expressed as the general mean applied to the vector of subgroup human development levels.[29] Consequently, when the development level of one subgroup rises and the other is unchanged, the overall development level must rise, and H_ε is subgroup consistent.[30]

We also noted above that the definition of H_ε does not rely on an arbitrary choice of sequencing: the same value is obtained whether aggregation takes place first over persons and then over dimensions, or vice versa. Finally, every member of the new class H_ε of human development indices (apart from the usual index $H = H_0$) satisfies the Kolm transfer principle. Indeed, suppose that B is obtained from A by a common smoothing M. Then $\mu_{1-\varepsilon}(xM) \geq \mu_{1-\varepsilon}(x)$, $\mu_{1-\varepsilon}(yM) \geq \mu_{1-\varepsilon}(y)$, and $\mu_{1-\varepsilon}(zM) \geq \mu_{1-\varepsilon}(z)$, with at least one strict inequality, since $\mu_{1-\varepsilon}$ satisfies the principle of transfers. Therefore, by the monotonicity of $\mu_{1-\varepsilon}$ we have

$$H_\varepsilon(B) = \mu_{1-\varepsilon}[\mu_{1-\varepsilon}(xM),\mu_{1-\varepsilon}(yM),\mu_{1-\varepsilon}(zM)] > \mu_{1-\varepsilon}[\mu_{1-\varepsilon}(x),\mu_{1-\varepsilon}(y),\mu_{1-\varepsilon}(z)] = H_\varepsilon(A)$$

and hence H_ε satisfies the Kolm transfer principle.

[29] In the special case where the two (equal sized) groups are defined according to gender, this formula is similar to the definition of the gender related development index $GR(A,B) = \mu_{1-\varepsilon}(H(A),H(B))$ of Anand and Sen (1995). The key difference is that GR ignores within-group inequalities through its use of the standard HDI over subgroups. Since H_ε includes within-group inequality, it follows that $H_\varepsilon(A,B)$ is smaller than $GR(A,B)$ (except when each of A and B is completely equal and the indices coincide).

[30] Where A and B have arbitrary population sizes, the decomposition formula in text is adjusted by population share weights s_A and s_B. So, for example, if $\varepsilon \neq 1$, the formula is $H_\varepsilon(A,B) = [(s_A(H_\varepsilon(A))^{1-\varepsilon} + s_B(H_\varepsilon(B)^{1-\varepsilon}))]^{1/(1-\varepsilon)}$ which is obviously strictly increasing in H(A) and H(B).

A highly distinctive aspect of measures from this class is their sensitivity to inequality *across* the dimensions of development.[31] Consider the dimension-specific development levels $\mu_{1-\varepsilon}(x)$, $\mu_{1-\varepsilon}(y)$, and $\mu_{1-\varepsilon}(z)$). As noted above,

$$H_\varepsilon(D) = \mu_{1-\varepsilon}[\mu_{1-\varepsilon}(x),\mu_{1-\varepsilon}(y),\mu_{1-\varepsilon}(z)]$$

$$= \mu[\mu_{1-\varepsilon}(x),\mu_{1-\varepsilon}(y),\mu_{1-\varepsilon}(z)](1 - I_\varepsilon[\mu_{1-\varepsilon}(x),\mu_{1-\varepsilon}(y),\mu_{1-\varepsilon}(z)])$$

or the arithmetic mean of the three development levels, $\mu_{1-\varepsilon}(x)$, $\mu_{1-\varepsilon}(y)$, and $\mu_{1-\varepsilon}(z)$, discounted by the inequality between them. Consequently, Country 1 with aggregate development levels of (0.70, 0.70, 0.70), for example, will have a higher level of development than Country 2 with levels of (0.95, 0.70, 0.55) since the arithmetic means are the same for the two, but there is more inequality across the dimensions of development in 2 than in 1. H_ε penalizes countries with uneven development and rewards countries that have more balanced achievements in the three dimensions, reflecting a view that while there is some substitutability between the dimensions of development, the degree of substitutability is not infinite.

An analogous observation can be made at the individual level, where the general mean is applied to obtain each person's level of development: $h_i = \mu_{1-\varepsilon}(x_i,y_i,z_i)$. Rather than regarding the various achievements as perfect substitutes of one another, $\mu_{1-\varepsilon}$ treats the three achievements as complements, with the degree of complementarity rising as ε rises.[32] The marginal rate of substitution between any two components is not constant, but rather diminishes along an indifference curve as the first component rises and the second falls. This is a natural assumption given the nature of the three dimensions under consideration.

[31] Note that this characteristic was a byproduct of our desire to satisfy subgroup consistency. It also ensures that the index is independent of the order of aggregation.
[32] The general mean has a well-known structure as a (symmetric) "constant elasticity of substitution" or CES function.

Finally, we have one more interpretation of the new index of human development using the usual index of human development and a new multidimensional measure of inequality. The family I_ε of Atkinson inequality measures can be extended to the multidimensional setting as follows:

$$I_\varepsilon(D) = 1 - [\mu_{1-\varepsilon}(D)/\mu(D)]$$

$$= [\mu(D) - \mu_{1-\varepsilon}(D)]/\mu(D).$$

Intuitively, $I_\varepsilon(D)$ measures the inequality among *all* the entries of D in just the same way as the original Atkinson index evaluates inequality in the concatenated vector (x,y,z). Given this broadened definition of the Atkinson inequality measures, we can now offer a multidimensional version of the well-known link between welfare and inequality:

$$H_\varepsilon(D) = H(D)[1 - I_\varepsilon(D)].$$

In other words, the level $H_\varepsilon(D)$ of human development according to the new class of indices is simply the original HDI level discounted by the level of inequality among all the entries in D as measured by the multidimensional Atkinson measure I_ε.

5 Empirical Illustration

This section provides an illustrative example of the usefulness of the new family of human development indexes.

5.1 The Data

For our empirical illustration we use data from Mexico. We choose this country because we are able to access a sample of the Population Census for the year 2000 from which we construct a data set including 10,099,182 individual records from 2.2 million households, each with information on incomes and education.

For the case of health, the Population Census does not include enough information in order to estimate either life expectancy or infant mortality/survival rates for each household. Therefore, as is commonly done, we impute individual levels from municipality-level data obtained from a different source (we use information from the National Population Council). Using data on municipalities guarantees that within each state, we are still able to capture inequalities across households living in different areas. However, not having truly individualistic data at this level will affect final outcomes for health, since no matter how narrowly the group is defined, the resulting levels are essentially averages that suppress intra-group variations and hence bias the human development index H_ε upwards. Unless or until individual level variables become available for health, this problem will remain.[33]

The fact that H_ε will be defined by aggregating within each dimension, and then by aggregating across dimensions, allows it to be meaningfully applied to this type of combined data, since the sample populations in the Census and that provided by the National Population Council, are indeed random. In addition, it will almost surely be the case that the sample populations will be of different sizes. However, the replication invariance of the general means ensures that the resulting levels, $\mu_{1-\varepsilon}$ (x), $\mu_{1-\varepsilon}$ (y), and $\mu_{1-\varepsilon}$ (z), are indeed comparable and can be meaningfully combined to obtain H_ε even when x, y and z are of different lengths.

The sample of the Mexican Population Census is representative for the 32 States of the country and each of the 2,441 municipalities. Since data on health is available only at the municipality level, we focus on State H_ε´s. State H_ε´s are obtained from individual records from

[33] As noted by Anand and Sen (1993), there is a second problematic aspect of using group-based variables. The level of a variable attributed to an individual can depend on the particular group partition that is employed. This is especially true of a variable like life expectancy, whose level for a given person depends crucially on the specific characteristics (e.g., race, gender, age) that are included in the definition of the groups.

the Census for income and education, while for health, they reflect averages from municipality level variables.

For income, we use a two-step procedure, since household and individual incomes from the Census are not comparable with the standard income measure in the HDI, which is GDP per capita. GDP per capita includes a large number of items, among which household income is one. In order to make our H_{ε}´s comparable with the HDI produced from aggregate data, the first step consists on comparing the State level GDP per capita for the year 2000 (provided by the National Statistical Institute), and compare it with the State annualized income per capita from the Census sample records.[34] In the second step, the ratio of per capita GDP to Census per capita income is used as a factor to blow up the per capita income of each individual in the sample. The H_{ε} is computed using the "adjusted" incomes rather than the original variable. Following the methodology in the HDI, for each individual we divide the difference between the log of the "adjusted" income and the log of the "adjusted" income for the individual with the lowest income in the country, over the difference between the log of the "adjusted" income for the individual, and the log of the "adjusted" income for the individual with the highest income in the country.

For the schooling variables, the Census sample includes data for each individual on illiteracy, school attendance and school attainment. Following the HDI standard methodology, for each household, we compute the share of literate individuals over 14 years of age over the total number of individuals that are older than 14. For school attendance, we obtain the

[34] The use of per capita income has its limitations. For instance, per capita values do not take into account intra-household allocations of income, which will diminish the measured level of inequality (in particular, much of the gender-based inequality will be unobserved) and therefore inflate the measured level of human development. We use per capita income as reference for two reasons. The first is that equivalence scales for Mexico are not available. The second is that income per capita is a natural variable to use in the context of the HDI, which uses GDP per capita for its computation.

proportion of 6-24 year old individuals that attend school. For each individual, we construct the schooling index by adding the literacy indicator weighted by .66 and the attendance variable weighted by .33.

In the case of the health dimension, as mentioned before, the HDI usually uses life expectancy at birth as first proxy. This variable is available for most countries at the national level. However, in country level analysis it is common to use infant mortality or infant survival rates as proxies for health conditions, when life expectancy is unavailable. Here we use infant survival rates because even though State life expectancies are available, the only way of introducing the inequality dimension into the measurement of this variable, is by using municipality level information.[35] Following the HDI standard procedure, we divide the difference between the estimated infant survival rate of each municipality and the lowest rate found in the country, over the difference between the survival rate of the municipality and highest survival rates in the country.

5.2 Empirical Results

If States *x* and *y* share the same value of H_0, and if *x* has an unambiguously more equal distribution of human development than *y* then distribution *x* must have a higher value of H_ε than *y* for every $\varepsilon>0$. More equality in the distribution of human development among individuals "flattens" out the graph of the (increasing) function $H_\varepsilon(x)$ in the parameter ε, so that in the limit, where human development is equalized, the graph becomes horizontal with all H_ε becoming equal.

[35] The National Population Council of Mexico finds a high correlation between infant survival rates and life expectancy across States, and recommends using life survival rates at the municipality as proxies for life expectancy.

A comparison between the values of H_ε for the States of Zacatecas, Guanajuato and Puebla illustrates this interpretation. Figure 1 plots H_ε for $\varepsilon = 0, 0.5, 1, 2$ and 3, respectively, for each State. According to the figure the value of the standard HDI (H_0) is practically the same in the case of Guanajuato and Zacatecas, so inequality comparisons can be made. The fact that for $\varepsilon>0$ the State of Guanajuato ranks higher than Zacatecas reveals that the distribution of human development is more equal in this State than in the other (the graph is relatively "flatter").

The Figure also reveals that the ranking between the States of Puebla and Zacatecas depends on the value of ε. When equal weight is placed on every individual observation within the State, Zacatecas ranks higher than Puebla. However, if greater weight is placed among lower values, the ranking is reversed, and Puebla appears to have higher human development than Zacatecas.

Table 1 presents State rankings using $\varepsilon=0$ and 3. The ranking changes considerably when comparing the ordering according to the traditional HDI (H_0), with the ordering when greater weight is placed among the lowest values. Therefore, when we include information into the HDI on the inequality with which human development is distributed, a very different picture emerges. Our impression about the level of human development across Mexican states is highly sensitive to attaching greater weight to individuals at the bottom of the distribution of human development.

The States of Guerrero (with low human development), Coahuila, and Baja California and the Federal District (with the highest human development) are among the few States that maintain their rankings regardless of the parameter value. Among the countries with the greatest re-rankings is Campeche, which goes from ranking as the State ranked at 18 (from lower to higher) with $\varepsilon=0$, to being ranked as the 7^{th} highest with $\varepsilon=3$. The relative position of Colima, Sonora, México, Chihuahua, Durango and Zacatecas deteriorates considerably when going from $\varepsilon=0$ to $\varepsilon=3$. In contrast, the relative position of Puebla, Hidalgo, Guanajuato, Querétaro and Campeche improves relative to other States when going from the standard to the bottom-sensitive HDI.

An additional question of interest is which of the three dimensions –income, education or health- is more sensitive to introducing inequality in the measurement of human development. Figure 2 presents the value of H_{ε} for each dimension of development separately –that is, before obtaining the general mean of the three indicators in order to produce the overall H_{ε} index. [36]

According to our results, the individual dimension where development is more unequally distributed is income. The general mean for this variable is much higher than for the other two when $\varepsilon=0$, and is considerably lower for $\varepsilon>0$. The second most unequal dimension appears to be education, followed by the health variables. These are somewhat expected results, since, on the one hand, income is the only of the three variables that does not have a natural upper bound - strictly speaking, it is possible that a single individual, or a small group of individuals, can concentrate all or most of the income available to society, and this prevents others from obtaining a larger share; for education and health, the accumulation by a single individual does not necessarily prevent others from having access to these forms of human capita, and there are limits inherent in the human condition that determine the maximum amount that a single individual can accumulate. On the other hand, data availability has forced us to compute the health indicator from municipality level rather than from individual data, which implies

[36] As mentioned before, H_{ε} is path independent with respect to aggregation within and across individuals and States. For this illustration we use the case where first we compute the general mean of each dimension per state separately using the individual data, and then aggregate across dimensions using the same general mean.

suppressing within-municipality inequality. This might be why health varies less when using an inequality-sensitive measure of human development.

Table 2 summarizes the number of places that each State moves when going from $\varepsilon=0$ to $\varepsilon=3$ for each of the three dimensions of development separately. When States are ranked according to the overall index of human development, which includes the three dimensions, 80 place changes occur when going from the standard to a bottom-sensitive measure -an average of 2.5 places per State.

With respect to the individual dimensions, income is the one where introducing information on inequality provokes more State re-rankings. When going from $\varepsilon=0$ to $\varepsilon=3$, 46 place changes are observed. For education, going from H_0 to H_3 produces 38 changes. The health dimension, which is the least sensitive to inequality, goes from H_0 to H_3 with 16 State place changes. This confirms the conclusions derived from Figure 2 with respect to the sensitivity of each individual dimension to inequality.

As already mentioned, a useful interpretation is that H_ε conveys information on the "loss" in development due to inequality among individuals. When we incorporate the sensitivity to inequality in the Mexican data, H_ε goes from a value of 0.6626 to a value of 0.4912 –a loss of 20%-. Thus, the reduction in inequality by itself would imply an important development gain.

Table 3 shows the dimension of the HDI that deteriorates the most when taking into account inequality among individuals. The education index turns out to be the one that is most severely affected by the inequality correction (18.5%). The life expectancy-child survival index is reduced by 0.7% as the aversion to inequality increases, while that income declines by 13.2%.

From the data in Table 1, we obtain the change by state when the importance of inequality increases with $\varepsilon=0$ to $\varepsilon=3$. The case of Oaxaca stands out with the largest percentage reduction in HDI by the sensitivity to inequality (37.8%). The states of Chiapas, Guerrero and Zacatecas also reduce their HDI by more than 30%. On the other hand, the Federal District is the entity with the smallest percentage decrease in its development index due to inequality (only 13.8%). This means that this entity is not only the one with the highest HDI, but also with the lowest within-inequality. States where the reduction is lower than 20% are Baja California, Aguascalientes, Nuevo León, Baja California Sur, Campeche, Coahuila and Sinaloa.

6 Conclusions

This paper presents a new methodology to incorporate the distributive dimension into the Human Development Index. One of the main limitations of the HDI is that by not including a distributional dimension, it is possible to have a country with a higher HDI than another, but where poverty is widespread or where large groups are left out of the development process. It is also possible to have improvements in the HDI simultaneously with stagnation or even deterioration in development for vast sectors of the population.

We present a new class of human development indices that include both, the traditional index, and a family of indices that are sensitive to the distribution of human development. This class of indices uses the general mean to summarize achievements within each dimension of development, and uses the same general mean to aggregate across dimensions.

The class of indices satisfies the following axioms: symmetry in dimensions, symmetry in people, replication invariance, and monotonicity, as well as linear homogeneity, normalization and continuity. Additionally it satisfies subgroup consistency, which guarantees that improvements or deteriorations in human development within a certain group of society (with human development remaining constant in the other groups) will be reflected in the overall measure of human development. Previous alternatives suggested in the literature violate this basic principle. The new class of indices also has the attractiveness of being path independent which guarantees that the order in which human development is aggregated across individuals, or groups of individuals, yields the same result –so, there is no need on relying on a particular choice of sequencing.

The methodology is applied to a sample of the Mexican Population Census for the year 2000, which offers a single, unified data set covering income and education for more than 10 million individuals. For health, we use municipality level data on child survival.

The empirical illustration shows that introducing the inequality dimension into the distribution of human development considerably changes our views about how States rank with respect to each other. A large number of re-rankings occur when placing more weight either at the lower or upper tails of the distribution. Among the three individual dimensions of development, income seems to be the most sensitive to inequality. If one considers the "loss" in human development due to inequality, it can reach up to 26% at the national level. Thus, reducing inequality would have an effect on the measurement of HDI by itself.

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Figure 1

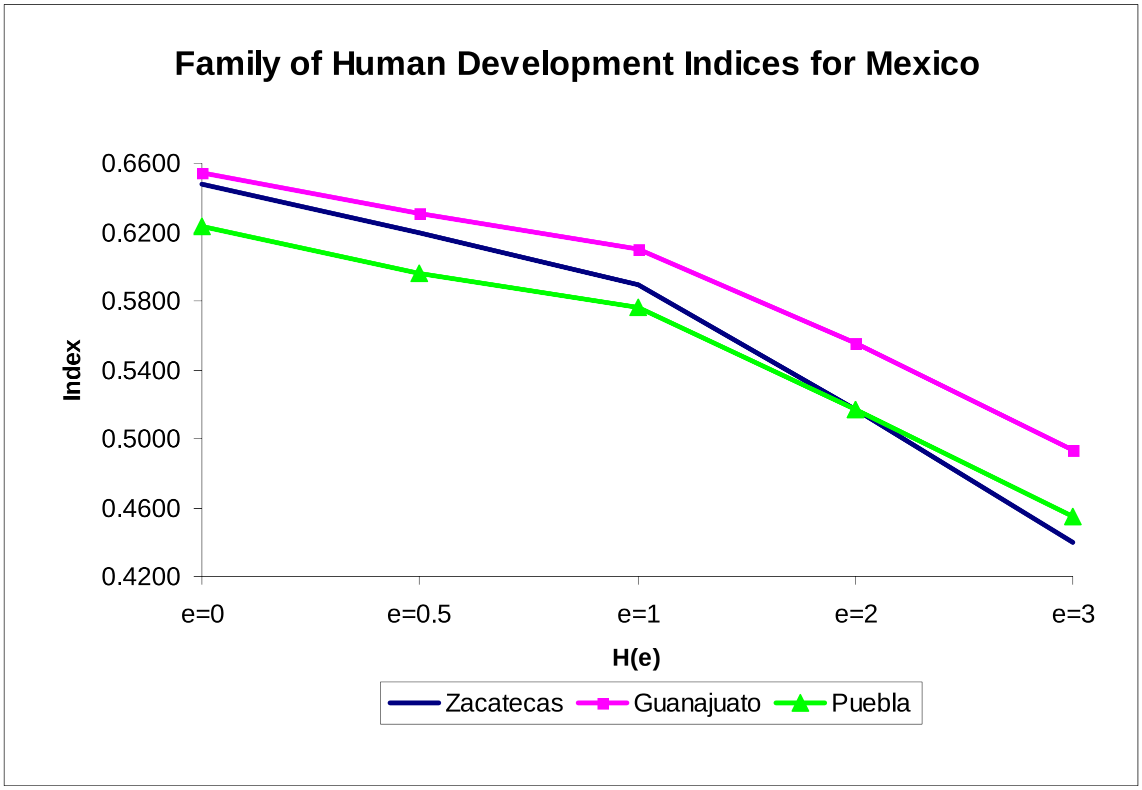

Figure 2

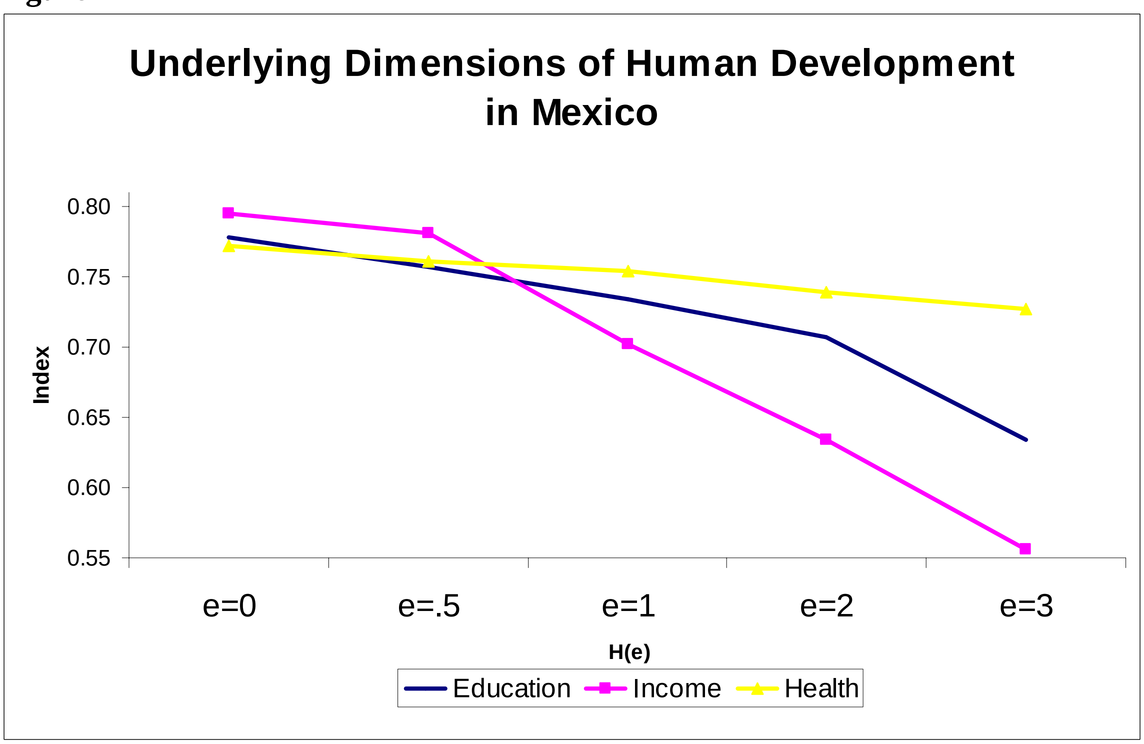

Table 1

HDI-GM correcting for Within Inequality by State, 2000					
	e=0		***e=3***		
	HDI-GM	**Ranking**	**HDI-GM**	**Ranking**	**Rank Change**
Aguascalientes	0.7001	5	0.5811	3	2
Baja California	0.7176	2	0.6150	2	0
Baja California Sur	0.7038	3	0.5787	4	-1
Campeche	0.6734	15	0.5473	7	8
Chiapas	0.5735	32	0.3797	31	1
Chihuahua	0.6739	14	0.5069	18	-4
Coahuila	0.6957	6	0.5637	6	0
Colima	0.6884	7	0.5428	10	-3
Distrito Federal	0.7403	1	0.6376	1	0
Durango	0.6608	20	0.4708	23	-3
Estado de México	0.6824	9	0.5185	14	-5
Guanajuato	0.6546	22	0.4937	19	3
Guerrero	0.5968	30	0.3995	30	0
Hidalgo	0.6449	24	0.4784	21	3
Jalisco	0.6772	12	0.5246	13	-1
Michoacán	0.6363	26	0.4509	26	0
Morelos	0.6691	16	0.5139	16	0
Nayarit	0.6638	18	0.4898	20	-2
Nuevo León	0.7021	4	0.5783	5	-1
Oaxaca	0.5881	31	0.3654	32	-1
Puebla	0.6232	28	0.4545	25	3
Querétaro	0.6637	19	0.5146	15	4
Quintana Roo	0.6798	11	0.5438	9	2
San Luis Potosí	0.6370	25	0.4641	24	1
Sinaloa	0.6817	10	0.5472	8	2
Sonora	0.6853	8	0.5256	12	-4
Tabasco	0.6646	17	0.5094	17	0
Tamaulipas	0.6752	13	0.5280	11	2
Tlaxcala	0.6600	21	0.4747	22	-1
Veracruz	0.6168	29	0.4337	29	0
Yucatán	0.6239	27	0.4497	27	0
Zacatecas	0.6482	23	0.4401	28	-5

Table 2

Changes in Country Rankings

Dimension of human development	From e=0 to e=3
Total number of place-changes in the overall H	80
Place changes in Income	46
Place Changes in Education	38
Place Changes in Health	16

Source: Authors´calculations from the sample of the Mexican Population Census 2000.

Table 3

Losses due to Inequality

Development dimension	% Change From e=0 to e=3
Change in the National HDI	26%
Health	-0.7
Education	-18.5
Income	-13.2

Source: Authors´calculations from the sample of the Mexican Population Census 2000.