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# Vibration Based Condition Monitoring: A Review

E. Peter Carden and Paul Fanning\*

Department of Civil Engineering, University College Dublin, Earlsfort Terrace, Dublin 2, Ireland

Vibration based condition monitoring refers to the use of in situ non-destructive sensing and analysis of system characteristics – in the time, frequency or modal domains – for the purpose of detecting changes, which may indicate damage or degradation. In the field of civil engineering, monitoring systems have the potential to facilitate the more economical management and maintenance of modern infrastructure. This paper reviews the state of the art in vibration based condition monitoring with particular emphasis on structural engineering applications.

**Keywords** condition monitoring  $\cdot$  damage detection  $\cdot$  vibration analysis  $\cdot$  model-updating

## 1 Introduction

Engineers and researchers, particularly in the aerospace and offshore oil industries, began to utilise vibration based damage detection during the late 1970s and early 1980s [41]. The early approaches used were based on correlating numerical models with measured modal properties from undamaged and damaged components. In the offshore oil industry, research objectives included the detection of near-failing drilling equipment and the prevention of expensive oil pumps from becoming inoperable [51]. Substantial practical problems included the influence of platform noise on measurements, instrumentation difficulties in hostile environments, varying mass-loading effects on the drilling pipe, changing platform mass caused by marine growth and the inability of wave motion to excite higher modes, which would enable localised damage to be identified. These difficulties hindered the development of frequency sensitivity methods and efforts in this area have

\*Author to whom correspondence should be addressed. E-mail: paul.fanning@ucd.ie

diminished considerably since the 1980s, although some researchers continue to advance these methods, (e.g. [80]).

According to Farrar and Doebling [41] the most mature and successful application of vibration-based damage detection technology has been in the monitoring of rotating machinery. The detection methodology is based on pattern recognition applied to time histories or spectra. Databases allow specific types of damage to be identified from the measured vibration signatures. Monitoring of rotating machinery enjoys benefits not shared with large structures such as oil platforms in terms of relative ease of access, greater control of environmental factors and their relative small scale. In efforts to extend this pattern recognition approach more widely, Farrar and Doebling [41] suggested that the vibrationbased damage detection problem is fundamentally one of statistical pattern recognition. In their opinion to advance the state of the art in vibration based damage detection the developments of

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non-model based pattern recognition methods are needed to supplement the existing model based techniques.

In the civil engineering community infrastructure monitoring is a vibrant area of current research. Health monitoring techniques for wind turbines were investigated by Ghoshal et al. [49], though a majority of the literature on infrastructural monitoring is focused on bridges and bridge management systems [2,3,5,18,46].

The time history response of a structure can be measured by a variety of sensors, e.g. accelerometers, strain gauges etc., and this data can then be converted from the time domain into the frequency domain using the Fourier transform. Further analysis of the frequency domain data is often undertaken to extract modal parameters to produce what is termed modal domain data. Techniques also exist to convert data directly from the time domain to the modal domain [78].

While measurements are always made in the time domain, the condition monitoring analyst may choose to analyse the data in the time, frequency or modal domains. Although conversion between the domains involves some compression of the data, Friswell and Penny [44] argue that for linear systems there is little loss of information between time and frequency domains. Furthermore, there may be some advantage in that the data may be averaged easily and thus the effects of random noise may be reduced.

The modal domain involves a further reduction in data volume compared to the frequency domain. While theoretically the frequency domain data (e.g. frequency response functions, FRFs), may contain information about a larger range of frequencies, Friswell and Penny [44] suggest a very practical point that unless the out of range mode(s) are very close, any response is dominated by the in-band modes and hence the frequency domain and modal domain essentially are equivalent for use in analysis.

Lee and Shin [68] point out, however, that the modal data can be contaminated by modal extraction error not present in the FRF data. Furthermore they suggest that FRFs can provide more information as the modal data is extracted from a very limited frequency range around resonance.

Several authors [9,41] question the suitability of modal data for damage detection arguing that modal information is a reflection of the global system properties while damage is a local phenomenon. For example the lower natural frequencies, which are those normally measured, are often relatively uninfluenced by local damage. The above is supported by the work of Alampalli et al. [4], who investigated the sensitivity of modal characteristics to damage in a laboratory scaled bridge span, and found that a local damage event does not necessarily change mode shapes more significantly at the damage location, or vicinity, than at other areas. A conclusion of an extensive literature survey by Doebling and coworkers [34] was that there is disagreement among researchers about the suitability of modal parameters for condition monitoring – one body of opinion suggests that they are sufficiently sensitive whereas the other disagrees. To date the opposing arguments have been demonstrated for specific test structures but have not been proven in a fundamental sense.

Nevertheless the majority of the literature to date has focused on methods based in the modal domain. This is probably because of two main reasons. Firstly, the early literature focused on the modal domain (e.g. [1]). Secondly, natural frequencies and mode shapes are easily interpreted and so are initially more attractive than somewhat more abstract features extracted in the frequency domains (e.g. the distortion identification function in [108,109]) and time domains (e.g. residuals of autoregressive models in [47]). Research into methods based in all three domains is however likely to continue, primarily because no single method has yet been found that identifies every type of damage in every type of structure.

As a result of the varying challenges offered by different structures and systems, significant research effort has been applied to condition monitoring with the emergence of a broad range of techniques, algorithms and methods. Rytter [105] classified the various methods based on the level of identification attempted:

Level 1: Determination that damage is present in the structure

- Level 2: Determination of the geometric location of the damage
- Level 3: Quantification of the severity of the damage
- Level 4: Prediction of the remaining service life of the structure

The emergence of shape memory alloys in the context of smart structures, Park et al. [97], suggests a fifth level in condition monitoring which would include so called 'self healing structures'.

A rich source of damage identification methods, such as matrix updating methods, has come from the finite element model updating literature [43]. Indeed many methods that do not use the matrix updating methods directly in the identification of damage rely on a correlated numerical model of the structure in its original state. Mottershead and Friswell [90] presented an extensive review of the literature in this area.

Although real damage in a structure can either be localised or distributed, model-updating techniques are generally more suitable for distributed damage events. The use of a large number of individual damage parameters, coupled with a limited amount of measured data, can lead to difficulties in convergence of, and non-uniqueness, of solutions in updating algorithms.

Damage identification techniques differ also by the number of sensors required for data acquisition. Natural frequencies may be measured using a single or few sensors whereas mode shapes or dynamic flexibility require multiple sensors. Issues of economic feasibility thus arise particularly if the intention is to monitor a network of structures. Some of the most successful results of damage identification have been achieved on laboratory scale truss structures, where the lack of rotational degrees of freedom (DOFs) and ease of accessibility allow entire mode shapes to be measured [63]. However, even with a large number of sensors the measurement of modal characteristics is usually incomplete.

Doebling et al. [34] presented an extensive survey of damage detection methods that use changes in modal properties (i.e. natural frequencies, modal damping factors and mode shapes). The literature reviewed concentrated primarily on Levels 1 to 3. Most of the literature focused on laboratory structures or controlled damage to field structures and did not attempt to predict the remaining service life of a structure. Level 4 identification is undoubtedly the ultimate aim of any condition monitoring system. Several different approaches were identified, from over 250 references, which they categorised as follows:

Natural Frequency Based Methods Mode Shape Based Methods Mode Shape Curvature/Strain Mode Shape Based Methods Dynamically Measured Flexibility Based Methods Matrix Update Based Methods Non-linear Methods Neural Network Based Methods Other Methods

In preparing this paper the authors decided to follow closely, but not identically, the categories of damage detection proposed by Doebling et al. with particular emphasis on papers and articles published after 1996 [34]. Readers interested in the state of the art in this area pre-1996 are referred to Doebling et al. [34].

# 2 Natural Frequency Based Methods

The physically tangible relation between stiffness and mass changes and natural frequency changes, coupled with ease of measurement of the natural frequencies (only a single sensor is required in many applications), was the impetus for using modal methods to identify damage. Salawu reviewed 65 publications dealing with the detection of structural damage through frequency changes [106].

Most of the early work was based on very simple structures and structural elements. Adams et al. [1] and Cawley and Adams [19] demonstrated that the ratio of the frequency changes in the two modes is only a function of damage location. The measurement of one pair of frequencies will yield a locus of possible damage sites. The loci for several pairs of modes may be superimposed, the actual damage site being given by the intersection of the curves. This method was successfully applied to several laboratory bars, using the axial resonances of the bars, for moderate damage levels. However, with more severe local damage (e.g. removing 60% of the cross-sectional area of a bar) a more distributed damage scenario was predicted. Additionally Banks et al. [9] showed that the geometry of the damage, and not solely the location and severity, affects the natural frequencies. For example, machined slots in test specimens produce different effects to real cracks.

Kessler et al. [57] study the effect on frequency response of various forms of damage (drilled through holes, delamination, impact damage, bending induced cracks and fatigue damage) on clamped composite plates and concluded that the only type of damage distinguishable from the others at low frequency ranges was fatigue damage. This was due to this form of damage producing many high-energy local modes that were not present in other specimens.

Chen et al. [22] questioned the effectiveness of using the changes in natural frequencies to indicate damage in a structure. The first four frequencies of a steel channel exhibited no shifts greater than 5%, due to a single notch severe enough to cause the channel to fail at its design load. Given that it is acknowledged that frequency variation due to incidental/ambient vibration and environmental effects can be as high as 5–10%, they argued that lower frequency shifts would not necessarily be useful damage indicators. Tests conducted on the I-40 Bridge [40] and on T-beam slab bridge decks [66,67] support this conclusion. When the cross-sectional stiffness at the centre of a main plate girder, on the I-40 Bridge, had been reduced by 96.4%, reducing the bending stiffness of the overall bridge crosssection by 21%, no significant reductions in the modal frequencies were observed.

Notwithstanding the above, a significant body of other researchers support the use of modal frequency shifts for damage identification. De Roeck et al. [31] monitored the Z24 Bridge in Switzerland over the course of a year. Environmental effects of air temperature, humidity, rain, wind speed and wind direction were monitored along with hourly readings from 16 accelerometers. Following a progressive damagetesting program it was demonstrated that once the effects of environmental influences were filtered out, stiffness degradations could be detected if the corresponding frequency shifts were more than just  $1\%$ .

The greatest success in the use of natural frequency shifts for damage identification, as evidenced by the number of published examples, is in small simple laboratory structures with only single damage locations. Lee and Chung [69] ranked the first four frequencies of a simulated cantilever beam to locate a single crack. The crack depth was then approximated iteratively to match the first frequency as closely as possible before the location of the crack was finally refined. Nikolakopoulos et al. [91] identified a single crack in an experimental single storey frame from shifts in the first three natural frequencies. The location and depth of the crack were determined from the intersection of the contour plots for all variations of location and depth against change in the natural frequencies. Chinchalkar [25] modelled a crack in a beam of varying cross-section using a rotational spring. Graphs of stiffness of the spring versus location were plotted for three natural frequencies and the point of intersection of the three curves was shown to give the crack location. Similarly, Yang et al. [130] used 3D plots of frequency change versus depth and location of a crack to identify a saw cut in an aluminium beam. The contour lines obtained from each frequency change plot were overlain and their intersection gave the true location and depth of the crack. In an approach similar to that of Adams et al. [1], Morassi [87] examined the identification of a single crack in a vibrating rod based on knowledge of the damageinduced shifts of a pair of natural frequencies. With free–free boundary conditions the shifts in the first two frequencies allowed the crack to be uniquely identified except for symmetrical positions. With cantilever or simply supported boundary conditions, this result does not hold good. Experimental identification of the location of a crack in a vibrating rod was successful, but an assessment of its depth proved unreliable when the crack was severe. Cerri and Vestroni [20] used

the shifts in the first three frequencies of a simulated beam to identify damage. Cracking was modelled as a rotational spring and parameterised using three variables representing location, intensity and extent of cracking.

The identification of multiple damage scenarios using frequency shifts, even for simple laboratory structures, is not as effective as evidenced by the scarcity of literature in this area relevant to single damage sites. Choy et al. [27] applied a damage identification methodology based on natural frequency changes to a numerical model of a beam on an elastic foundation. Damage was modelled as a reduction in the Young's modulus of a beam element and in a change in stiffness and damping of a Winkler spring used to model the elastic foundation. For a single fault, the change in each element was iteratively found which best matched the shift in each natural frequency. For two damage faults all possible combinations of two elements were calculated which matched the shift in the first two natural frequencies. The location and magnitude of damage was found through the intersection of these solution sets. These were then used to predict the third natural frequency and the closest match was deemed the true solution. Messina et al. [85] and Williams and Messina [127] presented a technique capable of tackling multiple damage location and identification based on natural frequency shifts. The method was based on using a linearised sensitivity of frequency shifts to damage and assumed changes in stiffness only. The algorithm was successfully verified experimentally on a three beam laboratory structure with upto two damage locations. Palacz and Krawczuk [94] showed that this method benefitted from the use of increasing numbers of frequencies but also that it was very sensitive to even small errors in measured frequencies  $(\pm 0.1\%)$ . However as the extent of damage, number of locations or severity, increases, the fundamental assumption of a linear relationship between frequency shifts and damage is no longer appropriate [15].

Here, the frequency shifts have concentrated on identifying the damage location. Surendra et al. [119] proposed that frequency shifts be used for predicting the fatigue life of a structure by correlating the rate of decrease of the first natural frequency with the fatigue life.

In conclusion, the focus on the use of frequency shifts for damage identification has been prolific, if inconclusive, in the literature. Successful identification algorithms have generally been limited to identification of a single or a few damage locations. Equally the most successful applications have been to small laboratory structures. The suitability of frequency shifts alone for identifying damage in full-scale structures, based on the literature to date, does not seem promising. However the work of De Roeck et al. [31], in filtering out ambient and environmental effects, is encouraging in this respect.

#### 3 Mode Shape Based Methods

Measurement of the mode shapes of a structure requires either a single excitation point and many sensors or a roving exciter with one or more fixed sensors. Many modal analysis techniques are available for the extraction of mode shapes from the data measured in the time domain [35,78]. Damage detection methods have been developed for the identification of damage based directly on measured mode shapes or mode shape curvatures.

#### 3.1 Direct Comparison of Mode Shapes

Two commonly used methods to compare two sets of mode shapes are the Modal Assurance Criterion, MAC [6] and the Coordinate Modal Assurance Criterion, COMAC [71].

The MAC value can be considered as a measure of the similarity of two mode shapes. A MAC value of 1 is a perfect match and a value of 0 means they are completely dissimilar. Thus, the reduction of a MAC value may be an indication of damage. Salawu and Williams [107] tested a reinforced concrete bridge before and after repair. Although the first seven natural frequencies shifted by less than 3% the MAC values showed substantial change leading the authors to argue that comparison of mode shapes is a more robust technique for damage detection than shifts in natural frequencies.

The COMAC is a pointwise measure of the difference between two sets of mode shapes and takes a value between 1 and 0. A low COMAC value would indicate discordance at a point and thus is also a possible damage location indicator. Frýba and Pirner [46] used the COMAC method for checking the quality of a repair to a prestressed concrete segment bridge after part of the superstructure had spontaneously slid off its bearings. The structure was lifted back on its bearings and cracks that had formed were sealed. COMAC analysis confirmed that the repaired segment responses were consistent with an undamaged segment.

Shi et al. [113] extended Messina et al.'s [85] method of using frequencies only in incomplete mode shapes for the location of damage. An attractive feature of this is that neither the expansion of an incomplete measurement set nor the reduction of the simulated stiffness and mass matrices is required. The method was demonstrated on a simulated 2-D planar truss and found to produce less false-positive identification than Messina et al.'s frequency shift method. As a further improvement in the algorithm's performance Shi et al. [114] optimised the sensor placement for the measurement of the incomplete mode shapes. The method was demonstrated on an experimental 8-bay truss structure measuring 20 of the 138 DOFs. Damage was represented as a loss of connection of bars to joints in up to two members.

A drawback of many mode shape based methods is the necessity of having measurements from a large number of locations. An interesting technique is the use of a scanning Laser Doppler Vibrometer (LDV), which allows for a dense grid of measurements. Khan et al. [58] used a scanning LDV to measure mode shapes in a steel cantilever beam, a steel cantilever plate and concrete beams. Cracks were located in the test specimens from localised mode shape discontinuities. It was found that in thick metal structures the defects are detectable only when they extend through more than half of the thickness. It was concluded that although the use of a scanning LDV has considerable potential, improvements in

speckle noise interference would be necessary for a successful application to actual field structures.

Araújo dos Santos et al. [7] described a damage identification algorithm based on the orthogonality conditions of the mode shape sensitivities. The algorithm was demonstrated on a simulated plate with up to three elements being reduced in stiffness to simulate damage. The results were compared with those obtained by using the mode shape sensitivities and were found to be more accurate. In two companion papers Ren and De Roeck [103,104] used Araújo dos Santos et al.'s idea of employing the orthogonality condition sensitivities. Their algorithm was demonstrated on a laboratory scale concrete beam. They concluded that, though mode shape based methods are well verified with simulated data, there are significant difficulties with fullscale structures – the predominant ones being noise and measurement errors, mode shape expansion of incomplete measurements and accurate well correlated modelling of test structures.

#### 3.2 Curvature

The use of mode shapes curvatures in damage identification is based on the assumption that the changes in the curvatures of mode shapes are highly localised to the region of damage and that they are more pronounced than changes in the displacements of the mode shapes although Alampalli et al. [4] showed that this is not necessarily the case, particularly for structures with redundancy. The curvature is often calculated from the measured displacement mode shapes using a central difference approximation,

$$
\phi_{ji}^{\prime\prime} = \frac{\phi_{(j+1)i} - 2\phi_{ji} + \phi_{(j-1)i}}{L^2} \tag{1}
$$

where  $i =$  mode shape number;  $j =$  node number;  $L =$  distance between the nodes.

Rathcliffe and Bagaria [102] used a gapped smoothing method to successfully locate a delamination in an experimental composite beam. The displacement mode shape was converted to a curvature shape using Laplace's difference equations. The curvature shape was then locally

smoothed using a gapped polynomial at each point. The damage index was defined as the difference between the curvature and the polynomial at each point. The largest index indicated the location of the delamination.

Wahab and De Roeck [122] applied a curvature-based method to the Z24 Bridge in Switzerland successfully. They introduced a damage indicator named the curvature damage factor, CDF, the difference in curvature before and after damage averaged over a number of modes. They concluded that the use of modal curvature to locate damage in civil engineering structures seemed promising.

Modal curvatures have also been used in conjunction with other measured data to identify damage. Oh and Jung [92] used both dynamic and static data from tests on a bowstring truss. Best results were achieved when mode shape curvatures and static displacements were used in combination. The authors argued that it was because the static displacements were obtained from loading conditions, which simulated higher modes of the structure, that their addition improved the damage assessment.

Wahab [124] used simulated curvature shapes of a beam in a sensitivity based model updating algorithm to identify damage. It was found that though curvature was more sensitive to damage than mode displacement shapes, convergence was not improved by the addition of the modal curvature.

The number of modal curvatures useable in damage identification routines is, naturally, limited to the number of displacement mode shapes available. In an effort to increase the amount of data available for input into damage identification routines, Sampaio et al. [110] extended the curvature approach to all frequencies in the measurement range by using FRF data. This method was tested with data from an intentionally damaged bridge. The method worked best with data before the first resonance or anti-resonance and was found to have higher performance than the curvature method. Further development of the method was needed to quantify and characterise the damage better.

In conclusion, mode shapes and their derivatives have been widely used to identify damage.

Some evidence [60,107,113] suggests that methods based on mode shapes are more robust than those based on natural frequency shifts. There are, nevertheless, some contradictions over the usefulness of mode shapes alone in damage detection even from the same authors. Ren and De Roeck [104] cast doubts on the use of mode shapes in large structures, while Wahab and De Roeck [122] presented promising results when applied to a bridge.

Such uncertainty has led to the investigation of other methods such as the use of operational deflection shapes, which have many similarities to mode shapes. More complex formulations involving the use of mode shapes have also been investigated, such as modal strain energy methods, which use modal curvatures and the dynamically measured flexibility and the residual force vector, which combine the use of natural frequencies and mode shapes.

# 4 Operational Deflection Shapes

Operational Deflection Shapes, ODS, depend on the location and relative magnitudes of the forces applied to the structure. If the structure is excited at a single location near resonance, then the mode shapes and ODS will be similar. However, non-modal behaviour may be easily seen when two excitations are applied. ODS provide a visual interpretation of the vibration patterns of a structure.

Waldron et al. [121] generated experimental ODS of a beam using piezoceramic actuators and measured them with a scanning Laser Doppler Vibrometer. Damage location was determined by the presence of a kink/discontinuity in the plot of the ODS. It was found that it was better not to excite several modes at once as this led to irregular ODS making damage identification difficult. Boundary conditions had an important impact. It was found easier to detect damage in fixed–fixed and pinned–pinned conditions than in fixed-free conditions. Location of damage was easier when an excitation location was closer to the damage location. The ODS was more sensitive to damage at higher frequencies than at lower frequencies.

#### 5 Modal Strain Energy

When a particular vibration mode stores a large amount of strain energy in a particular structural load path, the frequency and shape of that mode are highly sensitive to changes in that load path. Thus, changes in modal strain energy might also be considered as logical choice of indicator of the damage location. The literature has generally concentrated on 1-D strain methods, though these are applicable to 2-D and 3-D structures, which are decomposable into beam-like elements.

The strain energy in a Bernoulli–Euler beam associated with a particular mode shape may be calculated from,

$$
U_i = \frac{1}{2} \int_0^l EI \left(\frac{\partial^2 \phi_i}{\partial x^2}\right)^2 dx
$$
 (2)

The curvature required for this calculation is commonly extracted from the measured displacement mode shapes using a central difference approximation.

Kim and Stubbs [60] applied a damage identification algorithm to locate and size a single crack in an experimental plate girder. The method was also demonstrated to locate up to two damage sites in a simulated plate girder. The damage indicator was based on the ratio of modal strain energy of elements before and after the damage. A statistical hypothesis technique was used to classify the significance of the value of the damage indicator. Cubic spline functions were used to interpolate the incomplete mode shapes and produce a curvature function to calculate the modal strain energy. It was found that false-positive and false-negative predictions were strongly influenced by the quality and amount of modal information available. When applying the same algorithm to a laboratory stringer, Worden et al. [128] found evidence that damage location predictions were biased toward sensor locations and supports the importance of sufficient sampling of the mode shapes. Kim and Stubbs [61] derived a new damage index, which gave enhanced accuracy of damage localisation in a simulated two span beam compared to Kim and Stubbs [60].

By making the assumption that damage has only highly localised effects on mode shape curvature, Stubbs and Kim [118] used only postdamage data to localise and estimate severity of damage of an experimental two-span beam. A FE model was matched to three measured modes of the damaged beam. This produced a model with a uniform stiffness distribution, whereas the damaged beam had a single crack, which may be represented by a local stiffness reduction. The localisation of the crack was obtained with the same algorithm described in Kim and Stubbs [60]. False-positive locations were adjacent or closer to the true crack location, though the severity of damage was consistently over estimated.

Farrar and Doebling [41] were successful in using Kim and Stubbs [60] damage index in locating controlled damage in a bridge. They found that using this method outperformed the direct comparison of mode shape curvature before and after the damage.

Chen and Kiriakidis [24] identified damage in ceramic candle filters used in coal powered generation systems by comparing differences in the modal strain energy, calculated using measured modal curvature in damaged and undamaged specimens and from a correlated FE model, the location and size of the damage was estimated. Local damage was caused to the filters due to fire incidents, but no surface defects or visible damage was identifiable.

Park et al. [97] applied a modal strain energy method to a laboratory space truss modelled with 300 elements and 91 nodes. In 17 damage scenarios, 16 of 22 true-positive and 36 falsepositive identifications were made. It is interesting to note that unlike the frequency shift methods more severe damage scenarios were generally better identified than less severe scenarios. This was attributed to the masking effect of noise in the experimental data.

Law et al. [65] proposed the use of the elemental energy quotient (EEQ), defined as the ratio of the modal strain energy of an element to its kinetic energy. The difference in the EEQ before and after damage is normalised and averaged over several modes and used as a damage location indicator. The method was demonstrated on a simulated space frame. The full mode shapes were expanded from 40% of the total DOFs measured and the method was successful with 10% random noise added. The method was also applied successfully to an experimental two-storey plane frame with up to two joints loosened to simulate damage.

Hu et al. [54] presented two methods of damage assessment based on a relationship between modal strain energy and measured modal properties tailored to single damage cases. They suggest that methods involving the use of correlated stiffness and mass matrices are not ideal as the matrices are prone to error. The first method developed requires neither system matrix, while the second requires just the mass matrix, which they suggest is more accurate in general than the stiffness matrix. The second method performs better in more severe damage scenarios. The method was demonstrated on a fixed–fixed experimental beam with a single saw cut.

Shi et al. [115] presented an algorithm that was also based on the intuitive belief that mode shapes are not sensitive to local change in stiffness except when the measurement is made in, or close to, the damage domain. This implies that the largest changes in modal strain energy will occur in, or close to, the damage. The change in modal strain energy before and after damage was used to locate the damage in an experimental steel frame. Five partial modes were expanded and used to successfully locate the loosening of up to two semi-rigid bolted joints in the frame by calculating the change in the modal strain energy. The quantification of damage employed the calculated sensitivities of the modal strain energy. Shi et al. [116] improved the quantification of stiffness change by reducing the modal truncation error in the computation.

Assumptions of smooth curvature, except at damage locations, are based on uniform material properties. Peterson et al. [99,100] applied a modal strain energy method to identify and locate damage in a laboratory timber beam. One saw cut was made in the beam to represent damage. The variation of the material properties along the length of the beam, especially at locations of large knots, appeared as damage when using the localisation algorithm. However, as the saw cut was made deeper, confidence in the correct identification of its location increased.

In an effort to compare methods, Kim et al. [59] applied both a frequency based and a modal strain energy based method to identify damage (assumed to be single) location and size in a simulated beam. Two modes were used in each method. The frequency based method was based on the ratio of change in the eigenfrequencies. Curvature, for use in the modal strain energy method, was calculated by spline interpolating 289 modal displacements at the nodes of the beam modal. It was found that the modal strain energy method gave a more accurate prediction of location than the frequency based method.

Cornwell et al. [29] extended the 1-D strain method to a 2-D strain method for use in platelike structures. The 1-D strain method is applicable to plates by dividing them into strips and treating each strip individually. The largest error involved in this approach is that the torsional stiffness between slices is not preserved. Both the 1-D and 2-D methods were applied to an experimental aluminium plate with two saw cuts. Both methods performed comparably. At low levels of damage especially, both methods exhibited a tendency to produce false-positive results.

# 6 Dynamically Measured Flexibility Matrix Based Methods

The flexibility matrix is defined as the inverse of the stiffness matrix and, therefore, relates the applied static force to the resulting structural displacement. Thus, each column of the flexibility matrix represents the displacement pattern associated with a unit force applied at the associated DOF.

The dynamically measured flexibility matrix,  $[F]$ , is generally estimated from,

$$
[F] = [\Phi][\Lambda]^{-1}[\Phi]^T
$$
 (3)

where  $[\Phi]$  is the matrix of measured mode shapes and  $[\Lambda]$  is the diagonal matrix of associated measured modal frequencies squared. Due to the practical difficulties in measuring the higher modes, the flexibility matrix is generally estimated using just the lower modes. Furthermore, because of the inverse relationship to the square of the modal frequencies, the flexibility matrix is most sensitive to changes in the lower frequency modes.

Examples of use of a dynamically measured flexibility matrix are the work of Li et al. [70] and Dionisio [32]. Li et al. [70] proposed an approach for damage identification in slender structures, such as tall buildings and chimneys, based on modelling them as cantilevers and utilising the flexibility matrix in a least squares solution approach. The method assumed that damage in each storey of a building could be represented by just two variables and thus only a minimal number of modes were needed for successful identification. The authors did not tackle any issues of sparse measurements or compare cantilever models to more complex models or indeed experimental measurements.

In common with other damage detection methods there is no broad consensus on the merits of using dynamically measured flexibility as opposed to other structural parameters. Zhao and DeWolf [131] examined and compared sensitivity coefficients for natural frequencies, mode shapes and modal flexibility. On application to a simulated five DOF spring mass system it was found that the modal flexibility was the most sensitive to damage. Farrar and Doebling [41] did not come to the same positive conclusions when comparing the strain energy, the mode shape curvature and the changes in flexibility based methods in locating damage on the I-40 bridge over the Rio Grande in America. Four controlled damage states were investigated and it was found that the strain energy based method was the most successful one followed by the mode shape curvature based method. The change in flexibility method could only locate damage in the most severe damage scenario.

# 7 Residual Force Vector Method

With access to measured mode shapes, natural frequencies and an initial baseline model it is possible to formulate what is known as a

Residual Force Vector, RFV. Natural frequencies and mode shapes satisfy an eigenvalue equation, where considering the ith mode of the damaged structure,

$$
(K_d - \lambda_{d_i} M_d) \phi_{d_i} = 0 \tag{4}
$$

With  $\lambda_d$  being the square of the natural frequency, and  $\phi_d$ , the mode shape of the damaged structure being measured and therefore fully known for several modes. Assuming that the stiffness,  $K_d$ , and mass,  $M_d$ , matrices associated with the damaged structure are defined as,

$$
K_d = K_a + \Delta K \tag{5}
$$

$$
M_d = M_a + \Delta M \tag{6}
$$

Substituting Equations (5) and (6) into Equation (4) and rearranging arrives at the definition of the residual force vector  $R_i$  for the *i*th mode,

$$
R_i = (K_a - \lambda_{d_i} M_a) \phi_{d_i} \tag{7}
$$

Here, the right-hand side of Equation (7) is known. Each mode provides a single Residual Force Vector (RFV). This vector may be physically interpreted as the harmonic force excitation that would have to be applied to the undamaged structure, represented by  $K_a$  and  $M_a$ , at the frequency  $\sqrt{\lambda_{d_i}}$  so that the structure would respond with mode shape  $\phi_{d_i}$ .

Each row of the RFV represents a single degree of freedom of the numerical model of the structure. When damage occurs to an element connected to this degree of freedom, the entry in the RFV becomes very large compared to the other entries where no damage has occurred. Picking out these large terms therefore provides a method for identifying the location of damage. Subsequent algorithms are further required to quantify the damage.

Sheinman [112] presented several numerically simulated examples of a closed form algorithm for damage identification using this approach. Kosmatka and Ricles [63] identified single damage events (stiffness loss, connection loosening, lump mass addition) in a laboratory 135 member space truss. Measurements were made at each DOF to obtain complete mode shapes. The RFV was used to identify damage location. A weighted sensitivity algorithm estimated the magnitude of stiffness/mass change. As expected, it was found that increased correlation between the analytical model and the baseline modal properties improved the estimates of damage severity.

Farhat and Hemez [39] minimised the norm of the RFV by updating both stiffness and mass elemental parameters in a sensitivity based algorithm. Incomplete mode shapes were expanded by minimising the RFV. A large saving in computational effort was achieved at each iteration by only updating those elements with large entries in the RFV. This method was demonstrated on a simulated cantilever and a simulated plane truss. It was found important for identification to include modes that stored sufficient strain energy in the damaged elements. Brown et al. [14] extended the method to lightly damped structures. The mass and stiffness matrices are first updated and then the remaining RFV is absorbed by the damping matrix. The method worked well in simulated studies with damping less than 3%.

Castello et al. [17] proposed the use of a continuum damage model where a scalar parameter represented the local cohesion state of the material. The method was demonstrated on a simulated cantilever and a planar truss with up to two damage locations. Incomplete mode shapes were expanded by minimising the RFV, as in [39], while minimising the square of the RFV identified damage.

Doebling [33] used the RFV in an optimal matrix update method to identify damage in an 8-bay cantilevered truss. The stiffness matrix perturbation was matched to the RFV by updating the elemental stiffness properties with the condition that its rank be minimised. This was found to perform better than minimising the norm of the stiffness matrix perturbation. Furthermore, it was found that the optimal number of modes to be used in a minimumrank optimal update technique was equal to the expected number of elemental stiffness changes. This was considered a potential drawback, as in practice the expected order of damage will be typically unknown.

Kahl and Sirkis [56] located damage in a simulated cantilevered beam by using the RFVs from several modes and assessing the change in the stiffness matrix as a pseudo-output feedback controller.

Liu [75] identified the location and severity of a single damaged element in a simulated planar truss by minimising the square of the RFV. With sufficient modal data the element properties were directly obtainable without iteration. It was found that with the addition of noise, more modes needed to be included to improve the accuracy of the identification.

Chen and Bicanic [23] identified up to three damage locations in a simulated plate. A mode shape expansion technique was employed and two algorithms were used to identify damage, one involving the minimisation of the norm of the RFV and the second, the minimisation of the norm of the residual energy vector. Both methods were found to give similar convergence to the correct identification.

The formulation of the RFV relies on measurement of the mode shapes. If the mode shapes are sparsely measured, either reduction of the system matrices or expansion of the mode shapes is necessary. Reduction of the system matrices destroys the structure of the matrices and therefore the direct use of the RFV for location is negated. Expansion of the mode shapes from few measurements casts doubt on the ability to locate damage accurately. However, with sufficient measurements, the RFV seems to be a robust method for the location of damage and its use in sizing damage is certainly promising.

# 8 Model Updating Based Methods

The literature concerned with model updating has provided a rich source of algorithms adaptable to damage identification. Furthermore, many damage identification algorithms rely on a wellcorrelated numerical model of the structure in its initial state. Several issues arise when creating a correlated numerical model; the measured data chosen to be matched by the model, the accuracy of the initial model, the size and complexity of the model, the number of updating parameters and the non-uniqueness of resultant model in matching the measured data.

The accuracy of the initial model of a structure used to identify damage with an updating algorithm is important. Fritzen and Jennewein [45] used sensitivity based algorithms to locate and detect damage. It was found that even the use of Bernoulli–Euler beams instead of Timoshenko theory shifted the higher eigenfrequencies, so that no reasonable results were obtainable.

The size of the model to be updated is of concern, though with available computing power increasing constantly, it is now possible to tackle larger and more complex models than ever before. However as model-updating problems are usually solved by iterative methods that require the solution of the analytical problem at least once in each iteration, its application to large models can be very processor intensive. Möller and Friberg [86] proposed a method that reduced the problem by projection onto a subspace spanned by a reduced number of modes. This resulted in substantial computational timesavings.

Gola et al. [50] examined the number of parameters identifiable in sensitivity based updating methods. The theoretical number of parameters furnished by the matching of eigenvalues is equal to the number of measured resonant frequencies. When using mode shapes, the number of parameters has an upper limit of the number of modes times the number of degrees of freedom measured. This limit is further reduced depending on the structure of the mode derivatives.

The non-uniqueness of updated models is an important concern in damage identification as well as model updating. Berman [13] concluded that there can be no unique corrected dynamic model of a structure as long as the model has fewer DOF than the actual structure. He argued that as the true actual structure has an infinite number of DOF there exist an infinite number of physically reasonable models, which adequately predict the behaviour of the structure over an adequate frequency range. When such a model is applied to damage determination the true changes in the physical characteristics are required and this represents a far more onerous task than the prediction of model behaviour. In the same vein, Baruch [10] showed that simultaneous changes in the mass and stiffness matrices could not be identified using modal data alone. The reason identified was that mode shapes could not provide a reference basis. Methods that do use mode shapes as a reference basis may identify matrices quite different from the actual stiffness and mass matrices due to the identified matrices being nonunique. This has the important implication in damage identification that damage affecting both mass and stiffness properties are not uniquely identified when using modal measurements alone.

The concerns outlined above show why model updating has not been developed into a black box technology. Many engineering judgements are key to the success of any modelupdating project.

Many of the algorithms used to identify damage have attempted to match natural frequencies and mode shapes. Damping is generally neglected due to the difficulty in modelling it accurately. Casas and Aparicio [16] investigated the identification of cracking in laboratory concrete beams. Using a model updating technique, it was found that the shift in a single frequency could not alone distinguish between changes in bearing conditions, deformation modulus or cracking. It was also found that damping was not significantly different in the cracked beams compared to the uncracked beams and that there was no clear relation between crack growth and increase in damping. They suggested this to be an important conclusion as the ability to neglect damping, as a model updating parameter, is beneficial.

Measurement of natural frequencies alone is faster and more economical than measurement of mode shapes – hence damage detection with a model updating technique using only natural frequencies would be attractive.

A notch in a five-storey experimental frame was localised by Morassi and Rovere [88] using shifts in the first five frequencies related to the shear-type modes. It was found that the use of the hypotheses, of stiffness never being greater than its value in the reference configuration and the stiffness distribution being known within some regions of the model, played a crucial role

in the successful convergence of the damage identification algorithm. Maeck et al. [77] induced cracks in reinforced concrete beams by static loading and used a modal updating scheme to match the measured changes in the first five natural frequencies to an FE model. The assumption of particular damage patterns to reduce the number of updating parameters, however, would not be as easy in all, or more complex, structures.

The use of only natural frequencies curtails the number of possible updating parameters and therefore the type, number and location of damage that may be identified. For this reason it is beneficial to use measured mode shapes as well, if they have been measured. Wahab et al. [123] located damage in three reinforced concrete beams using four measured natural frequencies and mode shapes with good success. Jang et al. [55] identified controlled damage in upto three locations in a laboratory frame structure for which six modes were identified. Damage was successfully located although no attempt was made to quantify it. Cobb and Liebst [28] identified damage in an experimental 8-bay truss structure without computation of eigenvector and eigenvalue sensitivities and the corresponding eigenanalysis in an iterative updating algorithm. Koh et al. [62] applied a model updating approach to identify damage in a laboratory model of a six-storey steel frame building.

In full-scale tests on a 21.5 m bridge span scheduled for demolition, Halling [52] used model updating with three frequencies and two modes to identify controlled damage. Damage was represented by parameters modelling the moment of inertia at the top and bottom of the bridge columns, where damage was known to be located giving six updating parameters. Substantial changes in these parameters were successfully shown though their magnitudes were not correlated with actual changes observed.

Papadopoulas [96] presented a method of model updating and damage identification, which accounted for structural variability. The statistical properties of the healthy mass and stiffness parameters and the mean healthy natural frequencies and mode shapes of the system were first determined. The mean damaged natural frequencies and mode shapes of the system were

then simulated. The number of modes available was assumed to be equal to the number of damage parameters and these parameters were determined. The statistical properties of the damaged stiffnesses were then determined and probabilistically compared to the healthy stiffnesses to yield an estimate of the probability of damage.

The size of the model is important for computationally efficient updating. Law et al. [64] presented a damage detection oriented modelling methodology for large structures. The extensive number of DOFs in large, complex structures makes many updating algorithms computationally infeasible. To reduce the number of DOFs, superelements were formulated while the modal sensitivities to small physical changes were maintained for use in a sensitivity based updating algorithm. The method was demonstrated on a simulated bridge deck structure where the initial 5370 DOFs were reduced to 211 DOFs with reasonable success.

Model updating using FRF measurements directly has also been utilised for damage identification [30,82,21]. The initial, and obvious, advantage in using FRF data over modal data is that it negates the need to identify the modal parameters from measurements and to perform mode-pairing exercises. A further advantage in using FRF data over modal data in model updating is that FRF data can provide much more information in a desired frequency range than modal data which is limited to just a few FRF data points around resonance [74]. Grafe [51] also points out that, by using the many more data points available, systems of updating equations can be easily turned into over-determined sets of equations. Care should be taken to avoid ill-conditioned matrices, however, as adjacent points in the FRF are unlikely to contain significantly different information about the system.

In summary, model-updating methods have been used extensively in damage identification algorithms. In model updating, the engineer is forced to use his/her judgement to choose likely parameters and locations in the model that are in error. In damage identification the lack of knowledge of location leads to difficulties in

applying the same methods due to an increased number of parameters. Many authors have overcome this problem by making assumptions about the location and form of damage. Those that have not made these assumptions have found that they require large measured data sets to avoid ill-conditioning in the updating equation sets.

# 9 FRF Based Methods

Some literature has concentrated on the use of FRF measurements directly, as opposed to the modal data extracted from the FRF measurements. Lee and Shin [68] argued that there are two main advantages of using the FRF data. Firstly, modal data can be contaminated by modal extraction errors in addition to measurement errors, because they are derived data sets. Secondly, a complete set of modal data cannot be measured in all but the simplest structures. FRF data can provide much more information on damage in a desired frequency range compared to modal data that is extracted from a very limited range around resonances.

An unsophisticated but real time condition monitoring approach was proposed by Lim et al. [73]. A real time modal parameter identification algorithm was applied to identify damage in an experimental space truss. Changes in the continuously monitored FRF amplitude, damping and frequencies were interpreted as indications of damage. The advantage of continuous real time monitoring is that it gives an operator early warning, so that appropriate action may be taken before a catastrophic failure occurs.

Wang et al. [125] proposed an iterative approach to locate and size damage based on complete measurement of the receptance matrix at many frequency points. In practice, complete measurement of the receptance matrix is not possible, and those computed from a correlated numerical model were used for the unmeasured coordinates. The method was demonstrated experimentally on a 3-bay frame structure with two slot cuts. Three hundred and forty two frequency points were sampled from the FRF measurements. The iterative technique involved the computation of the complete receptance matrix of the numerical model at each of the 342 frequency points sampled. Inaccuracies in the damage detection were attributed to inaccurate modelling of the joint elements and of the slot cuts.

Lee and Shin [68] used both modal and FRF data from a simulated beam to identify upto three damage locations. Damage was simulated by the reduction in the Young's Modulus of sections of a Bernoulli–Euler beam. Modal data from the beam in the undamaged state and FRF data from the beam in the damaged state were used in the identification algorithm. It was found that using a multiple-excitation-frequency, multiple-measurement-point approach gave the most reliable results.

Fanning and Carden [36] proposed a damage detection methodology based on a single-input– single-output measurement based on a numerically efficient method of calculating a single FRF. The method requires a correlated numerical model of the structure in its initial state and a single measured FRF of the damaged system sampled at several frequencies to detect structural changes. The method was successful in detecting stiffness changes in a numerically simulated 2-D frame structure [37], and also in detecting mass changes in a numerically simulated 3-D lattice tower [15]. Subsequently, the method has been demonstrated experimentally in detecting additional lump masses in a lattice steel tower (Fanning and Carden [37]). The method is easily adaptable to make use of more than a single measured FRF.

# 10 Wavelet Transform Methods

Liew and Wang [72], Hou et al. [53] and Lu and Hsu [76] describe laboratory scale studies where wavelet theory has been used for damage detection. Wavelet transforms are based on the idea that any signal can be broken down into a series of local basis functions called wavelets. Any particular local feature of a signal can be analysed based on the scale and translation characteristics of wavelets. The transform may be applied and mapped to the space or time domain

of the structure. This is in contrast to the Fourier transform, which is generally used to map from the time domain to the frequency domain. Wavelet transforms are sensitive to singularities in a signal such as a step. Thus, they may be used to find an abrupt change in a mode shape, often indicative of damage, or locate a sudden change in response from an acceleration time response. When applied to the space domain, an important issue in the use of wavelet analysis is the number of DOFs measured. The finer the resolution of measurements in the space domain, the more information the wavelet analysis can provide.

#### 11 Neural Network Methods

Neural networks arose from the study of biological neurons and refer to a computational structure composed of processing units representing neurons. All neurons have multiple inputs and a single output. Neural networks have been applied successfully in many diverse applications including vibration based damage identification [42,79,126,132]. In general, neural networks are particularly applicable to problems where a significant database of information is available, but difficult to specify an explicit algorithm.

Ramu and Johnson [101] and Pandey and Barai [95] both applied back propagation neural networks to identify damage. In both cases the network was found to be effective except that the topology of the network was found to be critically important for performance. Barai and Pandey [8] compared the performance of a timedelay neural network, TDNN, to a back propagation network on the same 21-bar truss. In the TDNN both the original signal and the signal after certain time intervals are fed as input. The computational time involved in training the TDNN was greater, but the performance was found to be generally better.

An interesting aspect of the work of Marwala and Hunt [83] was the proposal of a committee of neural networks. Marwala [81] demonstrated the use of the committee approach on a damaged experimental cylinder. Three networks were trained and their outputs combined to give better predictions than by the three networks separately. Each network was trained with different data, namely FRFs, Modal data and Wavelet Transform Data. The improved performance of the committee was reasoned to be due to different structural alterations having different relative apparent effects in the frequency, modal and time domains. For example, a change in the damping due to the addition of a sponge had the greatest effect in the time domain, whereas the FRFs showed the greatest sensitivity to a drilled hole.

#### 12 Genetic Algorithm Methods

Genetic algorithms are methods for optimisation of functions based on the random variation and selection of a population of solutions. They are part of what may be described as evolutionary algorithms, which have been developed since the 1950s [84]. Their advantage over traditional hill-climbing optimisation algorithms is that they are capable of tackling multi-modal solution topologies that are typical of damage identification problems.

Many authors, for example Chiang and Lai [26] and Moslem and Nafaspour [89], describe a two-stage process where the RFV is used to locate damage initially and then in a second stage a GA is used to quantify the damage in the identified elements successfully. The method was demonstrated on a simulated truss structure of 13 elements with up to 3 elements damaged.

Ostachowicz et al. [93] identified the location and magnitude of an added concentrated mass on a simulated rectangular plate by using the shifts in the first four natural frequencies. A genetic algorithm was employed to overcome the problem of multiple peaks in the objective function.

#### 13 Statistical Methods

Farrar and Doebling [41] suggested that the vibration based damage detection problem is fundamentally one of statistical pattern recognition. In their opinion to advance the state of the art in vibration based damage detection the developments of non-model based pattern

recognition methods are needed to supplement the existing model based techniques.

This concept of non-model based identification has spurred interest in the use of novelty detection for condition monitoring. Novelty detection is concerned with the identification of any deviations in measured data relative to data measured under normal operating conditions. Features derived from measurements taken from a structure in its undamaged state will have a distribution with an associated mean and variance. If the structure is damaged, then there may be a change in the mean, the variance, or both. Statistical process control provides a framework for monitoring the distribution of the features and identifying new data that is inconsistent with the past – 'outlier analysis'. If all other variables can be eliminated then a change in the distribution characteristics of the features will indicate damage. It is important to note that the detection of damage, rather than location and quantification, is the objective of using statistical pattern recognition.

Worden et al. [129], Fugate et al. [47] and Fanning and Carden [38] all considered statistical process control approaches to damage detection. To create the features for monitoring, the authors used autoregressive functions fitted to history response data of an undamaged state. The mean and variance of the residuals of the autoregressive model were used to form the statistical process control charts. The same autoregressive model was then fitted to subsequent time history responses of damaged states. The resulting residuals were plotted on the control charts and those points lying outside of control limits were counted as outliers and used to indicate a change in the system. This approach was found to be effective in identifying damage in each case and, in the case of Fanning and Carden [38], it was demonstrated clearly that this statistical pattern recognition approach was clearly more effective than other single/few sensor algorithms.

Four waveform recognition techniques to distinguish between FRF waveforms of intact and damaged bridges were investigated by Samman and Biswas [108,109] in two companion papers. The first technique employed was the Waveform Chain Code (WCC) that characterises waveforms by their relative slope and curvature. Differences in slope and curvature were processed for use as features. The second technique was Adaptive Template Matching (ATM), which performed a point-by-point magnitude check for the detection of differences between two FRFs. The feature extracted from this technique was the tolerance required to keep a signal within the tolerance space of the reference signal. The second technique employed was an FRF assurance criterion (SAC) and was similar in formulation to the MAC. A maximum value of 1 indicated that the signals were identical, while a minimum value of 0 indicated that the signals were totally different. The third technique employed was called the Equivalent Level of Degradation System (ELODS). This technique was based on constructing a transformer that took as its input the FRF signal under examination and returned as output the very same signal if the structure was undamaged, but returned a distorted version of the input signal if the structure was damaged. This provided a distortion identification function as an identification feature. With simulated data in the absence of noise the ranking of effectiveness for the techniques were: ELODS, WCC, ATM and the least effective was the SAC. With data from a highway bridge, only the WCC method was successful in detecting a crack. All of the techniques can only detect the presence and not the location or severity of damage.

Non-model-based statistical pattern recognition techniques do not require the generation of a correlated numerical model to be a time consuming and difficult task. They are also suitable for data sets obtained through ambient excitation only, for example traffic or wind loading on a bridge structure. A disadvantage of these methods is that they are probably limited to Level 1 or possibly Level 2 identification.

INRIA in France [12] recently proposed a statistical model based damage detection and localisation method utilising a subspace based residual and a statistical analysis of aggregated sensitivities of the residual to damage. Damage is flagged when the value of the residual passes a statistically based decision rule. Damage localisation is determined using the sensitivities of the

residual to damage in elements of the structure calculated with the aid of an analytical model. The method is applicable to cases where only output data is available and was developed from subspace based stochastic identification methods [11]. The method gave satisfactory results when applied to a numerical two storey steel frame structure.

# 14 Other Methods

In the literature there are also techniques that do not fall easily into any of the categories described above. Gatulli and Romeo [48] proposed an adaptive, on-line control algorithm for both vibration suppression and damage detection. The method was demonstrated on a simulated three DOF system with one actuator. Large control efforts are required when an abrupt change in system properties occur, as the controller attempts to compensate and return the monitored responses to their initial values. These efforts may therefore be used as indicators of damage.

Sawyer [111] proposed a fuzzy logic based damage identification system. Static displacements, frequency shifts and mode shape MAC values were converted to fuzzy sets and input to the system. A fuzzy associative memory (FAM) then converted these into a fuzzy output set, which was finally defuzzified to produce a crisp data set containing the location and severity of damage. The FAM was trained using a set of FE simulations containing several damage magnitudes in each element separately. The system was set up for a single damage location and issues of computational feasibility arose in more complex structures or structures with multiple damage sites. The potential benefits of using fuzzy logic arise in its ability to deal with noisy or uncertain conditions.

Tan et al. [120] used strain gauges to monitor the dynamic response of reinforced concrete slabs. Plots of measured dynamic strain showed unique deflection signatures that varied with the internal state of the slab, thereby suggesting that these could potentially be used for condition monitoring and residual strength identification.

Sohn and Law [117] made use of Ritz vectors extracted from measured flexibility. The Ritz vectors are equivalent to a deflection pattern observed when an arbitrarily defined load vector is applied. The damage identification method employed a Bayesian probabilistic approach to match Ritz vector sets to give the most likely damage hypothesis. The search for the most likely damage hypothesis theoretically requires the examination of all possible damage scenarios. But, when damage is assumed to be localised in a few substructures and a branch-and-bound search algorithm is employed, the search becomes computationally feasible. The method was demonstrated on an experimental steel bridge model and it was found that the use of Ritz vectors outperformed the use of mode shapes in the algorithm. The authors attributed this to the better sensitivity of the Ritz vectors and to the increased amount of information obtained by employing multiple load patterns.

#### 15 Concluding Summary

A review of the state of the art in vibration based condition monitoring revealed numerous and diverse algorithms, which utilised data in the time, frequency and modal domains.

In general, the literature demonstrates that there is no universal agreement as to the optimum method for using measured vibration data for damage detection, location or quantification. Most notably, the sensitivity and measurability of the modal parameter shifts due to localised damage is a matter of disagreement amongst the research community.

Additionally no algorithm has yet been proposed, which can be applied universally to identify any type of damage in any type of structure. Equally given that no algorithm was found which attempted the prediction of the remaining service life of a structure, there is a clear challenge to the condition monitoring research community to tackle the so called Level 4 identification.

While some algorithms were capable of locating damage in only a single location, others were limited in the number of damage locations

identifiable only by the discretisation of a numerical model of the candidate structure. However, when applying damage identification techniques to test cases, the majority of the literature used structures with damage in only a few locations. This may be indicative of an implicit assumption that with remote and continuous monitoring only a limited number of independent damage events are likely to occur between successive assessments of the integrity of a structure, which would limit the number of damage locations an algorithm would be required to identify.

A significant feature of the reviewed literature is the balance between laboratory scale tests, numerical simulations and full-scale tests. A vast majority of the literature continues to focus on laboratory tests and numerical simulations. These tests and simulations, whilst beneficial in terms of testing proposed detection algorithms, cannot replicate the environmental effects to which real structures are subjected to. The issue of whether, or not, environmental effects can be reliably and confidently filtered from measured data for condition monitoring purposes has not yet been tackled comprehensively in the literature.

The algorithms identified varied also in the number of sensors required and in the level of damage identification attempted. Those that used data from a large number of sensors, such as the residual force vector based methods were generally successful. Those relying on a single or a few sensors, such as natural frequency based methods, were not consistently successful. The emergence of statistical pattern recognition techniques seem promising in the context of using few sensors although the ambition of these pattern recognition techniques is still limited to Level 1 identification. In the authors' view the issue of sensor affordability is one of the single most important decision making constraints facing the structural health monitoring community – there is clearly no argument that more sensors, and hence more measured data, lead to a greater success in damage detection – isn't it time that the structural health monitoring community begins to focus its attention on developing cheap robust sensors rather than more elaborate detection routines?

Finally, the entire set of algorithms reviewed attempts to reach a single optimised solution. However, is this realistic with a limited knowledge of a structure? Another approach, which has not received much attention in the literature, is the consideration of algorithms that use an intermediate number of sensors, which assume that damage in only a limited number of locations and recognise the limitations that this imposes by not seeking a unique solution but rather producing multiple diagnoses of varying probability.

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