ITERATIVE SOURCE-CHANNEL DECODING FOR ROBUST IMAGE TRANSMISSION

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ABSTRACT

In this paper we discuss the application of a joint source-channel decoding approach to image transmission over wireless channels. In addition to channel codes, also the implicit residual redundancy after source encoding in both horizontal and vertical direction is utilized for error protection. At the decoder we use an iterative ("turbo") source-channel decoder which can be obtained in the same manner as for serially concatenated channel codes. As a new result we show that this iterative decoding scheme in combination with a novel simplified joint source and channel coding rate allocation at the encoder can be successfully employed for protecting the image data, especially when the channel is highly corrupted. Furthermore, when the source correlations are approximated with a large training set at the decoder, only a small loss in performance is observed.

1. INTRODUCTION

Several joint-source channel coding approaches have already been proposed for the reliable transmission of still images over wireless channels. For example, some authors use a state-of-the-art source coder and add an elaborate error protection scheme for the highly sensitive source-encoded bitstream (e.g. [1,2]), often in combination with a joint source and channel coding rate allocation. These methods provide excellent results for moderately distorted channels, however, especially for low channel signal-to-noise ratios (SNRs) their performance highly depends on the properties of the used channel codes.

A second approach in the literature is, on the other hand, to design the source encoder such that the residual index-based redundancy in the resulting bitstream alone is sufficient to provide reasonable error protection at the decoder (e.g. [3–5]). These methods have less encoding delay and for very low channel SNR often yield similar or better performance compared to using channel codes.

In this paper we combine both approaches and present an iterative joint source-channel decoding approach for robust image transmission, where *both* the implicit residual index correlation after source-encoding and the explicit redundancies from channel codes are used for protecting the data. Furthermore, in the proposed experimental image transmission system the source and channel encoding rates are jointly allocated in a rate-distortion sense. For the source correlation a two-dimensional model [5] is used, and at the decoder side we utilize an iterative source-channel decoding scheme [6] analog to serially concatenated channel codes, where the two-dimensional source redundancy is exploited by a soft-input a-posteriori probability (APP) source decoder.

2. DESCRIPTION OF THE TRANSMISSION SYSTEM

The block diagram of the overall transmission system is depicted in Fig. 1. The two-dimensional (2-D) subband image is

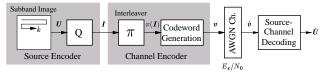


Fig. 1. Model of the transmission system

scanned in order to obtain the one-dimensional (1-D) subband vector $\boldsymbol{U} = [U_0, U_1, \dots, U_{N-1}]$ consisting of N source values U_k . After subsequent (vector-) quantization the resulting indices $I_k \in \mathcal{I}$ are represented with M bits, and thus can be described with the finite alphabet $\mathcal{I} = \{0, 1, \dots, 2^M - 1\}$. We can generally assume that there is a certain amount of residual redundancy in the index vector $\boldsymbol{I} = [I_0, I_1, \dots, I_{N-1}]$ due to delay and complexity constraints for the source encoder. In the following, the correlation between the indices I_k is modeled by a first-order stationary Markov-process with index transition probabilities $P(I_k = \lambda \mid I_{k-1} = \mu)$ for $\lambda, \mu = 0, 1, \dots, 2^M - 1$. For the sake of a more compact notation we will also use the expression $I_k^{(\lambda)}$ for the hypothesis $I_k = \lambda$.

In the channel encoder the bitstream is first interleaved in order to provide decorrelated bits for the following codeword generation step using an rate- R_C systematic channel code. We obtain the code bit vector $\mathbf{v} = [\pi(\mathbf{I}), \mathbf{c}]$ where $\mathbf{v} = [v_0, v_1, \ldots, v_{N_v-1}]$ with $v_k \in \{0, 1\}, \ N_v = N \cdot M/R_C$, and \mathbf{c} referring to the redundancy bits. The code bit vector \mathbf{v} is then transmitted over an AWGN channel, where coherently detected BPSK is used for the modulation. The conditional p.d.f. for the received soft bit $\hat{v}_m \in \mathbb{R}$ at the channel output given the transmitted bit $v_m, m = 0, 1, \ldots, N_v-1$, can be written as

$$p(\hat{v}_m \mid v_m) = \frac{1}{\sqrt{2\pi}\sigma_e} e^{-\frac{1}{2\sigma_e^2}(\hat{v}_m - v_m')^2} \text{ with } v_m' = 1 - 2v_m \quad (1)$$

and $\sigma_e^2 = \frac{N_0}{2E_s}$ denoting the channel noise variance. E_s is the energy to transmit each bit and N_0 corresponds to the one-sided power spectral density of the noise. Using conditional log-likelihood ratios (L-values) we may express (1) also as

$$L(\hat{v}_m \mid v_m) = \log \left(\frac{p(\hat{v}_m \mid v_m = 0)}{p(\hat{v}_m \mid v_m = 1)} \right) = 4 \frac{E_s}{N_0} \hat{v}_m = L_c \hat{v}_m. \quad (2)$$

The source-channel decoding step in Fig. 1 then provides an estimate of the input vector \boldsymbol{U} in such a way that the SNR at the decoder output is maximized.

3. JOINT ALLOCATION OF SOURCE AND CHANNEL CODING RATES

When considering the transmission model in Fig. 1 for every subband in a subband image compression scheme, we have the additional problem of choosing the quantizer bit rates and the channel coding rates for a given rate budget R_T such that the resulting distortion in the reconstructed signal is minimized. In the following, we propose a suboptimal strategy for jointly assigning the source and channel coding rates $\mathbf{r}_S = [R_S^{(0)}, \dots, R_S^{(j)}, \dots, R_S^{(K-1)}]$ and $\mathbf{r}_C = [R_C^{(0)}, \dots, R_C^{(j)}, \dots, R_C^{(K-1)}]$, with $R_{S/C}^{(j)}$ denoting the corresponding rates in the j-th subband. For simplicity reasons we restrict ourselves to the case where the only method of error protection is channel coding (i.e., we use source decoding by table lookups). The proposed method is based on the rate-distortion-optimal bit allocation algorithm from [7], which is modified such that also the error correction capabilities of the channel codes and the additional channel noise is incorporated in the distortion measure.

In order to obtain a transmission with bit error rate (BER) p_e over an AWGN channel we know from Shannon's channel coding theorem that under idealized conditions [8]

$$H(I) R_C f(p_e)/M \le C_{AWGN}(E_s/N_0)$$
 (3)
with $f(p_e) := 1 + p_e \operatorname{ld}(p_e) + (1 - p_e) \operatorname{ld}(1 - p_e)$

has to be satisfied. Herein, H(I) denotes the entropy of the source index vector I, and $C_{\mathrm{AWGN}}(\cdot)$ is the capacity of the binary-input AWGN channel, which depends on the parameter E_s/N_0 . By using the equality in (3) and applying it to every subband j we obtain a relation between the actual BER $p_e^{(j)}$ on the one hand, and the channel coding rate $R_C^{(j)}$, the quantized source signal $Q[U^{(j)},R_S^{(j)}]$ with rate $R_S^{(j)}$, and the channel capacity on the other hand:

$$f(p_e^{(j)}) = \frac{C_{\text{AWGN}}(E_s/N_0) \cdot M^{(j)}}{R_C^{(j)} \cdot H(Q[U^{(j)}, R_S^{(j)}])} \text{ for } j = 0, \dots, K-1.$$
 (4)

 $M^{(j)}$ refers to the word length in the j-th subband and depends on the actual rate $R_S^{(j)}$. A worst case estimate for the distortion $D(p_e^{(j)},R_S^{(j)},R_C^{(j)})$ in the j-th subband can be achieved in the following way: Let $P_{\nu}^{(j)},\,\nu=0,1,\ldots,M^{(j)}$, denote the probability that ν bits are received wrongly when the $M^{(j)}$ -bit codeword $I_k^{(j)}=\lambda$ is transmitted over a binary symmetric channel with error probability $p_e^{(j)}$. Then, $P_{\nu}^{(j)}$ can be specified as

$$P_{\nu}^{(j)} = \begin{pmatrix} M^{(j)} \\ \nu \end{pmatrix} \left(p_e^{(j)} \right)^{\nu} \left(1 - p_e^{(j)} \right)^{M^{(j)} - \nu} \tag{5}$$

where $p_e^{(j)}$ is derived by solving (4). A reconstructed distorted subband value $\bar{U}_k^{(j)}$ at time instant k can now be generated as

$$\begin{split} \bar{U}_k^{(j)} &= P_0^{(j)} \cdot U_q(\lambda, R_S^{(j)}) + \sum_{\nu=1}^{M^{(j)}} P_\nu^{(j)} \cdot U_q(\bar{\lambda}(\nu), R_S^{(j)}), \quad \text{with} \\ \bar{\lambda}(\nu) &= [\bar{\lambda}_0, \dots, \bar{\lambda}_{\nu-1}, \lambda_{\nu}, \dots, \lambda_{M^{(j)}-1}] \ \text{and} \ \bar{\lambda}_\ell = 1 - \lambda_\ell \ \ (6) \end{split}$$

for $\ell=0,1,\ldots,\nu-1$. $U_q(\lambda,R_S^{(j)})$ refers to the λ -th entry of the quantization table, which leads to a source coding rate $R_S^{(j)}$, and $\lambda_\ell \in \{0,1\}$ denotes a single bit of the index λ . The distorted value $\bar{U}_k^{(j)}$ can thus be obtained by successive flipping of bits in the binary representation of the index λ starting with the most significant bit. The corresponding entries of the quantization table

are then weighted with the probabilities $P_{\nu}^{(j)}$ from (5). Finally, the distortion $D(p_e^{(j)},R_S^{(j)},R_S^{(j)})$ can be calculated from the mean-squares error between $\bar{U}_k^{(j)}$ and the undistorted source value $U_k^{(j)}$.

The rate allocation scheme can now be summarized as follows:

- 1. Calculate $D(p_e^{(j)}, R_S^{(j)}, R_C^{(j)})$, $j = 0, 1, \ldots, K-1$ by using (4), (5) and (6) for all combinations of $R_S^{(j)}$, $R_C^{(j)}$ and the given E_b/N_0 on the channel. Here, $E_b = E_s/R_{C,\text{overall}}$, where $R_{C,\text{overall}}$ denotes the channel coding rate for the *whole* image.
- 2. Obtain an optimal allocation for \mathbf{r}_S and \mathbf{r}_C , given the rates and distortions from step 1 and the overall rate budget R_T , by using a modification of the bit-allocation algorithm from [7], where also the channel coding rates are included in the optimization.
- 3. In order to allow equal transmission energy for all bits in the whole image, update $R_{C,\text{overall}}$ by using \mathbf{r}_C from step 2. Go to step 1 and iterate until there is no further improvement for the vectors \mathbf{r}_C and \mathbf{r}_S .

4. SOFT-INPUT APP SOURCE DECODING

In the following, we assume that only the residual source redundancy in both horizontal and vertical direction is exploited for error-protection [5] and present a soft-input APP source decoder for the received source index sequence $\hat{I} = [\hat{I}_0, \hat{I}_1, \dots, \hat{I}_{N-1}]$. For simplicity the superscript "(j)" is again dropped in the notation. In order to limit the computationally complexity we restrict ourselves only to spatial correlations between adjacent pixels. This is shown in Fig. 2 where received source indices \hat{I}_k of a subband image and the corresponding first-order Markov models are displayed. The a-priori knowledge is restricted to all indices in the

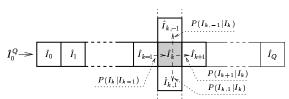


Fig. 2. Received subband values and associated Markov models boxes surrounded with bold lines. It is shown in [5] that an approximation for the 2-D APPs can be stated by using the source index transition probabilities $P(\cdot, I_k)$ from Fig. 2 according to

$$P(I_{k}^{(\lambda)} | \hat{I}_{0}^{k+1}, \hat{I}_{k,-1}, \hat{I}_{k,1}) \approx c_{k} \cdot P(I_{k}^{(\lambda)} | \hat{I}_{0}^{k}) \cdot \frac{1}{2^{M}-1} p(\hat{I}_{k+1} | I_{k+1}^{(\mu)}) P(I_{k+1}^{(\mu)} | I_{k}^{(\lambda)}) \cdot \frac{1}{2^{M}-1} \sum_{\nu=0}^{2^{M}-1} p(\hat{I}_{k,1} | I_{k,1}^{(\nu)}) P(I_{k,1}^{(\nu)} | I_{k}^{(\lambda)}) \cdot \frac{1}{2^{M}-1} \sum_{\kappa=0}^{2^{M}-1} p(\hat{I}_{k,-1} | I_{k,-1}^{(\kappa)}) P(I_{k,-1}^{(\kappa)} | I_{k}^{(\lambda)}), \quad (7)$$

for $\lambda=0,...,2^M-1$. The factor c_k ensures that true probabilities are given by (7). The 1-D APPs $P(I_k^{(\lambda)}|\hat{\pmb{I}}_0^k)$ are obtained by [9]

$$P(I_{k}^{(\lambda)} | \hat{\mathbf{I}}_{0}^{k}) = c_{k}' p(\hat{I}_{k} | I_{k}^{(\lambda)}) \cdot \frac{\sum_{\mu=0}^{2^{M}-1} P(I_{k}^{(\lambda)} | I_{k-1}^{(\mu)}) P(I_{k-1}^{(\mu)} | \hat{\mathbf{I}}_{0}^{k-1}), \quad (8)$$

which is a recursion. Furthermore, the variable c_k' denotes a normalization factor, and $P(I_k^{(\lambda)} | I_{k-1}^{(\mu)})$ refers to the 1-D index transition probabilities of the Markov model. The initialization for k=0 can be carried out with the source index probabilities $P(I_k=\lambda)$. The conditional p.d.f. $p(\hat{I}_k | I_k)$ corresponds to the channel term and can be calculated for a memoryless channel as

$$p(\hat{I}_k | I_k = \lambda) = \prod_{\ell=0}^{M-1} p(\hat{i}_{\ell,k} | i_{\ell,k} = \lambda_{\ell}).$$
 (9)

Herein, $i_{\ell,k} \in \{0,1\}$ is defined as the ℓ -th bit of the index I_k , and the p.d.f. $p(\hat{i}_{\ell,k} \mid i_{\ell,k} = \lambda_\ell)$ is defined analog to (1). A similar relation as in (9) holds for the indices I_{k+1} and $I_{k,\pm 1}$ in (7).

The 2-D APPs from (7) can be used for a mean-squares estimation of the reconstructed source value \hat{U}_k , which corresponds to the demand for maximal reconstruction SNR, according to

$$\hat{U}_k = \sum_{\lambda=0}^{2^M - 1} U_q(\lambda, R_S) \cdot P(I_k^{(\lambda)} | \hat{I}_0^{k+1}, \hat{I}_{k,-1}, \hat{I}_{k,1}).$$
 (10)

5. ITERATIVE SOURCE-CHANNEL DECODER

An error protection carried out by only using the residual source redundancy may not be enough in many transmission situations. Therefore, we assume that the output of the source encoder is protected by a systematic channel code, as it is depicted in Fig. 1. Note that this scheme is highly similar to a serially concatenated channel code. Therefore, we can apply an iterative ("turbo") decoding scheme, where the outer constituent channel decoder is replaced by the 2-D APP source decoder from Section 4.

The structure of the resulting decoder is depicted in At the beginning of the first iteration the channel decoder issues APPs $P(i'_{\ell,k} | \hat{v})$ for the information bits $i'_{\ell,k}$ of the interleaved source index sequence $\pi(I)$. These APPs are used to calculate the corresponding conditional L-values $L^{(C)}(i'_{\ell,k}) = L_c \, \hat{i}'_{\ell,k} + L_a^{(C)}(i'_{\ell,k}) + L_{\mathrm{extr}}^{(C)}(i'_{\ell,k})$ for $\ell = 0, \dots, M-1, \ k = 0, \dots, N-1$. The term $L_c \, \hat{i}'_{\ell,k}$ is defined analog to (2) for the interleaved index bit $i'_{\ell,k}, \; L_a^{(C)}(i'_{\ell,k})$ denotes the a-priori information for the index bit $i'_{\ell,k}$, and $L^{(C)}_{\text{extr}}(i'_{\ell,k})$ refers to the extrinsic information [10]. After subtraction of the a-priori term and after deinterleaving we obtain the L-values $L_{\rm e}^{(C)}(i_{\ell,k}) = L_c \, \hat{i}_{\ell,k} + L_{\rm extr}^{(C)}(i_{\ell,k})$, which are used as a-priori information $L_a^{(S)}(i_{\ell,k})$ for the VLC source decoder. In the following, we assume that all information bits are uncorrelated. Then, the corresponding index-based probabilities for the a-priori L-values $L_a^{(S)}(i_{\ell,k})$ can be obtained by bitwise multiplication of the probabilities for the index bits $i_{\ell,k} = \lambda_{\ell}$. By inserting this a-priori knowledge into the recursion (8) we obtain modified 1-D APPs as

$$\begin{split} \tilde{P}(I_{k}^{(\lambda)} \mid \hat{\pmb{I}}_{0}^{k}) &= c_{k}^{\prime\prime} \prod_{\ell=0}^{M-1} p(\hat{i}_{\ell,k} \mid i_{\ell,k} = \lambda_{\ell}) \, P_{\text{extr}}^{(C)}(i_{\ell,k} = \lambda_{\ell} \mid \hat{\pmb{v}}) \cdot \\ & \cdot \sum_{\mu=0}^{2^{M}-1} P(I_{k}^{(\lambda)} \mid I_{k-1}^{(\mu)}) \, P(I_{k-1}^{(\mu)} \mid \hat{\pmb{I}}_{0}^{k-1}) \end{split}$$

where $c_k^{\prime\prime}$ is a normalizing constant (as above). The 2-D APP source decoder now issues index-based modified APPs

 $ilde{P}(I_k^{(\lambda)} \,|\, \hat{\pmb{I}}_0^{k+1}, \hat{I}_{k,-1}, \hat{I}_{k,1}),$ which are obtained by replacing $P(I_k^{(\lambda)} \,|\, \hat{\pmb{I}}_0^k)$ with $ilde{P}(I_k^{(\lambda)} \,|\, \hat{\pmb{I}}_0^k)$ in (7). The corresponding bit-based L-values can be derived for $\ell=0,1,\ldots,M-1$ as

$$L^{(S)}(i_{\ell,k}) = \log \left(\frac{\sum_{\mu \in \mathcal{I}: \mu_{\ell} = 0} \tilde{P}(I_k^{(\mu)} | \hat{\mathbf{I}}_0^{k+1}, \hat{I}_{k,-1}, \hat{I}_{k,1})}{\sum_{\mu \in \mathcal{I}: \mu_{\ell} = 1} \tilde{P}(I_k^{(\mu)} | \hat{\mathbf{I}}_0^{k+1}, \hat{I}_{k,-1}, \hat{I}_{k,1})} \right). \tag{11}$$

By subtracting the source a-priori information $L_a^{(S)}(i_{\ell,k})$ from $L^{(S)}(i_{\ell,k})$ in (11) we finally obtain the extrinsic information $L_{\rm extr}^{(S)}(i_{\ell,k})$, which is used as a-priori information in the next channel decoding round.

6. RESULTS

An experimental image transmission system is now derived by applying the transmission model from Fig. 1 to every subband of an L-level wavelet octave filter bank and using the results from above. The K 2-D subband signals are first scanned in a meander-type fashion [5] and quantized with optimal scalar quantizers. Then, the resulting bitstream in each subband is interleaved using a random interleaver and channel encoded with a terminated memory-4 RCPC code [11]. In our setup we assume that sensitive side informations, as lowpass subband DC content, quantizer stepsizes and rate allocations, are protected by a sufficiently strong channel code, such that they can be transmitted without errors.

The experimental image transmission system is applied to the 512×512 pixel "Goldhill" test image for a 3-level wavelet decomposition and an overall target bit rate of $R_T=0.37$ bit per pixel (bpp) including channel coding and all side information. In the simulations we consider two techniques for obtaining the source transition probabilities at the decoder [5]. In the optimal case the transition probabilities are directly taken from the original image, which is denoted with "Or." in the simulations. Clearly, this method is infeasible in real transmission scenarios. Additionally, the transition probabilities are provided via a training set of 130 images ("Tr."), where the "Goldhill" image is not part of this set.

Fig. 4 shows the simulation results for different iterations of the iterative source-channel decoder ("SCD") from Section 5, where the peak-SNR (PSNR) values of the reconstructed images versus the channel parameter E_b/N_0 averaged over 100 simulated transmissions are displayed. For comparison purposes also the results for the soft-input source decoding technique presented in Section 4 are given ("2-D SISD"), where the residual (2-D) source redundancy alone is utilized for error concealment. We can see from Fig. 4 that after three iterations in the decoder and by using the original transition probabilities we have a gain of about 1 dB in the worst case compared to the "SCD, Tr." technique. However, for more than two iterations a slight gain is only observed for low channel SNR. Additionally, by comparing the results with the "2-D SISD" approach it can be observed that by adding explicit redundancy from channel codes in combination with a joint rate allocation and an iterative source-channel decoder the PSNR of the reconstructed image is improved by about 1 dB and more for $E_b/N_0 < 0$ dB. Since the rate allocation algorithm from Section 3 only searches for operation points on the convex hull in the rate-distortion plane [7], for some values of E_b/N_0 the algorithm yields an overall bit rate being slightly smaller than the target rate R_T . Therefore, the curves for the SCD technique in Fig. 4 lack the smooth behavior of those for the SISD approach.

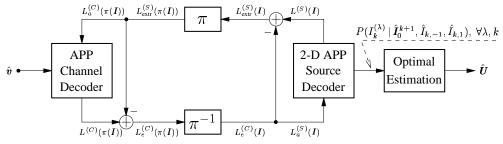


Fig. 3. Structure of the iterative source-channel decoder

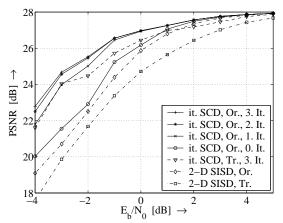


Fig. 4. Results for the "Goldhill" image $(R_T = 0.37 \text{ bpp}, L = 3)$

An example of the good reconstruction quality for a highly corrupted channel is depicted in Fig. 5.

7. CONCLUSIONS

By using the implicit two-dimensional residual source redundancy for error protection in conjunction with channel coding, an iterative decoding scheme is derived in a similar way as for serially concatenated channel codes: the difference is that a soft-input APP source decoder replaces the outer constituent channel decoder. As a new result we have shown that this source-channel decoding technique in combination with a novel joint source-channel rate-allocation can be used for robust image transmission over highly distorted AWGN channels. The advantage of the proposed method is that by incorporating residual redundancies into the decoding process less powerful channel codes can be chosen, which also reduces the complexity of the encoder and the encoding delay.

8. REFERENCES

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Fig. 5. Reconstructed image for $E_b/N_0 = -1$ dB (BER 10.4%), PSNR: 25.82 dB ($R_T = 0.37$ bpp, L = 3, it. SCD, three iterations).

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