An Optimized EV Charging Model Considering TOU Price and SOC Curve

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***Abstract*—Large-scale deployment of electric vehicles (EVs) is anticipated in the foreseeable future. Heavy intermittent charging load of EVs will create bottlenecks in supplying capacity and expose power system to severe security risks. In this paper, we propose an intelligent method to control EV charging loads in response to time-of-use (TOU) price in a regulated market. First, an optimized charging model is formulated to minimize the charging cost. Then, a heuristic method is implemented to minimize the charging cost considering the relation between the acceptable charging power of EV battery and the state of charge (SOC). Finally, the charging cost and energy demand in different time intervals are compared for both typical charging pattern and optimized charging pattern. Results show that the optimized charging pattern has great benefit in reducing cost and flatting the load curve if the peak and valley time periods are partitioned appropriately.**

***Index Terms*—Charging facility, charging load, electric vehicle, state of charge, time-of-use price.**

I. Introduction

THE development of EV is a major direction of modern automobile vehicles. EVs have zero emissions and low noise level; therefore, they have become an essential approach to solve environmental problems and offer energy shortages [1].

One of the bottlenecks that restrict the rapid growth of EV is the lack of the EV charging facilities [2]–[6]. Reference [2] analyzes the rapid charging station and its impact on the distribution systems. Reference [3] describes energy storage characteristics and economic value of the battery switching stations that combine the solar energy. Reference [4] establishes the model of the fuel cell charger and proposes the corresponding control strategy. A new concept of mobile charger and its optimal scheduling methods are presented in [5]. With incremental development in EV charging facilities, EV loads are expected to increase phenomenally in the near future. This will bring negative impacts on the stability of power grids [7]. EV loads are seldom considered in current practice of power system planning, which results in risks in system operations and management [8].

There are three main ways to enable EV-friendly access to power grid: 1) vehicle to grid (V2G); 2) use of energy management equipments (such as energy management concentrator and distributed energy management boxes); 3) the mechanism of electricity pricing. V2G means that the EV discharges the remaining energy stored in the battery into the grid when needed. References [9]–[11] discuss the power electronic converter technology that enables V2G. Energy management equipment can be used to maintain the balance between demand and supply, thus boosting the utilization of EV. Reference [12] describes local and global smart charging control strategies based on family energy control box. It shows that smart charging control strategy can reduce peak load and level the load curve. Reference [13] adopts PHEV management equipments to manage PHEVs in cities. The PHEV management equipments are able to determine the number of PHEVs connected to grid according to power flow calculation. Based on micro-simulation, it optimizes the capacity of PHEVs connected to power grid by introducing intelligent charging policy implemented in central and distributed locations. Reference [14] uses autostromboxes and the demand side management system to manage the charging load. A global optimization technique is presented to reduce the error between reference curve and the summarized load curve of all charging events. The electricity pricing mechanism serves stimulation and guide for power demand and consumption mode of customers. Customers will respond to variable electricity prices, decide whether they prefer charging or discharging, and actively adjust charging rate and time. For countries with mature electricity market environment, research has been focused in this area. For instances, [15] introduces an EV charging model based on real-time price information, while [16] optimizes the charging process by using the method of quadratic programming, considering the relationship between electricity price and load demand. The objective is to minimize charging cost and maximize discharging profit. Reference [17] uses linear programming model to respond to the real-price.

Scenarios are totally different for countries where electricity market is fully regulated. Electricity market in China may be a typical example of regulated market, where electricity prices are decided by the government and, once enacted, remain unchanged for a relatively long time. At present, the electricity pricing mechanism in China mainly includes the catalog price, the stepwise power tariff and the time-of-use (TOU) price. Unlike catalog price and the stepwise power tariff, TOU price is not the same in the different periods of one day, which makes it an important method for demand side management [18]. It is estimated that by 2050 the number of EVs in China will reach

Manuscript received February 25, 2011; revised May 31, 2011; accepted June 02, 2011. Date of publication August 08, 2011; date of current version February 23, 2012. Paper no. TSG-00067-2011.

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Digital Object Identifier 10.1109/TSG.2011.2159630

200 million, and the total charging load will be up to 330 million kilowatts [19]. With such a considerable capability, the EVs in China will play a significant role in balancing power supply and demand. Thereby, research on intelligent response to TOU price is of significance in market-regulated countries. Based on the existing research and the SOC curve, this paper proposes an optimized charging model for regulated market. By using the proposed method, EVs are able to adjust charging power and time, reduce the cost of costumers, thus "reduce peak and fill valley" in load demand.

The remainder of this paper is organized as follows. The optimized EV charging model considering TOU price and SOC curve is proposed in Section II. Section III provides a heuristic method to solve the optimized model. In Section IV, we present numerical simulations of optimized charging model and compare it with typical charging pattern. Finally, conclusions are made in Section V.

II. Optimized Model for Charging in Response to TOU Price

A. Problem Description

In regulated electricity market, TOU price is laid down by the government in advance, and the prices remain unchanged for a long time. The user can set the expected ending time of charging when EV is connected to the grid through the charger. To protect the battery from being damaged in charging process, the maximum charging power can be also set artificially. The charger with embedded TOU price module can intelligently formulate optimized charging scheme in consideration of the SOC curve and the maximum charging power set by user, the aim of which is to minimize the cost and realize peak clipping and valley filling.

B. Objective Function

Take the cost that EV users need to pay to charge once as objection function

$$\min C = \int_{t_0}^{t_0+T} M(t)P(t)\,dt. \tag{1}$$

In (1), t_0 is the starting time of charging, T is duration of charging, t_0+T is the ending time of charging. $M(t)$ and $P(t)$ represent the unit price and charging power in time t respectively.

C. Constraints

The initial SOC of various EV batteries are different as the driving mode and charging habits of different EV users are not the same. The energy demand of EV user with initial SOC considered is stated as below

$$\int_{t_0}^{t_0+T} P(t)\,dt = (1-S_{\mathrm{inl}})Q_r. \tag{2}$$

In (2), S_{inl} represents the initial SOC of EV battery and Q_r is the rated capacity of EV battery.

According to mass theory [20], in order to reduce the life loss of EV battery, the charging current should not surpass the acceptable charging current of EV battery $I = I_0 e^{-\alpha t}$ (I_0 is the maximum charging current at the starting time, α is ratio of acceptance). Theoretically I_0 is determined by initial SOC and internal resistance of EV battery [21]. The highest accepted voltage of EV battery should not exceed a certain limited value as well.

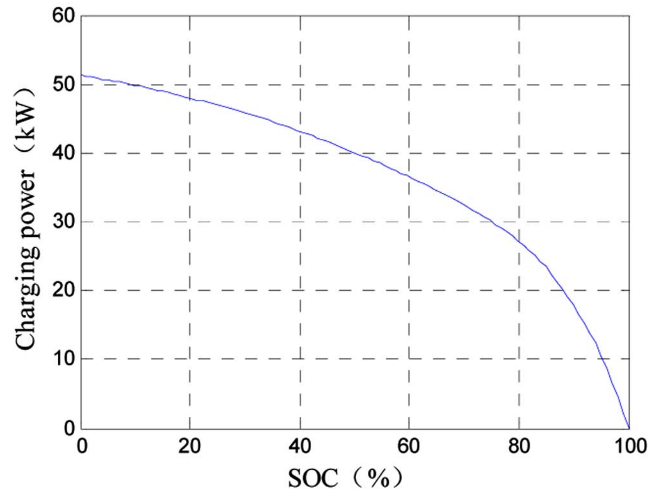

Fig. 1. The SOC curve.

From the power expression $P = VI$, the charging power of EV battery should not exceed a certain limited value. The charging power is constrained by (3)

$$0 \le P \le P_{\mathrm{battery}}(t). \tag{3}$$

In (3), $P_{\mathrm{battery}}(t)$ represents the maximum acceptable charging power of EV battery in time t, which is the function of SOC and temperature of battery. Temperature effect on $P_{\mathrm{battery}}(t)$ can be ignored when some measures are taken to keep the temperature of battery constant. Then the maximum acceptable charging power can be expressed as below

$$P_{\mathrm{battery}}(t) = f(S). \tag{4}$$

In (4), S is the current SOC of EV battery. The quantitative relationship between $P_{\mathrm{battery}}(t)$ and S can be described by SOC curve which is shown in Fig. 1 [22].

Except for the restriction P_{battery} which is the maximum acceptable charging power of EV battery, the maximum charging power is also restricted by: 1) the maximum power P_{user} set by EV user: 2) the maximum power P_{charger} EV charger can output. Therefore, the actual maximum power in charging process is

$$P_{\max} = \min\{P_{\mathrm{user}}, P_{\mathrm{charger}}, P_{\mathrm{battery}}\}. \tag{5}$$

Usually, both the maximum power P_{user} set by EV user and the maximum power P_{charger} EV charger can output are greater than the maximum acceptable charging power P_{battery} of EV battery. So the actual maximum charging power $P_{\max}$ is limited by P_{battery} in most cases. All constraints of optimized model presented above are consisted of expressions (2)–(5).

III. Algorithm

As a continuous mathematical model, in order to be convenient for calculation, the above optimized model is discretized.

The total charging time T is divided into N periods, and the length of each period is Δt. The discretized optimized model can be expressed as

$$\min C = \sum_{i=1}^{N} M(t_i)P(t_i)\Delta t. \tag{6}$$

The constraint of maximum charging power limited by SOC is nonlinear, so we design a heuristic algorithm.

1) Charge the EV battery using the maximum power $P_{\max}$ from the starting time $(t = t_1)$ until the battery SOC $= 1$ or $t = t_N$ (user-specified end time). If the battery is not filled until $t = t_N$, stop optimization. Otherwise, go to step 2).
2) An initial feasible solution $P_0 = [p_1, p_2, \ldots, p_N]$ (p represented the charging power) is obtained after step 1). Sorting the N periods based on TOU price is required. Symbols i and j $(i = 1, 2, \ldots, N;\ j = 1, 2, \ldots, N)$ are ascending sorted sequences, which means $M(t_{i+1}) > M(t_i)$, and $M(t_{j+1}) > M(t_j)$.
3) Set charge energy q as optimal step. The charge energy q transferred from high-price period to low-price period is defined as optimal step. The charge power e transferred from high-price period to low-price period can be expressed as $e = q/\Delta t$. In order to improve the precision of the simulation, q should take a small value such as 10^{-6}.
4) Assign $i = N$.
5) Assign $j = 1$.
6) Judge whether the energy left in period t_i is available for transferring. When $P(t_i) > e$, go to step 7); otherwise, go to step 11).
7) Judge the sequence of period t_i and period t_j. If $t_i < t_j$, go to step 8), which means it will not break the SOC constraint when transfer energy q from period t_i to period t_j directly. The reason is as follows: Assume $Q(t_n)$ and $Q(t'_n)$ indicate the energy stored in battery before and after the transference respectively. When $n < j$, there exist $Q(t'_n) = Q(t_n)$ $(n < i)$ or $Q(t'_n) = Q(t_n) - q$ $(n \geq i)$; when $n = j$, there exists $Q(t'_n) = Q(t_n)$. It can be seen clearly that the SOC of EV battery in each period does not increase. As the charging power does not exceed limit before the transference, the charging power cannot exceed limit after the transference either. If $t_i > t_j$, go to step 9).
8) Transfer the energy from the high-price period to the low-price period. It can be expressed using mathematical equations as: $P(t_i) = P(t_i) - e$; $P(t_j) = P(t_j) + e$. After that, the procedure goes to step 6).
9) Judge whether $P(t_j)$ is sufficient to achieve the maximum charging power $P_{\max}$. If $P(t_j) < P_{\max}$, go to step 8); Otherwise, go to step 10).
10) Set $j = j + 1$ and judge whether the price of period t_j is equal to the price of period t_i. If $M(t_i) = M(t_j)$, go to step 11); Otherwise, go to step 7).
11) Set $i = i - 1$ and judge whether i is equal to 1. It means there is no more energy can be transferred from the high-price period to the low-price period when $i = 1$, so stop optimization. Otherwise, go to step 5).

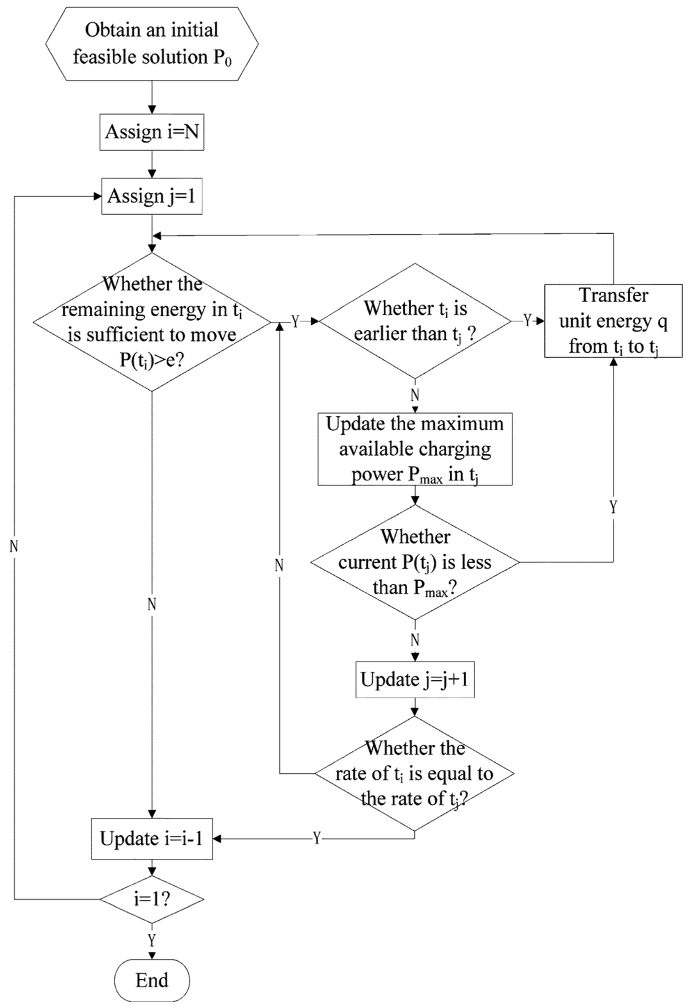

Fig. 2. The flowchart of optimization charge.

The flowchart of above optimization program is shown in Fig. 2.

IV. Case Study

In order to verify the effectiveness of optimized charging model presented in this paper, we adopt a typical charging pattern for comparison. The typical charging pattern is a common charging pattern of "plug and charge," the charging profile of which is consistent with the charging characteristics of battery. Nevertheless, the charging profile may not follow the charging characteristics of battery in optimized charging pattern, as it is determined by optimized charging algorithm which has been stated in previous section of this paper. The charging cost and energy demand in different times are compared separately in two cases: single EV and multi-EV. In the multi-EV case, the diversity of initial SOC and starting charging time among different EV should be considered. A probability model is introduced to describe this diversity, which will be discussed later. For single EV case, we neglect the randomness of initial SOC and starting charging time and assign them with specified value so as to get determined results, which is beneficial for us to understand the optimized charging model. In the following section, we will present settings and results of simulation.

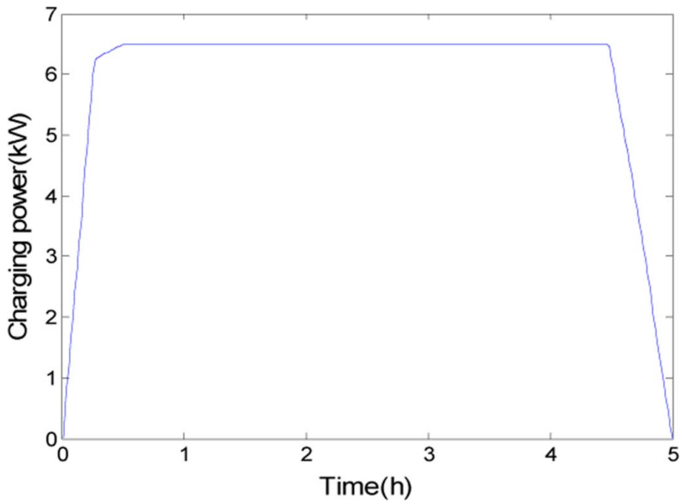

Fig. 3. The charging curve of the lithium-ion battery equipped in Nissan Altra EV.

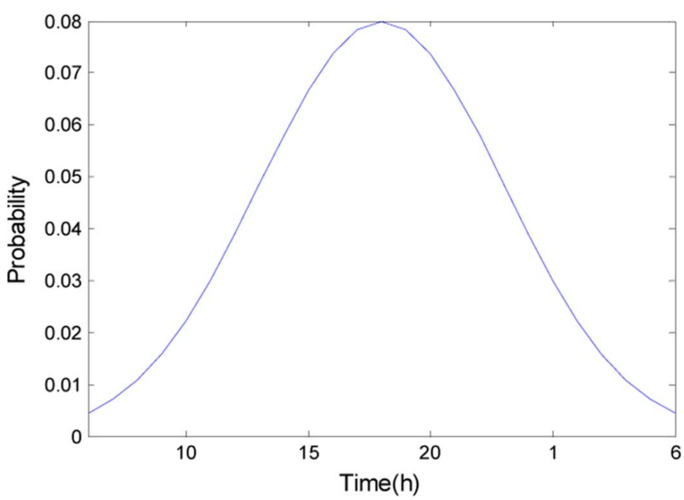

Fig. 4. The distribution curve of the starting time of charging.

A. Settings of Simulation

1) Typical Charging Characteristics of Lithium-Ion Battery: The charging curve of the lithium-ion battery equipped in an Nissan Altra EV is shown in Fig. 3 [23]. In completely discharging situations, the demand for energy is 29.07 kWh. In this study, we will employ the charging curve in Fig. 3 for typical charging pattern.

2) The Starting Time of Charging: It has much randomness at the starting time of charging. To describe the randomness, it established distribution model of the starting time. Assuming that the distribution of the starting time obeys a Gaussian distribution, that is,

$$f(t,\mu,\sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(t-\mu)^2/(2\sigma^2)}. \tag{7}$$

Most EV users start charging when return home from work at 18:00 and more than 90% of EV users' starting time for charging is between 13:00 and 23:00. Therefore, in this case, it takes μ for 18 and takes σ for 5 [17]. The probability distribution of the starting time is shown in Fig. 4.

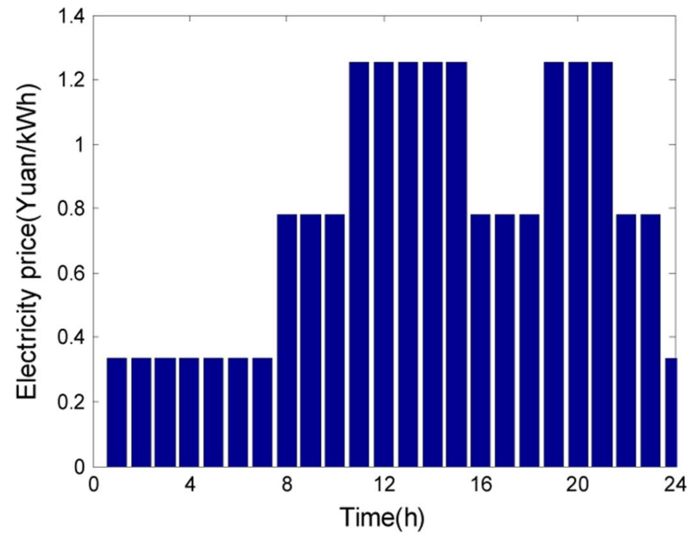

Fig. 5. The histogram of TOU price.

3) The Initial SOC: The initial SOC of EV battery also has some certain randomness. Using probability distribution model, it is described as

$$f(\mathrm{s},\mu,\sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(\mathrm{s}-\mu)^2/(2\sigma^2)}. \tag{8}$$

s represents the initial SOC of EV battery and it is commonly between 0.2 and 0.8. It takes μ for 0.5 and takes σ for 0.3 and μ is the average value of SOC and σ is standard deviation.

4) TOU Price: According to actual TOU price implemented in Beijing, the period of valley load is defined as 23:00–07:00, totally 8 h; the period of peak load is defined as 10:00–15:00 and 18:00–21:00, totally 8 h. The remaining time is the period of flat load. Adopting the actual price of electricity in the city, the prices of peak, flat and valley load period are 1.253 Yuan/kWh, 0.781 Yuan/kWh and 0.335 Yuan/kWh. The histogram of TOU price is shown in Fig. 5.

B. The Results and Analysis of Simulation

1) The Case of Charging for Single EV: In order to verify the effectiveness of optimized charging model and be convenient for observing the optimization process, the case of charging for single EV is implemented. Taking 20:00 as the starting time of charging, twelve hours as the total charging time, the energy demand of typical charging pattern and optimized charging pattern in different time are shown in Fig. 6. As it can be seen, optimized charging pattern can avoid peak demands and choose intelligently to charge in the time of valley demands which can reduce charging costs.

2) The Case of Charging for Multi-EV: The starting time of charging and initial SOC should be considered in the case of charging for multi-EV. Therefore, the Gaussian distribution function is implemented to describe the differences. According to data of 2010, the number of automobiles in Beijing was reached to 4.69 million [24]. Assuming the penetration of EVs (defined as the ratio of the number of EVs and the total number of automobiles [12]) is 5%, the number for EVs is about 234 500. In the process of computation, the average charging time of EV is about 6 h. Fig. 7 shows the energy demand of

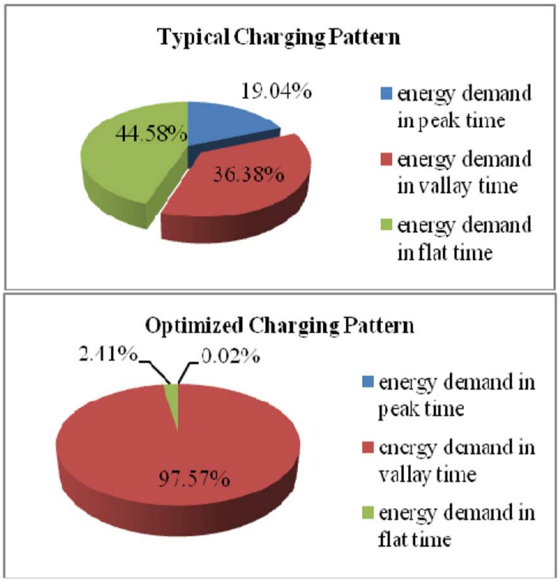

Fig. 6. The comparison of different charging patterns for single EV.

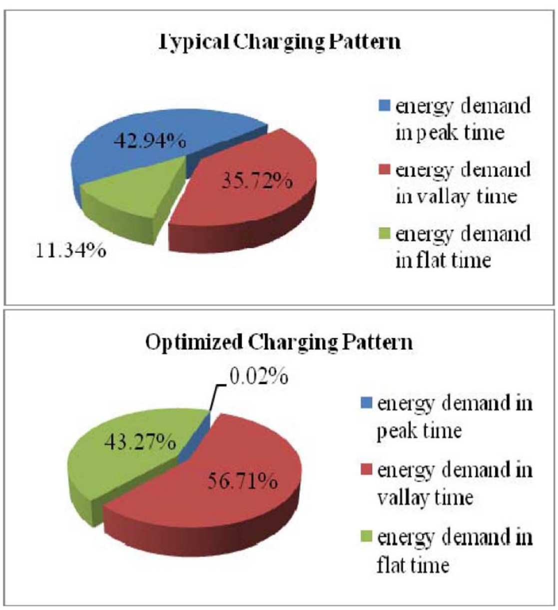

Fig. 7. The comparison of different charging patterns for multi-EV.

different charging patterns in multi-EV charging case. As it can be seen, the optimized charging pattern in multi-EV charging case can shift a mass of peak load to valley load, which is similar with that of single charging case.

Table I lists the charging cost of different charging patterns in different cases. It is clearly seen that the optimized charging pattern can bring a significant reduction in the cost of EV users' charge. And it is evident that the performance of single EV case shown in the table is better than that of multi-EV case. The reason may be attributed to the available optimal space. In single EV case, the time span for charging covers more low-price periods, which means the optimal space is relatively large. So, through the optimization, almost all of the charge will be concentrated in low-price periods, which results in a better performance. From another perspective, it is implied that the charging

TABLE I
THE CHARGING COST OF DIFFERENT CHARGING PATTERNS IN DIFFERENT CASES

	Typical Charging Pattern	Optimized Charging Pattern	Reduction in cost
The charging cost of single EV(Yuan)	20.673	10.02	51.52%
The charging cost of Multi-EV($10^4\times$Yuan)	2179.4	1314.9	39.67%

cost will be less if EV users can consciously arrange appropriate plug-in time.

V. Conclusion

This paper proposes an intelligent charging method for EV charging facilities in response to TOU price. The purpose is to alleviate the stress in power grid under peak demand and to meet the demand response requirements in regulated market.

A comparative analysis of typical charging pattern and optimized charging pattern for charging performance in different case is also presented. Simulation results of both single EV and multiple EVs have validated the effectiveness of the proposed approach. This work serves as a useful reference for research on charging strategies in open electricity market.

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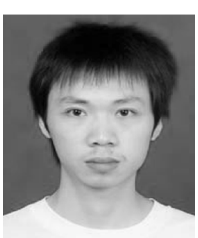

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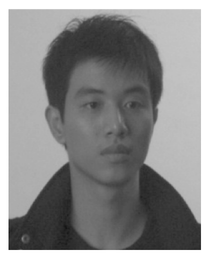

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