**A New Insertion-based Construction Heuristic for Solving the Pickup and Delivery Problem with Time Windows**

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## Abstract

In this paper we present a new insertion-based construction heuristic to solve the multi-vehicle pickup and delivery problem with time windows. The new heuristic does not only consider the classical incremental distance measure in the insertion evaluation criteria but also the cost of reducing the time window slack due to the insertion. We also present a non-standard measure, *Crossing Length Percentage,* in the insertion evaluation criteria to quantify the visual attractiveness of the solution. We compared our heuristic with a sequential and a parallel insertion heuristic on different benchmarking problems, and the computational results show that the proposed heuristic performs better with respect to both the standard and non-standard measures.

## Keywords

Construction heuristic; pickup and delivery problem; time windows;

## 1. Introduction

Many core problems arising in logistics and public transit can be modeled as a pickup and delivery problem with time windows (Savelsbergh and Sol, 1995). In the pickup and delivery problem with time windows (PDPTW), each transportation request has a pickup and delivery point and the completion of servicing these points must be performed within a given time window. Since exact approaches can only solve relatively small sized problems due to the problem being NP-hard, researchers have resorted to heuristic algorithms to solve the larger sized problems (Savelsbergh and Sol, 1995; Lu and Dessouky, 2004).

Insertion heuristics construct feasible schedules by iteratively inserting undetermined nodes into existing routes. A new route is created if no undetermined node can be inserted into any existing route. Two decisions need to be made by any insertion heuristic: the selection of the next insertion node and the selection of the next insertion spot. Most insertion heuristics use some criteria function, typically based on the incremental of distance or cost, as a selection rule. By applying different selection rules, variant insertion heuristics may be generated. Insertion-based procedures are used to create initial/final solutions in most heuristics for PDPTW because they are fast and still can produce a quality solution. Hunsaker and Savelsbergh (2002) and Campbell and Savelsbergh (2004) outline the importance of developing efficient insertion heuristics for routing and scheduling problems. As stated above, they are the preferred method for deriving an initial solution in improvement heuristics or meta-heuristics. Furthermore, because of their computational efficiency and ability to operate in real-time, many of the commercial routing and scheduling software packages use an insertion-based heuristic for their solution approach (Palmer et al., 2004).

The focus of this paper is to derive a new insertion-based heuristic for the PDPTW. Our procedure differs from the classical insertion methods in two aspects. First, the classical insertion methods typically choose the next insertion by selecting a feasible insertion that has the minimal increase in travel distance or time, with respect to both the time window and capacity constraints. They do not directly take into consideration the degree of feasibility when determining which node and location to insert next. This myopic characteristic prevents the insertion-based heuristic from constructing higher quality solutions, especially when more restricted feasibility constraints are considered such as time window constraints. To overcome this myopic characteristic, in this paper, we propose a new insertion evaluation function, which takes into consideration the increase of travel time as well as the reduction in the slack in the time window due to the insertion operation. We refer to the time difference between the time window and the service time as the slack in the time window. For example, instead of always choosing the node and location with the lowest cost as the next insertion, it may be better to select an insertion, which does not use much of the available slack so that more opportunities are left for future insertions.

Second, in practice, operational planners tend to prefer more visually attractive solutions. This has been observed and confirmed by researchers who have implemented commercial routing software for industry. For example, see Poot et al. (2002), Rahimi and Dessouky (2001), and Sahoo et al. (2005). On the other hand, an investigation of human performance on the Traveling Salesman Problem (TSP) made by MacGregor and Ormerod (1996) reports that untrained adults, solving "by eye," are able to find solutions that are as good as, or even better than, those produced by a number of well-known heuristics, not subject to individual differences. Their work reveals that more visually attractive solutions tend to have less total length of distance. We believe that this may also be true for more restrictive problems, such as the PDP and PDPTW. To account for this effect, we develop a non-standard measure to evaluate the visual attractiveness for the existing routes, called *Crossing Length Percentage* (CLP), to evaluate the visual attractiveness. A crossing is defined as a point where two routes intersect each other. Our proposed insertion heuristic incorporates this non-standard measure to create more visually acceptable solutions.

The reminder of this paper is organized as follows. The literature on the PDPTW is reviewed in Section 2. In Section 3, we give the formal definition of the problem studied in this paper. The whole procedure of the proposed insertion heuristic, including the non-standard quality measure, CLP, is given in Section 4. The computational results are provided in Section 5. Finally, some concluding comments are made in Section 6.

## 2. Literature Review

Jaw et al. (1986) adapt the traditional insertion algorithm to the multiple-vehicle dial-a-ride problem with time windows. The heuristic selects the customers in the order of the increase of the earliest pickup time, inserts the selected customer into the cheapest feasible position in the existing routes, or adds an unused vehicle to the problem if no feasible position is found. Madsen et al. (1996) develop an alternative insertion algorithm for the dial-a-ride problem.

For the single-vehicle PDPTW, Van der Bruggen et al. (1993) develop a two-phase local search heuristic algorithm. The construction phase starts with an initial route obtained by visiting the locations in order of increasing centers of their time window, taking prior and capacity constraints into account. The initial route may be time infeasible. Then, the initial route is made feasible by interactively applying an objective function that penalizes the violation of time windows. The feasible routes obtained in the construction phase are continually improved in the improvement phase. Both phases use a variable-depth search built up out of seven basic types of arc-exchange procedures. The method has been tested on problem instances with up to 50 clients.

Ioachim et al. (1995) propose an approximation algorithm for the multiple-vehicle PDPTW. The algorithm first creates a set of clusters by solving a reduced multiple-vehicle PDPTW problem exactly using column generation. Each cluster is a small trip that visits a set of customers. Finally, a multiple traveling salesman problem is solved to sequence the clusters into the final routes. The proposed approach is tested on a problem set with up to 250 customers.

Toth and Vigo (1997) describe a fast and effective parallel insertion heuristic to determine the schedule of transporting handicapped persons using a fleet of heterogeneous vehicles. The problem can be considered as a multiple-vehicle PDPTW with additional real-life operational constraints. They also present a tabu procedure to improve the solution obtained by the insertion algorithm. The proposed approach has been applied in a DSS (Decision Support System) to plan the vehicle service schedule for the city of Bologna, Italy.

Savelsbergh and Sol (1998) propose three approximation algorithms derived from their branch-and-price based optimal algorithm. They use the approximation algorithms to generate columns for the linear programming relaxation. They observe that creating a good set of columns in the root is the key issue in finding good solutions. Their approximation algorithms are tested on problem sets with up to 50 customers.

Nanry and Barnes (2000) present a reactive tabu search approach to solve the multiple-vehicle PDPTW. The initial routes are created through a greedy insertion method. Then, a reactive tabu search method with three proposed neighborhoods is used to improve the initial route. The approach is tested on instances with sizes of 25, 50 and 100 customers, which were constructed from Solomon's C1 benchmark problems.

Landrieu et al. (2001) present a probabilistic tabu search to solve the single-vehicle PDPTW. The algorithm was tested on a class of randomly generated instances.

Li and Lim (2001) propose a metaheuristic for the multiple-vehicle PDPTW. First, a local optimal is found based on three defined neighborhoods. Then, a simulated annealing like multiple-restart strategy is applied to escape from the local optimal and a tabu-list is used to avoid cycling in the search process. The algorithm is tested on the modifications of each of Solomon's benchmark problem for VRPTW (56 instances).

Lao and Liang (2002) present a two-phase method for the multiple-vehicle PDPTW. In the first phase, a new hybrid heuristic that is based on a standard insertion procedure and sweep procedure is applied to construct the initial solution. In the second phase, a tabu search method is proposed to improve the solution. The algorithm is also tested on the instances generated from Solomon's benchmark problems.

Cordeau and Laporte (2003) describe a tabu search heuristic for the multiple-vehicle dial-a-ride problem with time windows. They propose a new neighborhood evaluation procedure and give extensive computational results based on randomly generated data sets.

Dessouky, Rahimi, and Weidner (2003) add an environmental component to the criteria function of an insertion algorithm for the PDPTW problem in addition to the standard cost component. Simulation results show that adding the environment component to the insertion criteria component significantly improves the environmental effect of the schedule without significantly degrading the cost component.

Diana and Dessouky (2004) present a parallel regret insertion heuristic to solve a large-scale dial-a-ride problem with time windows. The proposed algorithm is tested on data sets of 500 and 1000 requests built from data of paratransit service in the Los Angeles County.

## 3. Problem Definition

We consider the multiple-vehicle PDPTW as follows. We have $n$ customers that need to be served, each with a pickup location $v_i$ and a delivery location $v_{i+n}$, $i = 1, 2, \ldots, n$. The pickup and delivery requests from a customer must be served by the same vehicle, which we refer to as the *pairing* constraint. Each customer's pickup request must be served before its delivery request, which we refer to as the *prior* constraint. There are $m$ identical vehicles available in the fleet. The capacity of each vehicle is $Q$, and at any time the total load in a vehicle cannot exceed $Q$, which is the *capacity* constraint.

A time window $(E_i, L_i)$ is specified for each location $v_i$, $i = 1, 2, \ldots, 2n$, where $E_i$ denotes the earliest time a service can take place and $L_i$ denotes the latest time a service can start. All vehicles must start from a central facility with location $v_0$ and return to location $v_{2n+1}$ during a common time interval. Location $v_0$ and $v_{2n+1}$ may be physically the same. Assume $E_0$ is the common earliest time that all vehicles are ready to depart from depot $v_0$ and $L_0$ is the latest time that a vehicle can leave depot $v_0$. The departure time window for a vehicle from depot $v_0$ is $[E_0, L_0]$ and the return time window for a vehicle at depot $v_{2n+1}$ is $[E_{2n+1}, L_{2n+1}]$, where $E_{2n+1} = L_{2n+1} = L_0$. We assume that waiting at any location $v_i$ before

$E_i$ is allowed but starting the service after time $L_i$ is not allowed. That is, the *time window* constraint is violated if the arrival time at any location $v_i$ is later than $L_i$.

To summarize, the problem we consider has the above four types of constraints: pairing, prior, capacity, and time window. The objective is to minimize the total cost, which includes the fixed vehicle cost and travel cost that is proportional to the travel distance.

In reality, the application of the pickup and delivery problems often involves other service related constraints, especially when they are applied to transport people. The common service related constraints include the maximum total duration of the route, the maximum wait time at a stop, and the maximum ride time that limits the time between pickup and delivery. Notice that the problem defined above has implicitly taken into account the maximum total duration of the routes, because by enforcing that a vehicle cannot leave depot $v_0$ earlier than the time instant $E_0$ and a vehicle has to return to depot $v_{2n+1}$ no later than time instant $L_0$, the total duration of any route is no more than $L_0$ - $E_0$. The maximum ride time constraint is indirectly accounted for by having time windows for both the pickup and delivery points.

## 4. Construction Heuristic

Consider the problem defined in Section 3. Assume the travel time between any pair of locations $v_i$ and $v_j$ is $t_{i,j}$, $i = 0, 1, 2, \ldots, 2n, 2n+1$. We denote by $A_i$ the earliest arrival time of a vehicle at location $v_i$ and $D_i$ the earliest departure time of a vehicle at location $v_i$, where $D_i = \max(E_i, A_i) + d_i$ and $d_i$ is the vehicle's service time at location $v_i$. Note that $A_i$ and $D_i$ are not constants for customer $i$, and they are relative to the solution under construction. The time window constraint is violated if $A_i > L_i$ at any location $v_i$. We maintain two quantities for each assigned location $v_i$: waiting time, $W_i$, $W_i = \max(0, A_i – E_i)$ and the maximal postponed time interval, $Y_i$, where $W_i+Y_i$ indicates the maximal additional time that can be induced by feasibly inserting other locations before $v_i$ in the current route. Assume all $k$ locations in a route $R$ in sequence are $v_{r_1}$, $v_{r_2}$, … , $v_{r_k}$, we compute $Y_{r_j}$ at each location $v_{r_j}$, $j$=1,2,..., $k$ as follows. Figure 1 illustrates different $W$ and $Y$ values in a route with four locations.

$$Y_{r_j} = \begin{cases} L_{r_k} - \max(A_{r_k}, E_{r_k}) & \text{for } j = k \\ \min\{L_{r_j} - \max(A_{r_j}, E_{r_j}), Y_{r_{j+1}} + W_{r_{j+1}}\} & \text{for } i = k-1, k-2, \ldots, 1 \end{cases}$$

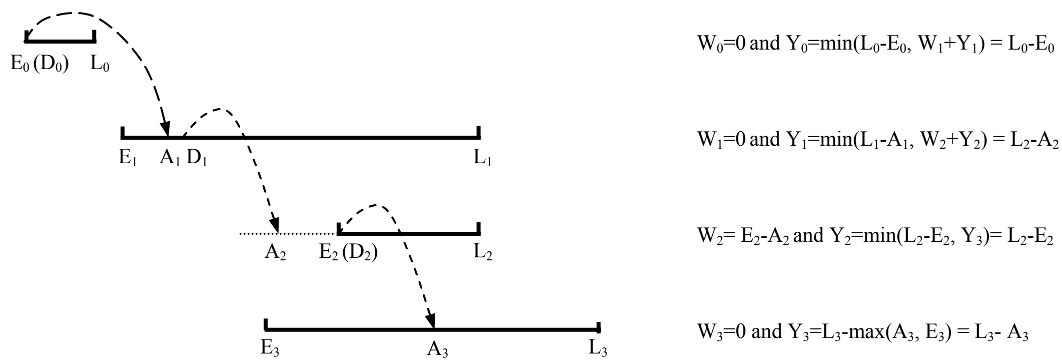



**Figure 1. An Example of Different Y and W values in a route with four locations**

From the definition of $Y_i$, we know that if $v_i$ and $v_j$ are two adjacent locations in a route and location $v_i$ precedes location $v_j$, then we have $Y_i \le Y_j + W_j$.

Next, we give the feasibility checking functions including variables $Y_i$ and $W_i$ at each location $v_i$. Assume that location $v_i$ and $v_j$ are two adjacent locations in an existing route. Define $A_k^{'}$ as the earliest arrival time at location $v_k$ if inserting it into the current route between locations $v_i$ and $v_j$, $A_k^{'} = \max(A_i, E_i) + \mathrm{d}_i + t_{i,k}$.

Checking whether it is feasible to insert location $v_k$ between location $v_i$ and $v_j$ amounts to verifying the following two equations: $A_k^{'} \le L_k$, and $\max(A_k^{'}, E_k) + d_k + t_{k,j} \le A_j + W_j + Y_j$.

## 4.1. Initial Route Selection

The m-PDPTW is generally a highly constrained problem, because of the existence of additional constraints such as pairing constraints, prior constraints, and time window constraints. In a highly constrained problem, each route can contain only a limited number of customers. There exist pairs of customers such that the customers in each pair cannot be in the same route even if the route includes only these two customers.

Some researchers have shown the importance of obtaining the right initial routes (e.g. Liu and Shen, 1999). A set of "*right*" initial routes can help to obtain a better assignment for the remaining customers. When the fleet includes homogeneous vehicles and all the vehicles are based at a common depot, we create the initial routes by finding a maximal set of customers, where it is impossible to serve any two customers in this set with a single vehicle.

To compute the maximal set, we first generate a graph $G(G_N, G_A)$, where $G_N$ is the node set and $G_A$ is the arc set. Each node in $G_N$ represents a customer to be served, $|G_N| = n$. An undirected arc connects the nodes representing customer $c_1$ and $c_2$ in graph G, if and only if no vehicle can serve customers $c_1$ and $c_2$ in a single route. Then, the problem of finding a maximal set of exclusive customers equals the problem of finding a maximum size clique in *G,* a clique being a subset of the nodes such that each node is connected to all other nodes of the subset. The Max-Clique problem is a well-known NP-Complete problem (Brelaz, 1979). We use a simple greedy algorithm to find the largest clique as follows.

Algorithm 4.1.

Step 1. If all nodes in $G_N$ are marked, stop and $G_N$ is the largest clique. Otherwise, arbitrarily select an unmarked node $i$, $i \in G_N$, which has the highest number of edges incident with it and set node $i$ as marked.

Step 2. Remove all the nodes from set $G_N$, where there is no edge linking node $i$ and these nodes directly. Remove all the edges from set $G_A$ that are incident with the removed nodes. Go to step 1.

Figure 2 gives an example of finding the maximal clique using Algorithm 4.1. The original problem is shown in Figure 2(a). First, we select node 1, set it as marked, and remove node 5 as well as the two arcs incident with node 5. The resulting graph $G$ is shown in Figure 2(b). Next, we select and mark node 3, remove node 6 as well as the arc incident with node 6, and the result is shown in Figure 2(c). Finally, we select and mark nodes 2 and 4 in sequence and get the graph consisting of node 1, node 2, node 3, and node 4, and they are all marked. Therefore, nodes 1, 2, 3, and 4 are the maximal clique in the original graph.

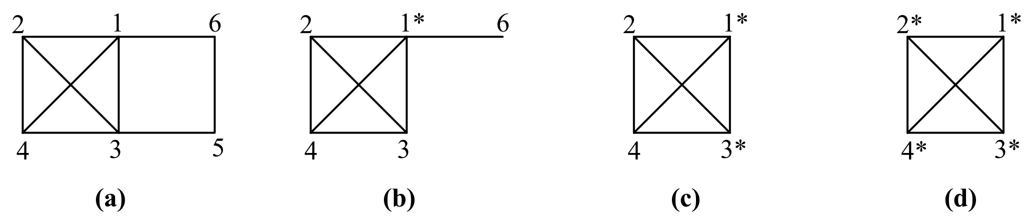



**Figure 2. An Example of Using Algorithm 4.1 to Get the Largest Clique**

The computational complexity of Algorithm 4.1 is $O(n^2)$. Since Algorithm 4.1 is fast, we run Algorithm 4.1 $n$ times with choosing the first marked node as each node belonging to $G_N$ and we select the maximal clique in these $n$ obtained cliques as the final result.

Finally, a set of initial routes is created with each route containing one customer, whose corresponding node in graph G is in the maximal clique found by Algorithm 4.1.

## 4.2. Computation of the Insertion Cost

Our objective is to minimize the total travel distance as well as the number of used vehicles, while obeying the pairing, prior, vehicle capacity and time window constraints. Hence, we need to take into account both the spatial and temporal aspects of the problem. The amount of the insertion cost should reflect both the increment of the distance as well as the amount of the reduced slack in the time windows. From our knowledge, there has been no work that considers including in the insertion cost the reduction of the time window slack due to the insertion. In this section, we propose a new methodology to compute the insertion cost for the routing problems with hard time window constraints.

Recall that two quantities, the waiting time ($W$) and the maximal postponed time ($Y$), are associated with each location in a route. Inserting a new location in a route may affect the $W$ and $Y$ values of the locations before and after the insertion position in the route. For example, Figure 3(a) gives the $W$ and $Y$ values for each location in a route, which we refer it as the $W$-$Y$ graph of the route. Figure 3(b) shows the $W$-$Y$ graph after inserting a new location between the third and fourth location in the route. We can conclude the following three observations from Figure 3.

1. Inserting a new location may decrease the $Y$ value of the locations before the insertion but not change the $W$ value of the locations before the insertion.

2. The increase of the travel time caused by the insertion is offset by the waiting time at the locations after the insertion or the maximal postponed time at the last location in the route.

3. If there is waiting time at the new inserted location, the waiting time can be considered as being moved from some waiting time after the insertion or the maximal postponed time from the last location in the route.

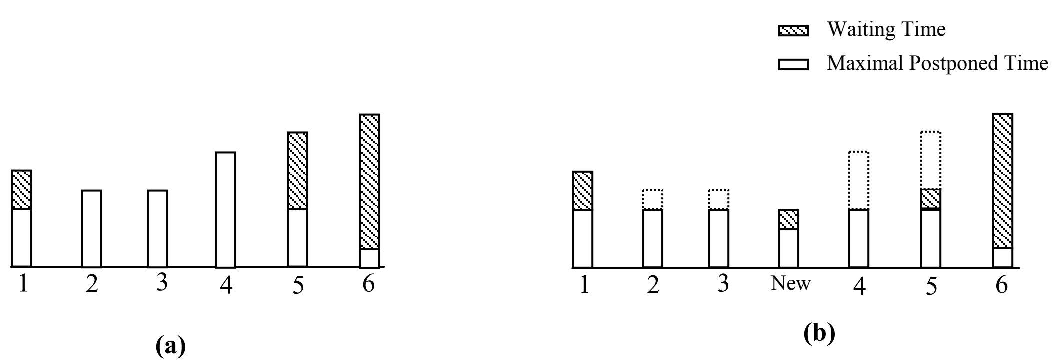



**Figure 3. An Example Showing the *W-Y* Graphs for the Routes before and after the Insertion**

Next, we describe how to evaluate the insertion cost. Assume that a route $R$ contains $k$ locations: $v_{r_1}$, $v_{r_2}$, …, $v_{r_k}$. And it is feasible to insert location $v_{r_j}$ between location $v_{r_{i-1}}$ and $v_{r_i}$ in route $R$. Let $A'_{r_j}$ be the vehicle's earliest arrival time at location $v_{r_j}$ after inserting location $v_{r_j}$ into route $R$, $A'_{r_j}$ = $\max(A_{r_{i-1}}, E_{r_{i-1}}) + d_{i-1} + t_{i-1,j}$. Let $W'_{r_j}$ = $\max(0, E_{r_j} - A'_{r_j})$ and $Y'_{r_j} = \min\ \{ L_{r_j} - \max(A'_{r_j}, E_{r_j}),\ W_{r_i} + Y_{r_i} - \Delta \}$, where $\Delta = t_{i-1,j} + t_{j,i} - t_{i-1,i} + d_j + W'_{r_j}$. Let the cost of this insertion operation be $C$. Then, we have $C = c_1 + c_2 + c_3$, where $c_1$ represents the decrease of the time window slack of locations $v_{r_1}$, $v_{r_2}$, …, $v_{r_{i-1}}$, after inserting location $v_{r_j}$ after location $v_{r_{i-1}}$ in route $R$, $c_2$ represents the decrease of the time window slack of $v_{r_j}$ itself after the insertion, and $c_3$ represents the decrease of the time window slack of locations $v_{r_i}$, $v_{r_{i+1}}$, …, $v_{r_k}$, after inserting location $v_{r_j}$ before location $v_{r_i}$ in route $R$. To compute $c_1$, we use the following algorithm.

Algorithm 4.2.

Step 0. Let $\beta = W'_{r_j} + Y'_{r_j}$, $c_1 = 0$, and $k = i-1$.

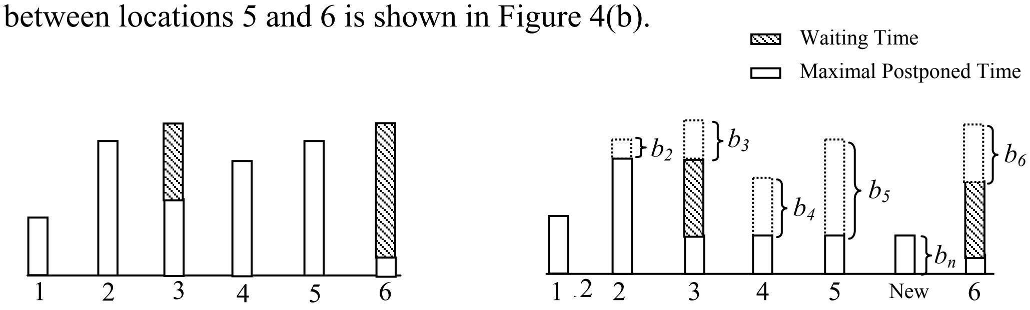
Step 1. If $\beta \geq Y_{r_k}$ or $k = 1$, return $c_1$ and stop the algorithm.

Step 2. If $k = i-1$, let $c_1 = Y_{r_k} - \beta$ and go to step 4. Otherwise, go to step 3.

Step 3. If $W_{r_{k+1}} > 0$, let $c_1 = c_1 + \min(Y_{r_k} - \beta,\ W_{r_{k+1}})$.

Step 4. Let $\beta = W_{r_k} + \beta$ and $k = k-1$. Go to step 1.

We illustrate Algorithm 4.2 with the example shown in Figure 4. Figure 4(a) gives the $W$-$Y$ graph of a route with six locations. The $W$-$Y$ graph after inserting a new location between locations 5 and 6 is shown in Figure 4(b).



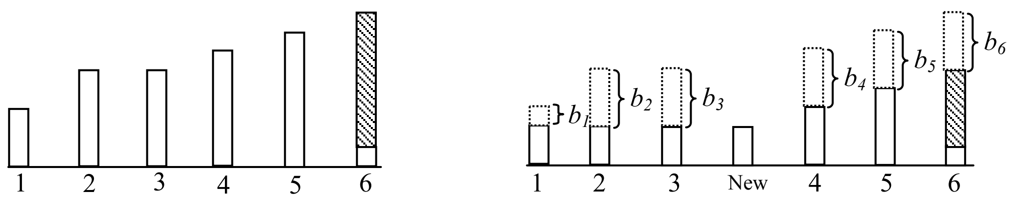

(a) (b)

Inserting a new location in Figure 4(b) reduces $Y_5$ by $b_5$, $Y_4$ by $b_4$ and $Y_3$ by $b_3$. But we only count $b_5$ in $c_1$, because reducing $Y_4$ by $b_4$ causes $Y_5$ to decrease by $b_4$ and reducing $Y_3$ by $b_3$ causes both $Y_4$ and $Y_5$ to decrease by $b_3$. Thus, we use $b_5$ to represent the reduction of the time window slack at locations 3, 4 and 5. Furthermore, the insertion also reduces $Y_2$ by $b_2$. If $Y_2$ is reduced by $b_2$, this $b_2$ amount of time will be offset by $W_3$ and will not affect $Y_3$, $Y_4$ and $Y_5$. Therefore, $b_2$ should be considered as an additional lost of slack and we have $c_1 = b_2 + b_5$**.**

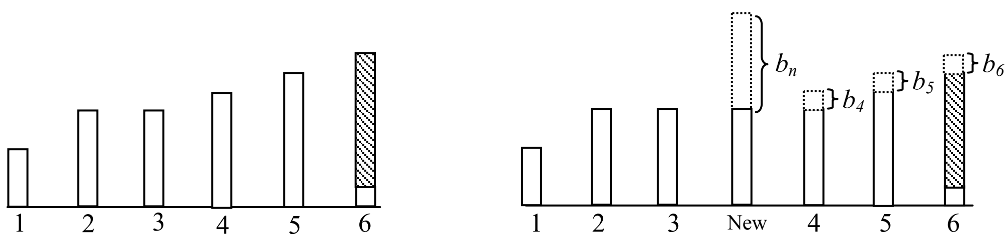
Next, we compute the cost of $c_2$ using the following equations. Remember that $W'_{r_j}$ = $\max(0, E_{r_j} - A'_{r_j})$ and $Y'_{r_j} = \min\{L_{r_j} - \max(A'_{r_j}, E_{r_j}), W_{r_i} + Y_{r_i} - \Delta\}$, where $\Delta = t_{i-1,j} + t_{j,i} - t_{i-1,i} + d_j + W'_{r_j}$ and $A'_{r_j} = \max(A_{r_{i-1}}, E_{r_{i-1}}) + d_{i-1} + t_{i-1,j}$. Define $c_2 = L_{r_j} - \max(A'_{r_j}, E_{r_j}) - Y'_{r_j}$. The cost of $c_2$ measures the reduction of the time window slack for location $v_j$ due to the small slack in the succeeding locations after the insertion.

Finally, the value of $c_3$ represents the increment of the insertion distance. If the vehicle's velocity is constant, the travel distance is proportional to the travel time. Therefore, we unify the units of all the costs, $c_1$, $c_2$ and $c_3$, to be time here. Formally, we define $c_3 = t_{i-1,j} + t_{j,i} - t_{i-1,i}$. In the example in Figure 4, we have $c_3 = b_6$.

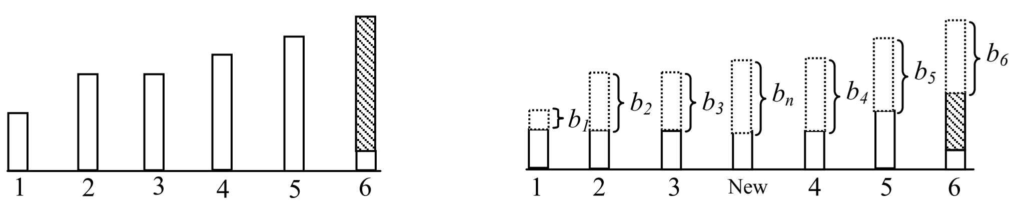
A large value of $c_1$ implies that from the time windows point of view, it is better to insert $v_j$ at some position before location $v_{i-1}$, because inserting location $v_j$ after location $v_{i-1}$ makes the arrival time at location $v_j$ so close to the time window $L_j$ that the $Y$ values of some locations before location $v_{i-1}$ are diminished. On the contrary, a large value of $c_2$ implies that from the time windows point of view, it is better to insert $v_j$ at some position after location $v_i$, because the insertion of $v_j$ before $v_i$ wastes the significant slack at $v_j$, $L_j - \max(A'_j, E_j)$, due to a small slack in the succeeding nodes. Therefore, we have the following property for $c_1$ and $c_2$. If both $c_1 > 0$ and $c_2 > 0$, it implies that the insertion of $v_j$ between $v_{i-1}$ and $v_i$ has significantly increased the travel time that both the $Y$ values at $v_{i-1}$ and $v_i$ are diminished. We illustrate the above three different cases in Figure 5 (a), (b) and (c), respectively. Assume that a new location will be inserted between the third and fourth locations in the route.




**(a) $c_1 = b_3 > 0$, $c_2 = 0$, and $c_3 = b_4$.**




**(b) $c_1 = 0$, $c_2 = b_n > 0$, and $c_3 = b_4$.**




**(a) $c_1 = b_3 > 0$, $c_2 = b_n$, and $c_3 = b_4$.**

**Figure 5. Three Different Cases for Computing the Insertion Cost**

## 4.3. Consideration of the Measure for Visual Attractiveness

In this section, we describe how to embed the non-standard quality criteria, CLP, into our proposed construction heuristic to improve the quality of the solution.

Poot et al. (2002), based on their experience on developing the SHORTEREC Distriplanner® a commercial vehicle routing software for industry, point out that visually attractive plans seem to be more logical to the operational planners and closer to those plans created by the human dispatcher. Therefore, producing such plans not only generates more reasonable solutions for practical routing problems but also helps to create trust among the planners and drivers in a route planning system, which leads to fast acceptance of the system.

We can evaluate the visual attractiveness of a solution from two aspects: the visual attractiveness of each single route and the visual attractiveness of different routes' geographical relationship. The experiments made by MacGregor and Ormerod (1996) suggest that visual attractiveness of a single route implicitly reflects the route's length. However, no research supports that such a relationship also exists between the geographical relationship of different routes and the total distance of the solution. For example, it is visually more attractive to have less requests served by one route contained in the convex hull of another route, but there is no evidence that indicates that solutions with less overlap of the convex hulls from different routes are more probable to have less total distance. An instance of a solution with less overlap from different routes' convex hulls having longer total distance can be found in Sahoo et al. (2005). Thus, we only consider measuring the visual attractiveness of each individual route.

MacGregor and Ormerod (1996) propose the convex-hull hypothesis, which advocates the use of the convex hull as part of a strategy to create a TSP tour, to explain why a high quality TSP solution can be found by trying to obtain a visual attractive solution. Rooij et al. (2003) try to justify the same phenomenon with the hypothesis that people achieve this by trying to avoid crossings. Namely, the convex-hull hypothesis and the crossings-avoidance hypothesis are not mutually exclusive. We can always find a more visually attractive TSP solution by removing the crossings from a non-crossing-free TSP solution. Comparing two crossing-free solutions for the same TSP, the one closer to the convex hull is more visually attractive and more probable to have shorter total distance.

Next, we propose a measure *Crossing Length Percentage* (CLP) to evaluate the degree of crossings with a single trip. Note that the number of crossings alone is not a proper quantity measure to evaluate the crossing level of a trip, since the crossing level also depends on how deep the crossings are and whether the multiple crossings entangle each other. The CLP of a trip is defined as the sum of the crossed length of all the crossings divided by the total length of the trip. The crossed length of a crossing equals the minimum total length of the crossing. The CLP value of a trip can be calculated using the following Algorithm 4.3.

Algorithm 4.3.

Step 1. Assume the total length of a trip is $\lambda$. Compute all the $n$ crossing points in the trip. Let them be $cs_1$, $cs_2$, …, $cs_n$.

Step 2. For each crossing point $cs_i$, $i = 1,2, \ldots n$. Calculate the distance $\beta_i$, where $\beta_i$ equals the route's length of the portion within crossing point $cs_i$.

Step 3. The CLP value of the trip can be computed as $\sum_{i=1}^{n} \min(\beta_i, \lambda - \beta_i) / \lambda$.

Next, we illustrate how to apply Algorithm 4.3 to compute the CLP value for the two trips shown in Figure 6, where we depict each segment's length next to the segment.

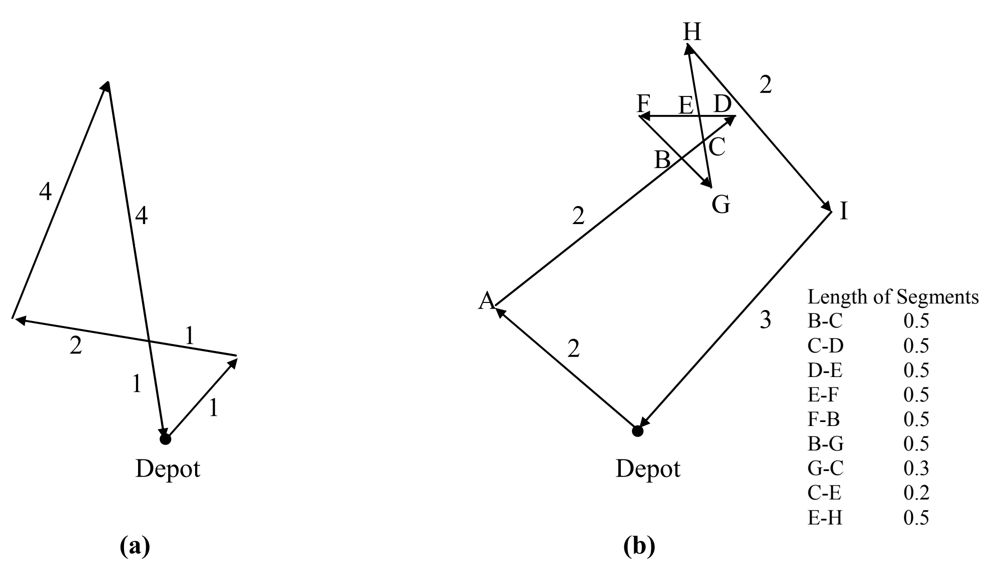



**Figure 6. Examples of Computing Crossing Length Percentage**

The trip in Figure 6(a) contains only one crossing. The CLP value of the trip in Figure 6(a) is min {4+4+2, 1+1+1}/13 = 0.23. The trip in Figure 6(b) contains multiple crossings. The crossed lengths for the three crossings at points B, C and E in Figure 6(b) equal BC+CD+DE+EF+FB, CD+DE+EF+FB+BG+GC, and EF+FB+BG+GC+CE, respectively. And, the sum of the crossed lengths for all these three crossings is BC+2*CD+2*DE+3*EF+3*FB +2*BG+2*GC+CE = 7.3. Note that the lengths of segments CD, DE, EF, FB, BG and GC are counted more than once to represent the entanglement of the crossings. For instance, the length of segments EF and FB are counted three times because they are contained in all three crossings. The CLP value of the trip in Figure 6(b) is 7.3/13 = 0.56.

The CLP value measures the visual attractiveness of a single route with crossings. Maintaining routes with lower CLP at each iteration in an insertion heuristic can avoid the acceptance of unattractive insertions in the earlier stage of creating interim routes. Not only does the CLP value indicate whether a trip is visually attractive or not, but it also indicates the most entangled portion in a visually unattractive trip. A portion in a trip, which comprises of segments counted the highest number of times in the computation of the CLP, is more likely to have entanglements. Therefore, to improve a trip, instead of randomly exchanging the visiting sequence of all the customers in the entire trip, we need to focus on adjusting the customers' sequence within the most entangled portion of the trip. This makes the behavior of each improvement step more reasonable and closer to what human dispatchers will do.

Next, we discuss in detail how to incorporate the CLP to guide the insertion process. As pointed out before, crossing avoidance is a common logic behind the human dispatcher while creating routes. It is easy to find locations to insert without causing any crossings at the beginning of the insertion because each route contains only a few locations and many locations are available to be added. As more and more locations are assigned and the routes become more and more complicated, the dispatching logic changes from emphasizing the visual attractiveness to utilizing the capacities of the existing routes (e.g. assigning as many locations as possible to the existing routes).

We use the following strategy to represent the above logic. First, determine a set $\Pi = \{(p_1, q_1), (p_2, q_2), \ldots, (p_h, q_h)\}$, where $h$ is a predetermined constant, $p_i$ represents the CLP threshold value at level $i$, and $q_i$ is the percentage of the assigned locations among the total locations at level $i$. Assume the current percentage of the assigned locations among all the locations is $q'$ and $q_{i-1} \leq q' < q_i$. Then, we consider only feasible insertion positions that will have CLP values smaller than $p_i$ after the insertion is made. For example, if we set $\Pi = \{(5\%, 40\%), (30\%, 60\%), (\infty, 100\%)\}$ and currently only 12 of the 100 customers have been assigned to the routes. We consider only feasible insertion positions that will not lead the CLP value of the routes greater than 5%, since $q' = 0.12 < 0.40$.

## 4.4. Overall Approach

In this section, we describe the entire construction heuristic algorithm for the m-PDPTW problem based on the discussions in Sections 4.1 to 4.3. The flowchart for the algorithm is shown in Figure 7.

First, we compute the initial routes using Algorithm 4.1. Next, we find a location $v$ and a route $r$, where inserting $v$ to $r$ is feasible and has the minimal incremental cost $C = c_1 + c_2 + c_3$. Furthermore, inserting $v$ to $r$ will not make the CLP value of route $r$ greater than the threshold value $p_k$ at $\Pi$, where $k$ represents the current level in set $\Pi$. We build an incremental cost table in which the rows represent the unassigned customers and the columns the used routes. Each cell $(i, j)$ represents the best incremental cost of inserting customer $i$ to route $j$. Each cell is associated with four values, $c_1$, $c_2$, $c_3$ and $p$, the CLP value of route $j$ after inserting customer $i$.

If cell $(i, j)$ has the minimal value of $c_1 + c_2 + c_3$ and the value of $p$ of cell $(i, j)$ is less than the current $p_k$ in set $\Pi$, we insert customer $i$ into route $j$ at the computed position. If the $p$ value of any cell in the incremental cost table is greater than $p_k$, we increase the level $k$ in set Π by 1 and reselect the cell with the minimal incremental cost. This strategy ensures that in selecting the next insertion customer and route pair the CLP value criteria does not dominate the incremental insertion cost. If there exists a customer that cannot be inserted into any existing route, assign an unused vehicle for this customer. The above steps are iterated until there are no unassigned customers or there are still some unassigned customers but all the vehicles in the fleet have been used. In the latter case, the problem is infeasible.

Assume there are $n$ customers and $m$ vehicles. The construction heuristic requires checking all the insertion positions for each unassigned customer in each existing route. The computational complexity of the common insertion heuristic for the pickup and delivery problem is $O(n^3)$ on the condition that the computational complexity to compute the insertion cost is a constant, O(1). However, in our insertion heuristic the computational complexity of computing the costs is $O(n)$, where computing the cost includes computing $c_1$, $c_2$, $c_3$ and $p$. To compute $c_1$, we use Algorithm 4.2 whose complexity is $O(n)$. The complexities to compute $c_2$ and $c_3$ both are $O(1)$. The complexity to compute the new CLP value after inserting two locations is $O(n)$. Therefore, the complexity of computing the cost for each customer at each location in each route is $O(n)$ and the total computational complexity for the construction heuristic is $O(n^4)$.

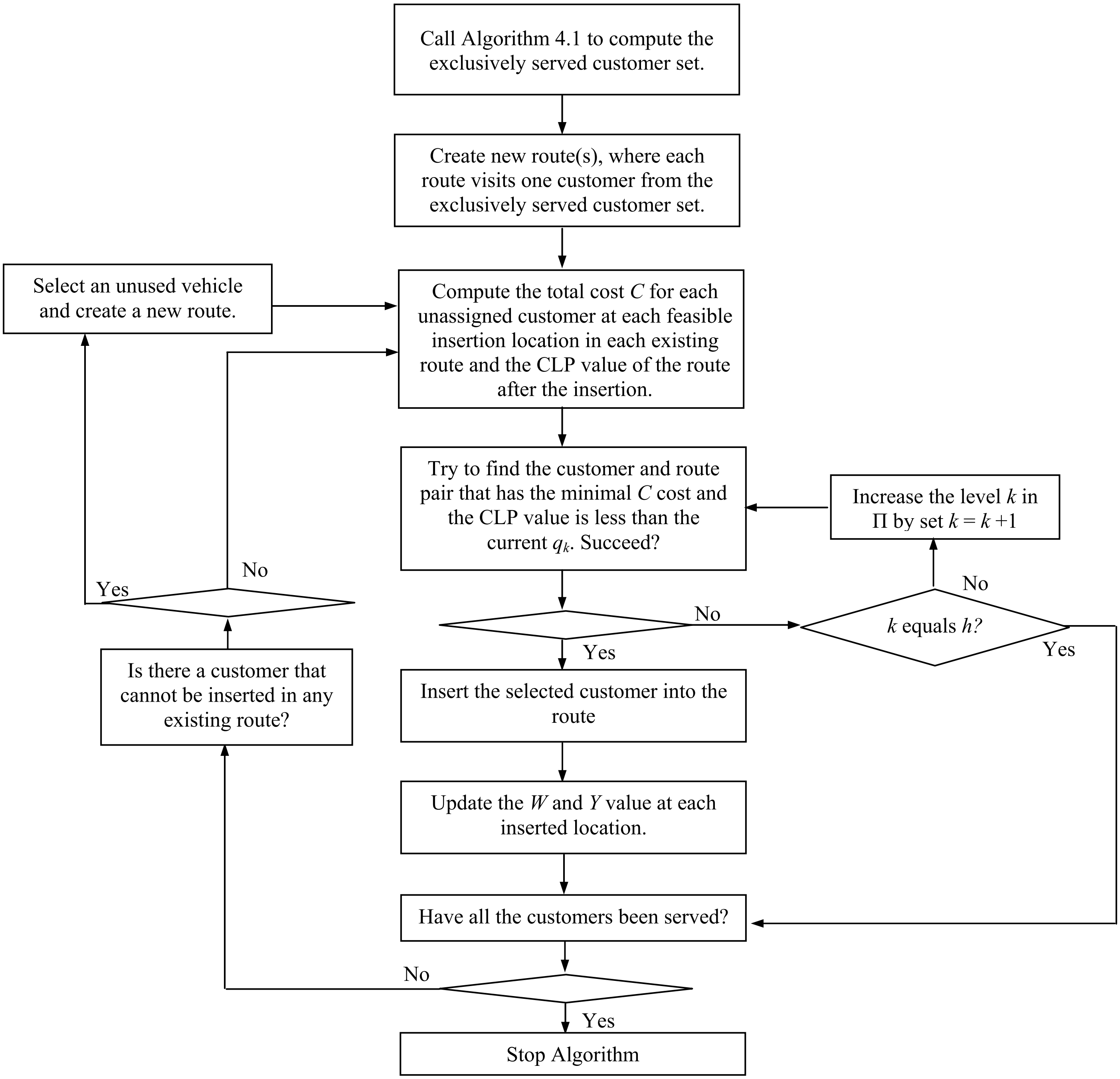



**Figure 7. Flow Chart of the Overall Construction Heuristic**

## 5. Computational Results

Our algorithm was coded in C++ with Standard Template Library and executed on a personal computer using Pentium III 1.2G processor. We set $\Pi = \{(5\%, 40\%), (15\%, 60\%), (50\%, 80\%), (\infty, 100\%)\}$. We tested the proposed construction heuristic on the benchmark problem instances provided by Li and Lim (2001). They generated the data sets from Solomon's (1987) vehicle routing benchmark problems consisting of 56 instances by randomly pairing up the customer locations. Li and Lim solved these new created pickup and delivery instances using a tabu-embedded simulated annealing algorithm. Since our objective is to develop a new insertion heuristic that can efficiently construct solutions quickly, we are not trying to beat their results for the following two reasons. First, their algorithm is much more complex than ours (e.g. their average computational time is 730 seconds, but ours is 13 seconds). Secondly, their algorithm is not bounded and sensitive to the data set, but our proposed heuristic is computationally bounded (their computational time varies from 33 seconds to 4106 seconds, but our computational time varies from 10 seconds to 23 seconds).

We compared our algorithm with a sequential insertion heuristic algorithm and a parallel insertion heuristic. The sequential insertion algorithm starts with an initial route, which consists of only one customer that is closest to the vehicle's depot. It then searches for all feasible insertions of the unassigned customers and finds the insertion that results in the minimum increase of the objective function value. If such an insertion is found, the customer will be added to the route at the position with the minimal additional cost. If there exists any customer that is infeasible to insert into any existing routes, create a new route for this customer. The parallel insertion differs from the sequential insertion algorithm only at the initial routes generation stage. From our knowledge, there is no good strategy in the literature on how to choose pivot customers to create the initial routes for the m-PDPTW. Toth and Vigo (1997) develop a parallel insertion procedure for the m-PDPTW, in which the number of initial routes is calculated as the minimum number of vehicles needed to serve 60% of all the requests. However, this strategy is not suitable for our test data. Therefore, we still use Algorithm 4.1 to select the pivot customers for the initial routes. Notice that for the m-PDPTW, multiple routes are primarily created because of the time window constraints.

The computational results are shown in Tables 1 and 3. We compared and listed three measures of the solution quality obtained by the sequential insertion, parallel insertion, and our proposed construction heuristic: number of used vehicles, total travel time and average crossing length percentage (CLP) value. We also listed the number of vehicles and total travel time of the results reported by Li and Lim (2001). We cannot compute the CLP value from the Li and Lim results since they did not present the complete final solution (routes) in their paper. Table 1 shows the results on the 19 clustered instances generated from the C1 and C2 data sets in Solomon's benchmarking problem. Each instance in Solomon's problem has 100 locations, but some instances here have more than 100 locations because some dummy nodes are added for pairing purposes.

| Prob | Sequential Insertion | | | Parallel Insertion | | | Proposed Algorithm | | | Li&Lim's Tabu Search | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | # of Veh. | Travel Time | Avg. CLP | # of Veh. | Travel Time | Avg. CLP | # of Veh. | Travel Time | Avg. CLP | # of Veh. | Travel Time |
| LC101 | 11 | 867.3 | 0.00 | 10 | 828.9 | 0.00 | 10 | 828.9 | 0.00 | 10 | 828.9 |
| LC102 | 13 | 1129.1 | 0.11 | 12 | 1001.9 | 0.08 | 11 | 912.0 | 0.01 | 10 | 828.9 |
| LC103 | 12 | 1172.1 | 0.16 | 12 | 1205.5 | 0.14 | 11 | 882.5 | 0.00 | 10 | 827.8 |
| LC104 | 11 | 966.1 | 0.06 | 11 | 915.5 | 0.01 | 9 | 861.9 | 0.01 | 9 | 861.9 |
| LC105 | 12 | 1037.6 | 0.07 | 11 | 828.9 | 0.00 | 10 | 828.9 | 0.00 | 10 | 828.9 |
| LC106 | 13 | 1139.6 | 0.16 | 13 | 1035.6 | 0.15 | 10 | 931.5 | 0.03 | 10 | 828.9 |
| LC107 | 11 | 924.8 | 0.00 | 11 | 899.8 | 0.01 | 10 | 828.9 | 0.00 | 10 | 828.9 |
| LC108 | 12 | 1247.5 | 0.33 | 11 | 1251.9 | 0.33 | 11 | 1063.5 | 0.00 | 10 | 826.4 |
| LC109 | 11 | 1293.8 | 0.05 | 11 | 1273.8 | 0.05 | 11 | 924.5 | 0.00 | 10 | 827.8 |
| LC201 | 3 | 591.5 | 0.03 | 3 | 591.5 | 0.03 | 3 | 591.5 | 0.03 | 3 | 591.5 |
| LC202 | 5 | 972.3 | 0.13 | 4 | 759.9 | 0.05 | 3 | 663.5 | 0.02 | 3 | 591.5 |
| LC203 | 4 | 849.5 | 0.06 | 4 | 822.2 | 0.06 | 4 | 753.5 | 0.01 | 3 | 585.5 |
| LC204 | 4 | 954.9 | 0.02 | 4 | 954.9 | 0.02 | 3 | 591.2 | 0.00 | 3 | 591.2 |
| LC205 | 4 | 792.0 | 0.00 | 4 | 688.9 | 0.01 | 3 | 592.4 | 0.00 | 3 | 588.9 |
| LC206 | 4 | 867.9 | 0.09 | 4 | 841.9 | 0.07 | 4 | 618.3 | 0.00 | 3 | 588.5 |
| LC207 | 4 | 709.3 | 0.00 | 4 | 684.5 | 0.01 | 3 | 588.3 | 0.00 | 3 | 588.3 |
| LC208 | 4 | 761.5 | 0.00 | 4 | 719.9 | 0.00 | 3 | 629.4 | 0.00 | 3 | 588.3 |

**Table 1. Computational Results of the Four Heuristics on the Clustered Problems.**

From Table 1, our proposed heuristic outperforms the sequential heuristic and parallel heuristic in all the LC100 and LC200 problem instances on each performance measure: number of used vehicles, total travel time and average CLP values. We summarize the results in Table 2. The difference of the LC100 and LC200 problems is that the LC100 problems have a shorter scheduling horizon while the LC200 problems have a longer scheduling horizon (larger time windows). Comparing with the sequential insertion in the LC100 problems, our proposed heuristic reduced the total travel time by 17%, and the total CLP value by 94%. Comparing with the parallel insertion in the LC100 problems, our proposed heuristic reduced the total travel time by 12%, and the total CLP value by 71%. Comparing with the sequential insertion in the L200 problems, our proposed heuristic reduced the total travel time by 22%, and the total CLP value by 81%. Comparing with the parallel insertion in the L200 problems, our proposed heuristic reduced the total travel time by 15%, and the total CLP value by 60%. Therefore, our heuristic gave a significant improvement for the LC problems regarding the visual attractiveness measurement, the CLP value. Note that the sequential insertion method gave better CLP values for the LC200 problems than the LC100 problems because the sequential insertion heuristic does not take into account the time windows while performing insertions. Therefore, it performed worse and generated more crossings for the data sets with tighter time windows (e.g. LC100). Since our insertion procedure directly accounts for the time windows, the CLP values do not deteriorate with tighter time windows.

| Prob | Sequential Insertion | | | Parallel Insertion | | | Proposed Algorithm | | | Li&Lim's Tabu Search | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | # of Veh. | Travel Time | Avg. CLP | # of Veh. | Travel Time | Avg. CLP | # of Veh. | Travel Time | Avg. CLP | # of Veh. | Travel Time |
| LC100 | 11.8 | 1086.4 | 0.11 | 11.3 | 1026.9 | 0.085 | 10.3 | 895.8 | 0.005 | 9.9 | 832.0 |
| LC200 | 4 | 812.3 | 0.04 | 3.88 | 757.9 | 0.031 | 3.3 | 628.5 | 0.007 | 3 | 589.2 |

**Table 2. Summary of the Computational Results of the Three Heuristics on the Clustered Problems.**

We also compared our solutions with the solutions reported by Li and Lim (2001). Since they did not report the CLP value, we only compared the number of used vehicles and the total travel time. Li and Lim's heuristic is a tabu-based metaheuristic and their computational time is sensitive to the data of the problem. For example, their computational time varies from 33 seconds to 4106 seconds and the average computational time is about 730 seconds. On the contrary, our proposed heuristic is very quick and computationally bounded. The longest computational time is only 23 seconds and the average computational time is around 13 seconds. Furthermore, comparing with the Li and Lim's results in the L200 problems, our proposed heuristic's results are within 8% regarding the total travel time. Comparing the results in the L200 problems, our proposed heuristic's results are within 6% regarding the total travel time. Therefore, our proposed heuristic gave very close results to the Tabu search on the cluster problems with both loose and tight time window constraints.

In the following Table 3, we provide the computational results of using the three algorithms in solving the LR100 and LR200 problems. The LR type problems are randomly distributed problems. Similar as before, the LR100 problems have short scheduling horizon while the LR200 problems have a longer scheduling horizon.

| **Prob** | **Sequential Insertion** | | | **Parallel Insertion** | | | **Proposed Algorithm** | | | **Li&Lim's Tabu Search** | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | **# of Veh.** | **Travel Time** | **Avg. CLP** | **# of Veh.** | **Travel Time** | **Avg. CLP** | **# of Veh.** | **Travel Time** | **Avg. CLP** | **# of Veh.** | **Travel Time** |
| LR101 | 21 | 1818.4 | 0.16 | 19 | 1767.4 | 0.05 | 19 | 1749.3 | 0.05 | 19 | 1650.8 |
| LR102 | 19 | 1725.9 | 0.12 | 19 | 1602.2 | 0.03 | 19 | 1561.0 | 0.03 | 17 | 1487.6 |
| LR103 | 16 | 1683.6 | 0.36 | 14 | 1552.2 | 0.31 | 13 | 1333.2 | 0.01 | 13 | 1292.7 |
| LR104 | 12 | 1316.4 | 0.08 | 12 | 1166.8 | 0.05 | 11 | 1109.5 | 0.02 | 9 | 1013.4 |
| LR105 | 19 | 1697.0 | 0.25 | 16 | 1449.2 | 0.04 | 16 | 1427.3 | 0.04 | 14 | 1377.1 |
| LR106 | 17 | 1619.6 | 0.39 | 16 | 1547.2 | 0.28 | 13 | 1298.1 | 0.04 | 12 | 1252.6 |
| LR107 | 13 | 1344.1 | 0.06 | 11 | 1193.6 | 0.06 | 10 | 1111.3 | 0.00 | 10 | 1111.3 |
| LR108 | 12 | 1339.5 | 0.05 | 12 | 1306.6 | 0.05 | 10 | 1037.4 | 0.01 | 9 | 968.9 |
| LR109 | 14 | 1390.2 | 0.03 | 12 | 1376.6 | 0.02 | 12 | 1263.4 | 0.00 | 11 | 1239.9 |
| LR110 | 15 | 1463.9 | 0.07 | 12 | 1227.1 | 0.07 | 10 | 1170.4 | 0.02 | 10 | 1159.3 |
| LR111 | 13 | 1306.8 | 0.09 | 12 | 1282.6 | 0.06 | 12 | 1184.7 | 0.02 | 10 | 1108.9 |
| LR112 | 13 | 1265.6 | 0.03 | 11 | 1237.4 | 0.03 | 10 | 1058.4 | 0.01 | 9 | 1003.7 |
| LR201 | 7 | 1571.5 | 0.43 | 6 | 1492.4 | 0.28 | 4 | 1298.8 | 0.01 | 4 | 1263.8 |
| LR202 | 7 | 1611.2 | 0.58 | 6 | 1581.4 | 0.52 | 5 | 1351.4 | 0.00 | 3 | 1197.6 |
| LR203 | 6 | 1404.7 | 0.11 | 6 | 1372.5 | 0.10 | 4 | 1067.7 | 0.00 | 3 | 949.4 |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| LR204 | 5 | 1314.9 | 0.09 | 4 | 1186.8 | 0.04 | 2 | 849.0 | 0.01 | 2 | 849.0 |
| LR205 | 5 | 1566.6 | 0.39 | 4 | 1417.7 | 0.06 | 4 | 1210.4 | 0.02 | 3 | 1054.0 |
| LR206 | 4 | 1465.0 | 0.08 | 4 | 1524.5 | 0.07 | 4 | 1098.8 | 0.00 | 3 | 931.6 |
| LR207 | 4 | 1493.7 | 0.09 | 4 | 1461.7 | 0.09 | 4 | 1135.6 | 0.03 | 2 | 903.0 |
| LR208 | 4 | 1437.4 | 0.00 | 4 | 1353.1 | 0.01 | 3 | 908.1 | 0.01 | 2 | 734.8 |
| LR209 | 4 | 1378.2 | 0.31 | 3 | 1008.2 | 0.11 | 3 | 937.0 | 0.00 | 3 | 937.0 |
| LR210 | 5 | 1532.5 | 0.44 | 4 | 1381.6 | 0.35 | 4 | 1106.4 | 0.05 | 3 | 964.2 |
| LR211 | 4 | 1151.4 | 0.07 | 4 | 1162.9 | 0.07 | 3 | 1107.0 | 0.01 | 2 | 907.8 |

**Table 3. Computational Results of the Three Heuristics on the Randomly Distributed Problems**

From Table 3, we can conclude that our proposed heuristic outperforms the sequential heuristic and parallel heuristic in all the LR100 and LR200 problem instances. The summary of the computational results in Table 3 is shown in Table 4. From Table 4, we can conclude that comparing with the sequential and parallel insertion algorithm our proposed heuristic significantly reduces the average CLP values for both the LR100 and LR200 problem sets. It also reduced the average travel time by 14% and 24% for the LR100 and LR200 problem sets comparing with the sequential insertion, respectively. Comparing with the parallel insertion, it reduced the average travel time by 7% and 18% for the LR100 and LR200 problem, respectively. Comparing with the Li and Lim's meta-heuristic, our proposed heuristic's results are within 4.3% regarding the total travel time for the LR100 problem sets and are within 12% regarding the total travel time for the LR200 problem sets.

| **Prob** | **Sequential Insertion** | | | **Parallel Insertion** | | | **Proposed Algorithm** | | | **Li&Lim's Tabu Search** | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | **# of Veh.** | **Travel Time** | **Avg. CLP** | **# of Veh.** | **Travel Time** | **Avg. CLP** | **# of Veh.** | **Travel Time** | **Avg. CLP** | **# of Veh.** | **Travel Time** |
| LR100 | 15.3 | 1497.6 | 0.14 | 13.8 | 1392.4 | 0.09 | 12.9 | 1275.3 | 0.02 | 11.9 | 1222.1 |
| LR200 | 5 | 1447.9 | 0.23 | 4.5 | 1358.4 | 0.16 | 3.63 | 1097.2 | 0.03 | 2.72 | 972.0 |

**Table 4. Summary of the Computational Results of the Three Heuristics on the Randomly Distributed Problems**

## 6. Conclusion

We proposed a new insertion-based construction heuristic for solving the m-PDP with time windows. Our objective was to design a computationally bounded heuristic that generates high quality solutions in a short time. Furthermore, we investigated how to quantitatively measure the visual attractiveness of the generated solutions. Visual attractiveness is a key logic behind the strategy of a dispatcher and a visual attractive solution can improve the trust between the routing software and planners. A measure, *Crossing Length Percentage*, was presented to quantify the visual attractiveness of the solution. This measure was incorporated in the proposed heuristic to improve the quality of the solution.

The new heuristic considered the insertion cost that not only includes the classical incremental distance but also the cost of the reduction of the time window slack due to the insertions. We compared our heuristic with a insertion heuristic and a parallel heuristic on different benchmarking problems, and the computational results showed that the proposed heuristic performed better with respect to both the standard and non-standard measures. We also compared our heuristic's results with the results obtained from a tabu search by Li and Lim (2001). The number of used vehicles and the total travel time of our heuristic were both close to the best solutions from their tabu search in both cluster and randomly generated instances.

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