Combined WDM and SONET Network Design

O. Gerstel
Optical Network Group
Tellabs Operations Inc.
15 Saw Mill River Rd
Hawthorne, NY 10532
ori@tellabs.com

P. Lin
Tellabs Res. Ctr.
Tellabs Operations Inc.
One Kendall Square Bldg. 100
Cambridge, MA 02139
pjlin@trc.tellabs.com

G. Sasaki
Dept. Elec. Engr.
U. Hawaii
2540 Dole St.
Honolulu, HI 96822
sasaki@spectra.eng.hawaii.edu

Abstract

This paper considers grooming of low speed traffic into high speed lightpaths in a WDM based optical ring with a primary goal of reducing the cost of the entire system, which is dominated by the cost of the SONET transmission equipment connected to the optical ring. The paper attempts to enumerate the architectural options provided by SONET to arrive at a cost-effective solution, including UPSR and BLSR rings, use of back-to-back connections between SONET ADMs to reduce the overall cost, and use of different ring speeds (OC-48 and OC-12).

To demonstrate each of the architectures, a uniform traffic is considered and its grooming and resulting SONET architecture demonstrated. The paper deviates from earlier approaches which break the problem into two steps: traffic grooming and assignment of lightpaths to rings, in that it looks at the problem as a whole and tries to solve it in a single step. The paper also considers the characteristics of SONET UPSR and BLSR rings and how these affect the grooming. The paper derives lower and upper bounds to these problems for uniform traffic and shows how these improve on known results.

I. Introduction

Wavelength division multiplexing has enjoyed quick and very successful commercial acceptance due to its simplicity and low cost in comparison to the alternative TDM solutions. The next step in the same direction, of optical networking based on wavelength routing, is believed to take a similar path from laboratories and test-beds into commercial deployment in the near future. However, the success of this step is very dependent on the economical case for it: will optical networking provide a substantial cost saving over deployment of point-to-point WDM links, interconnected by TDM equipment?

The key to the answer is the amount of optical passthrough that is possible in a given network. Such passthrough provides the means for potentially large savings since it reduces the number of TDM line terminals needed. If this passthrough is provided via an optical add-drop multiplexer, its cost is very low in comparison to the TDM equipment it replaces. This still holds true (albeit to a lesser extent) even if the passthrough is supported via point-to-point WDM multiplexers connected back-to-back via transponders.

The amount of optical passthrough depends on the traffic pattern itself, but also on how it is *groomed.* In other words, the traffic can be grouped into *lightpaths* so that the traffic streams avoid being dropped at intermediate nodes. (Here, a *lightpath* is an optical communication connection between two TDM line terminals.) This means that a large fraction of the traffic streams groomed through a lightpath should terminate where the lightpath terminates.

As noted in [1], in order to optimize the cost of a network it does not suffice to determine the lightpaths needed for a given traffic pattern. One also needs to take into account the higher layer that will use these lightpaths and its particular topological needs. This is especially relevant for the higher layer being SONET — which is the most likely option, at least for the short term. Due to its stringent protection requirements, SONET's base topology is a ring (or a protected point-to-point system, which is equivalent to a ring). Furthermore, SONET rings are divided into two main architectures: bidirectional line-switched rings (BLSR) and unidirectional path-switched rings (UPSR) and these architectures impose additional constraints on how the lightpaths are used. SONET rings can also be interconnected and traffic routed from one ring to another. While this is beyond the scope of the optical network design, taking it into account can improve the grooming and the overall cost of the network.

In this work we take all of the above considerations into account while determining the grooming of low-speed traffic streams into high-speed lightpaths. A simpler form of this question has been discussed in a number of recent works [2], [1], [4], [5], however, none of these works provides a comprehensive framework for dealing with the issue which takes into account the entire gamut of parameters with respect to the SONET architecture.

In particular, [2], [1] break the entire problem into

This work was sponsored in part by grant F30602-97-1-0342 by DARPA and Rome Laboratory, Air Force Materiel Command, USAF. The U.S. Government is authorized to reproduce and distribute reprints for Governmental purposes notwithstanding any copyright annotation theorin. The views and conclusions contained therein are those of the authors and should not be expressed or implied, of the Defense Advanced Research Projects Agency (DARPA), Rome Laboratory, or the U.S. Government.

two steps: (1) grooming the low speed traffic into high speed lightpaths; and (2) grouping the lightpaths into SONET BLSR rings. While this approach serves to simplify the problem, there may be a potential 20% cost savings in ADMs if both problems are considered in one step.

In [4], uniform traffic patterns in UPSR networks are considered. An implicit assumption in [4] is that traffic has to be serviced by a single UPSR from source to destination. While this is certainly an option, it is also possible to route traffic between UPSR rings either via digital cross-connects (DXCs) or by connecting low speed (tributary) interfaces of the participating SONET ADMs back-to-back. The potential cost savings of this added flexibility is up to 37.5%, as demonstrated in Section IV.

BLSR rings are considered in [5] as well with the objective to minimize ADM cost. Again, the uniform traffic pattern is considered. However, no comparison is made with UPSR networks, as we will do in Section V.

A. Overview of the paper

The paper is structured as follows. In Section II we provide a brief introduction to SONET architectural aspects including the rings architecture, the interconnection patterns between rings, and the cost model.

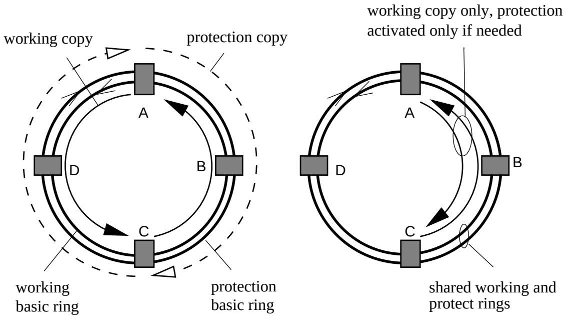
In Section III we describe the general network design goal and the restricted design problem we are studying in the current paper. In Section IV we explore the use of UPSR rings to accommodate uniform traffic. In particular, we compare our results to those of [4] and present a network designs that performs better than their bounds. The reason for the improvement stems from the fact that we allow interconnection of rings in support of the lower speed traffic whereas [4] assumes the low speed traffic is routed through the same SONET ring from source to destination. In Section V we explore the use of BLSR/2 rings to accommodate uniform traffic. We provide an example which demonstrates that the two step approach presented in [2], [1] can lead to a 20% less cost-efficient design. We also compare BLSR/2 with UPSR, and show there exist a case where a UPSR is a better approach while in other cases BLSR/2 performs better.

In Section VI we revisit our earlier results and modify them to support a mix of OC-12 and OC-48 rings. This demonstrates the merit of not using the highest possible line rate on each lightpath. We also discuss criteria for using UPSR vs. BLSR rings and to mix the two on a single WDM ring. We summarize the paper in Section VII.

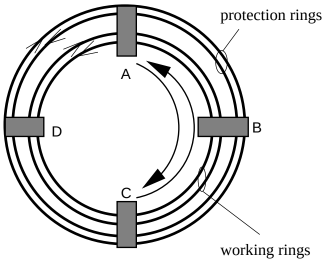
II. SONET network architectures

A. Self healing rings

The standard bodies have defined three types of SONET self-healing rings in addition to point-to-point (linear) protected links (see e.g., [3] for a survey). A *unidirectional path-switched ring* (UPSR), shown in Figure 1(a) is based on a pair of fibers between each pair of adjacent nodes, each running half-duplex traffic. These fibers constitute two unidirectional counter-propagating “basic rings”. Transmitter A sends data to C on one of the basic rings in the clockwise direction. C also sends data to A on that ring in the clockwise direction and at the same rate. In this way, A and C have full-duplex communication. Simultaneously A and C both send another copy of the same data on the second basic ring in the counter-clockwise direction. The receiver of A receives 2 copies of the data and selects the one that is better. If that copy fails, the receiver switches to the other copy.



(a) UPSR (b) BLSR/2



(c) BLSR/4

Fig. 1. The different types of SONET self-healing rings

In Figure 1(b) is shown a *two-fiber bidirectional line-switched ring* (BLSR/2). This ring comprises two unidirectional counter-propagating basic rings as well. In this case, to implement full-duplex communication between A and B, A transmits to C along the shortest path in the ring in the clockwise direction but C transmits to A in the counter-clockwise direction along the same route on the other ring. (Thus, the transmissions for the full-duplex communication use the same side of the ring which is how BLSR/2 is implemented in practice.) 50% of capacity in each ring is reserved to handle failures. If a link fails, the nodes at the ends of the link switch the traffic on to the spare capacity on the other ring. The traffic is looped back around the ring back to the link after the fault.

A 4-fiber BLSR (BLSR/4) as in Figure 1(c), is similar to a BLSR/2 except that it uses four basic rings (a pair in each direction) and provides a higher degree of protection. Two of the basic rings are working rings and the other two are protection rings. The traffic is normally sent along shortest paths on the working rings, up to the full utilization of these fibers (as opposed to the BLSR/2 case). If a working fiber on a

Criterion	**UPSR**	**BLSR/2**	**BLSR/4**
Protection mechanism	1+1 protection	ring protection	ring and linear protection
Protected entity	each connection (path) separately	the entire line	the entire line
Max working capacity per connection	100% of line speed	50%	100%
Max aggregate working capacity in a ring of N nodes	$1\times$ line speed	$N\times$ 50 % line speed	$N\times$ 100 % line speed
Traffic pattern	Ideally to a single-hub	Ideally between close by nodes	Ideally between close-by nodes
Typical application	Access ring	inter-office ring	Long distance inter-office ring
Management	Easy	Harder	Similar to BLSR/2
Cost	X	Slightly more	twice more

Fig. 2. A comparison of different SONET ring architectures

link fails, the nodes at the end of that link switch the traffic to the protection fiber on the same link. If the protection fibers have failed as well (or a node has failed), then the traffic is switched to the protection fiber around the ring.

A simpler form of SONET protection exists in point-to-point SONET links. Such links use a diversely routed fiber to provide for 1+1 linear protection. This protection is based on sending two copies of the data on two disjoint routes. The receiving side determines which copy is better and receives it, in similarity to UPSR. To simplify the presentation we assume this option to be identical to a two node UPSR and do not discuss it further. These SONET rings are briefly compared in Table 2.

B. Ring interconnections

A closer look at a SONET ADM and on how these ADMs are interconnected is provided in Figure 3. In it, a pair of OC-48 ADMs are depicted. The high-speed lines of these ADMs are 2.5 Gbps and are depicted in bold lines. These lines are fed into lightpaths and are multiplexed together onto a fiber via WDM (not depicted). The tributary (port side) interfaces are of lower speed (155 Mbps in the figure). The number of port side interfaces corresponds to the amount of traffic that can be add-dropped at the ADM. Also depicted in the figure are four possible interconnection patterns in and between ADMs:

- Connecting the ADM to lower-speed equipment, such as other SONET multiplexers, IP routers or ATM switches — see the connection marked (a) in the figure,
- Connecting the tributary interfaces to each other, as depicted by interconnection (b),
- Cross-connecting the tributary interfaces via a digital cross-connect as depicted in option (c), and
- Passing a traffic stream through an ADM without dropping it, as depcited in option (d).

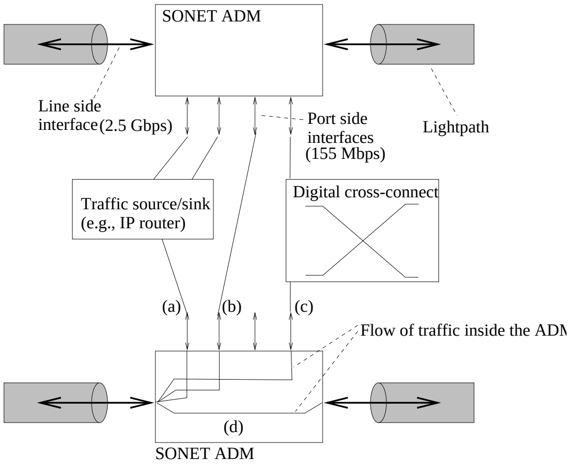


Fig. 3. The interfaces of a SONET ADM and how they may be interconnected (all interfaces comprise of a pair of simplex cables).

C. Cost model

As for the cost model for the different options, we assume the following rules, approximating realistic costs:

1. The cost ratio between an OC-4n ADM and an OC-n ADM is 2.5. In other words, a 4-fold growth in the capacity corresponds to a 2.5 growth in the cost.
2. The cost of a BLSR/4 is about twice the cost of a BLSR/2. It follows that the cost of passing X bits of traffic is very similar whether one is using a UPSR, BLSR/2 or BLSR/4. Given the similarity between BLSR/2 and BLSR/4 in both the traffic they can handle and the cost per bit, we ignore BLSR/4 from now on.
3. The cost of a lightpath is low w.r.t. the cost of the terminating equipment. This is especially true if optical passthrough is supported since the relatively high cost of transponders can be considered as part of the cost of the terminating equipment (since they are needed only to terminate the lightpath),
4. The cost of a tributary interface is low w.r.t. the cost of the line side interface. Again, this is accentuated with WDM because the added transponder costs can be considered part of the line side interface (for cost purposes only),
5. The cost of cross-connecting traffic through a digital cross-connect is non-negligible, however, given our static traffic pattern assumption, we will not be using this interconnection option. In-

stead we cross-connect rings by using the static interconnection type (b) as depicted in Figure 3.

III. Design goals

We first describe the general design goal and then restrict it to a more specific one, which enables us to provide analytical results and reasonable comparisons.

A. General goal

Given a physical topology comprising of sites interconnected via WDM links, a traffic pattern between sites (projected or real traffic, expressed as a number of lower speed traffic streams between the sites), and the amount of expected dynamism in the traffic, determine the following:

1. The type of optical nodes at each site: an optical line terminal (OLT), fixed optical ADM (OADM), switchable optical ADM or optical cross-connect (OXC),
2. The set of lightpaths that are routed through the optical network,
3. The set of SONET ADMs and LTEs to terminate the lightpaths,
4. The protection strategy to be used (UPSR, BLSR, linear protection etc.),
5. The line speed of each of the rings (OC-12, OC-48, or OC-192),
6. How the SONET ADMs are interconnected to service the given traffic streams (back-to-back, via DXCs).

The goal is to minimize the overall cost of the solution, including the following components:

1. Optical nodes, including transponders, optical amplifiers, optical switches, etc.,
2. SONET ADM costs, including line and port interfaces, and
3. Digital cross-connect costs.

B. Restricted design problem

While the above design goal is desired, it will not facilitate simple analysis and comparison. Therefore we focus on the following more limited goal, which is still a reasonable approximation of the general goal.

We will consider a single physical WDM ring comprising of N nodes, where the nodes are numbered $0, 1, ..., N-1$ going clockwise. Since we are only considering UPSR and BLSR/2 systems, we assume that the physical WDM ring comprises two unidirectional counter-propagating basic fiber rings. Let W denote the number of wavelengths in the network numbered $0, 1, ..., W-1$. If the WDM ring is a WDM UPSR network then each wavelength supports a UPSR ring. In other words, the WDM UPSR network operates as W UPSR rings. Traffic between the different UPSR rings may be cross-connected at nodes if the ADMs are available. Similarly, if the WDM ring network is a WDM BLSR/2 network then each wavelength supports a BLSR/2 ring. Again, traffic between different BLSR/2 rings may be cross-connected at nodes if the ADMs are available.

The traffic is assumed to be static and have a *uniform* pattern. In particular, each pair of nodes have r low speed (tributary) traffic streams between them. Thus, they have full-duplex communication between them. For example, if the low speed traffic streams are OC-3 then each pair of nodes will have r full duplex OC-3 communication. Each wavelength has a line speed indicated by a parameter g (for *granularity*). g is the number of low speed streams that can fit into a line. For example, if each wavelength is a SONET OC-48 ring then $g = 16$ since $16 \times$ OC-3 rate = 1 OC-48 rate (recall, OC-3 = 155 Mb/s and OC-48 = 2.5 Gb/s). Notice that a UPSR ring (at a wavelength) can support g unidirectional traffic streams on each link because transmission is at full rate in one direction around the ring. On the other hand, a BLSR/2 ring (at a wavelength) can support $g/2$ bidirectional (or full duplex) traffic streams on each link because transmission on a link is at half rate (the other half is for protection).

The goal is to minimize the cost of the SONET rings, while neglecting the costs of the optical layer — this is a valid assumption as long as the number of optical amplifiers is low. Therefore it holds for metropolitan networks more than for long-distance networks. The cost of port side interfaces and the interconnection between them are also neglected herein. The primary cost of interest is the number of ADMs.

IV. Grooming in WDM UPSR

In this section, we will describe traffic grooming in UPSR WDM rings. We will describe the *single-hub ring* which leads to efficient use of ADMs. The architecture assumes that low speed traffic may be cross-connected at nodes, which make it more efficient in utilizing ADMs. It will be compared to the results in [4], which assumes no cross-connection of low speed traffic streams.

Before discussing the single-hub ring, we present a lower bound on the number of ADMs in a WDM UPSR ring.

Theorem 1: If $r \leq g$ then the number of ADMs in a UPSR WDM ring for the uniform traffic is at least

$$\max\{\left\lceil 2N(N-1)\frac{r}{g+r}\right\rceil, N\}.$$

Proof. The second term in the maximum is trivial, so for this proof we will only consider the first term. Consider a low speed traffic stream. If it traverses only one lightpath then we say that it is fully *supported* by the lightpath. More generally, if it traverses a multiple m lightpaths then we say that each lightpath *supports* an amount $\frac{1}{m}$ of it. (For example, if $m = 3$ then each lightpath supports a third of it.)

Now consider a lightpath. We will determine an upper bound on the amount of traffic streams it supports. Notice that some of the traffic streams only traverse this lightpath. Let n_1 denote their number. The rest of the traffic streams traverse other lightpaths as well. Let n_2 denote their number, and notice that the lightpath supports at most $\frac{1}{2}$ of these streams. Thus, the lightpath supports at most $n_1 + \frac{1}{2}n_2$ amount of traffic streams. This is at most $r + \frac{1}{2}(g-r)$ because $n_1 \leq r$

(from the uniform traffic assumption) and $n_1+n_2 \leq g$. Thus, the amount of traffic streams per lightpath is at most $r+\frac{1}{2}(g-r)$, which can be rewritten as $(r+g)/2$.

The total number of traffic streams is $N(N-1)r$. Therefore, the number of lightpaths is at least $2N(N-1)r/(g+r)$. The lemma is proven because the number of ADMs is equal to the number of lightpaths. □

The single-hub UPSR WDM ring architecture has one node, say node 0, as the *hub*, where traffic is cross-connected. The hub has ADMs at every wavelength to cross-connect all traffic going through it. The other nodes (i.e., the non-hubs) route their traffic streams to and from the hub. All traffic streams go from their sources to the hub, and from the hub to their destinations. Thus, each traffic stream takes two hops, unless either its source or destination is the hub and then it takes one hop. It turns out that if r is much less than g then the architecture has a number of ADMs that is close to the lower bound in the theorem. Intuitively, this follows from the fact that if r is much less than g then an efficient architecture has most of the traffic streams traversing two lightpaths.

Each of the non-hub nodes have $k = \lfloor (N-1)r/g \rfloor$ wavelengths dedicated to it. On each of the k wavelengths there are two ADMs, one at the non-hub node and the other at the hub. On these wavelengths the non-hub sends and receives $g \cdot k$ traffic streams. Since the non-hub sends and receives $(N-1)r$ traffic streams, there may be some traffic left over. In particular, this amounts to $T_f = (N-1)r - g \cdot k$ traffic streams being sent to and from the hub which we refer to as the *fractional traffic* for the non-hub.

To efficiently use resources, fractional traffic from different non-hubs share wavelengths (and the ADM at the hub). The number of non-hubs that can share a wavelength is $\lfloor g/T_f \rfloor$. The number of wavelengths used for fractional traffic is

$$\lambda_f = \begin{cases} 0, & \text{if } T_f = 0 \\ \left\lceil \frac{(N-1)}{\lfloor g/T_f \rfloor} \right\rceil, & \text{otherwise} \end{cases}$$

Note that the number of wavelengths for the single-hub ring is

$$(N-1)k + \lambda_f,$$

where the first term is the number of wavelengths dedicated to non-hubs and the second term is the number of wavelengths for fractional traffic. Also note that the number of ADMs for the ring is

$$(N-1)\lceil (N-1)r/g \rceil + ((N-1)k + \lambda_f),$$

where the first term is the number of ADMs at the non-hubs and the second term is the number of ADMs at the hub.

For example, consider the case when $N = 6$, $r = 1$, and $g = 4$. Each non-hub node must source and sink 5 traffic streams. Each non-hub node has its own wavelength that carries 4 traffic streams to and from the hub. The wavelength has an ADM at the non-hub node and hub only. Each non-hub node has one remaining traffic stream to and from the hub. Four of the non-hubs can share a wavelength to carry their fractional traffic. The wavelength has five ADMs: one at each non-hub that uses it and one at the hub. Another wavelength is used by the remaining non-hub for its fractional traffic, and it has an ADM at the non-hub node and hub. Thus, the network uses a total of 7 wavelengths, and a total of 17 ADMs.

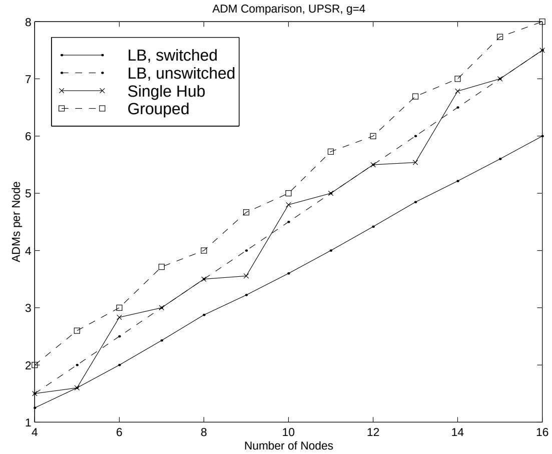


Fig. 4. ADM cost per node when $g = 4$ and $r = 1$.

Figure 4 shows the average number of ADMs per node versus N for the case when $r = 1$ and $g = 4$ for the single-hub network. Also plotted is the lower bound of Theorem 1. The lower bound curve is labeled “LB,switched”. There are two other curves in the figure which are taken from formulas in [4]. The curve labeled “LB,unswitched” is a lower bound on the number of ADMs assuming that traffic streams cannot be cross-connected. The curve labeled “Grouped” is the number of ADMs required by a network architecture proposed in [4]. We will refer to this architecture as the *Grouped* architecture because it is based on the nodes being partitioned into groups, and then pairs of groups share wavelengths to communicate with one another. The Grouped architecture disallows cross-connected traffic streams.

As you can see in the figure, the single-hub network always has less ADMs than the Grouped architecture, and frequently has less ADMs than the lower bound that assumes no cross-connection. Of course, the price to be paid using the single-hub ring is a large number of wavelengths, which is approximately twice that of the Grouped architecture. For example, if $N = 8$ then the number of wavelengths for the single-hub is 14, while the number of wavelengths for the Grouped architecture is 8. What Figure 4 demonstrates is that if ADM cost is dominant and wavelengths are plentiful then cross-connection can lower network cost.

Figure 5 shows the number of ADMs per node as a percentage of the lower bound (of Theorem 1) for $r = 1$ and $g = 4$. Figure 6 plots the average number of ADMs per node when $g = 16$ and $r = 1$. Notice that the difference between the curves that allow and disallow cross-connection is greater, especially for large N. For example, when $N = 16$, the number of ADMs

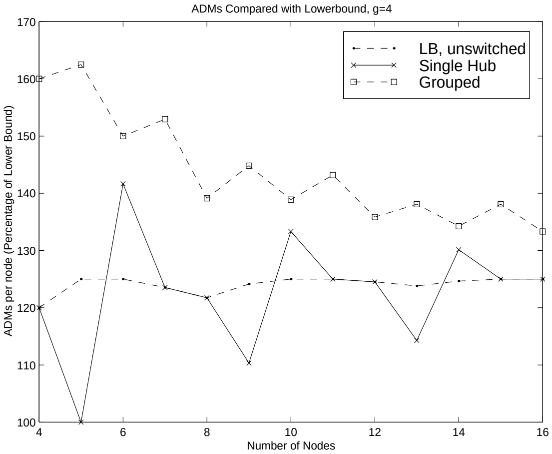


Fig. 5. Percentage of ADMs per node over the lower bound assuming traffic may be cross-connected, for $g = 4$ and $r = 1$.

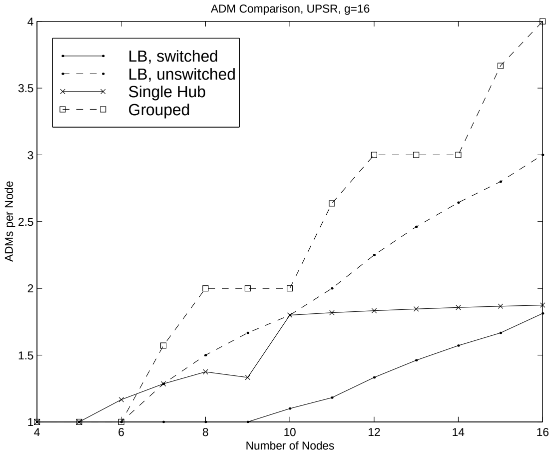


Fig. 6. ADM cost per node when $g = 16$ and $r = 1$.

per node for "LB,unswitched" is 3, while the number of ADMs per node for the single-hub network is 1.875. Thus, a network with cross-connection of traffic, such as the single-hub network, has a savings of 37.5% over any network without any cross-connection. Figure 7 plots the number of ADMs per node as a percentage of the lower bound (of Theorem 1) for $g = 16$ and $r = 1$.

V. Grooming in WDM BLSR/2

In this section, we will consider BLSR/2 WDM ring networks. We will first give an example where two step approach to designing a network is less cost efficient. Then we will compare BLSR/2 with UPSR.

A. Two-step approach is not always cost efficient

Consider a WDM BLSR/2 network with $N = 5$ nodes, $r = 1$, and $g = 8$. If the low speed (full duplex) traffic streams followed shortest hop paths, the number of streams across any link would be 3. Since $g/2$ (= 4) streams can be supported on each wavelength on each link, the number of wavelengths required to support the traffic is $W = 1$, and the number of ADMs is 5.

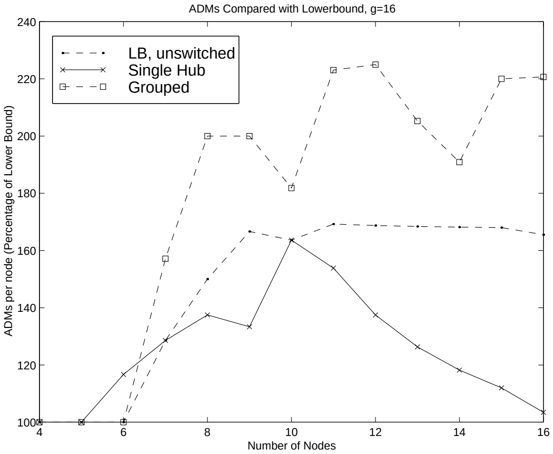


Fig. 7. Percentage of ADMs per node over the lower bound assuming traffic may be cross-connected, for $g = 16$ and $r = 1$.

Now suppose the WDM BLSR/2 network were designed using the two-step approach. The first step is to find a virtual topology that will carry all the traffic, where the links of the topology can each carry $g/2$ traffic streams. The virtual topology must be a tree because (a) a tree has the minimum number of links of any connected topology, and (b) there is at least one tree (the star) that can carry the traffic (in particular, one node is designated the star's hub and the other nodes send their $N-1$ traffic streams directly to it on a virtual link).

The second step is to layout the virtual topology on the WDM BLSR/2 ring. Now suppose that the virtual topology can be layed out on one BLSR/2 ring (i.e., on one wavelength). Since the virtual topology is a tree, there must be some link in the ring without traffic. This implies there is a link with $2\times 3 = 6$ traffic streams passing through it because it is between two sets of nodes, where one set has two nodes and the other set has the remaining three nodes. The link cannot support 6 traffic streams (its limit is $g/2 = 4$), so we can conclude that the virtual topology cannot be layed out in a single BLSR/2 ring.

The number of ADMs must be at least 6 because (i) each node must have at least one ADM, and (ii) there must be some node with more than one ADM, otherwise nodes with ADMs at different wavelengths (i.e., different BLSR/2 rings) cannot communicate. Therefore, the two-step approach can lead to twice as many wavelengths and 20% more ADMs. Note that these arguments can be easily extended to integer multiples of the above, yielding a more general result.

B. BLSR/2 vs. UPSR

We will now compare the costs of UPSR with BLSR/2. One would expect that a well designed WDM BLSR/2 ring will have lower cost than a well designed WDM UPSR ring because traffic in BLSR/2 is bidirectional and can take advantage of spatial reuse of bandwidth, while UPSR is unidirectional and does not allow spatial reuse. This would seem to be the case for most instances. However, we have one example where UPSR does a little better than BLSR/2.

Example 1: In this example we have a $N = 4$ and $g = 2$. We assume that the traffic is nonuniform, which is a departure from the uniform traffic pattern assumption that we have made throughout the paper. In particular, nodes 0 and 2 have a full duplex traffic stream between them, and nodes 1 and 3 have a full duplex traffic stream between them as well. A well designed WDM UPSR will require only one wavelength and an ADM per node. A WDM BLSR/2 network requires at least two wavelengths since the two traffic streams will overlap at some link. Thus, a well designed BLSR/2 network will have two wavelengths and an ADM per node. Thus, compared to BLSR/2, UPSR has the same number of ADMs (i.e., primary cost), but only half as many wavelengths (i.e., secondary cost).

For the remainder of this section, we will compare UPSR and BLSR/2 using the uniform traffic. In general, for this traffic, BLSR/2 is less costly than UPSR. To illustrate this for $r = g/2$, we will use the following lower bound on the number of ADMs required for a BLSR/2 ring. The proof of the lower bound will be omitted because it follows the same arguments in the proof of Theorem 1. (We should note that there is another lower bound presented in [5] although no general formula was given. However, the bound is different since it can be checked that for the special case of $g/4 = r$, their bound is tighter for small to moderate N, while our bound is tighter for large N.)

Theorem 2: If $r \leq g/2$ then the number of ADMs in a BLSR/2 WDM ring for the uniform traffic is at least

$$\max\{\left\lceil N(N-1)\frac{r}{(g/2)+r}\right\rceil, N\}.$$

Notice that the bound in this theorem is different from the bound in Theorem 1. In fact, if $r = g/2$ and N is sufficiently large, a BLSR/2 WDM ring will have smaller numbers of wavelengths and ADMs than are possible by any UPSR WDM ring. To see this suppose the BLSR/2 WDM ring has a single lightpath connecting every pair of nodes. This is sufficient to support the traffic because each pair of nodes has $g/2$ traffic streams between them. The lightpaths can be arranged into

$$\begin{cases} \frac{N^2}{8} + \frac{N}{4}, & \text{if } N \text{ is even} \\ \frac{N^2-1}{8}, & \text{if } N \text{ is odd} \end{cases}$$

wavelengths ([2] shows how this is done). This BLSR/2 WDM ring requires

$$\frac{N(N-1)}{2} + \begin{cases} \frac{N}{2}, & \text{if } N \text{ is even} \\ 0, & \text{if } N \text{ is odd} \end{cases}$$

ADMs.

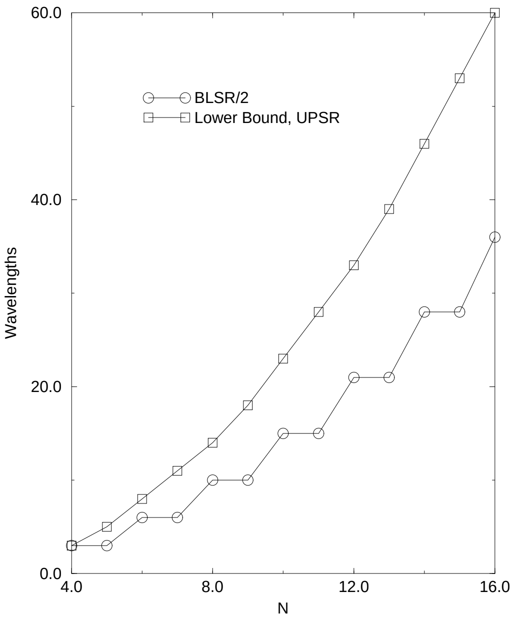


Fig. 8. Comparison of the lower bound on wavelengths for UPSR and the number of wavelengths for BLSR/2 when $r = g/2$.

Now for any UPSR ring, a lower bound on the number of wavelengths is $\left\lceil \frac{rN(N-1)}{2g} \right\rceil = \left\lceil \frac{N(N-1)}{4} \right\rceil$. In addition, from Theorem 1, a lower bound on the number of ADMs is $\left\lceil \frac{3N(N-1)}{2} \right\rceil$. For large N, BLSR/2 has about half the wavelengths and about three-fourths the ADMs of UPSR. Figure 8 compares, for moderate values of N, the lower bound on wavelengths for UPSR and the number of wavelengths for the BLSR/2 network. Figure 9 compares, for moderate values of N, the lower bound on ADMs per node for UPSR and the number of ADMs per node for the BLSR/2 network. Notice that for these moderate values of N, the BLSR/2 network has significantly lower numbers of wavelengths and ADMs than any UPSR WDM ring.

This shows that BLSR/2 can be significantly better than UPSR in both wavelengths and ADMs, and it is possible in part because r was chosen to make the bounds in Theorems 1 and 2 significantly different. However, if r is much smaller than g then the bounds become closer. Then if ADMs are the dominant cost, UPSR cannot be much more costly than BLSR/2. However, BLSR/2 can be used to significantly lower the secondary cost of numbers of wavelengths. For example, [5] presents some results of efficient network constructions for cases when N is odd.

VI. Using SONET rings with different line speeds

In this section we will consider WDM networks with SONET rings with different line speeds, e.g., OC-12 and OC-48. Note that the jump from OC-12 line speed to the OC-48 is a factor of four, but the jump in cost

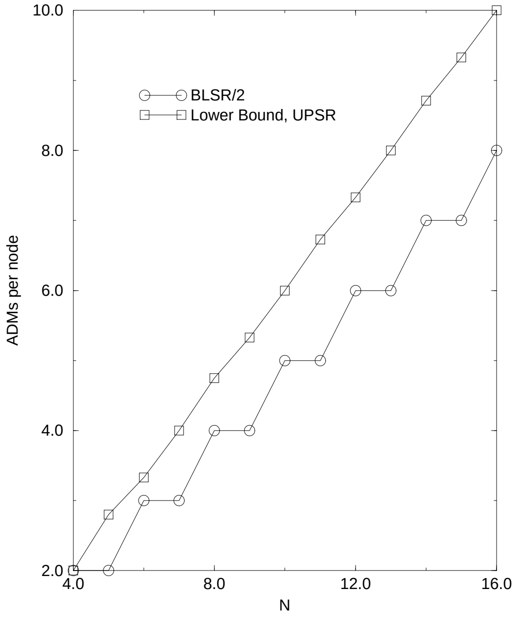


Fig. 9. Comparison of the lower bound on ADMs per node for UPSR and the number of ADMs per node for BLSR/2 when $r = g/2$.

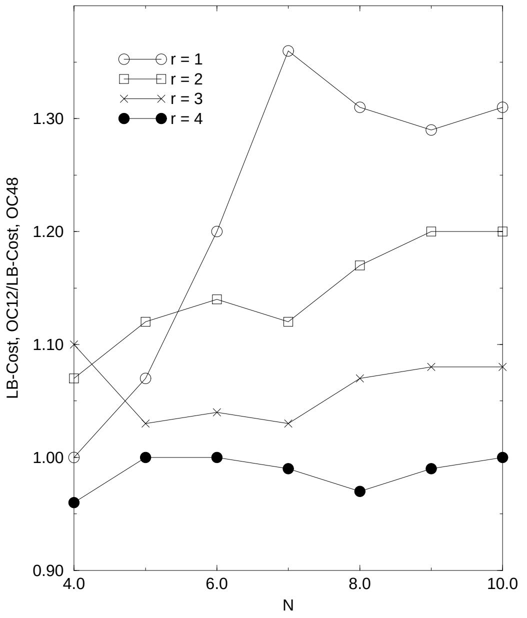


Fig. 10. Ratio of lower bound of cost of OC-12 fixed line speed over lower bound of cost of OC-48 fixed line speed.

is by a factor of 2.5. Thus, the cost per bandwidth decreases by a factor of $5/8 = 0.625$. (A similar factor can be observed for OC-12 vs. OC-3 costs.) What may prevent this potential decrease in cost is inefficient use of the bandwidth. This could be due to lack of traffic to utilize all the bandwidth or the network may not be configurable to the traffic pattern efficiently.

To simplify the discussion, we will only consider OC-12 and OC-48 UPSR SONET rings. We will also assume that traffic streams are at the OC-3 rate. Thus, if a SONET ring is OC-12 then $g = 4$, or if a SONET ring is OC-48 then $g = 16$. The WDM rings will have a *fixed* line speed at all wavelengths (either all OC-12 or all OC-48), or have *mixed* line speeds (both OC-12 and OC-48). We will next compare costs between WDM rings with fixed line speeds. Subsequently, we will consider a WDM ring with mixed line speeds.

To compare costs of WDM rings with fixed line speeds, we will first derive lower bounds. We will employ Theorem 1 which states that the number of ADMs in a WDM ring is at least $\left\lceil \frac{2N(N-1)r}{g+r} \right\rceil$. (Note that the bound is valid only when $r \leq g$.) Thus, a lower bound cost for a WDM UPSR OC-12 ring is

$$\left\lceil \frac{2N(N-1)r}{r+4} \right\rceil \tag{1}$$

if $r \leq 4$; and a lower bound cost for a WDM UPSR OC-48 ring is

$$2.5 \left\lceil \frac{2N(N-1)r}{r+16} \right\rceil \tag{2}$$

if $r \leq 16$. We can compare these lower bounds for the range $r \leq 4$ (the case $r > 4$ is not considered because then the bound for OC-12 does not hold). The ratio of the lower bound cost for OC-12 over the lower bound cost for OC-48 is plotted in Figure 10 for $r = 1, 2, 3, 4$. The cost for OC-48 is often less than OC-12, and can sometimes be greater but then only slightly. An example when OC-12 is cheaper is when $N = 9$ and $r = 4$. Then the lower bound cost for OC-12 is 72, while the lower bound cost for OC-48 is 72.5. Also notice that for $r = 4$ and $N = 9$ (and more generally for all odd N), the lower bound cost for OC-12 can be realized by having an OC-12 lightpath connect each pair of nodes and arranging the lightpaths as in [2]. We refer to this type of network as a *direction connection network*. Thus, this is an example when a network with fixed line speed of OC-12 will have lower ADM cost than any network with fixed line speed of OC-48.

The comparison above was with lower bound costs for OC-12 and OC-48. Figure 11 compares the lower bound cost for OC-12 with an upper bound cost for OC-48 for $r = 4$, where the upper bound cost is realized by an OC-48 single hub network. Also shown is the cost of the single-hub architecture for OC-12 for $r = 4$. (Note that the figure has two additional cost curves to be discussed later.) In addition, note that the lower bound cost for OC-12 can be realized by the direct connection network (for N odd).

Intuitively, the cost curves are determined by the cost per bandwidth of OC-12 vs. OC-48, bandwidth utilization, and the amount of traffic cross-connection that is required. Traffic cross-connection will lead to ADM inefficiencies because traffic streams may be forced to drop before termination. In the figure, the

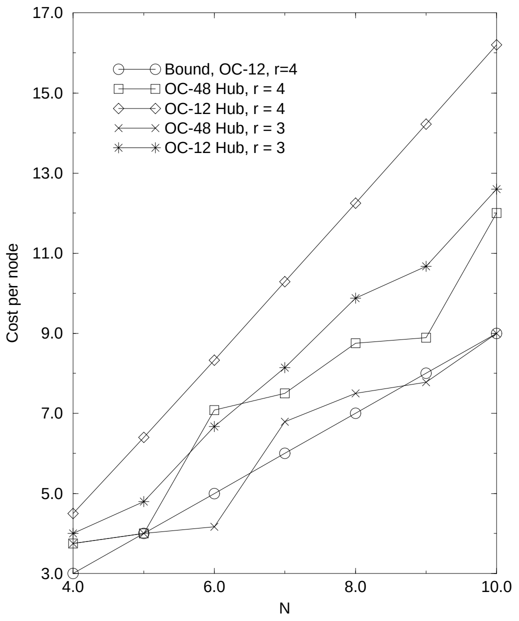


Fig. 11. Ratio of lower bound of cost of OC-12 fixed line speed over lower bound of cost of OC-48 fixed line speed.

lower bound cost for OC-12 is the lowest cost since it corresponds to an OC-12 direct connection network with no cross-connection and high bandwidth utilization. Its bandwidth and ADM efficiencies offset the lower cost per bandwidth for OC-48. The highest cost is for the OC-12 single-hub network, which has traffic cross-connection (and ADM inefficiencies) and high cost per bandwidth.

The figure also shows the costs for the single-hub OC-48 and the single-hub OC-12 for $r = 3$. As expected, the OC-48 single-hub is less expensive than the OC-12 single-hub. Furthermore, the cost of the OC-48 single-hub ring becomes comparable to the OC-12 direct connection network (corresponding to the lower bound cost for OC-48 when $r = 4$). This is due to the fact that the OC-12 direct connection network is less efficient in using its bandwidth [$g = 4$ for each lightpath connection] for $r = 3$.

Finally, we discuss a network architecture that has a mix of OC-48 and OC-12 SONET rings. To simplify the discussion, the network is assumed to be the single-hub architecture. The network will be designed similar to a single-hub ring with OC-48 line speeds, except that its *fractional* traffic may be assigned to OC-12 SONET rings. In particular, each non-hub node has $k = \lfloor (N-1)r/16 \rfloor$ wavelenths assigned to it. Each of these k wavelengths is an OC-48 SONET ring with an ADM at the non-hub node and an ADM at the hub. On these wavelengths, the non-hub node sends and receives $16 \cdot k$ traffic streams to the hub. Since the non-hub sends and receives $(N-1)r$ traffic streams, there may be some left over. This amounts to $T_f = (N-1)r - 16k$ traffic streams which we refer to as the *fractional traffic* for the non-hub.

If the fractional traffic of the non-hubs are handled by SONET OC-48 rings then we have a single-hub WDM network with fixed line speed of OC-48. If the fractional traffic of the non-hubs are handled by SONET OC-12 rings then we have a single-hub WDM network with mixed line speeds. The number of OC-12 SONET rings (i.e., wavelengths) required is described next. Each non-hub has $k_1 = \lfloor T_f/4 \rfloor$ OC-12 wavelengths, where each wavelength has an ADM at the non-hub and hub. The remaining traffic is $T_1 = T_f - 4k_1$, and it shares a wavelength with other non-hubs. The number of non-hubs that can share a wavelength is $\lfloor 4/T_1 \rfloor$. The number of OC-12 wavelengths used for sharing is

$$\lambda_1 = \begin{cases} 0, & \text{if } T_1 = 0 \\ \left\lceil \frac{N-1}{\lfloor 4/T_1 \rfloor} \right\rceil, & \text{otherwise} \end{cases}$$

Then the total number of OC-12 ADM for the fractional traffic is

$$(N-1) \lceil T_f/4 \rceil + ((N-1)k_1 + \lambda_1). \qquad (3)$$

The first term is the number of ADMs at non-hubs, and the second term is the number of ADMs at the hub. Therefore, the total cost is

$$2.5 \cdot 2 \cdot k(N-1) + (N-1) \lceil T_f/4 \rceil + ((N-1)k_1 + \lambda_1), \quad (4)$$

where the first term is the cost due to the OC-48 rings, and the rest of the terms are due to the OC-12 rings.

Figure 12 shows the cost for the single-hub networks for $r = 4$ with fixed line speed of OC-48, fixed line speed of OC-12, and mixed line speeds. Notice that the mixed line speed network and the fixed OC-48 network have the lowest costs, and alternate being the lowest as N increases. To explain the cross-overs, we will provide an approximate calculation to predict when they occur. Recall that the difference between the fixed OC-48 network and the mixed network is in the cost of supporting the fractional traffic. Let T_f denote the amount of fractional traffic at a non-hub node. The approximate cost per non-hub node to take care of this traffic in the fixed OC-48 network is $2.5(1 + T_f/16)$, where the "2.5" term is the cost of an OC-48 ADM, the "1" term is the ADM at the non-hub node, and "$T_f/16$" is the contribution of the ADM at the hub (since approximately $16/T_f$ non-hub nodes share a wavelength). The approximate cost per non-hub node to take care of the fractional traffic using OC-12 rings is $2 \cdot (T_f/4)$ because each non-hub node requires approximately $T_f/4$ OC-12 wavelengths, and each wavelength has (approx.) an ADM at the hub and non-hub. Thus, a cross-over should occur when $2.5(1 + T_f/16) = 2(T_f/4)$, or $T_f = 7.3$. In Figure 12, $T_f = 8$ (approx. 7.3) when $N = 7$. In addition, $T_f = 0$ (no fractional traffic) when $N = 5$ and 9. Notice that the cost curves for fixed OC-48 networks and mixed networks cross-over when $N = 5, 7, 9$.

Figure 13 shows the costs for single-hub rings for fixed OC-48, fixed OC-12, and mixed line speeds for $r = 2$. Here, $T_f = 8$ when $N = 5$ and 13. Also $T_f = 0$ when $N = 9$. Again, notice that the cost curves for fixed OC-48 and mixed networks cross-over at $N = 5, 9, 13$.

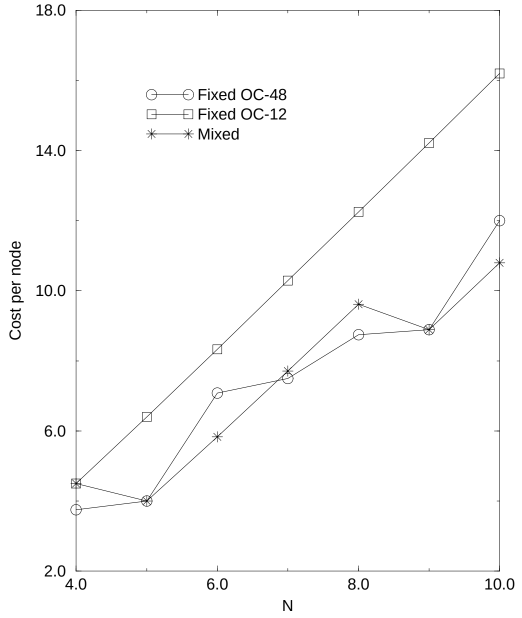


Fig. 12. Cost per ADM of single-hub rings for fixed OC-48, fixed OC-12, and mixed line speeds, and $r = 4$.

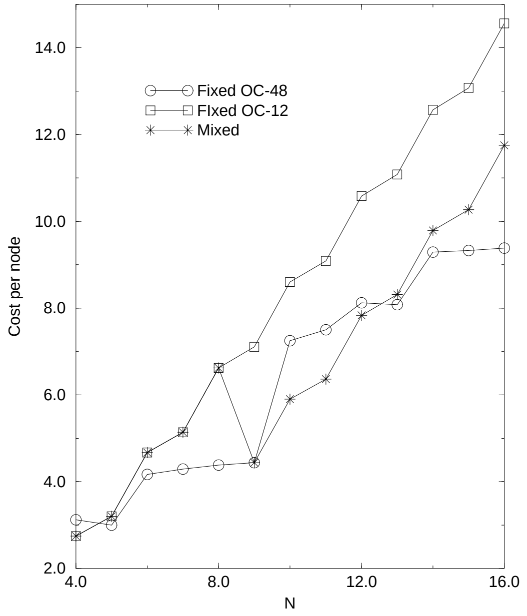


Fig. 13. Cost per ADM of single-hub rings for fixed OC-48, fixed OC-12, and mixed line speeds, and $r = 2$.

VII. Summary

In this paper, we discussed a number of network design and traffic grooming issues in WDM rings that are particular to SONET architectures. We discussed WDM for UPSR, and demonstrated that cross-connection can improve cost. We compared WDM for BLSR/2 with USPR to illustrate that BLSR/2 can typically yield better designs, but not always. Finally, we explored how having the flexibility to choose line speeds on different wavelengths can further lower network costs.

Acknowledgments

The authors gratefully acknowledge the helpful comments of the reviewers.

References

[1] O. Gerstel, P. Lin, and G. Sasaki, "Wavelength assignment in WDM rings to minimize system cost instead of number of wavelengths," *Proc. IEEE Infocom '98*, (San Francisco, CA), pp. 94-101, April 1998.

[2] O. Gerstel, R. Ramaswami, and G. Sasaki, "Cost effective traffic grooming in WDM rings," *Proc. IEEE Infocom '98*, (San Francisco, CA, USA), pp. 69-77, April 1998.

[3] I. Haque, W. Kremer, and K. Raychauduri, "Self-Healing rings in a synchronous environment," *SONET/SDH: a sourcebook of synchronous networking*, Eds. C.A. Siller and M. Shafi, IEEE Press, New York, pp. 131-139, 1996.

[4] E. Modiano and A. Chiu, "Traffic grooming algorithms for minimizing electronic multiplexing costs in unidirectional SONET/WDM ring networks, *Proc. CISS '98*, (Princeton, NJ, USA), Mar. 1998.

[5] J. Simmons, E. Goldstein, and A. Saleh, "On the value of wavelength-add/drop in WDM rings with uniform traffic," *Proc. OFC '98*, (San Jose, CA, USA), pp. 361-362, Feb. 1998.