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Modeling Coupled Surface–Subsurface Flow Processes: A Review

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Surface and subsurface flow systems are inherently unified systems that are often broken into sections for logical (e.g., time scales) and technical (e.g., analytical and computational solvability) reasons. While the basic physical laws are common to surface and subsurface systems, spatial and temporal dimensions as well as the continuum approach used for the subsurface lead to different formulations of the governing partial differential equations. While in most applications such decoupling of the systems works well and allows a very accurate and efficient description of the individual system by treating the adjacent system as a boundary condition, in the case of water flow over a porous medium, it does not. Therefore coupled models are in increasing use in this field, led mostly by watershed and surface irrigation modelers. The governing equations of each component of the coupled system and the coupling physics and mathematics are reviewed first. Three different coupling schemes are identified, namely the uncoupled (with the degenerated uncoupled scheme being a special case of the uncoupled), the iteratively coupled, and the fully coupled. Next, the different applications of the different coupling schemes, sorted by field of application, are reviewed. Finally, some research gaps are discussed, led by the need to include vertical momentum transfer and to expand the use of fully coupled models toward surface irrigation applications.

Abbreviations: PDE, partial differential equation.

Hydrologic systems and closely related engineered systems (such as irrigation systems) are naturally continuous. That is, fluxes of water are continuous between the atmosphere, the surface, and the subsurface, regardless of the water state (phase). Nevertheless, large differences in the time scales associated with the flow of water within these three flow domains leads to breaking of the hydrologic cycle into three individual components. The major components and the interaction between them are shown schematically in Fig. 1. Atmospheric water, in the form of precipitation, is the source for both surface runoff and vadose zone water through direct infiltration. Surface water also infiltrates into the vadose zone or, in cases of shallow groundwater, directly into the saturated zone. Groundwater, whether perched or regional, is a source for surface streams through springs. Other connections illustrated in Fig. 1 are out of the scope of this review.

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To achieve this system breakdown, neighboring systems are simplified dramatically to simple boundary conditions. For most applications, this breaking into subsystems is beneficial for science, both for its ability to simplify systems to an acceptable level of understanding and for practical reasons such as the mathematical ability to solve problems.

The oversimplification of neighboring systems is natural, but leads to incompleteness of the models. For example, surface water modeling largely depends on the subsurface conditions at the headwater for both soil moisture and spring flow. It becomes clear that to achieve higher accuracy in the modeling of the systems, a higher level of interaction with neighboring systems must be taken into account. Of the three major components of the hydrologic cycle, namely the atmospheric or oceanic, the surface, and the subsurface, we focus in this review on the way surface

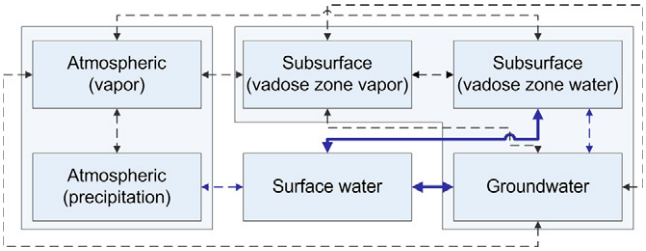

Fig. 1. Relations between different components of the hydrologic cycle. Black lines indicate vapor processes, blue lines indicate non-vapor processes. The solid, bold lines are the focus of this discussion.

and subsurface flows interact and how this interaction is modeled mathematically.

In recent years, the use of mega-models (in terms of scale as well as of scope) has flourished (see e.g., Dudhia, 1993). Typically, these computational tools primarily model the atmosphere, but also include surface and subsurface components. The last two are important for the atmospheric system as boundary conditions; however, the surface (and certainly the subsurface) systems are typically simplified as a model of buckets, and physical aspects of the systems, other than rough mass balance, are ignored. Furthermore, the development of remote sensing tools (see, e.g., Crosson et al., 2002) allows the physics of the surface–subsurface systems (with regard to modeling atmospheric processes) to be completely ignored. This review does not touch these models at all.

A surface hydrology computational model typically includes a temporal portion where rainfall is the water supply to a surface–subsurface system. Infiltration to the subsurface system at this stage depends on water availability (i.e., rain rate) and soil infiltration capacity (without completely defining the ability of the soil surface to absorb water). The point where the infiltration capacity becomes smaller than the water availability is the point where runoff starts to be generated. Infiltration models (such as, e.g., Horton, 1933, 1939) are often used to detect that point. In many of the models presented here, this stage, or rain-controlled infiltration, is an inherent part of the model. That is, many models cover temporally both the prerunoff and runoff times. The focus here, however, is in the surface–subsurface coupling (i.e., when runoff exists) and therefore the review barely touches this point.

This review starts with the governing equations (partial differential equations) that describe the flow in both the surface and subsurface components of the coupled system. First, a relatively complex (and complete) system is presented and some frequently used simplifications are shown. Next, the physical alternatives for internal (interfacial) boundary conditions and numerical alternatives for coupling of the systems are discussed. This is essentially the heart of the review. An overview of the different applications of the different coupling methods is also presented.

Although the subject of coupled surface–subsurface flow is covered well in this review, it is clear that with the enormous volume of literature available, such a review can never cover all the research that was performed. Several recent reviews address the same field but have different focuses: Khanna and Malano (2006) reviewed the basin irrigation technique with a good section on empirical infiltration functions; Singh and Woolhiser (2002) reviewed the mathematics of watershed hydrology; Aksoy and Kavvas (2005) covered the field of watershed and hillslope erosion and sediment transport, which is closely related to the focus of this review; Sophocleous (2002) covered the conceptual models of surface–groundwater interaction, but with very little mathematical and numerical notation; and finally, Morita and Yen (2000) covered similar themes as this review, but focused more on the numerical aspects of modeling.

Governing Equations

A complete coupled surface–subsurface flow system includes the surface component, the subsurface component, interfacial (i.e., between the surface and the subsurface) boundary conditions, external boundary conditions, and initial conditions. External and initial conditions, as well as system parameters, are generally a function of the specific problem of concern, and therefore will not be discussed here in detail. This section reviews the partial differential equations used to describe the two major systems, i.e., the surface and the subsurface components. Interfacial boundary conditions will be discussed separately.

Water flow in both domains, i.e., above and below surface, obeys the basic physical laws of conservation of mass and conservation of momentum. Therefore, a common ground would be at the level of fluid mechanics, i.e., the Navier–Stokes equation and the mass conservation equation. Each domain has seen a different development through history, however, leading the surface component to the Saint-Venant equations and the subsurface component to the aquifer equation or the Richards equation.

Surface Flow Equations

Equations for describing unsteady flow in natural and nonnatural channels mostly result directly from the Saint-Venant equations or one of its approximations (often called the *shallow water equation*). The Saint-Venant equations are originally one dimensional and include a mass conservation equation and a momentum conservation equation. The way two-dimensional flow is modeled is discussed below. Three-dimensional treatment of surface flow practically does not exist in the literature, other than the full Navier–Stokes equation. The most complete version of the equations (Yen and Tsai, 2001) is

$$\frac{\partial A}{\partial t}+\frac{\partial Q}{\partial x}=q_{\mathrm{r}}-q_{\mathrm{s}} \qquad [1]$$

$$\begin{aligned} &c_1\frac{1}{gA}\frac{\partial Q}{\partial t}\\ &+c_2\left\{\frac{1}{gA}\frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right)+\frac{1}{gA}\left[U_{\mathrm{r}x}q_{\mathrm{r}}-U_{\mathrm{s}x}q_{\mathrm{s}}-\left(q_{\mathrm{r}}-q_{\mathrm{s}}\right)\frac{Q}{A}\right]\right\}\\ &+c_3\frac{\partial H}{\partial x}-c_4\left(S_0-S_{\mathrm{f}}\right)=0 \end{aligned} \qquad [2]$$

where Q [$L^3\ T^{-1}$] is the flow discharge in the channel, A [L^2] is the flow cross-sectional area, H [L] is the flow depth, q_s [$L^2\ T^{-1}$] is a sink term (e.g., infiltration or lateral losses, in terms of volume per unit length per unit time), q_r [$L^2\ T^{-1}$] is channel sources (due to, e.g., rain, in terms of volume per unit length per unit time), U_{sx} [$L\ T^{-1}$] and U_{rx} [$L\ T^{-1}$] are x components of the channel sinks and sources, x [L] is the direction along the channel, S_0 and S_f (both dimensionless) are channel and friction slopes, respectively, t [T] is the time, g [LT^{-2}] is the acceleration due to gravity, and c_1 through c_4 (dimensionless) are Boolean parameters used to reduce the general equation to its subversions. Note that Eq. [1] is a mass balance equation, while Eq. [2] is a momentum conservation equation (sometimes known as a *force equilibrium* equation). Also note that there are nonconservative versions of these equations, expressed in terms of the water level (rather than discharge).

The sink–source terms are worth a short discussion. While for the mass balance equation both rain and infiltration are well understood, this is not the case for momentum transfer. When horizontal surface flow is considered, as is almost always the case, the horizontal component of the rain (due to wind speed) can be significant, especially if the surface area of the surface water is large (i.e., shallow flow). On the other hand, the infiltration

horizontal component of momentum transfer is hardly evident and indeed hardly accounted for. It is speculated that it should be used only when modeling very steep flow.

By setting different values to the Boolean parameters, different versions of the surface flow models are obtained, namely, the kinematic wave:

$$S_0 - S_f = 0 \quad [3]$$

which is obtained by setting $c_1 = c_2 = c_3 = 0$ and $c_4 = 1$; the non-inertial wave (also known as *zero inertia* and sometimes mistakenly [see Yen and Tsai, 2001] referred as the diffusion wave) approximation:

$$\frac{\partial H}{\partial x} - \left(S_0 - S_f\right) = 0 \quad [4]$$

obtained by setting $c_1 = c_2 = 0$ and $c_3 = c_4 = 1$; the gravity wave approximation:

$$\frac{1}{gA}\frac{\partial Q}{\partial t} + \left\{\frac{1}{gA}\frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right) + \frac{1}{gA}\left[U_{rx}q_r - U_{sx}q_s - \left(q_r - q_s\right)\frac{Q}{A}\right]\right\} + \frac{\partial H}{\partial x} = 0 \quad [5]$$

which is obtained by setting $c_1 = c_2 = c_3 = 1$ and $c_4 = 0$; the quasi-steady dynamic wave:

$$\left\{\frac{1}{gA}\frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right) + \frac{1}{gA}\left[U_{rx}q_r - U_{sx}q_s - \left(q_r - q_s\right)\frac{Q}{A}\right]\right\} + \frac{\partial H}{\partial x} - \left(S_0 - S_f\right) = 0 \quad [6]$$

which is obtained by setting $c_2 = c_3 = c_4 = 1$ and $c_1 = 0$; and the (hydro) dynamic wave, which is the full partial differential equation (Eq. [2] with $c_1 = c_2 = c_3 = c_4 = 1$). This classification is commonly accepted throughout the literature, though in some cases the terminology differs (compare, e.g., Yen and Tsai [2001] to Lamberti and Pilati [1996]).

All equations leave large space for empiricism—especially with regard to the way the friction slope, S_f, is modeled, where Manning's equation is typically used.

As for the one-dimensional case, there exist several versions of the governing Saint-Venant-like equations for two-dimensional cases. The version used by Morita and Yen (2002) is introduced first (depth averaged, horizontal in the x, y plane, where y [L] is the second horizontal coordinate). The differences found when compared with the one-dimensional case are primarily when breaking the cross-sectional area into components and when velocities are considered in two directions:

$$\frac{\partial H}{\partial t} + \frac{\partial (uH)}{\partial x} + \frac{\partial (vH)}{\partial y} = q_r - q_s \quad [7]$$

$$\frac{\partial (uH)}{\partial t} + \frac{\partial \left(u^2 H\right)}{\partial x} + \frac{\partial (uvH)}{\partial y} + q_r u_{rx} + gh\frac{\partial H}{\partial x} - gH\left(S_{0x} - S_{fx}\right) = 0 \quad [8]$$

$$\frac{\partial (vH)}{\partial t} + \frac{\partial (uvH)}{\partial x} + \frac{\partial \left(v^2 H\right)}{\partial y} + q_r u_{ry} + gH\frac{\partial H}{\partial y} - gH\left(S_{0y} - S_{fy}\right) = 0 \quad [9]$$

where H [L] is the flow depth, u and v [L T^{-1}] are velocities in the x and y directions, respectively, and the additional subscripts x and y indicate bottom and friction slopes in the x and y directions, respectively. Note that the dimensions of q_r and q_s here are [L T^{-1}], different than the definition in Eq. [1].

Playan et al. (1994a), following Akanbi and Katopodes (1988), used similar version of the partial differential equations but instead of the direct momentum transfer used by Morita and Yen (2002) ($q_r u_{rx}$ and $q_r u_{ry}$), they introduced extra terms D_{1x} and D_{1y} [L^2 T^{-2}] to take the place of the zero on the right-hand side of Eq. [8] and [9], respectively. In both cases, however, only horizontal components of the momentum are transferred as the models are depth averaged. Note also that Playan et al. (1994a) developed their model for surface irrigation and therefore did not consider the effect of additions to the stream by (for example) rain (i.e., q_s is omitted)—in such cases, extra terms would need to be added to account for momentum transfer between the rain and the stream. Note that the momentum transfer is a function of both the flow rate and its velocity. The momentum transfer function (horizontal only) is approximated (Akanbi and Katopodes, 1988; Strelkoff, 1969) for one-dimensional flow by

$$D_{1x} = \frac{uq_r}{2}, \quad D_{1y} = \frac{vq_r}{2} \quad [10]$$

The different versions of the Saint-Venant equations shown above assume that the interface between the surface water and the subsurface does not change. In practice, shear forces may cause soil particles to detach from the streambed and move with the possibly turbulent water. This review strictly does not discuss cases where significant interface geometry change is involved (e.g., a dam break), but does cover cases of minor change such as watershed erosion. For completeness, refer to the appendix , which describes a formulation (by Bradford and Katopodes, 1998) for two-dimensional vertical flow.

Subsurface Flow Equations

Subsurface systems typically are modeled by embedding Darcy's law into a mass conservation operator. For the sake of completeness, however, we start with the volume-averaged Stokes equation:

$$q + \frac{k\rho}{\phi\mu}\frac{\partial q}{\partial t} - \frac{k\tilde{\mu}}{\phi\mu}\nabla^2 q = -k\frac{\gamma}{\mu}\nabla\Phi \quad [11]$$

(1) (2) (3) (4)

where q is the flux [L T^{-1}], k [L^2] is the permeability, and Φ [L] is the hydraulic head. Without going into further detail, this equation is composed of four components, where the middle two typically are not seen in Darcy's law. The second term is typically neglected as it lasts only for a short fraction of a second when conditions change. The third term is called the *Brinkman term* (Brinkman, 1947). While this term is (with practically no excep-

tion) omitted when pure porous media are of concern, it is often used in problems that include an interface between a porous medium and free water (see, e.g., Shavit et al., 2002; COMSOL, 2007).

As mentioned above, for most subsurface problems (and for practically all such problems at regions far from interfaces), the simpler version of Darcy's law can be used:

$$q = -K\nabla\Phi \quad [12]$$

where K is the hydraulic conductivity [L T^{-1}]. Darcy's law generally applies for saturated flow, but it is widely accepted that the Buckingham (1907) extension can be used to apply it to unsaturated flow with a correction for the hydraulic conductivity:

$$K = K_s K_r \quad [13]$$

where K_s is the saturated hydraulic conductivity [L T^{-1}] and K_r (dimensionless) is the relative hydraulic conductivity, generally a function of the water content or the pressure head. The relative hydraulic conductivity is typically described using semiempirical models such as Gardner (1958), Brooks and Corey (1964), van Genuchten (1980), and others.

Two major forms of the continuity equation result from embedding Darcy's law into a mass conservation scheme—the aquifer equation:

$$S\frac{\partial\Phi}{\partial t} = \nabla K\nabla\Phi \quad [14]$$

and the Richards equation, which is used for variably saturated conditions:

$$\frac{\partial\theta}{\partial t} = \nabla\left[K(h)\nabla h\right] + \frac{\partial K}{\partial z} \quad [15]$$

where S (dimensionless) is a storage coefficient composed of media and fluid elasticity, θ (dimensionless) is the volumetric water content, and h [L] is the (possibly negative) pressure head ($\Phi = h + z$). In Eq. [14], the storage coefficient takes different forms depending on the dimensionality of the equation (one, two, or three dimensional), its orientation (horizontal or vertical if fewer than three dimensions are considered), and the type of aquifer (confined or phreatic). There are several mixed formulations that allow the use of a single partial differential equation for both unsaturated and aquifer systems (see, e.g., Dogan and Motz, 2005). Also, the Richards equation may be presented in terms of the head, the water content, or in a mixed form (which is the one presented here; see, e.g., Warrick, 2003). Naturally, both equations have different forms when fewer than three dimensions or a different coordinate system are used.

The aquifer equation (Eq. [14]) is generally linear. In some cases, however, such as when a phreatic aquifer is considered, it may become nonlinear due to the identity between the boundary and the unknown equation, the head. Further nonlinearity is due to vertical averaging when fewer than three dimensions are considered. The Richards equation (Eq. [15]) is inherently nonlinear. Its solution requires formulation of head–water content relations, definition of an unsaturated conductivity function, and linearization.

Conceptual Models for Coupling: Physics

As discussed above, although of similar origin (the Navier–Stokes equations) and of similar concept (mass and momentum conservation), the surface and subsurface systems follow different partial differential equations. Therefore, boundary conditions need to be defined at the interface between the systems.

In general, boundary conditions are defined at the interface between domains, whether external interfaces, bounding one system from the exterior world, or internal ones; however, subsurface systems, which are naturally two- or three-phase systems, are modeled as a continuum using the representative elementary volume (REV) concept (see, e.g., Bear, 1972; Warrick, 2003). Therefore, the exact location of the interface between the surface and subsurface systems is somewhat vague. Furthermore, the effective value of some REV-averaged parameters (such as porosity or hydraulic conductivity) near the interface may deviate from its value in locations that are far from interface. It is clear that the interface between the two flow systems is not "mathematically sharp." Nevertheless, for most engineering problems (i.e., of significant scale), this issue can be ignored and the boundary can be considered to be sharp.

This discussion generally follows Alazmi and Vafai (2001). The problem of interfacial boundary conditions between a porous medium and a homogenous fluid has been investigated by many researchers. Beavers and Joseph (1967) pioneered the performance of experimental studies that created the foundation for research in this field. The most important finding was the detection of a slip condition at the interface. That is, the porous medium cannot be treated as an impervious boundary as it had been (and to some degree still is) treated in the past. Neale and Nader (1974; we rely here on Alazmi and Vafai [2001]) proposed a continuity in both the velocity and its gradient at the interface by introducing the Brinkman term in the momentum equation for the porous side (see Eq. [1]). Several other formulations for boundary continuity, following the same approach using the extended Darcy form, were presented by, e.g., Vafai and Tien (1981), Vafai and Kim (1990b), and Kaviany (1991). Other significant advances were made by Larson and Higdon (1987a,b), and by Sahraoui and Kaviany (1992). Whitaker (e.g., Whitaker, 1992; Ochoa-Tapia and Whitaker, 1995a,b, 1997) used the volume-averaging method to define a jump in the velocity across the interface.

Alazmi and Vafai (2001) summarized the different approaches for treating boundary conditions between porous and free fluid domains. They distinguish between five different categories of boundary conditions as follows.

The first category, which is perhaps the most intuitive, requires the continuity of velocity and its gradient across the interface:

$$u_{PM} = u_{FF} \quad [16]$$

$$\left(\frac{du}{dn}\right)_{PM} = \left(\frac{du}{dn}\right)_{FF} \quad [17]$$

where u [L T^{-1}] is the water velocity (perpendicular to the interface), n is the direction normal to the interface, PM denotes porous medium, and FF denotes free fluid. This type of boundary condition was used by, e.g., Neale and Nader (1974), Vafai and Kim (1990a,b), and Jang and Chen (1992).

The second category still requires the continuity of velocity across the interface (Eq. [16]), but the velocity gradients are weighted by the kinematic viscosity, μ [M L^{-1} T^{-1}], where on the porous medium side an effective viscosity term, μ_{eff}, is used:

$$\mu_{\text{eff}}\left(\frac{\mathrm{d}u}{\mathrm{d}n}\right)_{\text{PM}} = \mu\left(\frac{\mathrm{d}u}{\mathrm{d}n}\right)_{\text{FF}} \quad [18]$$

This category was used by Vafai and Thiyagaraja (1987), Poulikakos and Kazmierczak (1987), and later by Kim and Choi (1996).

The third category, used by Whitaker and Kuznetsov (e.g., Ochoa-Tapia and Whitaker, 1995a,b; Kuznetsov, 1996, 1997, 1998a,b, 1999), includes a jump condition in the gradient, while the velocity is assumed to be continuous across the interface:

$$\frac{\mu}{\phi}\left(\frac{\mathrm{d}u}{\mathrm{d}n}\right)_{\text{PM}} = \mu\left(\frac{\mathrm{d}u}{\mathrm{d}n}\right)_{\text{FF}} + \beta_1 \frac{\mu}{\sqrt{k}} u \quad [19]$$

where β_1 is a dimensionless coefficient, ϕ (dimensionless) is the porosity, and k is the permeability [L^2]. This jump condition allows for a difference between the porous medium and the free fluid, and therefore allows treatment of both domains with their classical partial differential equations.

The fourth category, presented by Ochoa-Tapia and Whitaker (1998), is very similar to the third category, i.e., continuity of the velocity and a jump in its derivative. The jump here has two terms, however—one proportional to the velocity (as in the third category) and an additional term proportional to the velocity squared:

$$\frac{\mu}{\phi}\left(\frac{\mathrm{d}u}{\mathrm{d}n}\right)_{\text{PM}} = \mu\left(\frac{\mathrm{d}u}{\mathrm{d}n}\right)_{\text{FF}} + \beta_1 \frac{\mu}{\sqrt{k}} u + \beta_2 \rho u^2 \quad [20]$$

where β_2 is a dimensionless coefficient and ρ [$M\,L^{-3}$] is the fluid density.

The fifth and last category deals only with the velocity derivative from the free-fluid side, which is defined by

$$\left(\frac{\mathrm{d}u}{\mathrm{d}n}\right)_{\text{FF}} = \frac{\alpha^*}{\sqrt{k}}\left(u_{\text{int}} - u_{\text{inf}}\right) \quad [21]$$

where an interface velocity u_{int} [$L\,T^{-1}$] is defined independently of the free-fluid velocity, u_{inf} [$L\,T^{-1}$]. The velocity gradient is a function of these two velocities and a dimensionless slip coefficient, α^*. This approach, originally used by Beavers and Joseph (1967), was later used by Sahraoui and Kaviany (1992).

In general, it is important to understand that two conditions must be satisfied—the continuity of the momentum and the conservation of mass. It is customary to represent these two conditions as velocity and its normal gradient; however, other representational options are equally valid. In many of the cases that will be presented below, boundary conditions have been represented in terms of flux and energy (head), which is more customary in fields such as porous media internal boundaries (see, e.g., Janković and Barnes, 1999; Furman and Neuman, 2003; Furman and Warrick, 2005). The different approaches presented above result from (i) the different formulations of the flow equations in the two regions (i.e., mostly due to the treatment of the porous medium as a continuum); (ii) the difficulty in defining a sharp interface; and (iii) the desire to simplify the boundary conditions to allow practical application.

Conceptual Models for Coupling: Numerics

Generally, a complete system of coupled surface–subsurface flow systems involves a mathematical description of the surface process, a mathematical description of the subsurface process, a mathematical description of external boundary conditions, and a mathematical description of the internal boundary conditions (i.e., at the surface–subsurface interface). The surface system is often described as a hyperbolic or parabolic partial differential equation (PDE) and the subsurface system is described by a parabolic PDE.

Three different levels of coupling between surface and subsurface processes, illustrated in Fig. 2, can be distinguished. These include the no-coupling, the iterative coupling, and full coupling. All three components are described below. In theory, the higher the level of coupling, the higher the accuracy. As the coupling involves numerics (rather than analytics), however, this is not necessarily true. The three different coupling levels are briefly described here. A review of the different applications of the three levels is provided farther below.

The first level of numerical coupling actually means no coupling. That is, at each time step, each system is solved independently, where typically the surface water component is solved first (mostly because of the faster dynamics). Based on the solution, an internal boundary condition value is specified and then the other system is solved. No feedback is used to correct the first system. As the boundary condition at the interface between the systems applies (in general) to both systems, an approximation is necessary. It is convenient to use conditions from the previous time step as a guess of boundary conditions for the system that is solved first. We refer to this level of coupling, following Morita and Yen (2000), as *uncoupled*. Note that this coupling level was referred to in the past as *externally coupled* by Freeze (1972).

The first coupling level can actually be split into two subgroups, the first of which is described above. The second subgroup includes the mimicking of one of the involved systems (surface or subsurface) into an algebraic formulation (typically a specific solution for one of the systems). This approach is popular among surface irrigation modelers, and is referred to here as *degenerated uncoupled*.

The second level of coupling involves feedback between the two systems. The first stages are similar to the uncoupled level: one system is solved, interfacial boundary conditions are formulated, and the second system is solved using these boundary conditions. The difference is that the solution of the second system is now used to update the internal boundary condition within the same time step. The first system is solved again using this updated boundary condition, and the whole process is repeated until convergence criteria are achieved (typically until there is no significant change in one of the solved components). Morita and Yen (2000) referred to this coupling level as *alternating iterative*. We refer to this coupling level as *iterative coupling*.

The third coupling level, which is the most complete, involves solving the two systems and the internal boundary conditions

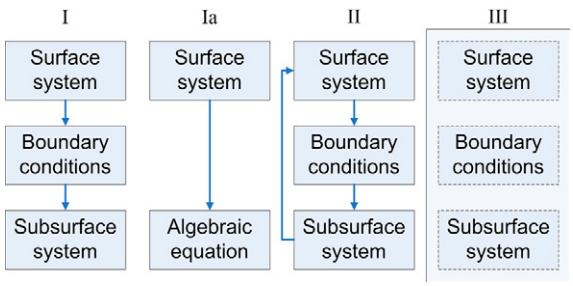

FIG. 2. The four different numerical coupling schemes: (I) uncoupled; (Ia) degenerated uncoupled; (II) iterative coupling; and (III) fully coupled.

together. That is, the two PDEs and the interface equation (which may be an ordinary differential equation) are solved simultaneously. This coupling level is referred to here as *fully coupled* (note that Morita and Yen [2000], although at the time they could not identify a single application of this coupling level, referred to it as *coupled simultaneous solution*).

It is important to note that it is not clear that full coupling is superior to iterative coupling. Full coupling involves discretizing and solving parabolic and hyperbolic equations in a single set. This may lead to numerical difficulties resulting from the different natures of the equations. The full coupling also leads to larger systems that need to be solved. As a numerical solution typically involves the iterative solution of spatially and temporally discretized equations, at the most basic level the difference (from the computational efficiency point of view) may be between external (in the case of iterative coupling) and internal (in the case of full coupling) iterations.

Overview of Computational Tools and Applications

As discussed above, coupling surface and subsurface flow processes is a difficult mathematical task. Therefore, in most practical cases, the attempt is to decouple the two processes, focus on one of the two, and simplify the other process to a set of (typically) empirical parameters and functions. At present, the higher levels of coupling (*fully coupled* and *iterative coupling*) codes tend to be associated with academic studies with lesser practical application, although this "sharp boundaries" observation has tended to blur in recent years.

The overview of computational codes begins by reviewing the way coupled flow processes are treated in some of the most widely used codes: MODFLOW and HEC-RAS. Following the review of these megacodes, other commercial and noncommercial applications in different disciplines associated with coupled surface–subsurface flow are reviewed. Some representative examples of the models described are summarized in Table 1.

MegaCodes

MODFLOW

Several popular codes are in intensive use in the field of groundwater, clearly led (in terms of popularity) by MODFLOW (McDonald and Harbaugh, 1988) and its many descendants (e.g., MODFLOW 96 [Harbaugh and McDonald, 1996]; MODFLOW 2000 [Harbaugh et al., 2000]) and interfaces (e.g., Argus ONE [Winston, 2000]; GMS [Environmental Modeling Systems, 2007]; Groundwater Vistas [Environmental Simulations

Table 1. Selected coupled surface–subsurface flow models.

Reference	Commercial name or nickname	Surface†	Subsurface‡	Coupling§
Irrigation				
Strelkoff et al. (1998); U.S. Arid-Land Agricultural Research Center (2006)	SRFR/WinSRFR	1D ZI	EA	DU
Utah State University (1999, 2003)	SIRMOD	1D KW/ZI/HD	EA	DU
Katopodes and Strelkoff (1977)		1D HD	EA	DU
Singh and Bhallamudi (1996, 1997)		1D HD	EA	DU
Akanbi and Katopodes (1988)		2D HD	EA	DU
Zerihun et al. (2003)		1D ZI	EA	DU
Zerihun et al. (2005a,b,c)		1D ZI	1D R	IC
Schwankl et al. (2000)		1D ZI	EA	DU
Abbasi et al. (2003a,b)	ZIMOD	1D ZI	EA	DU
Wöhling et al. (2004, 2006)		1D ZI	2D R	IC
Watershed				
U.S. Army Corps of Engineers (2000)	HEC-HMS	1D KW/TL/UH	EA	DU
VanderKwaak (1999)	InHM	2D	3D R	FC
Panday and Huyakorn (2004)	MODHMS	1D+2D HD	3D R	FC/IC
Refsgaard and Storm (1995)	MIKE-SHE	1D+2D ZI	1D R/EA	UC/DU
Bixio et al. (2002)		1D ZI	3D R	UC
Fiedler and Ramirez (2000)		2D HD	EA	DU
Gandolfi and Savi (2000)		2D HD	1D R	UC
Singh and Bhallamudi (1998)	1DS1DSS/1DS2DSS	1D HD	1D/2D R	UC
Smith and Woolhiser (1971)		1D KW	1D R	UC
Govindaraju and Kavvas (1991)		1D ZI	2D R	IC
Liang et al. (2007)		2D HD	GW	FC
Gunduz and Aral (2005)		1D HD	GW	FC
Subsurface				
McDonald and Harbaugh (1988)	MODFLOW	VB	GW	DU
Cheng and Anderson (1993)	MODFLOW	VB	GW	DU
Council (1999)	MODFLOW	VB	GW	DU
Other				
Kouznetsov et al. (2007)		1D HD	3D R	IC

† 1D, one-dimensional; 2D, two-dimensional; ZI, zero inertia; KW, kinematic wave; HD, dynamic wave (hydrodynamic, full inertia); TL, time lag; UH, unit hydrograph; VB, mass (volume) balance.

‡ EA, empiric or algebraic; R, Richards' equation; 3D, three-dimensional; GW, groundwater.

§ DU, degenerated uncoupled; IC, iterative coupling; FC, full coupling; UC, uncoupled.

Incorporated, 2007]; and others). To some degree, MODFLOW codes can be considered as a de facto standard. The MODFLOW codes primarily consider the saturated zone. Some, but purely vertical, interaction (i.e., interaction between a MODFLOW cell and the area just above it) with surface water is allowed through some of the original and later developed packages. The number of external packages that were developed for MODFLOW is enormous. Only a few of the packages, and mostly the "mainstream" ones (i.e., that were developed in, or in close cooperation with, the USGS) are discussed here.

Two levels of surface–subsurface interaction within the MODFLOW framework can be differentiated: (i) prescribed recharge; and (ii) linear dependence of recharge in groundwater and surface water heads. For the second case, several modifications exist but with no significant change in the recharge calculations. In the first case, surface (or atmospheric) processes affect the groundwater system in a manner that (can be approximated as one that) does not depend on the state of the groundwater system. Within this category fall the recharge package (RCH, McDonald and Harbaugh, 1988); the river package (RIV, McDonald and Harbaugh, 1988); the reservoir package (RES1, Fenske et al., 1996); and several "out of the mainstream" (i.e., out of the USGS framework) modifications (e.g., Osman and Bruen, 2002) to the river package. These modifications were primarily designed to achieve a more accurate estimation of infiltration in the case where the water level is significantly below the stream (i.e., $h <$ RBOT), but treatment of the streams as a dynamic hydrologic unit is made only (to my knowledge) through the streamflow packages. Note that for most packages the recharge function includes (at least) two cases: (i) cases where groundwater level affects recharge, and (ii) cases where it does not. In this zero-level interaction category, we include only those cases where groundwater is deeper than the bottom level of the surface water body (e.g., the lower end of the streambed).

The next level of interaction models the intermediate flux as linear to the difference between surface conditions (that are assumed to be prescribed) and groundwater level. That is, the flux between the surface water and the groundwater system is of the form $Q = C(h_S - h_{GW})$, where Q [$L^3 T^{-1}$] is the interaction flux, C [$L^2 T^{-1}$] is a constant (that typically is composed of geometry factors and hydraulic properties), h_S [L] is the head at the surface system (typically prescribed but not necessarily constant in time), and h_{GW} [L] is the head in the groundwater system. In this group are included, e.g., RIV, for the case where the groundwater level is above the river bottom level (McDonald and Harbaugh, 1988), and RES1, for the case of shallow groundwater (Fenske et al., 1996).

In the same category (i.e., of first-order interaction) are some packages that include a more sophisticated treatment of the surface system but still assume a linear interaction form for the flux between the surface and the subsurface systems. The lake packages (LAK1 [Cheng and Anderson, 1993] and LAK2 [Council, 1999]) allow computation of the lake (or reservoir) level as a function of all its inputs and outputs (a lake may cover more than one MODFLOW cell, hence the water level depends on the inflows and outflows from more than a single cell). The computation is based on a simple mass-balance equation (assuming pseudo-steady state), hence at each time step the water level of the lake can be assumed to be prescribed. In a similar way, the streamflow routing package (SFR1 and its predecessor STR1, Prudic et al., 2004) computes the head at each segment of the river, assuming steady flow conditions and that the stage can be computed using, e.g., Manning's equation. The outflow from each stream segment is computed in a manner similar to the basic river package (i.e., using the aggregated Darcy's law), with the limitation that the infiltration cannot exceed the saturated hydraulic conductivity.

Another alternative for a groundwater model that includes streamflow routing is the DAFLOW package (Jobson and Harbaugh, 1999). The approach to modeling the streamflow here is different than the one used in previous (and later) packages in that not only is the mass balance solved, but momentum is also conserved. The conceptual model of the surface water assumes that the dynamic behavior of the stream system is composed of steady segments (i.e., segments in which steady flow occurs) and transition zones. These transition zones are approximated as shock waves (i.e., discontinuities in the flow rate and depth) and are routed through the channel system in velocities that are computed using the normal (steady) velocities in the channel before and after the shock. While this solution is interesting in terms of the modeling of the surface water, the coupling between the surface and subsurface systems is still first order.

The MODBRANCH model is another combination of MODFLOW, this time with a surface channel flow model. In this case, Swain and Wexler (1996) coupled the channel flow model BRANCH (Schaffranek, 1987; Schaffranek et al., 1981) with MODFLOW. The BRANCH model solves the Saint-Venant equations. Since the time scales between the two components are significantly different, the computational interface "calls" the BRANCH module several times during each MODFLOW time step. The interaction between the two modules uses a simple leakage term similar to that discussed, e.g., for the SFR1 module. To account for the changing head in the aquifer, an iterative scheme is necessary.

The SFR1 package, while a great advance for MODFLOW in terms of modeling a surface system (as are the DAFLOW and most other MODFLOW-related surface interaction models), still does not deal with the vadose zone, and most importantly (from the groundwater system point of view) does not consider the temporal delay in the arrival of recharge. That is, any drop of water leaving one system (in all previously discussed packages) was assumed to arrive immediately in the other system. The streamflow-routing package (SFR2, Niswonger and Prudic, 2005) is specifically designed to answer the question of timing the arrival of recharge to groundwater when there is no direct (i.e., saturation) connection between the river and the aquifer. Its action can be described in several stages. First, the recharge rate is computed in a manner similar to that used in the river package; however, the infiltration rate is limited by the saturated hydraulic conductivity of the vadose zone. At the second stage, the infiltration is routed through the vadose zone. Vadose zone flow is modeled using a kinematic-wave approximation of the one-dimensional Richards equation, assuming that flow is driven only by gravitational forces:

$$\frac{\partial \theta}{\partial t} + \frac{\partial K(\theta)}{\partial z} = 0 \qquad [22]$$

This approximation of the Richards equation is solved using the method of characteristics, where the Brooks and Corey (1966)

function ($K = K_s S_e^{\alpha}$, where S_e [dimensionless] is the effective saturation) is used to specify the unsaturated hydraulic conductivity function. Infiltration waves created by a change in the infiltration rate at each time step (an increase in the rate will create the wave lead and a decrease will create the wave tail [*trailing wave* in MODFLOW language], with infiltration fluxes computed by Darcy's law as noted above) are routed through the vadose zone. Boundary conditions for this method of characteristics solution are given in terms of water content, and for that purpose the flux is translated using

$$\theta = \theta_r + \left(\frac{q}{K_s}\right)^{1/\alpha} S_y = 0 \qquad [23]$$

where θ_r (dimensionless) is the residual water content and S_y (dimensionless) is the specific yield, taken here as a more realistic evaluation of the difference between saturated and residual water content.

It is important to understand that in the case of the MODFLOW SFR1 module and especially the SFR2 module, although the surface water and the unsaturated zone are modeled in a significantly nonempirical way, the level of interaction between the surface and the subsurface system is limited to a simple computation of fluxes through a direct solution of the aggregated Darcy's law. Coupling therefore, at least in terms of mass, between surface water and groundwater, bypasses the vadose zone. A higher level of coupling does not exist at this stage in the framework of MODFLOW (but see MODHMS below). Note, however, that two recently developed programs already couple the vadose zone flow with MODFLOW (the VSF package [Thoms et al., 2006] that couples the three-dimensional Richards equation with MODFLOW and the HYDRUS package for MODFLOW 2000 [Seo et al., 2007; see also Twarakavi et al., 2008] that couples the one-dimensional Richards equation with MODFLOW). The availability of such programs suggests that full coupling of surface flow and MODFLOW through full simulation of the vadose zone flow is near.

HEC-RAS

The U.S. Army Corps of Engineers' River Analysis System (HEC-RAS) is software that allows performance of one-dimensional steady and unsteady flow river hydraulics calculations. The HEC-HMS (U.S. Army Corps of Engineers, 2000) is a modified version (of HEC-RAS) designed to deal with watershed hydrology (while the original HEC is a more hydraulic version) utilizing a simplified version of the full Saint-Venant equation:

$$\frac{\partial Q}{\partial t} + \frac{\partial QV}{\partial x} + gA\left(\frac{\partial z}{\partial x} + S_f\right) = 0 \qquad [24]$$

Several alternatives are available for routing rainfall to runoff (including several unit hydrographs, but also a kinematic wave option), and to route flow through the watershed channels. The HEC-HMS model includes several loss functions, including constant, exponential (with time), Green and Ampt (1911), Soil Conservation Service (1971) curve number loss, and soil moisture accounting loss. The latter means that the whole system is divided into five layers (including canopy, surface, vadose, and two groundwater layers), and losses are computed in a hierarchy according to abstraction coefficients and allowed rates.

Irrigation Models

Modeling surface irrigation is among the most difficult tasks for modeling surface flow. This is primarily because of the high nonlinearity of the flow tip (the front of the surging water wave). Researchers in the field of surface irrigation have primarily focused their attention on descriptions of the surface flow, and approximated the infiltration losses by empirical formulas. It is only in recent years that applications of a higher level of coupling in surface irrigation modeling have been developed significantly.

Irrigation techniques can be split into two major categories: volume irrigation and precise irrigation (sometimes referred to as pressurized and nonpressurized, or surface irrigation and microirrigation). The first group includes primarily surface (flood) irrigation techniques (furrow, basin, and border), while the second group includes drippers, sprinklers, and spray techniques. Our distinction between these two groups is based primarily on the fact that when a precise irrigation scheme is properly designed, it is relatively easy to decouple the surface and subsurface processes. That is, when the drip or sprinkler discharge is designed in accordance with the soil properties, very little surface runoff is generated (see, e.g., Cuenca, 1989; Benami and Ofen, 1983). In such cases, the hydraulic problem (of flow in pipes and drip maze) and the fluid mechanics problem (in the case of sprinkler irrigation, the path of the drops through the air) barely affect the subsurface flow. Boundary conditions for the subsurface problem are of the prescribed flux (Neumann) type. This is especially true with modern discharge-regulated (pressure-compensated) devices and for buried drippers.

Flood irrigation systems are commonly used in most countries. In the United States, an estimated total area of roughly 115,000 km^2 is irrigated using flood techniques (USGS, 2000), accounting for roughly half of the irrigated land. In India, >500,000 km^2 of land are flood irrigated (FAO, 2007). Three techniques are used: furrow, mostly in row crops and in the absence of modern machinery; borders, for close-growing crops; and basins, mostly for field crops but also orchards. Although some claim that high efficiency can be achieved in flood irrigation, it is commonly accepted that these systems are inherently inefficient in their water use.

Most flood irrigation modeling tools apply the most basic coupling scheme, i.e., the degenerated uncoupled approach. This approach includes reduction of the subsurface flow process into a relatively simple algebraic equation that depends directly or indirectly on the solution of the surface system, most commonly the modified Kostiakov equation and its variations (Kostiakov, 1932; Haverkamp et al., 1988; Clemmens, 1981; FAO, 1989—note that the modified Kostiakov equation is also known as Kostiakov–Lewis or Mezencev):

$$I = k^* \tau^a + f_0 \tau \qquad [25]$$

where I [L] is the cumulative infiltration, τ [T] is the opportunity time, and k^* [L T^{-a}], a (dimensionless), and f_0 [L T^{-1}] are empirical parameters. Some variations of this equation exist, mostly by omitting the second term (which is linear with time) or splitting between using the first and second terms (aka the branch method). The addition of a constant (e.g., Walker et al., 2006) is useful to account for crack filling or other nontemporal abstractions. The opportunity time indicates the cumulative time

that the surface (at a specific location) was flooded. Therefore, when the Kostiakov equation is used, the "coupling" parameter is the time, and it is activated at a location depending on whether the solution for the surface component indicated that location as flooded.

Quite a few researchers and engineers have used the Kostiakov equation to model "uncoupled" surface–subsurface flow. Only a small portion of many are listed here. The two most commonly used codes (at least in the USA) are SRFR (Strelkoff et al., 1998; U.S. Arid-Land Agricultural Research Center, 2006) and SIRMOD (Utah State University, 1999). The SRFR program considers a one-dimensional zero inertia flow for simulating the flow in a single furrow, border, or basin. The interaction with the subsurface is calculated through the use of an algebraic infiltration function, where the user has the option to choose between several subversions of the Kostiakov (1932) infiltration function, as well as the Soil Conservation Service (1974) method and the time-rated intake families (Merriam and Clemmens, 1985). Applications of the SRFR program are numerous. For example, Alazba (1999) used SRFR in its zero-inertia mode to examine the influence of the inlet signal (inflow hydrograph) on irrigation performance.

The SIRMOD (Utah State University, 1999, 2003) program solves several versions of the Saint-Venant equations: a reduced version of the full dynamic wave equation (omitting mostly sinks and sources), the zero inertia equation, and the kinematic wave equation. It uses the Preissmann (roughly equivalent to the Crack–Nicolson method, which is better known in the subsurface literature) double-sweep algorithm (Liggett and Cunge, 1975) in conjunction with deformable control volume. As with most irrigation modeling schemes, different phases of the irrigation (i.e., advance, wetting, depletion, and recession) are solved using slightly different numerical simplifications, listed in Walker and Skogerboe (1987). Similar to SRFR, the subsurface system is considered as an algebraic equation, utilizing several versions of the Kostiakov (1932) equation and the Soil Conservation Service (1974) method (but not the original Soil Conservation Service curves; see Walker et al., 2006).

In addition to the previously mentioned codes, quite a variety of surface irrigation codes have been developed, in most cases for research purposes. Schwankl et al. (2000) used the zero inertia formulation, combined with the modified Kostiakov infiltration, to examine furrow irrigation performance under spatially varying parameters (the spatially variable parameters included furrow geometry [embedded with the Kostiakov infiltration], roughness, and infiltration coefficients). Katopodes and Strelkoff (1977) solved a one-dimensional zero inertia (called diffusion wave by them) equation (see further details in Walker and Skogerboe, 1987). Abbasi et al. (2003a,b) used a zero inertia model and the branch-modified Kostiakov equation to model water flow and solute transport in the surface system. Zerihun et al. (2003) used the Kostiakov infiltration function to solve the zero inertia equation with several simplifications of the difficult phases of irrigation (advance and recession). Singh and Bhallamudi (1996, 1997) used both the Kostiakov equation and the Parlange–Haverkamp equation (see below; the user has the option of choosing between the models) to solve the one-dimensional diffusion wave equation. Akanbi and Katopodes (1988) used the Kostiakov equation to solve a two-dimensional hydrodynamic wave surface flow system. Zapata and Playan (2000), following Playan et al. (1994a,b), used two-dimensional full inertia with the Kostiakov equation to model basin irrigation. Among other issues, they focused on the microtopography of the basin.

Alternatives to the Kostiakov equation (but still in the framework of representing the subsurface system as an algebraic equation) include the classic Green and Ampt (1911) equation:

$$i = K_s\left(1+\frac{h_0}{I}\right) \qquad [26]$$

where i [L T^{-1}] is the infiltration rate and h_0 [L] is the effective pressure head at the dry part of the soil. The Green and Ampt (1911) model is implicit in that the infiltration rate depends on the cumulative infiltration. This approach (of using the Green and Ampt model) was used by, e.g., Savabi (1993) in the framework of the Water Erosion Prediction Project. Warrick et al. (2005) used the Green and Ampt approach to examine the temporal variability of surface infiltration under ponding conditions, but used actual field records and not model results. The other (relatively) used alternative to an algebraic equation for simulating the subsurface infiltration is the Parlange equation (Haverkamp et al., 1990):

$$I = K_0\tau + \frac{(h-h_{str})(\theta_s-\theta_0)K_s}{q_s-K_s} + \frac{S^2+2h_{str}K_s(\theta_s-\theta_0)}{2\delta(K_s-K_0)}\ln\left[1+\frac{\delta(K_s-K_0)}{q_s-K_s}\right] \qquad [27]$$

where S [L T$^{-0.5}$] denotes sorptivity, δ is a dimensionless shape parameter that roughly scales the ratio between advection and diffusion, and h_{str} [L] is the minimum allowed pressure head. The advantage of this equation over the Kostiakov or other equations is in its use of subsurface accepted parameters.

Zerihun et al. (2005a,b,c) combined the zero-inertia surface flow model of Zerihun et al. (2003) with multiple modules of HYDRUS-1D (Šimůnek et al., 2005) and also added solute transport for the surface and subsurface components. Wöhling et al. (2004, 2006) and Wöhling and Schmitz (2007a,b) combined an analytical solution of the one-dimensional zero inertia wave equation with the popular HYDRUS-2D code (Šimůnek et al., 1999). The coupling between the surface and the subsurface modules is done iteratively. Unique in this model is that the subsurface component includes root water uptake, making it suitable for whole-season modeling. Furman and Zerihun (unpublished data, 2004) did the same with the numerical zero inertia model of Zerihun et al. (2003), where the output of the surface component are water levels. These are passed to the subsurface component as boundary conditions. After solution of the subsurface component, the surface fluxes are passed back to the surface component as sink terms.

A fixed infiltration rate is obviously less accurate than a full solution of the Richards equation; however, the use of such a rate has several advantages. One advantage is the reduction of the need to iterate between surface and subsurface modules, as demonstrated below (the use of zero depth means practically that). Another advantage is that it allows an analytical solution of the surface component. Rivlin and Wallach (1995) used a fixed infiltration rate to derive an analytic solution to the kinematic wave equation, and used the solution as a base for a coupled surface–subsurface solute transport model. They solved the problem for both the rising and receding stages of irrigation. Earlier similar

solutions are by Emmerich et al. (1989), also for solute transport, and by Lane et al. (1988) for erosion.

Iteratively coupling surface and subsurface flow is computationally demanding. This is especially true for surface irrigation where the nonlinearity associated with surge flow, and especially with the surge tip, is extreme. An interesting new approach (introduced by Schmitz et al., 2005; Schütze et al., 2005) is to replace the full solution of the subsurface component with a trained neural network. By that, the computational effort is dramatically reduced but the advantages associated with the physical modeling of subsurface flow are essentially maintained.

As discussed above, many of the irrigation models apply the degenerated uncoupling approach. That is, the subsurface system is simplified to an algebraic equation that depends on a parameter that is directly (e.g., water level) or indirectly (e.g., time) related to the solution of the surface system. I am not aware of the reverse alternative (that the surface system is embedded as an algebraic equation in the subsurface system), except perhaps in cases where a surface reservoir is considered. In these cases, the whole surface system is treated as a simple algebraic equation (as momentum is not considered and global [whole reservoir] mass balance is applied). Such mass balance is applied within the framework of HYDRUS-1D (Šimůnek et al., 2005). The simple MODFLOW packages (RES1 and LAK1) can also be considered as degenerated uncoupled models.

Watershed Models

Other than irrigation, coupled surface–subsurface flow is mostly used in watershed applications. Variations range from flood forecasting to erosion modeling. In comparison to irrigation practices, the spatial (and therefore also the temporal) scale is larger. While irrigation models are typically used for fields ranging in length from several 10s to several 100s of meters, with time scales of minutes to several days, watershed applications may cover much larger areas and lengths, with the minimum scale at the 100s of meters (and maximum at the 1000s of kilometers), resulting in simulation times that may well span more than a season (and sometimes decades). Nevertheless, the associated physics is the same. The spatial, and in particular the temporal, scales should allow higher validity to empirical infiltration equations. Natural systems, however, typically show higher heterogeneity.

River flow routing is one of the most common hydrologic practices. Techniques for river routing are frequently based on the Saint-Venant equations and often simplified to the kinematic wave equation. See Chow et al. (1988) for a brief description of classical methods such as the Runge–Kutta and the Muskingum–Conge methods. Significant attention is not paid here to the solution methods (which are the focus of the work of many); focus is on the interaction with the subsurface.

Singh and Woolhiser (2002) provided a comprehensive overview of models used to simulate watershed hydrology. A smaller portion of the available models is covered here but discussion is of the whole scope in terms of the coupling between the surface and subsurface.

Smith and Woolhiser (1971) were perhaps the first to couple the problems of surface–subsurface flow without embedding one of the two as a simplified algebraic equation. They solved the kinematic wave with a one-dimensional Richards equation. As long as there is no ponding on the surface, water flux (rain) is used as a boundary condition for the subsurface model. Once ponding starts, the boundary condition becomes a zero pressure head (expressed as saturation). This simplification reduces the need to iterate or to solve equations simultaneously.

Govindaraju and Kavvas (1991) solved a combination of the one-dimensional zero inertia equation for a hillslope, the one-dimensional zero inertia equation for a stream, and the two-dimensional Richards equation. For the surface–subsurface, they used an iterative scheme (although they called it *internal coupling*).

Fiedler and Ramirez (2000) solved the two-dimensional full inertia equations using a finite differences scheme by considering the Green and Ampt (1911) infiltration capacity and correcting for actual infiltration according to time-step available water. Gandolfi and Savi (2000) also solved the two-dimensional full inertia version of the Saint-Venant equations but considered a one-dimensional model for the Richards equation for the subsurface. To avoid iterations, it was assumed that the pressure head boundary condition for time steps and locations where ponding occurs was zero.

Singh and Bhallamudi (1998) solved the one-dimensional full inertia equation for the surface part, combined with either multiple one-dimensional modules (a model named 1DS1DSS) or a two-dimensional module (model name 1DS2DSS) of the Richards equation. To avoid iterations, subsurface conditions from the previous time step are used to compute infiltration. Woolhiser et al. (1996) combined the kinematic wave equation with the Smith and Parlange (1978) infiltration function, which explicitly includes the saturated hydraulic conductivity, to examine its effect on the flow.

Together with the HEC family of models, the MIKE-SHE system (Refsgaard and Storm, 1995) is one of the most comprehensive watershed models. It includes modules for estimation of snowmelt, canopy interception, evapotranspiration, overland and channel flow, unsaturated and saturated flow, solute transport, erosion, and more.

The MIKE-SHE model solved the two-dimensional zero inertia version of the Saint-Venant equations. The interface between the surface and the subsurface waters is different for overland and for channel flow. The channel interaction is simpler conceptually and computes the fluxes using a simple Darcy's law equation (similar to that done in the MODFLOW RIV package, above) where streambed flux resistance may be considered. In the case of overland flow, a full solution of the vertical one-dimensional Richards equation is used to compute fluxes, where the upper boundary condition may switch between prescribed flux (in the case where there is no ponding) and prescribed head (for ponding conditions).

The Richards equation is primarily used to compute the time lag for water arrival to groundwater. A secondary algorithm can be used to represent faster flowing water (through macropores). In such a case, a percentage of the net rainfall is simply defined as recharge and is transferred instantaneously to the groundwater.

Bixio et al. (2002) combined a three-dimensional Richards' equation module with a one-dimensional zero inertia surface flow module (Muskingum–Cunge; see, e.g., Chow, 1964; Chow et al., 1988) to study some effects of the morphogenic depression on the infiltration of radionuclides originated by the Chernobyl disaster. They used a noniterative scheme that computes surface flows, and "passes" water levels (as boundary conditions) to the subsurface

component. The fluxes are computed by the subsurface system only for answering the question of eventual surface storage (that is important for switching between soil- and atmosphere-limiting boundary conditions).

Modeling fully coupled surface–subsurface systems was not available until recently (see, e.g., Morita and Yen, 2002) for actual applications. Recently, VanderKwaak (1999) developed the Integrated Hydrological Model (InHM), which fully couples a two-dimensional zero inertia formulation with the three-dimensional dual-continua Richards' equation formulation. His formulation, leading to functions that resemble the formulation for the subsurface (i.e., linear flow law) is based on, for example, Vieira (1983). The InHM model also couples solute transport. The interaction between the surface and subsurface systems is done through a secondary soil layer, with thickness proportional to the hydraulic properties of the actual subsurface system. Water fluxes (solute fluxes are treated in a similar way) are considered to vary linearly with the pressure head gradient across the secondary soil layer. That is, the intermediate layer acts as a one-dimensional variably saturated Darcy's law layer, with the conductivity determined by the wetness of the layer.

The InHM system was further developed and applied, primarily at Stanford University. Several of these modifications and some applications are listed at http://www.inhm.org (verified 30 Mar. 2008).

The second fully coupled surface–subsurface watershed model is by Panday and Huyakorn (2004). The MODHMS model includes a two-dimensional zero inertia overland flow, a one-dimensional zero inertia channel flow, and a three-dimensional Richards' equation-based subsurface flow. All three components are coupled (that is, direct flow is allowed between all three systems without the need to go through another system). The two surface components are modified to include storage (i.e., in surface depressions). Flow between the surface and the subsurface components are assumed to be a linear function of the head difference between the systems, but without the use of an intermediate layer. (Therefore, fluxes are computed as functions of head differences and not head gradients, making it practically the same as in InHM.) Fluxes between the surface systems (i.e., the one- and two-dimensional components) are assumed to be through weirs, with possible submergence.

The MODHMS model allows several levels of coupling: (i) full coupling of all three systems (i.e., surface–subsurface boundary conditions are explicitly solved by the model); (ii) full coupling of only the two surface systems, where interaction between the surface and the subsurface is achieved through iterative (or time lagging) coupling passing the fluxes as boundary conditions to the subsurface system; (iii) coupling only the surface components and iterating (or time lagging) in a similar manner to ii, but the head is passed as a boundary condition to the subsurface system instead of the flux.

A third application of full coupling is by Gunduz and Aral (2005). They combined the one-dimensional full inertia model with the two-dimensional (horizontal) phreatic aquifer equation. The link between the models is through a Darcy-like difference equation across the river bed thickness (similar to the MODFLOW approach).

The last, somewhat simplified, application of fully coupled surface–subsurface flow is by Liang et al. (2007). They combined a horizontal two-dimensional surface flow (full inertia) with the two-dimensional (horizontal) Boussinsq equation for groundwater flow in an unconfined aquifer. The coupling between the submodels here is horizontal (i.e., there is no location where vertical coupling occurs).

Erosion and Sediment Transport Models

Modeling erosion and sediment transport is closely related to surface flow. The detachment of soil particles from the soil body can occur via two different mechanisms. The first is related to drag forces between the surface water and the soil surface. These forces are highly affected by water velocity. The second mechanism is related to water seepage and typically occurs in regions of high surface curvature.

The first mechanism is highly related to both the creation and mobility of eroded materials, while the second may be related more closely to the creation of the materials. For this reason (and for higher complexity in modeling), greater emphasis has typically been given to the overland flow than to the subsurface flow, where the subsurface flow is typically considered only as a means to make the surface component more accurate. Several surface flow models were developed with the specific goal of modeling erosion processes. A comprehensive review of these models can be found in Aksoy and Kavvas (2005). Here we present two examples: the Water Erosion Prediction Project (Lane and Nearing, 1989; Savabi, 1993) used the one-dimensional kinematic wave formulation uncoupled with the Green and Ampt (1911) solution for infiltration; the KINEROS (Smith et al., 1995; Woolhiser et al., 1990) combines a one-dimensional kinematic wave formulation with a more complex subsurface algebraic equation.

Groundwater Models

Direct groundwater interaction with surface water was discussed above in the context of the MODFLOW code. In general, surface water is considered groundwater's upper boundary condition; however, as groundwater is often modeled as a horizontal two-dimensional system, interaction with surface water becomes a source term in the governing equation.

The way boundary conditions (or source terms) related to surface water are modeled depends primarily on the vertical distance between surface water and groundwater, and the type of flow between them (i.e., whether there is a full saturation regime or not). In the latter case, obviously the vadose zone needs to be modeled to preserve the physical nature of the problem. This is not always the case, however, and very often (see MODFLOW above for examples) a simple empirical first-order difference equation is used to model the flow, in most cases without any consideration of temporal delays.

In the second case, surface–subsurface interaction is modeled in a manner somewhat closer to physics. Even in the simple MODFLOW interactions (RIV and LAK modules), a retarding layer (riverbed) is considered. Hunt et al. (2003) and Sophocleous (2002) discussed several ways that lake–groundwater is modeled, from the groundwater model point of view. In general, there are three ways to approach lake–groundwater interaction: (i) through a prescribed head boundary condition; (ii) through definition of the lake as a high-conductivity (and high-storage) zone; (iii) by coupling its mass balance equation with the groundwater system (as done in the LAK MODFLOW packages). Theoretically, the

second option means that the surface system is solved in a coupled way; however, as Darcy's law is used (and not the actual hydrodynamics) this is meaningless.

Langevin et al. (2005) combined the two-dimensional full inertia model with the three-dimensional anisotropic groundwater flow equation for a wetland (the southern Everglades of Florida). Coupling, however, was based on lagging so that previous time step solutions for the subsurface system could act as boundary conditions for the surface one.

The analytic element method (for basics and applications of the method, see, e.g., Strack, 1989, 2003; Haitjema, 1985) is often used for combined surface–groundwater interactions. In most (if not all) cases, however, the surface system is treated as prescribed (see, e.g., Bakker, 2007).

Other Models

There are several important applications of surface–subsurface coupled flow that do not fall under the discussions above.

"The EPA Storm Water Management Model (SWMM) is a dynamic rainfall–runoff simulation model used for single event or long-term (continuous) simulation of runoff quantity and quality from primarily urban areas" (USEPA, 2005). This model uses Manning's equation to compute flow in conduits. That is, steady flow conditions are assumed. The SWMM model is a hydraulic model and coupling with the subsurface is allowed through simplified infiltration. Three infiltration models are available: Horton (1933) infiltration, Soil Conservation Service (1971) curve number abstraction, and Green and Ampt (1911).

Kouznetsov et al. (2007) combined the one-dimensional full inertia with the three-dimensional Richards equation (using FEMWATER for the subsurface) to investigate bacteria transport above and below the surface.

Coupling Error Estimation

Different coupling methods are subject to different errors; however, since different error patterns are also associated with the numerical schemes used to solve the surface and subsurface components, it is practically impossible to attribute error to the coupling method. Without intensively investigating the numerical schemes, several points associated directly or indirectly with the numerical coupling of surface and subsurface modules can be highlighted.

First, we refer to the "uncoupled" coupling group. This group includes full solution of the governing PDE (i.e., Saint-Venant or one of its simplifications for the surface and the Richards equation for the subsurface). As iterations are avoided, however, very often a specific water level (often of zero depth) is used as a boundary condition for the subsurface at times when the surface is flooded. Using a fixed level regardless of the specific event causes error in the estimation of the boundary fluxes. At the same time, the surface component is computed using a biased infiltration (typically a value from a previous time step is used).

The error associated with the fixed boundary condition was investigated by several researchers. Wallach et al. (1997) compared infiltration using a one-dimensional numerical solution of Richards' equation considering water depths (determined from a one-dimensional kinematic wave model) and considering zero level depth. Wallach et al. (1997) bounded the infiltration error for three different cases by roughly 5%. The error in water level (found by considering the correct or approximated infiltration) is bounded by 1.2%, except for locations where the difference in infiltration means that the surface may get dry (e.g., near the flow tip).

Warrick et al. (2005) examined the error in infiltration when considering a constant (temporally averaged and spatially–temporally averaged) water depth compared with a variable one. They also examined the zero-depth option. Warrick et al. (2005) used actual field data (water levels from border and basin irrigation experiments) and computed the infiltration by the Green and Ampt (1911) infiltration function (but also verified the approach using a numerical solution of the Richards equation). They concluded that the error in infiltration is highest for the zero-depth assumption (bounded by roughly 15% for the cumulative event infiltration) and lower for the spatially–temporally averaged depth (roughly 7%) and the temporally averaged depth (2%). Note, however, that the maximal relative error is roughly at the end of the advance phase. Furman et al. (2006) examined the error associated with using Kostiakov parameters in an event that is different than the calibration event. They found errors similar to those found by Warrick et al. (2005) and suggested a simple correction method that can theoretically reduce most of the error.

Side Aspects: State-Dependent Boundary Conditions

System-dependent boundary conditions are a side aspect that relate to the coupling of surface and subsurface flow systems. That is, boundary conditions that cannot be treated as prescribed but depend on the state of the modeled system. Such boundaries frequently occur when two systems are naturally coupled but the modeler tries to avoid the need to model a fully coupled system.

Here are several examples where such boundary conditions are used. Šimůnek et al. (2005) included three different boundary conditions in the framework of the HYDRUS-1D program: (i) evaporative fluxes that depend on soil surface wetness; (ii) buildup of water on top of the surface due to the soil's inability (due to low hydraulic conductivity) to pass flux; and (iii) seepage face—a boundary condition that changes its nature (from no flow to prescribed head) depending on the solution. Some of these boundary conditions were also embedded in the other models of the HYDRUS family (see www.pc-progress.cz; verified 30 Mar. 2008).

Furthermore, in most watershed applications there is a time when infiltration capacity becomes smaller than the quantity of applied water. At that temporal point, the boundary condition for the subsurface governing equation needs to be switched (typically from second [prescribed flux] type to first [prescribed head] type). The same problem occurs in surface irrigation, at the time when the surging wave arrives at a location. Boundary conditions need to be switched from no flow to prescribed head. At the recession phase of irrigation, the reverse process happens. When including the furrow shape (e.g., FIM, Wöhling et al., 2006; Furman and Zerihun, unpublished data, 2004), nodes of the boundary need to be switched according to the water level in the furrow. That is, the type of the boundary condition needs to be changed from prescribed head to prescribed flux, and vice versa. This switching often causes numerical difficulties that need to be addressed (Jirka Šimůnek, personal communication, 2006). In FIM (Wöhling et al., 2006), numerical instabilities were overcome by an efficient second-order convergent iterative solution. A wide range of

system-dependent boundary conditions were embedded in the HYDRUS (2D/3D) code (Šimůnek et al., 2006).

Discussion and Conclusions

In the literature presented above, there is not yet a full consensus regarding the right mathematical formulation for interfacial boundary conditions. Several of the alternatives provided in the literature were presented. It is clear, however, that conservation both of mass and of momentum need to be addressed. While the conservation of mass is simple to formulate (see Eq. [16]), the conservation of momentum is not. It is possible that some of the discrepancies result from the inherent problem of defining a sharp boundary to an averaged based (continuum approach) system such as the subsurface system, which leads to a difference between observations and theory.

Regardless of the discrepancy in the formulation of momentum across the free surface–porous medium interface, momentum transfer is generally ignored in practice. This is because most of the formulations for water flow over a porous medium consider the depth-averaged formulation of the Navier–Stokes equations that lead to the Saint-Venant equations (Eq. [1] and [2]), with no momentum transfer component across the lower boundary (a sink–source term in the depth-averaged formulation). While for large-scale applications (e.g., watersheds), this seems to be a more than reasonable approximation (as large spatial and temporal scales allow smooth averaging), it may be less accurate for small-scale applications. For example, in surface irrigation (or in surge flow in desert rivers), it is clear that the tip of the advancing water is experiencing more complex physics than the already-wetted locations along the water path. Furthermore, as calibration of these models is often performed based on the temporal behavior of the tip (see, e.g., Zerihun et al., 2005b), it is clear that ignoring the momentum transfer may lead to inherent error. In fact, only a single application (Bradford and Katopodes, 1998) was found of a coupled surface–subsurface process that includes vertical components. Of the different aspects of the coupled surface–subsurface flow covered here, this is believed to be the one that needs the most attention in research. It is important to note, however, that higher accuracy of models can probably be achieved through tackling the problems associated with the different components, e.g., inclusion of surface microtopography or subsurface heterogeneity.

Earlier applications of surface–subsurface interactions focused mostly on the surface component, embedding the subsurface as a simple, typically empirical, algebraic equation for the infiltration. Recent years have seen a systematically increasing number of models with higher level coupling, starting with lagged solutions and later iterative solutions. While computational effort becomes more and more significant, both the accuracy and the applicability of the solutions increase. This is especially true for surface irrigation methods, where until recently the subsurface distribution of water (and nutrients) was not considered at all, while the purpose of irrigation is to bring it to the root zone.

Very recent years have seen several applications of fully coupled processes (i.e., solution of both the surface and the subsurface components as one system), led by VanderKwaak (1999) and Panday and Huyakorn (2004). While this is a big step forward, primarily in terms of robustness, it is not yet clear if the coupled formulation is beneficial in terms of accuracy and computational effort. In a discrete numerical world, the tradeoff is between external and internal iterations (in terms of computational effort) and between convergence criteria and numerical errors (in terms of accuracy). It is assumed that in the near future these models will also penetrate the field of surface irrigation.

Appendix

For completeness, a formulation by Bradford and Katopodes, 1998) of the surface flow governing equations for a vertical two-dimensional case is presented:

$$\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}=0 \qquad [A1]$$

$$\frac{\partial u}{\partial t}+\frac{\partial\left(u^2\right)}{\partial x}+\frac{\partial(uw)}{\partial z}+\frac{1}{\rho}\frac{\partial p}{\partial x}-\frac{\partial}{\partial x}\left(2\nu\frac{\partial u}{\partial x}-\frac{2}{3}k\right)-\frac{\partial}{\partial z}\left[\nu\left(\frac{\partial u}{\partial z}-\frac{\partial w}{\partial z}\right)\right]-g_x=0 \qquad [A2]$$

$$\frac{\partial w}{\partial t}+\frac{\partial\left(w^2\right)}{\partial z}+\frac{\partial(uw)}{\partial x}+\frac{1}{\rho}\frac{\partial p}{\partial z}-\frac{\partial}{\partial z}\left(2\nu\frac{\partial u}{\partial x}-\frac{2}{3}k\right)-\frac{\partial}{\partial x}\left[\nu\left(\frac{\partial u}{\partial z}-\frac{\partial w}{\partial z}\right)\right]-g_z=0 \qquad [A3]$$

where (mostly consistent with the original notation) p [M L^{-1} T^{-2}] is pressure, ρ [M L^{-3}] is the water density, and g_x and g_z [L T^{-2}] are components of the gravity force (due to the slope of the channel) in the x- and z-coordinate directions, respectively (z [L] is the vertical coordinate). The term k [L^2 T^{-2}] here is the turbulent kinetic energy and ν [L^2 T^{-1}] here is the eddy viscosity given by $\nu = C_\mu k^2/\varepsilon$, where ε [L^2 T^{-3}] is the rate of kinetic energy dissipation and C_μ is a dimensionless empirical coefficient. The solution of this equation requires solution of k – ε diffusion-like equations that will not be discussed here. See Bradford and Katopodes (1998) and Rodi (1980) for further detail.

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