Numerical Evaluation of the Lambert W Function and Application to Generation of Generalized Gaussian Noise With Exponent 1/2

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***Abstract*—We address the problem of synthesizing a generalized Gaussian noise with exponent 1/2 by means of a nonlinear memoryless transformation applied to a uniform noise. We show that this transformation is expressable in terms of a special function known under the name of the Lambert W function. We review the main methods for numerical evaluation of the relevant branch of the (multivalued) Lambert W function with controlled accuracy and complement them with an original rational function approximation. Based on these methods, synthesis of the generalized Gaussian noise can be performed with arbitrary accuracy. We construct a simple and fast evaluation algorithm with prescribed accuracy, which is especially suited for Monte Carlo simulation requiring large numbers of realizations of the generalized Gaussian noise.**

***Index Terms*—Generalized Gaussian noise, Lambert W function, noise synthesis.**

I. INTRODUCTION

MANY modern engineering processes, including signals, images and communication systems, often have to operate in complex environments dominated by non-Gaussian noises [1], [2]. Efficient design, control, and performance evaluation in these contexts depend on the capability of modeling and synthesizing such non-Gaussian noises. A random variable X with prescribed cumulative distribution function $F(x)$ can be generated [3] as $X = F^{-1}(U)$, from a random variable U uniform over [0, 1) and universally available with most scientific-computation softwares through congruential recurrences [3]. In this paper, we address the synthesis of X as a generalized Gaussian noise with exponent 1/2 and show that the corresponding inverse $F^{-1}(x)$ is expressable by means of a special function known under the name of the Lambert W function. We provide numerical methods for explicit evaluation of the relevant branch of the (multivalued) Lambert W function with controlled accuracy. Based on these methods, we construct a simple and fast algorithm with prescribed accuracy that is especially suited for computation requiring large numbers of realizations of the generalized Gaussian noise.

Manuscript received July 28, 2000; revised May 30, 2002. The associate editor coordinating the review of this paper and approving it for publication was Dr. Ta-Hsin Li.

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Publisher Item Identifier 10.1109/TSP.2002.801912.

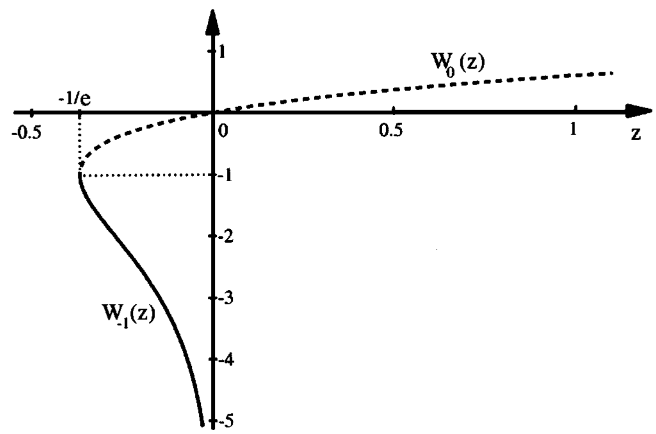


Fig. 1. Two real branches of the Lambert W function. Solid line: $W_{-1}(z)$ defined for $-1/e \leq z < 0$ (of interest to us in this paper). Dashed line: $W_0(z)$ defined for $-1/e \leq z < +\infty$. The two branches meet at point $(-1/e, -1)$.

II. LAMBERT W FUNCTION

The Lambert W function is defined to be the inverse of the function $w \mapsto we^w = z$. This function $W(z)$, which thus verifies $W(z)e^{W(z)} = z$, is a multivalued function defined in general for z complex and assuming values $W(z)$ complex. If z is real and $z < -1/e$, then $W(z)$ is multivalued complex. If z is real and $-1/e \leq z < 0$, there are two possible real values of $W(z)$: The branch satisfying $-1 \leq W(z)$ is denoted by $W_0(z)$ and called the *principal branch* of the W function, and the other branch satisfying $W(z) \leq -1$ is denoted by $W_{-1}(z)$. If z is real and $z \geq 0$, there is a single real value for $W(z)$, which also belongs to the principal branch $W_0(z)$. It is the real branch $W_{-1}(z)$ for $-1/e \leq z < 0$ that will be useful to us in the sequel. Both real branches $W_{-1}(z)$ and $W_0(z)$, for z real, are represented in Fig. 1.

Consideration of the Lambert W function can be traced back to J. Lambert around 1758, and later, it was considered by L. Euler [4]. This function has progressively been recognized in the solution to many problems in various fields of mathematics, physics, and engineering, up to a point at which the authors of [4] convincingly argued to establish the Lambert W function as a special function of mathematics on its own. These elements also motivated the introduction of the Lambert W function in the Maple mathematical software [5]. Here, we will present an additional application of the Lambert W function, which is especially relevant for signal processing, for the synthesis of a generalized Gaussian noise with exponent 1/2.

III. Generalized Gaussian Noise

Generalized Gaussian noise is characterized by a probability density function of the form

$$f(x) = A\exp\left(-|bx|^{\alpha}\right) \tag{1}$$

with parameters expressed in terms of the Gamma function: $A = (\alpha/2)[\Gamma(3/\alpha)]^{1/2}/[\Gamma(1/\alpha)]^{3/2}$ and $b = [\Gamma(3/\alpha)/\Gamma(1/\alpha)]^{1/2}$, and an exponent $\alpha > 0$. Equation (1) characterizes a zero-mean unit-variance random quantity X; in the sequel we restrict our attention to these conditions knowing that arbitrary mean m and variance σ^2 can straightforwardly be restored by the change of variable $X \leftarrow \sigma X + m$.

For $0 < \alpha < 2$, $f(x)$ is a heavy-tailed density [1], [2], yet with all its moments finite (in contrast to other heavy-tailed densities like stable densities [6]), making it suitable for physical modeling of many processes with heavy tails. Applications have recently been found for speech, audio, or video signals [7]–[9], images [10]–[13], or turbulence [14]. Efficient simulation of generalized Gaussian noise can thus benefit to better understanding and control of such processes.

We address here the case $\alpha = 1/2$ associated with the probability density

$$f(x) = \frac{\sqrt{30}}{2}\exp\left(-\left|2\sqrt{30}x\right|^{1/2}\right) \tag{2}$$

and cumulative distribution function

$$F(x) = \begin{cases} \frac{1}{2}\left(1+\left|2\sqrt{30}x\right|^{1/2}\right)\exp\left(-\left|2\sqrt{30}x\right|^{1/2}\right) \\ \quad \text{for } x \le 0 \\ 1-\frac{1}{2}\left(1+\left|2\sqrt{30}x\right|^{1/2}\right)\exp\left(-\left|2\sqrt{30}x\right|^{1/2}\right) \\ \quad \text{for } x \ge 0. \end{cases} \tag{3}$$

We want to invert $F(x)$ of (3) and show that this inverse can be expressed in terms of the Lambert function W_{-1}. We first consider the branch for $x \le 0$ of $F(x)$ of (3), and for its inversion, this branch is written under the form

$$F = \tfrac{1}{2}(1-y)e^{y} \tag{4}$$

where $y = -(-2\sqrt{30}x)^{1/2}$. Equation (4) leads to

$$-\frac{2F}{e} = (y-1)e^{y-1} \tag{5}$$

invertible under the form

$$y - 1 = W_{-1}(-2F/e) \tag{6}$$

to yield

$$x = -\frac{1}{2\sqrt{30}}\left[1 + W_{-1}(-2F/e)\right]^2. \tag{7}$$

The correct branch W_{-1} of the Lambert function is identified since the mapping passes through the points $(x = 0,\ F = 1/2)$ and $(x = -\infty,\ F = 0)$.

Similar arguments can be applied to invert the branch for $x \ge 0$ of $F(x)$ of (3). This finally leads, for (3), to the inverse function

$$F^{-1}(x) = \begin{cases} -\frac{1}{2\sqrt{30}}\left[1 + W_{-1}(-2x/e)\right]^2 \\ \quad \text{for } 0 < x \le 1/2 \\ \frac{1}{2\sqrt{30}}\left[1 + W_{-1}(-2(1-x)/e)\right]^2 \\ \quad \text{for } 1/2 \le x < 1. \end{cases} \tag{8}$$

Equation (8) solves the problem of synthesizing a generalized Gaussian noise with $\alpha = 1/2$ from a uniform noise as exposed in the Introduction, provided we are able to handle the numerical evaluation of the function W_{-1}. We now address this problem.

IV. Numerical Evaluation of W_{-1}

We come to the numerical evaluation of $W_{-1}(z)$ for $-1/e \le z < 0$. As we mentioned, this numerical evaluation is directly accessible in the Maple mathematical software [5], [15]. Beyond, the Lambert W function, although it is useful in many domains, is not yet available in standard mathematical software libraries. We will thus describe here explicit elements for its numerical evaluation, allowing us to write Fortran or C routines, for instance.

A. Arbitrary-Precision Evaluation

A specificity of the Lambert W function is that it is defined as an inverse function. As a consequence, arbitrary-precision evaluations can be obtained by means of iterative root-finding methods. For a given z, one can find $W_{-1}(z)$ as the root (in w) of the equation $we^{w} = z$ while also keeping track of the correct branch. Numerous iterative methods are available for this purpose. A choice has to trade off between complexity of implementation, conditions, and number of iterations to convergence at given precision. These properties are usually controllable via the order of the method (the highest order of the derivatives of the function to be zeroed used by the algorithm). Newton's method [3] is a simple first-order method that is appropriate but with relatively slow convergence. A better compromise is realized by Halley's method, which is a third-order method that constitutes the choice implemented by Maple and leads to high-precision evaluation in reasonable time [4], [5]. It is based on the iteration scheme

$$w_{j+1} = w_j - \frac{w_j e^{w_j} - z}{(w_j+1)e^{w_j} - \frac{(w_j+2)(w_j e^{w_j} - z)}{2w_j+2}}. \tag{9}$$

Fourth-order methods, which are faster but more complicated, are proposed in [16] and [17] but are limited as they are exposed to the principal branch $W_0(z)$.

Again, since the Lambert function is an inverse function, a simple way is available for controlling the error, or accuracy, of a given numerical evaluation. At a given z in $(-1/e,\ 0)$, the value $W_{-1}(z)$ is evaluated as the (approximated) root w_{eval} of $\varphi(w) = z$ with the function $\varphi(w) = we^{w}$. If the exact root is denoted w_{true} [it is the "true" value of $W_{-1}(z)$], then the residue $z - w_{\text{eval}}e^{w_{\text{eval}}}$ provides access to the evaluation error

$w_{\rm true}-w_{\rm eval}$. For any reasonable root-finding algorithm, $w_{\rm eval}$ is very close to $w_{\rm true}$, and one has as a very good approximation

$$\frac{z-w_{\rm eval}e^{w_{\rm eval}}}{w_{\rm true}-w_{\rm eval}}\approx\varphi'(w_{\rm eval})=(1+w_{\rm eval})e^{w_{\rm eval}} \qquad (10)$$

whence the evaluation error

$$w_{\rm true}-w_{\rm eval}\approx\frac{z-w_{\rm eval}e^{w_{\rm eval}}}{(1+w_{\rm eval})e^{w_{\rm eval}}}. \qquad (11)$$

With (9) controlled by (11), arbitrary-precision evaluation of the Lambert W function can be realized, as done by Maple.

B. *Fast Approximation*

In addition to iterative schemes, more direct evaluation methods based on series or asymptotic expansions exist, offering a different tradeoff between accuracy and speed.

1) Series Expansion for $z\geq -1/e$: For z in the vicinity of $-1/e$, one has the series expansion [4]

$$W_{-1}(z)=\sum_{\ell=0}^{+\infty}\mu_\ell p^\ell \qquad (12)$$

where $p=-\sqrt{2(ez+1)}$. This series can be computed to any desired order from the recurrence relations

$$\mu_k=\frac{k-1}{k+1}\left(\frac{\mu_{k-2}}{2}+\frac{\alpha_{k-2}}{4}\right)-\frac{\alpha_k}{2}-\frac{\mu_{k-1}}{k+1} \qquad (13)$$

$$\alpha_k=\sum_{j=2}^{k-1}\mu_j\mu_{k+1-j} \qquad (14)$$

$$\mu_0=-1,\quad \mu_1=1,\quad \alpha_0=2,\quad \alpha_1=-1. \qquad (15)$$

The series of (12) converges for $-\sqrt{2}<p\leq 0$, that is, for $-1/e\leq z<0$, which covers the whole domain of existence of $W_{-1}(z)$. The first terms of the series of (12) are

$$W_{-1}(z)=-1+p-\tfrac{1}{3}p^2+\tfrac{11}{72}p^3-\tfrac{43}{540}p^4+\tfrac{769}{17\,280}p^5-\tfrac{221}{8505}p^6+\cdots. \qquad (16)$$

2) Asymptotic Series for $z<0$: Now, for z approaching 0^-, the Lagrange inversion theorem provides an asymptotic series expansion, with $L_1=\ln(-z)$ and $L_2=\ln[-\ln(-z)]$, which reads [4]

$$W_{-1}(z)=L_1-L_2+\sum_{\ell=0}^{+\infty}\sum_{m=1}^{+\infty}C_{\ell m}L_1^{-\ell-m}L_2^m. \qquad (17)$$

The coefficients $C_{\ell m}$ are expressable as $C_{\ell m}=(-1)^\ell S(\ell+m,\,\ell+1)/m!$, where $S(\ell+m,\,\ell+1)$ is a non-negative Stirling number of the first kind [18], computable via the generating function

$$x(x-1)\cdots(x-n+1)=\sum_{m=0}^{n}(-1)^{n-m}S(n,\,m)x^m,\qquad n\geq 1 \qquad (18)$$

and $S(n,\,m)=0$ for $m>n$. It follows from (18) that $\forall\, n\geq 1$, one has $S(n,\,0)=0$ and $S(n,\,n)=1$. The recursion

$$S(n,m)=S(n-1,m-1)+(n-1)S(n-1,m),\qquad n>1 \qquad (19)$$

is also available.

The first terms of the series of (17) are

$$\begin{aligned}W_{-1}(z)=&\,L_1-L_2+\frac{L_2}{L_1}+\frac{(-2+L_2)L_2}{2L_1^2}\\&+\frac{(6-9L_2+2L_2^2)L_2}{6L_1^3}\\&+\frac{(-12+36L_2-22L_2^2+3L_2^3)L_2}{12L_1^4}\\&+\frac{(60-300L_2+350L_2^2-125L_2^3+12L_2^4)L_2}{60L_1^5}\\&+\cdots.\end{aligned} \qquad (20)$$

3) Taylor Series for $-1/e<z<0$: Now, for $z\in(-1/e,\,0)$, the Lambert function $W_{-1}(z)$ remains bounded, with derivatives existing at all orders n and easily expressable (since W_{-1} is an inverse function) as [4]

$$W_{-1}^{(n)}(z)=\frac{d^nW_{-1}(z)}{dz^n}=\frac{\exp[-nW_{-1}(z)]P_n[W_{-1}(z)]}{[1+W_{-1}(z)]^{2n-1}}\qquad \text{for } n\geq 1 \qquad (21)$$

where the polynomials $P_n(u)$ are defined by the recurrence relation

$$P_{n+1}(u)=-(nu+3n-1)P_n(u)+(1+u)P_n'(u)\qquad \text{for } n\geq 1 \qquad (22)$$

and the initial polynomial $P_1(u)=1$.

For any z_0 and z both in $(-1/e,\,0)$, one thus has the Taylor series

$$W_{-1}(z)=W_{-1}(z_0)+\sum_{n=1}^{+\infty}\frac{1}{n!}W_{-1}^{(n)}(z_0)\times(z-z_0)^n. \qquad (23)$$

Equation (21) reveals an interesting property: Although the Lambert function $W_{-1}(z)$ is a special function, the value of any of its derivatives $W_{-1}^{(n)}(z_0)$ at any point $z_0\in(-1/e,\,0)$ can be expressed solely by way of standard functions applied to $W_{-1}(z_0)$. Thus, approximation of $W_{-1}(z)$ with (23) in the vicinity of z_0 only requires knowledge of $W_{-1}(z_0)$, which is obtainable, for instance, from Section IV-A.

4) Rational Function Approximation for $-1/e<z<0$: Rational functions are usually able to provide a more compact expression for a given level of accuracy, compared with the polynomial expansion of the Taylor series. Since $W_{-1}(z)$ is an inverse function, such a rational approximation is easy to construct based only on standard (avoiding special

If $-1/e \leq z < -0.333$

$p = -\sqrt{2(ez+1)}$

$W_{-1}(z) = -1 + p - \frac{1}{3}p^2 + \frac{11}{72}p^3 - \frac{43}{540}p^4 + \frac{769}{17280}p^5 - \frac{221}{8505}p^6$

Else If $-0.333 \leq z \leq -0.033$

$W_{-1}(z) = \frac{-8.0960 + 391.0025z - 47.4252z^2 - 4877.6330z^3 - 5532.7760z^4}{1 - 82.9423z + 433.8688z^2 + 1515.3060z^3}$

Else If $-0.033 < z < 0$

$L_1 = \ln(-z)$

$L_2 = \ln[-\ln(-z)]$

$W_{-1}(z) = L_1 - L_2 + \frac{L_2}{L_1} + \frac{(-2+L_2)L_2}{2L_1^2} + \frac{(6 - 9L_2 + 2L_2^2)L_2}{6L_1^3} + \frac{(-12 + 36L_2 - 22L_2^2 + 3L_2^3)L_2}{12L_1^4} + \frac{(60 - 300L_2 + 350L_2^2 - 125L_2^3 + 12L_2^4)L_2}{60L_1^5}$

End If

Fig. 2. One-shot evaluation algorithm for the Lambert function $W_{-1}(z)$ over its domain of existence $z \in [-1/e,\, 0)$, realizing an approximation with relative error strictly below 10^{-4} for any $z \in [-1/e,\, 0)$.

functions) computations. One first settles a rational function of prescribed orders L and M under the parametric form

$$W_{-1}^{\text{rfa}}(z) = \frac{\sum_{\ell=0}^{L} a_\ell z^\ell}{1 + \sum_{m=1}^{M} b_m z^m}. \tag{24}$$

One then constructs a set of points $\{(w_j,\, w_j e^{w_j} = z_j)\}$, and the discrepancy $\sum_j [W_{-1}^{\text{rfa}}(z_j) - w_j]^2$ is minimized by any standard method to find a set of coefficients a_ℓ and b_m defining an acceptable $W_{-1}^{\text{rfa}}(z)$. The quality of the resulting approximation $W_{-1}^{\text{rfa}}(z)$ can then be assessed via (11) or against a high-precision evaluation from Section IV-A, and it can be controlled through the orders L and M and through the range and resolution covered by the training points $\{(z_j,\, w_j)\}$.

A possible rational function approximation is (25), shown at the bottom of the page, which we have constructed from 100 points $(w_j,\, w_j e^{w_j} = z_j)$ with the w_js equispaced over the interval $[-5, -1.5]$. We have devised the rational function $W_{-1}^{\text{rfa}}(z)$ of (25) to provide a relative approximation error for $W_{-1}(z)$ of less than 10^{-4} for any $z \in [-0.333,\, -0.033]$ (see Fig. 3).

5) Fast Algorithm With Prescribed Accuracy: For fast one-shot evaluation of $W_{-1}(z)$ over its full domain of existence $z \in [-1/e, 0)$, we have devised the numerical algorithm presented in Fig. 2. This algorithm, for $z \in [-1/e,\, -0.333)$, implements the series expansion of (16). For $z \in [-0.333,\, -0.033]$, it implements the rational function approximation of (25). For $z \in (-0.033,\, 0)$, it implements the asymptotic expansion of (20).

The algorithm of Fig. 2 evaluates the approximation $W_{-1}^{\text{approx}}(z)$ for $W_{-1}(z)$. We have devised it to provide a relative approximation error $|W_{-1}(z) - W_{-1}^{\text{approx}}(z)|/|W_{-1}(z)|$ strictly below 10^{-4} for any $z \in [-1/e,\, 0)$. We have evaluated the behavior of this relative error of $W_{-1}^{\text{approx}}(z)$ against a high-precision evaluation for $W_{-1}(z)$ based on the iterative scheme of Section IV-A, and the result is presented in Fig. 3.

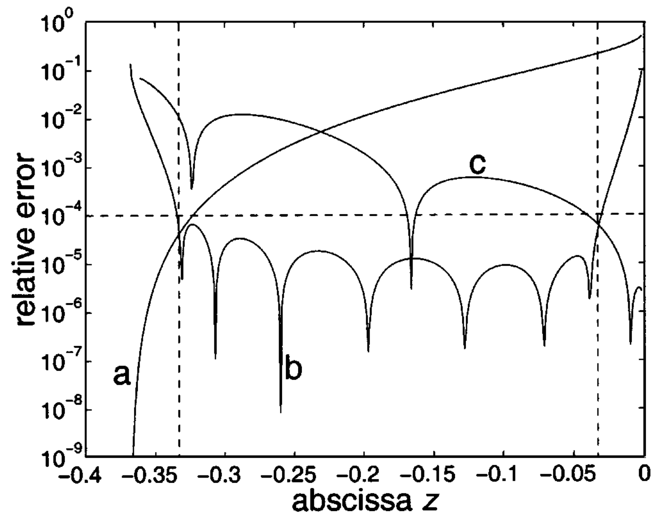


Fig. 3. Relative error $|W_{-1}(z) - W_{-1}^{\text{approx}}(z)|/|W_{-1}(z)|$ for one-shot evaluations $W_{-1}^{\text{approx}}(z)$ of the Lambert function $W_{-1}(z)$ given by (a) the series expansion of (16), (b) rational function approximation of (25), and (c) asymptotic expansion of (20). The vertical dashed lines have abscissa $z = -0.333$ and $z = -0.033$. Over the domain of existence $z \in [-1/e \approx -0.368,\, 0)$ of $W_{-1}(z)$, the one-shot evaluation algorithm of Fig. 2 realizes a relative approximation error everywhere below 10^{-4} by implementing branch (a) for $z \in [-1/e,\, -0.333)$, branch (b) for $z \in [-0.333,\, -0.033]$, and branch (c) for $z \in (-0.033,\, 0)$.

V. Generation of Generalized Gaussian Noise With Exponent 1/2

We have implemented, as a C routine, the one-shot evaluation of the Lambert function $W_{-1}(z)$ realized by the algorithm of Fig. 2. When run on a Pentium III 500-MHz processor, this routine typically can perform 10^7 evaluations of $W_{-1}(z)$ in about 10 sec, whereas only 10^3 evaluations can be performed in the same time by Maple with its standard implementation of $W_{-1}(z)$. A speed increase by a factor of order 10^4 can thus be obtained with the limited-precision one-shot evaluation of $W_{-1}(z)$ of Fig. 2. A realization of the resulting generalized Gaussian noise with exponent 1/2 is shown in Fig. 4.

We have performed an estimation of its probability density based on 10^7 values drawn for the generalized Gaussian noise and collected into bins of width $\Delta x = 0.1$. The estimated density presented in Fig. 5 is compared with the theoretical model of (2) and shows very good agreement. The region of the tails of the density $f(x)$, for large arguments $|x|$, corresponds to values of the generalized Gaussian noise that are produced, according to (8), when the Lambert function approaches $W_{-1}(0^-)$. In this region, the Lambert function is expressed by the asymptotic expansion of (17), which is absolutely convergent [4]. The terms left out when this expansion is truncated as in Fig. 2 are

$$W_{-1}^{\text{rfa}}(z) = \frac{-8.0960 + 391.0025z - 47.4252z^2 - 4877.6330z^3 - 5532.7760z^4}{1 - 82.9423z + 433.8688z^2 + 1515.3060z^3} \tag{25}$$

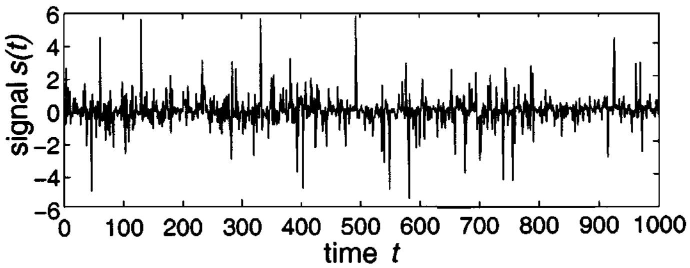


Fig. 4. Realization of a generalized Gaussian noise with exponent 1/2 generated with the algorithm of Fig. 2 to implement the function of (8) performing the transformation of a uniform noise.

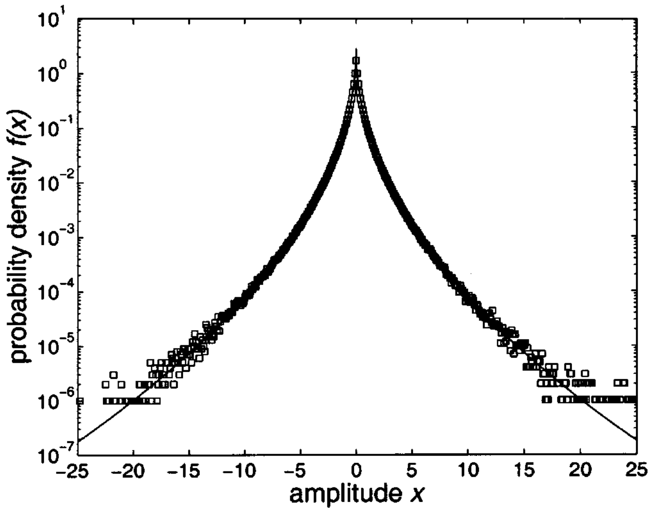


Fig. 5. Probability density function estimated for the generalized Gaussian noise with exponent 1/2 synthesized as in Fig. 4, superimposed to the theoretical model of (2) (continuous solid line).

$O[(L_2/L_1)^6]$ [4], [18] and clearly form an approximation error vanishing asymptotically. This translates into an approximation scheme capable of an accurate representation of the tails of the density $f(x)$, even in the far tails asymptotically, in principle. This capability is visible in Fig. 5, up to the limitation due to finite counts that get sparse in the tails.

This fast synthesis of generalized Gaussian noise based on the algorithm of Fig. 2 is especially suited for Monte Carlo simulation requiring large numbers of noise realizations. We have used it, for instance, to estimate the probability of error of nonlinear detectors designed for generalized Gaussian noise [19].

VI. Conclusion

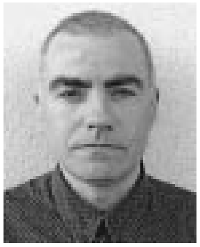
We have shown that a generalized Gaussian noise with exponent 1/2 can be generated from a uniform noise subjected to a nonlinear transformation expressed in terms of the Lambert function $W_{-1}(z)$. We have reviewed the main methods for numerical evaluation of $W_{-1}(z)$ with controlled accuracy. We have complemented these methods with an original rational function approximation for $W_{-1}(z)$. By collecting these methods, we have constructed a simple and fast algorithm for numerical evaluation of $W_{-1}(z)$ over its full domain of existence with a prescribed level of accuracy. This algorithm can be straightforwardly coded in any programming language with standard algebra. It is especially suited for Monte Carlo simulation, requiring large numbers of realizations of the generalized Gaussian noise. Such a tool can contribute to better understanding and control of heavy-tailed processes.

Acknowledgment

The authors would like to thank R. Corless for valuable informations on the Lambert W function.

References

[1] M. Grigoriu, *Applied Non-Gaussian Processes*. Englewood Cliffs, NJ: Prentice-Hall, 1995.
[2] R. J. Adler, R. E. Feldman, and M. S. Taqqu, Eds., *A Practical Guide to Heavy Tails: Statistical Techniques and Applications*. Boston, MA: Birkhäuser, 1998.
[3] W. H. Press, S. A. Teukolsky, W. T. Vetterling, and B. P. Flanery, *Numerical Recipes: The Art of Scientific Computing*. Cambridge, MA: Cambridge Univ. Press, 1992.
[4] R. M. Corless, G. H. Gonnet, D. E. G. Hare, D. J. Jeffrey, and D. E. Knuth, "On the Lambert W function," *Adv. Comput. Math.*, vol. 5, pp. 329–359, 1996.
[5] R. M. Corless, G. H. Gonnet, D. E. G. Hare, and D. J. Jeffrey, "Lambert's W function in Maple," *Maple Tech. Newsletter*, vol. 9, pp. 12–22, 1993.
[6] G. Samorodnitsky and M. S. Taqqu, *Stable Non-Gaussian Random Processes*. New York: Chapman & Hall, 1994.
[7] K. Sharifi and A. Leon-Garcia, "Estimation of shape parameter for generalized Gaussian distributions in subband decompositions of video," *IEEE Trans. Circuits Syst. Video Technol.*, vol. 5, pp. 52–56, Jan. 1995.
[8] F. Chen, Z. Gao, and J. D. Villasenor, "Lattice vector quantization of generalized Gaussian sources," *IEEE Trans. Inform. Theory*, vol. 43, pp. 92–103, Feb. 1997.
[9] B. Aiazzi, L. Alparone, and S. Baronti, "Estimation based on entropy matching for generalized Gaussian PDF modeling," *IEEE Signal Processing Lett.*, vol. 6, pp. 138–140, Apr. 1999.
[10] K. A. Birney and T. R. Fischer, "On the modeling of DCT and subband image data for compression," *IEEE Trans. Image Processing*, vol. 4, pp. 186–193, Apr. 1995.
[11] P. Moulin and J. Liu, "Analysis of multiresolution image denoising schemes using generalized Gaussian and complexity priors," *IEEE Trans. Inform. Theory*, vol. 45, pp. 909–919, Apr. 1999.
[12] G. V. Wouwer, P. Scheunders, and D. V. Dyck, "Statistical texture characterization from discrete wavelet representations," *IEEE Trans. Image Processing*, vol. 8, pp. 592–598, Apr. 1999.
[13] J. R. Hernández, M. Amado, and F. Pérez-González, "DCT-domain watermarking techniques for still images: Detector performance analysis and a new structure," *IEEE Trans. Image Processing*, vol. 9, pp. 55–68, Jan. 2000.
[14] U. Frisch, *Turbulence: The Legacy of A. N. Kolmogorov*. Cambridge, MA: Cambridge Univ. Press, 1995.
[15] D. Redfern, *Maple V Handbook*. Berlin, Germany: Springer-Verlag, 1997.
[16] F. N. Fritsch, R. E. Shafer, and W. P. Crowley, "Algorithm 443: Solution of the transcendental equation $we^w = x$," *Commun. ACM*, vol. 16, pp. 123–124, 1973.
[17] D. A. Barry, P. J. Cullingan-Hensley, and S. J. Barry, "Real values of the W-function," *ACM Trans. Math. Software*, vol. 21, pp. 161–171, 1995.
[18] D. J. Jeffrey, R. M. Corless, D. E. G. Hare, and D. E. Knuth, "Sur l'inversion de $y^\alpha e^y$ au moyen des nombres de Stirling associés," *Comptes Rendus de l'Académie des Sciences, Paris, Série I*, vol. 320, pp. 1449–1452, 1995.
[19] F. Chapeau-Blondeau, "Nonlinear test statistic to improve signal detection in non-Gaussian noise," *IEEE Signal Processing Lett.*, vol. 7, pp. 205–207, July 2000.



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