Intertemporal Incentives Under Loss Aversion

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Abstract

This paper studies the intertemporal allocation of incentives in a repeated moral hazard model with reference-dependent preferences as in Kőszegi and Rabin (2009). Besides consumption utility, the agent experiences utility from the change in his beliefs about present and future effort and income. When effort plans are rational, the prospect of being disappointed by the outcome realization equals that of being pleasantly surprised and hence the loss-averse agent is on net hurt by news. In contrast to the prediction with classical preferences but consistent with real-world contracts, if consumption utility is not too concave and news about present income carries sufficiently larger weight in utility than news about future income, a contract setting a current fixed wage is optimal as it minimizes the agent's compensation for bearing risk. Despite this, I further prove that several standard features of the contract with classical preferences–such as no rents to the agent, conditions to achieve first-best cost and the non-optimality of random contracts–still hold.

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1 Introduction

Classical models of dynamic moral hazard predict that in the optimal contract both future and present payments should be contingent on current performance (Lambert (1983), Rogerson (1985), Murphy (1986), Malcomson and Spinnewyn (1988), Chiappori, Macho, Rey, and Salanié (1994)). This prediction, however, is at odds with the observation that real-world contracts seldom use current payments to motivate the agent. For instance, MacLeod and Parent (1999) and Parent (2001) find that in the U.S the percentage of workers paid through commissions or piece-rates varies between 4 to 10%.[1]

On the contrary, contracts completely deferring incentives into future payments are common. Wage revisions, promotions (Baker, Jensen, and Murphy (1988), Baker, Gibbs, and Holmstrom (1994), Treble, Van Gameren, Bridges, and Barmby (2001)), dismissal threats (MacLeod and Malcomson (1988, 1989), Sen (1996), Kwon (2005)) and yearly productivity bonuses (Joseph and Kalwani (1998), Steenburgh (2008)) are widely used in real-world contracts and are usually combined with a fixed-present wage. For instance, based on the Executive Compensation Surveys for 1974-1986, Jensen and Murphy (1990) find that for each $1000 change in the shareholder's wealth, the CEO's present and next year salary and bonus increase by 2 cents. In contrast, their wealth from salary revisions, outstanding stock options and performance-related dismissals increase in about 75 cents, suggesting that present incentives are negligible relative to incentives deferred into future payments.

One possible explanation for this puzzle relies on the difficulty to find objective performance measures. When a formal output-based contract is not verifiable, incentives must be set through–possibly informal–contracts rewarding long-term performance rather than short-term output (Eaton and Rosen (1983), Huck, Seltzer, and Wallace (2004)).[2] Evidence, however, suggests that the lack of verifiable output-based measures cannot fully account for the widespread use of contracts using current fixed wages with deferred incentives.[3]

[1]In Parent (2001) the percentage depends on the survey used. Adding a productivity bonus component and excluding profit sharing plans, the percentage increases to 20% using the National Longitudinal Survey of Youth. Barkume (2004) finds similar results. Using data for 2001 on the Bureau of Labor Statistics Employment Cost Index (ECI), he estimates that only 7% of the average total employment in skilled production use commission, piece-rates or productivity bonuses to motivate employees. These numbers, however, are lower bounds since the incentive-pay definition of the ECI only includes individual payments (commission, piece-rates and production bonuses), omitting profit-sharing distributions, all-employee payments and other non-production bonuses.

[2]The observation that finding verifiable performance measures is difficult because of monitoring costs (Barkume (2004), Lazear (1999)), incentives towards skill acquisition (for a survey see Gibbons (1998)) and action complexity (Holmström and Milgrom (1991), MacLeod (2000)), shifted away the contract theory literature from the risk-incentive trade-off towards reviewing the assumptions over the economic environment. As a result, the literature developed a whole array of models that greatly improved the match between theoretical contracts and those observed in reality such as those on relational contracts (MacLeod and Malcomson (1989), Levin (2003)), multitasking (Holmström and Milgrom (1991)) and skill acquisition and career concerns (Holmström (1999), Gibbons and Murphy (1992)).

[3]Hutchens (1987) using data from the National Longitudinal Survey tests the hypothesis that delayed payments to motivate the agent are less likely to occur in jobs involving repetitive tasks as a proxy for low monitoring costs. He finds that workers in these type of jobs have a 9% smaller probability of a pension, 8% smaller probability of

This paper shows that contracts fully deferring incentives into the future can be optimal in a moral-hazard setting when agents have dynamic reference-dependent preferences. To model these preferences, I adapt and integrate to a classical moral-hazard environment the general model of individual decision-making by Kőszegi and Rabin (2009). Besides consumption utility, agents with these preferences experience utility from the change in his beliefs about present and future effort and income. When effort plans are rational, the prospect of being disappointed by the outcome realization equals that of being pleasantly surprised and hence the loss-averse agent is on net hurt by news brought by the outcome realization. As a result, if the agent is more sensitive to receive news about his present payments relative to receive news about his future payments, a fixed-first-period-wage contract using second-period consumption utility–rather than changes in income beliefs–to replace incentives is optimal. Such contract implements the action path and saves the agent from the most painful source of income-belief fluctuations minimizing thus the compensation the principal must provide the agent for bearing risk.

After a review of the related literature, Section 3 presents the basic framework. The set up presents a two-period model with a finite set of outcomes and effort levels, where the agent is paid at the end of each period once the common-knowledge performance measure has been realized. The model further assumes that the agent has neither access to credit nor can save between periods and that both the agent and the principal can credibly commit to stay in the relationship until the end of period two. This environment mimics that in Rogerson (1985).

In an initial period zero, given the contract offered by the risk and loss-neutral principal the agent forms effort plans to be executed in the upcoming two periods. Following Kőszegi and Rabin (2009), the model assumes the agent has rational expectations in forming these plans: given the contract offered by the principal and given knowledge of the relationship between effort and the conditional probability distribution of outcomes, the agent only forms effort plans he knows he will be willing to follow given the income beliefs they generate and hence the future preferences they induce. With these effort plans and related income beliefs, the agent decides whether to accept or reject the contract.

In the beginning of period one, the agent executes an action, observes the outcome realization

mandatory retirement and 18% shorter length of tenure. Even though suggestive that monitoring costs do play a role in the intertemporal allocation of incentives, these magnitudes indicate that other effects must also be in place. Lazear and Moore (1984) contrasts the steepness of the earnings temporal profiles of self-employed workers against those of wage and salary workers to investigate whether the evolution of earnings can be completely explained by human capital accumulation and skill acquisition. They find that salaries grow faster than productivity, suggesting that skill acquisition alone cannot account for the temporal behavior of wages. Similar results are found in Kotlikoff and Gokhale (1992) and Ilmakunnas, Maliranta, and Vainiomäki (2004). From the theoretical perspective, Akerlof and Katz (1989) explicitly show that in a classical framework future incentives are not a perfect substitute for incentives coming from present payments. As a consequence, models such as those in career concern and termination threats (Stiglitz and Weiss (1983), Shapiro and Stiglitz (1985), MacLeod and Malcomson (1988), Sen (1996), Banks and Sundaram (1998)) simply assumed that incentives were totally deferred into future payments. Lazear (1981) is the only exception. See Section 2.

and receives his period-one payment. Following Kőszegi and Rabin (2009), upon the outcome realization the agent experiences consumption utility from the period-one payment and effort as well as "contemporaneous gain-loss utility" from changes in his period-one effort plans and income beliefs. To compute this utility, the agent compares the distribution induced by the payment he actually received and the effort he actually exerted with the distribution induced by the income beliefs and effort plans he formed in period zero about period one's payments and effort. In these comparisons the agent is loss averse: receiving a payment lower than expected hurts more than receiving an income higher than expected is pleasant and exerting an effort higher than expected is more costly than the gain of working less than planned.[4] Besides contemporaneous gain-loss utility, whenever second-period contracts depend on first-period performance, the agent also experiences "prospective gain-loss utility" from changes in his period-one beliefs and plans about period-two payments and effort, i.e., from comparing the beliefs he now holds about the second-period payment with those he inherited from period zero (equivalently for effort).[5] In all these computations it is thus assumed that preferences are separable across consumption domains and across time. Finally, at the end of the period, the agent updates his effort plans and the associated income beliefs for the second period.

In a second and final period, the agent executes effort, receives the corresponding payments, and the relationship ends. Just as in period one, after the outcome realization, the agent experiences consumption utility from the exerted effort and the received payment along with contemporaneous gain-loss utility from comparing the income beliefs and effort plans inherited from the end of period one to the payment actually received and the effort actually executed.[6]

Two parameters index the total instantaneous utility. First, a reference-dependent parameter $\eta \geqslant 0$ shows the importance of gain-loss utility–contemporaneous plus prospective–relative to consumption utility. When $\eta = 0$, this preference structure reduces to classical reference-independent preferences. Second, a parameter $\gamma \geqslant 0$ shows the relative importance of contemporaneous gain-loss utility relative to prospective gain-loss utility, i.e, how sensitive the agent is to receive information about his present payments relative to receive information about his future payments. Besides

[4]In general, gain-loss utility captures the idea that news changing previous beliefs about present and future effort and income generate a present utility flow that is part of the agent's decision utility. This basic idea that agents get utility from changes in consumption levels rather than level themselves was introduced in the seminal prospect theory paper by Kahneman and Tversky (1979). They further stated that decreases are more heavily felt than same-sized increases, phenomenon they called loss aversion.

[5]Prospective gain-loss utility is a form of anticipatory utility in the spirit of Caplin and Leahy (2001) and many others. The key difference is that this is a referent-dependent feeling of anticipation where the reference point corresponds to the previous beliefs for the corresponding period. Thus, the use of this type of utility should not be regarded as a radical departure from the behavioral literature.

[6]Notice that this description implicitly defines the reference point as previous-period beliefs. Since the agent forms his effort plans and income beliefs based on the contract offered by the principal, this reference point specification is in the same spirit as Hart and Moore (2008). It differs from their model in that the reference point is endogenously determined and considers all payments rather than one particular scenario to which–for exogenous reasons–the agent feels entitled to.

assuming that γ is non-negative, the model does not impose restrictions over the value of this parameter.[7] Finally, to isolate the role of γ, the model assumes that the agent maximizes the undiscounted sum of his instantaneous utilities.

Section 4 characterizes some basic properties of the optimal long-term contract with dynamic reference-dependent preferences. I start exploring the shape of the optimal contract when the state perfectly reveals the action. I prove that when the action is observable, the optimal contract does not depend on η. Intuitively, when the outcome perfectly reveals the agent's action there is no uncertainty and thus the agent cannot be surprised by the payment he actually gets: under certainty, reference-dependent agents are behaviorally equivalent to consumption-utility maximizers.[8] Next, I provide a basic characterization of the second-best contracts. Broadly, I show that several features of the optimal contract with classical preferences still hold when agents are reference-dependent. After proving that the optimal contract exists when agents are reference-dependent, I prove that such a contract is unique if the consumption utility function is strictly concave, just as with classical preferences. Furthermore, and because of a very similar argument to that with classical preferences, I show that the optimal contract does not leave rents to the agent. Then I prove that the second-best contract achieves the first-best cost when the principal wants to implement the minimum effort in every period and when there is at least one outcome with probability zero under the chosen action but positive under the lower cost actions. However, and most importantly, in general first-best is not achieved when the consumption utility function is linear because the agent is loss averse and thus the risk-incentive trade-off is still present.

Section 5 presents the main prediction: when contemporaneous gain-loss utility is strong enough relative to prospective gain-loss utility and consumption utility is not too concave, the optimal contract defers all incentives into future payments, i.e. the contact uses a fixed-first-period wage. To understand this prediction, assume that consumption utility is linear. Because the agent has rational expectations and thus only forms effort plans that he will actually execute, the prospect of being disappointed by the outcome realization equals that of being pleasantly surprised by it. As a consequence, because the agent is loss averse–bad news about his income hurt him more than good news please him–he gets expected disutility out of the changes on his income beliefs caused by the income news brought by the outcome realization. Intuitively, the agent dislikes risk from the reference-dependent component of his utility function because news makes beliefs fluctuate. When the agent is less hurt by receiving news about his future payments relative to receive news about his present payments (γ sufficiently small), the principal can save the agent the most unpleasant source of income-beliefs fluctuations by fixing the first-period wage, increasing thus the total expected utility the agent gets from the contract. Setting a fixed-first-period wage,

[7] Kőszegi and Rabin (2009) assume $\gamma < 1$ from the start.

[8] The fact that reference-dependent preferences reduce to classical preferences under certainty in a general model of consumption, is explicitly derived in Kőszegi and Rabin (2006).

however, is done at the expense of shutting down incentives coming from first-period consumption utility and contemporaneous gain-loss utility since income beliefs do not fluctuate when agents have rational expectations and are paid a fixed wage. To replace the incentives lost the principal increases as much as necessary incentives coming from second-period payments. In particular, she changes the level of all payments within each continuation contract so to create incentives from second-period consumption utility. Increasing second-period incentives, however, increases the gain-loss disutility the agent gets from prospective gain-loss utility, i.e, from fluctuations in his beliefs about future payments. Nevertheless, since the agent is less sensitive to receive news about his future payments relative to receive news about his preset payment, the gains from shutting down contemporaneous gain-loss disutility outweigh the losses from increasing prospective gain-loss disutility. As a consequence, the agent gets a higher total period-zero expected utility and thus the principal can decrease the level of payments and still implemented the effort path.

When consumption utility is concave the principal can keep the agent's marginal consumption utility high by spreading incentives across periods.[9] In such a case, a sufficient condition for the present-period-fixed-wage contract to be optimal is the consumption utility function being not too-concave so that the reference-dependent effect pushing towards deferring incentives into future payments–to minimize the disutility the agent gets from fluctuation in income beliefs–dominates. Rabin (2000) shows that, for reasonable risk taking behavior, the agent's consumption utility function is compatible with risk neutrality for wealth levels of the order of an average monthly salary.[10]

To further investigate the role of γ, I also explore the shape of the optimal long-term contract when the agent is more sensitive to news about future consumption than to news about present consumption. I prove that in this case, the optimal contract must rely on first period contingent payments to induce a desired level of effort in the first period. Even more, in this case there is always a γ big enough such that it is not convenient for the principal to make second-period payments contingent on first-period performance. Intuitively, with a big γ is too expensive to use deferred incentives because it triggers expensive prospective gain-loss disutility. By making payments only contingent in first-period performance the principal increases the agent's total utility by saving him painful fluctuations in his beliefs about future income. This result contrasts with the prediction of classical reference-independent preferences, where the optimal long term contract can only reduce to a sequence of one-period contracts if the agent is risk neutral.

Section 7 explores the convenience of using performance measures that are uncorrelated with the agent's unobserved action. I prove that, just as in the case of classical preferences, using ex-post-random contracts is never optimal since a contract paying in each period the certainty equivalent

[9] Such an intertemporal risk-sharing argument drives the prediction of classical-repeated-moral-hazard models where is optimal to make present and future payments contingent on current performance.

[10] A reasonable length period for our model is a month because such period is long enough so that the agent can form plans, but not so long that the agent gets used to his income beliefs.

of the period-random contract allows the principal to decrease the expected cost of implementing the desired action. Contrary to the classical case, this result is valid even if consumption utility is linear.

Finally, to capture the intuition that the information the agent gets in period one about period-two payments may be relevant to review the agent's effort plans, Section 7 assumes prospective gain-loss utility is realized after second-period effort plans are updated. In a two-action set up, I show it is harder to motivate the agent after a first-period success relative to the case of a first-period failure. This is because, even though the realization of the prospective gain-loss utility gain is increasing in the second period effort plans–high effort maximizes the distance between the reference distribution and the actual one–and the realization of the prospective gain-loss utility loss is decreasing in the second period effort plans–high effort minimizes the distance between the reference distribution and the actual one–loss aversion implies that the latter effect is stronger than the former. As a consequence, the optimal contract needs to set stronger incentives after a first-period success relative to a first-period failure. Even though a similar phenomena is occurs with classical preferences, the latter rationale differs from the low marginal utility explanation effect driving the effect in such case. Moreover, with reference-dependent preferences the prediction depends crucially on the agent having planned to exert high effort and also in the timing between the review of plans and the actual execution of the action. If the principal allows enough time after X_1 as been observed and the action execution, the agent may get used to his effort beliefs, rendering the realization of prospective gain-loss utility irrelevant for planning

Section 8 summarizes, discusses the model's limitations and presents a future research agenda extending the two-period model.

2 Related Literature

This paper builds on the classic contract theory literature on dynamic moral hazard and the behavioral literature on reference-dependent preferences. Lambert (1983) and Rogerson (1985) were the first to prove that when the principal and the agent engage in a repeated interaction in which the agent is paid at the end of every period, the optimal contract displays "memory in wages": second-period payments must be contingent on the first-period outcome realization. This result was latter generalized to T periods by Murphy (1986). There has been a vast amount of work done on dynamic moral hazard exploring several features of the optimal contract, such as the role of commitment and the conditions for the long-term contract to be implementable as a series of one-period contracts (Malcomson and Spinnewyn (1988), Fudenberg, Holmstrom, and Milgrom (1990), Rey and Salanie (1990), Chiappori, Macho, Rey, and Salanié (1994)) or the role of the discount factor in achieving efficiency when the principal-agent relationship is infinitely repeated (Radner (1986), Spear and Srivastava (1987)). For the purposes of this paper, however, all this literature predicts that optimal payments spread incentives across periods or concentrates them into present payments if the long-term contracts is spot implementable.

The literature on reference-dependent preferences started with the seminal work of Kahneman and Tversky (1979) who proposed that agents experience utility from changes in levels of consumption where losses hurt more than same-sized gains are pleasant, a phenomenon they denominated loss aversion. Subsequently, a whole array of reference-dependent models were developed. Bell (1985), Loomes and Sugden (1986), Gul (1991), Munro and Sugden (2003), Sugden (2003), Matthey (2005), Kőszegi and Rabin (2006) and De Giorgi and Post (2008) among others have presented static models of reference-dependent preferences. The model presented in Kőszegi and Rabin (2009) builds on prospect theory in two ways. First, they provide a generalization of the gain-loss utility function to a dynamic setting by introducing the notion of prospective gain-loss utility. Second, they endogenize the reference point by assuming the agent has rational expectations, and thus the reference point corresponds to the agent's rational beliefs derived from the economic environment in which the problem is embodied.[11]

There is in fact a small but growing body of literature on behavioral contract theory exploring the role of reference-dependent preferences on hidden-action models. deMeza and Webb (2007) develop a model of static moral hazard using several models of reference-dependent preferences. They find that optimal contracts have intervals where payments are insensitive to performance and that carrots are more frequently used than sticks. The model of Herweg, Müller, and Weinschenk (2008) is the closest to this work. They build a model of static moral hazard with static reference-dependent preferences as in Kőszegi and Rabin (2006). They conclude that (1) the optimal contract has two levels of payments when the agent has linear consumption utility and (2) that the contract under concavity is significantly simpler than the one predicted by classical theory. My model differs from theirs mainly in that I am interested in different phenomena: I focus on the intertemporal allocation of incentives rather than the contract's complexity, for which I rely on a dynamic setting. Moreover, while they use continuous actions with a linear specification of the conditional probability distribution and the first-order approach to solve the principal's problem, I use the discrete approach of Grossman and Hart (1983) and a completely standard environment.[12]

Lazear (1981) is to the best of my knowledge the only paper that has explored the intertemporal allocation of incentives. His motivation was to explain why wage's age-tenure profiles are steeper than productivity tenures and the existence of compulsory retirement policies. His model builds on the observation that by overpaying the agent in the late stage of his career while underpaying him

[11] Kőszegi and Rabin (2009) is to the best my knowledge the only one that explicitly considers reference-dependent preferences in a dynamic setting besides the models in Barberis and Huang (2001) and Barberis, Huang, and Santos (2001) which are specific to the financial market. The model of Matthey (2005) is the closest as she also introduces the notion that agents derive utility from changes in beliefs about future consumption.

[12] Iantchev (2005) and Daido and Itoh (2005) also build models of static moral hazard with reference-dependent preferences to explain the Pygmalion Effect and contract complexity respectively.

during his early years, the firm can increase the workers' effort because he will work hard to avoid the possibility of being dismissed. This compensation structure, however, increases the incentives of the firm to terminate the contract before the stipulated date. The optimal wage-profile thus trade-offs the cost for the firm of not honoring the contract while maximizes the worker's lifetime earnings subject to a zero-profit condition. The compulsory retirement policy arises thus to avoid the possibility of the agent staying longer than needed in the firm. This model, greatly differs from ours. First, nothing in the model's basic logic prevents the optimality of present-contingent payments, a very convincing argument with classical preferences from a consumption smoothing perspective (see Akerlof and Katz (1989)). Moreover, it gives up on the risk-incentive trade-off to explain deferred payments.

3 The Model

The principal-agent relationship lasts three periods, $t \in \{0, 1, 2\}$. In period zero the agent (he) receives a take-it-or-leave-it offer (TIOLI) from the principal (she) and decides whether to accept or reject it. If he accepts, both parties credibly commit to stay in the relationship until the end of period two. In each of the two subsequent periods the agent privately chooses an effort level e_t from a finite set $\mathcal{E} = \{e_1, \ldots, e_J\} \subset \mathbb{R}$. Next, a verifiable *iid* performance measure X_t is drawn from the finite set $\mathcal{X} = \{x_1, \ldots, x_N\} \subset \mathbb{R}$, where $x_1 < x_2 < \cdots < x_N$. Let $\Pi^X(x_n|e_j)$ be the associated distribution function conditional on effort and let $\pi_n^j \equiv \Pi^X(x_n|e_j) - \Pi^X(x_{n-1}|e_j) > 0$, $\forall n, j$ represent its density. I assume that the agent is paid at the end of each period and that the agent has no access to credit nor can he save between periods. Even though unrealistic, relaxing this assumption does not affect the characteristics of the optimal contract with standard preferences that concerns us, i.e., second-period wages depending on first-period outcome and contingent payments in every period (Chiappori, Macho, Rey, and Salanié 1994, Malcomson and Spinnewyn 1988).[13]

The principal is assumed to be risk and loss neutral. Her gross benefit equals the realization of the performance measure X_t, and her only cost corresponds to the payment made to the agent.[14]

[13]Since my model focuses on formal or written incentive schemes, verifiability is sufficient. However, along the paper, we will refer interchangeably between verifiability and observability. The assumption of X_t being *iid* is standard in the literature and avoids complications due to learning. Extending the analysis to X_t in the Euclidean space $\mathbb{R}^N$, N>1 and $e_j \in \mathbb{R}^J$, J>1 may not be innocuous since issues related to effort substitution and task conflict arise, as the literature in multitasking has acknowledged (see for example Dewatripont, Jewitt, and Tirole (2000) and Holmström and Milgrom (1991)). Even though generalizations of my model to learning or multitasking may be interesting, they are not the focus of this paper. Finally, the assumption of the agent being able to commit to stay in the relationship for any period one outcome realization is relaxed in Section 5.

[14]The assumption of the principal being risk neutral can be justified in our interest in employment contracts. In such environment it is reasonable to assume that employers can diversify risk as they have several employees. Extending the analysis though to a risk-averse principal, where her gross benefits are represented by the existence of a well behaved function $b : \{x_1, \ldots, x_N\} \longrightarrow \mathbb{R}$, $b''(\cdot) < 0$ is straightforward when establishing the existence of conditions needed for the main result, but it may not be innocuous to establish the strength of those conditions. See a discussion in Section 5.

A long-term contract denoted by $S = (S_1(X_1), S_2(X_1, X_2))$, $S_t \subset \mathbb{R}$ for $t = 1, 2$, is a set of N+N^2 contingent payments governing the principal-agent relationship during periods one and two. I refer to S_1 as the first period contract. Let s_n represent its realization if $X_1 = x_n$ is observed in the first period. I refer to S_2 as the second period contract or "continuation contract" possibly depending on both X_1 and X_2. Let s_{nm} denote its realization if $X_1 = x_n$ and $X_2 = x_m$. If $s_{nm} \neq s_{n'm}$ for at least one $n \neq n'$ and m, it is said that the contract displays *memory in wages* (Chiappori, Macho, Rey, and Salanié (1994)).

I depart from the standard framework solely by assuming the agent has reference-dependent preferences. To set up these preferences, I use the model of dynamic reference-dependent preferences in Kőszegi and Rabin (2009). Accordingly, agents experience two types of utilities, standard consumption utility and utility from changes in beliefs about their present and future consumption over the two standard consumption domains in hidden action models: payments and effort. Period t utility flow corresponds to:

$$v_t = V(s_t, e_t) + \eta \sum_{\tau=t}^{2} \gamma_{t,\tau} G(\widetilde{s}_{t,\tau}, \widetilde{e}_{t,\tau} | \widetilde{s}_{t-1,\tau}, \widetilde{e}_{t-1,\tau}) \tag{1}$$

The first component, $V(s_t, e_t)$, represents classical reference-independent consumption utility from current payment s_t and current executed effort e_t. Assumption 1 presents the assumptions over this function.

Assumption 1 *(Consumption Utility)*
The consumption utility function is of the form $V(s_t, e_t) = u(s_t) - c(e_t)$ where $u(s_t)$ is twice continuously differentiable with $u'(s) > 0$, $u''(s) \leqslant 0$ for all $s \geqslant \underline{s}$ where $\lim_{s \downarrow \underline{s}} u(s) = -\infty$ and $c(e_j) > c(e_k)$ for all e_j, e_k satisfying $e_j > e_k$.

Assumption 1 is standard in the contract theory literature. First, it states that the agent's preferences are separable across effort and income. Second, it says that consumption utility in the payments domain is strictly increasing and weakly concave, whereas consumption utility in the effort domain is strictly increasing and always non-negative. Finally, Assumption 1 ensures the existence of a payment such that the agent gets sufficiently low disutility.

The second term in equation (1), $G(\widetilde{s}_{t,\tau}, \widetilde{e}_{t,\tau} | \widetilde{s}_{t-1,\tau}, \widetilde{e}_{t-1,\tau})$, corresponds to the "gain-loss" utility function. It represents the utility the agent gets today from departures of present and future consumption from a reference point, which corresponds to the effort plans and income beliefs the agent made in the previous period about present and future effort and income. Formally, at the end of periods $t = 0, 1$, the agent forms an effort plan $\widetilde{e}_{t,\tau} \in \mathcal{E}$ for *all* future periods $\tau > t$. Notice that since the continuation contract S_2 depends on X_1, second period effort plans must be contingent on the first period realization, i.e., $\widetilde{e}_{t,2} \equiv \widetilde{e}_{t,2}(X_1)$, $t = 0, 1$. Given the contract, and given knowledge of the probability distribution over outcomes, these effort plans fully induce beliefs about future payments. For $t = 0$ and $\tau = 1, 2$ and for $t = 1$ and $\tau = 2$, let $\widetilde{\Pi}^S_{t,\tau} \equiv \prod_{k=t+1}^{\tau} \Pi^S_k(\widetilde{e}_{t,k})$, correspond to the income probability distribution the agent formed in period t about the future period $\tau > t$. Let $\widetilde{s}_{t,\tau}$ denote a realization of such distribution. Thus, $G(\widetilde{s}_{t,\tau}, \widetilde{e}_{t,\tau}|\widetilde{s}_{t-1,\tau}, \widetilde{e}_{t-1,\tau})$ represents the utility the agent gets today from departures of present and future consumption $(\widetilde{s}_{t,\tau}, \widetilde{e}_{t,\tau})$ from the consumption beliefs the agent formed in the previous period about present and future consumption $(\widetilde{s}_{t-1,\tau}, \widetilde{e}_{t-1,\tau})$ for all $\tau \geqslant t$.[15]

To develop the intuition behind the results, it is useful to distinguish between gain-loss utility arising because of departures of present consumption from the plans made for the present period in the previous period ($\tau = t$) and utility arising from changes in plans about future consumption ($\tau > t$). As in Kőszegi and Rabin (2009), I refer to the term $G(s_t, e_t|\widetilde{s}_{t-1,t}, \widetilde{e}_{t-1,t}) \equiv G(\widetilde{s}_{t,t}, \widetilde{e}_{t,t}|\widetilde{s}_{t-1,t}, \widetilde{e}_{t-1,t})$ as "contemporaneous gain-loss utility" where the first expression keeps the notation of s_t and e_t as the actually realized payment and actually implemented effort and to $G(\widetilde{s}_{t,\tau}, \widetilde{e}_{t,\tau}|\widetilde{e}_{t-1,\tau}, \widetilde{s}_{t-1,\tau})$ $\tau > t$ as "prospective gain-loss utility".

Two parameters index this utility function. First, a parameter $\gamma_{t,\tau}$ representing the relative importance of the two types of gain-loss utility in the current utility flow, i.e., the relative weight the agent assigns to changes in his beliefs about present consumption relative to changes in plans about his future consumption. I normalize $\gamma_{t,t} = 1$ $\forall t$ and let $\gamma_{1,2} \equiv \gamma > 0$ and make no further assumptions over the value of this parameter. Second, the parameter $\eta \geqslant 0$ is a "reference-dependence parameter" indicating the strength of the gain-loss utility component of the total utility flow relative to consumption utility. If $\eta = 0$, these preferences reduce to classical reference-independent preferences.

To illustrate gain-loss utility, consider the following example for the payment domain (the effort domain is analogous). A worker, who lives only to consume one-dollar caramel-filled chocolate bars has historically been paid every Friday a salary of 100 dollars and expects to be paid the same in the future. For simplicity, assume he has linear consumption utility. This Friday he shows up and finds out that his weekly salary has been permanently reduced to 80 dollars. If the worker were to have classical reference-independent preferences, that same Friday utility would decrease by 20 bars. If the worker were to have dynamic reference-dependent preferences, that same Friday utility flow diminishes from two extra sources. First, he experiences a painful sensation of loss from comparing the 80 bars he will actually eat today instead of the 100 he had expected. On top of this, that same Friday, the agent also experiences a utility decrease from knowing that in all *upcoming* Fridays he will be consuming twenty fewer caramel-filled bars. If η =0.1 these two effects imply a

[15] The assumption that the agent experiences utility directly over money and not over the related consumption is standard in moral hazard models but is not neutral in the Kőszegi and Rabin (2009) model. In fact, the original model is capable of explaining why agents derive utility form money: because money brings news about future consumption. For simplicity, though, I omit such interesting observation.

total decrease in that Friday's utility flow of 20+0.1[G(80|100)+γ99G(80|100)]>20.[16]

Assumption 2 presents the assumptions made over the gain-loss utility function. Consider the payment domain (see Appendix A for the analogous specification of gain-loss utility in effort). For $p \in (0,1)$, let $s_{\widetilde{\Pi}^S_{t,\tau}}(p;\widetilde{e}_{t,\tau})$ be the payment level at percentile p of the lottery of payments $\widetilde{\Pi}^S_{t,\tau}$ given the effort belief $\widetilde{e}_{t,\tau}$, which is defined as usual by $\widetilde{\Pi}^S_{t,\tau}(s_{\widetilde{\Pi}^S_{t,\tau}}) \geqslant p$ and $\widetilde{\Pi}^S_{t,\tau}(s) < p \; \forall s < s_{\widetilde{\Pi}^S_{t,\tau}}$.

Assumption 2 *(Gain-loss Utility)*

(i) $G(\widetilde{s}_{t,\tau}, \widetilde{e}_{t,\tau}|\widetilde{e}_{t-1,\tau}, \widetilde{s}_{t-1,\tau}) = G(\widetilde{s}_{t,\tau}(\widetilde{e}_{t,\tau})|\widetilde{s}_{t-1,\tau}(\widetilde{e}_{t-1,\tau})) + G(\widetilde{e}_{t,\tau}|\widetilde{e}_{t-1,\tau})$

(ii) for $t = 1$ and $\tau = 1, 2$ and for $t = 2$ and $\tau = 2$, the $G(\cdot|\cdot)$ function corresponds to:

$$G(\widetilde{s}_{t,\tau}(\widetilde{e}_{t,\tau})|\widetilde{s}_{t-1,\tau}(\widetilde{e}_{t-1,\tau})) = \int_0^1 \mu\big(u(\widetilde{s}_{\widetilde{\Pi}^S_{t,\tau}}(p;\widetilde{e}_{t,\tau})) - u(\widetilde{s}_{\widetilde{\Pi}^S_{t-1,\tau}}(p;\widetilde{e}_{t-1,\tau}))\big)dp \qquad (2)$$

(iii) where $\mu(x) = x$ if $x \geqslant 0$ and $\mu(x) = \lambda x$ where $\lambda \geqslant 1$.

Assumption 2 part (*i*) states that the gain-loss utility function is separable between consumption domains where the notation for gain-loss utility in payments makes explicit the effort plan generating the corresponding income belief. Following Kőszegi and Rabin (2009), part (*ii*) assumes the agent makes a rank order comparison between his beliefs at the end of the previous period with those held at the beginning of it. Intuitively, the agent compares the wort percentile of his old beliefs with the worst percentile of his new beliefs, the second worst percentile of his old beliefs with the second worst percentile of his new beliefs, and so on for all percentiles. Notice that for the (t, τ) pairs (1,1) and (2,2), equation (2) corresponds to contemporaneous gain-loss utility. In such a case, given x_n, the agent compares a particular payment against a non-degenerate payment distribution (the distribution of payments before x_n is realized corresponding to S_1 or S_2 depending on whether t is 1 or 2). For the pair (t,τ)=(1,2) equation (2) corresponds to prospective gain-loss utility. In such case, given x_n, the agent compares two non-degenerate distributions: the distribution of second-period payments before x_n is observed with that after x_n is observed.

To illustrate how the agent makes the rank order comparison to compute gain-loss utility, consider the following example. A worker, who does not experience disutility from effort, lives and works during two periods. His payments are governed by the following long-term contract: the first-period contract and the continuation contract after a first-period success pay 100 or 80 dollars depending on whether the period outcome is high or low; meanwhile the continuation contract after

[16]Evidence of this preference structure can be found in Loewenstein (1988), who finds that people are willing to pay more to avoid delaying the delivery of a good when they expect to receive it immediately than to speed up the delivery when they expected to receive it later. For a more detailed description of the evidence for the relevance of prospective gain-loss utility as decision utility, see Kőszegi and Rabin (2009) and Matthey (2005). For recent evidence of expectations as the reference point, see Abeler, Falk, Götte, and Huffman (2009).

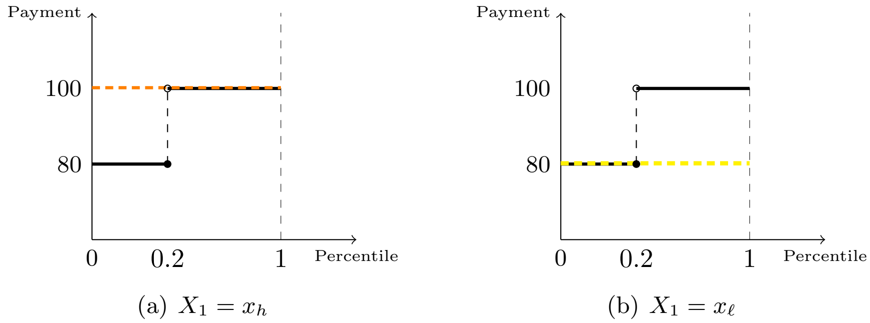


Figure 1: First period contemporaneous gain-loss utility realization.

a first period failure pays 60 or 40 if second-period outcome is high or low respectively. Assume further that in every period observing a high or low outcome is equally likely and that η =0.1, γ =1. First, consider contemporaneous gain-loss utility for period one illustrated in Figure 1. If a high outcome is observed (panel (a)), the agent compares the percentiles in the degenerate lottery putting probability one on the realization of a hundred dollars against the percentiles of the original non-degenerate distribution, i.e., he gets 0.5μ(100-80)+0.5μ(100-100)=10. Equivalently, if a low outcome was realized (panel (b)), the gain-loss utility loss corresponds to 0.5μ(80-80)+0.5μ(80-100)=0.5×2(-20)=-20.[17] A similar analysis is valid for second-period gain-loss utility under any continuation contract.

Consider now prospective gain-loss utility. Before the first period outcome is realized, the agent expects to be paid 40, 60, 80 or a 100 dollars, each with probability 0.25 because outcome realizations are independent. Panel (a) in Figure 2 illustrates the computation if the high outcome is observed in the first period. In such case, the agent will be paid with the high-payment continuation contract and thus he will experience a prospective gain-loss utility gain from comparing each percentile of the old distribution (solid line) of second-period payments with those of the new distribution (dashed line). This prospective gain-loss utility gain equals 0.25μ(80-40)+0.25μ(80-60)+0.25μ(100-80)+0.25μ(100-100)=0.25×(40+20+20)=20 since under the old distribution the first 25 percentiles of payments equal 40 dollars, meanwhile under the new distribution the first 25 percentiles pay 80 dollars; the second percentiles of the old distribution pay 60 dollars meanwhile the second 25 percentiles pay 80 dollars and so on. Equivalently, panel (b) in Figure 2 illustrates the computation if a low outcome is observed in the first period. Whenever λ = 2, if a low outcome is observed in the first period, the agent experiences a loss equal to 0.25μ(40-40)+0.25μ(40-60)+0.25μ(60-80)+0.25μ(60-100)=-0.25×2×(20+20+40)=-40.

[17] This specification of the gain-loss utility is equivalent to that used in Kőszegi and Rabin (2006, 2007). It is also worth noticing that in the effort domain things are simpler. Because of the assumption made on $\widetilde{e}_t \in \mathcal{E}$, that is, conditional on the past, the agent only forms effort plans in pure strategies, contemporaneous gain-loss utility in effort reduces to $\mu(c(\widetilde{e}_{t-1,\tau}) - c(\widetilde{e}_{t,\tau}))$. As a consequence, in period one, given a chosen effort or in period two given

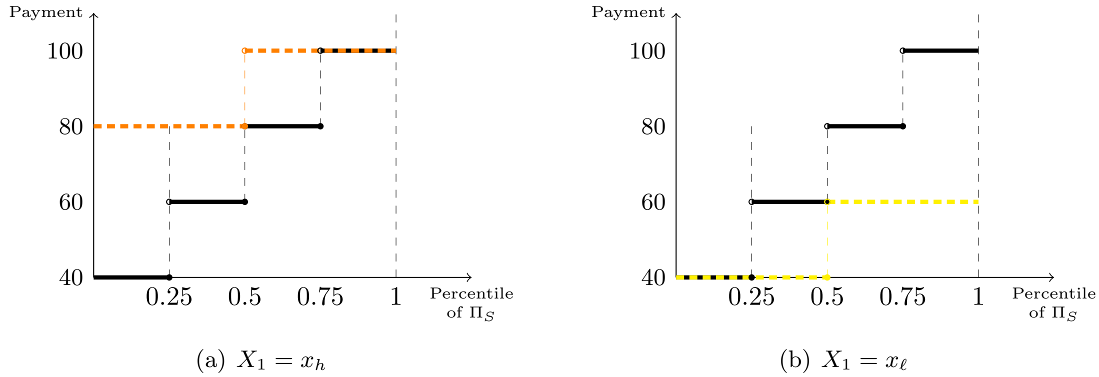


Figure 2: Prospective gain-loss utility realizations.

Part (*iii*) in Assumption 2 defines the function $\mu(\cdot)$. It corresponds to the equivalent of the "value function" in Kahneman and Tversky (1979). To capture the intuition that the strength of the reference-dependent utility is proportional to the importance of consumption utility, present and past beliefs are stated in their consumption utility equivalents.[18] The value function is assumed to be continuous over the whole domain and differentiable everywhere, except at the origin. For simplicity, and to isolate the effects of loss aversion, it is assumed to be a piece-wise linear function, that is, flat but with different slopes in the gain and loss domains. The parameter λ is the "loss-aversion parameter", representing how intense losses are felt relative to gains. Notice that if $\lambda = 1$ gains are equally felt relative to losses and for a piece-wise value function, preferences reduce to classical preferences.[19]

Finally, the model assumes the agent in period t makes his decisions to maximize the sum of his instantaneous utilities:

$$U_t = \sum_{\tau=t}^{2} v_\tau$$

where there there is no discounting to highlight the role of γ.[20] If there is uncertainty in the agents

the realization of x_n, all quartiles of the (degenerated) effort lottery make the same comparison.

[18]To see this, consider the following example in Kőszegi and Rabin (2006). When choosing between whether to accept or reject a 50% lottery of winning or loosing 100 dollars, versus a 50% lottery of winning or loosing a paper clip, it is reasonable to assume that the agent will be more loss-averse over the first lottery rather than the second.

[19]Whenever a piece-wise linear value function is assumed, loss aversion becomes a necessity to experience reference-dependent preferences. See formulas in Appendix A. In a more general specification of the value function, in which the gain domain is assumed to be concave and the loss domain is assumed to be convex–phenomenon called diminishing sensitivity–the agent can experience reference-dependent preferences without being loss averse.

[20]It is worth considering the difference between a traditional discount factor and γ. Following Kőszegi and Rabin (2009) one may justify the first in terms of a decision maker putting lower weight on future utility flows because something may cause those future dates to be irrelevant. Time discounting is thus a discounting measure *between* utility flows. Contrarily, the gain-loss utility weight γ is a *within* utility flow "discounting". It may justified in terms of the decision maker putting a different weight into events happening in the future, not necessarily because they may not take place but simply because awareness is decreasing in the date of experience. See Section 8 for a discussion on the implications of including an exponential discount factor.

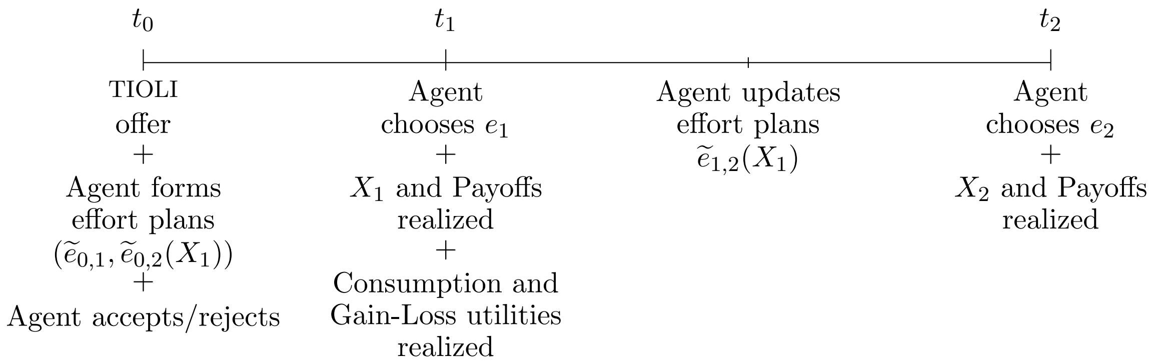


Figure 3: Timing of the Principal-Agent Interaction

takes expectations accordingly.

Figure 3 presents the time line of the principal-agent interaction. After the principal offers the contract to the agent, the agents forms effort plans $(\widetilde{e}_{0,1}, \widetilde{e}_{0,2}(X_1))$ for the two upcoming periods. Given these plans, he accepts or rejects the contract.[21] In the beginning of period one the agent chooses effort e_1. Immediately thereafter X_1 is realized and the agent experiences consumption and gain-loss utilities. At the end the same period, and with $X_1 = x_n$ at hand, the agent reviews his effort plans for the upcoming period and forms $\widetilde{e}_{1,2}$. At the beginning of period two the agent chooses second-period effort, and immediately after the realization of X_2 is observed and payoffs are realized.[22] The last ingredient to the preference specification is the assumption that the agent has rational expectations: he correctly anticipates the consequences of his effort decision and cannot form effort plans he knows he will not carry out given the implications these plans have over payments and future preferences. The agent's equilibrium behavior is thus a complete contingent effort plan for period one and two, from which the agent will never want to deviate when actually executing it–given that the effort plan constitutes the reference–nor when updating it as information arrives. Moreover, notice that because of rational expectations, when updating his effort plans, the agent will only consider deviating to plans that he knows he is willing to follow, that is, to rational or consistent plans.[23]

[21] Notice that this specification abuses a bit of the word "plan", because if the agent does not accept the contract he will never get to execute these actions. Thus, one can think of the period-zero plans are "conditional plans". We do so to avoid introducing further notation to denote the agent's intended actions.

[22] The length of the period is neutral in models with classical preferences. However, this is not the case with reference-dependent preferences. In fact, an implicit assumption of the model is that the periods are long enough so that the agent can form plans but not that long so that the agent does not consider deviations from the references as in the "choice acclimating equilibrium" in Kőszegi and Rabin (2007). A month is a credible long-term period for this model.

[23] Notice that assuming the agent has rational expectations implies no contradiction with the assumption that agents have reference-dependent preferences. In fact, reference-dependent preferences is by no means a form of irrationality but simply a different preference structure, which the completely rational agent correctly maximizes given the economic environment and restrictions he faces.

To formally introduce the agent's equilibrium behavior, I make an explicit distinction between the total expected utility at the beginning of $t = 1, 2$, when the agent actually implements a period effort, and the total expected utility the agent gets at the end of the period, once the performance measure has been realized and the agent is updating his effort plans for the upcoming periods. In periods zero and one, before X_1 is realized, $\widetilde{e}_{t,2}(X_1)$, $t = 0, 1$, corresponds to a function mapping from $\mathcal{X}$ to $\mathcal{E}$. At the end of period one, once x_n has been observed, plans for second-period effort are denoted using this realization, i.e. $\widetilde{e}_{1,2}(x_n)$ and the expected utility function is denoted contingent on x_n.

Definition 1 *(Agent's Equilibrium Behavior)*
Given a contract S, the effort path $(e_1^{PE}, e_2^{PE}(X_1))$ is a "preferred personal equilibrium" (PE) for the agent if

$$
\begin{array}{llcll}
(i) & EU_0(e_1^{PE}, e_2^{PE}(X_1)|e_1^{PE}, e_2^{PE}(X_1)) & \geqslant & EU_0(e_1, e_2(X_1)|e_1, e_2(X_1)) & \forall (e_1, e_2(X_1)) \in \mathcal{E}^{PE} \\
(ii) & EU_1(e_1^{PE}, e_2^{PE}(X_1)|e_1^{PE}, e_2^{PE}(X_1)) & \geqslant & EU_1(e_1, e_2^{PE}(X_1)|e_1^{PE}, e_2^{PE}(X_1)) & \forall e_1 \in \mathcal{E} \\
(iii) & EU_2(e_2^{PE}(x_n)|e_2^{PE}(x_n); x_n) & \geqslant & EU_2(e_2|e_2^{PE}(x_n); x_n) & \forall e_2 \in \mathcal{E}, \forall x_n
\end{array}
$$

where $\mathcal{E}^{PE} \equiv \{e \in \mathcal{E}, e_2(X_1) \in \mathcal{E} \times \cdots \times \mathcal{E} | (ii)$ and (iii) hold for $e = e_1^{PE}$ and $e_2 = e_2(X_1)^{PE}\}$.

Definition 1 says that an effort path is a personal equilibrium for the agent if (1) given x_n, in the second period the agent does not want to deviate from his second-period planned effort to any other second-period rational plan (2) in the first period the agent does not want to deviate from his planned effort nor change his plans for the second period to another second-period effort he knows he is willing to follow and (3) in period zero he knows that he will be willing to plan and implement the effort plan in the upcoming periods since it is the rational plan that gives him the highest utility among all rational plans.[24,25]

To actually compute the equilibrium, the agent must proceed backwards: in period zero, he considers which action will be convenient to implement in period two given the continuation contract and each realization of the first period performance measure x_n. This will constitute the effort plan he will hold at the end of period one, $\widetilde{e}_{1,2}(x_n)$. Because of rational expectations, however, he must also check that, given this plan, once in period two he will be willing to carry this plan through, that is, $e_2 = \widetilde{e}_{1,2}(x_n)$. With e_2 and $\widetilde{e}_{1,2}(x_n)$ the agent can consider which effort will be optimal to

[24] This definition of personal equilibrium slightly differs to that in Kőszegi and Rabin (2009). In part (i) I assume the agent forms his period zero reference point at the same time he forms his plans, as in the static model in Kőszegi and Rabin (2006), meanwhile Kőszegi and Rabin (2009) assume that in period zero the agent has an arbitrary set of plans for future periods. They show that the predictions of the model are robust to this specification. This should not make any difference for the model predictions.

[25] Notice that because of the assumption that plans are updated after gain-loss utility is realized, the condition that the agent does not want to deviate to other second-period credible plans at the end of period one is ensured by (i). We relax this assumption in Section 7.

implement in period one. This will constitute his effort plan for the period $\widetilde{e}_{0,1}$. Then, just as before, he must make sure that, given this belief and those for period two, he will actually implement the action, that is $e_1 = \widetilde{e}_{0,1}$. Thus, since in period zero he knows that $\widetilde{e}_{1,2}$ is a rational plan, then he sets $\widetilde{e}_{0,2} = \widetilde{e}_{1,2}$. An effort path $(e_1^{PE}, e_2^{PE}(X))$ thus will be an equilibrium for the agent if $\widetilde{e}_{0,1} = e_1^{PE}$ and $\widetilde{e}_{0,2}(x_n) = \widetilde{e}_{1,2}(x_n) = e_2^{PE}(x_n)\ \forall x_n$.

I finally turn to the definition of an implementable path and the principal's problem.

Definition 2 *(Implementable Effort Path)*
An effort path $(e_1, e_2(X_1))$ is said to be implementable by a contract S if
(i) $EU_0(e_1, e_2(x_n)) \geqslant U_R$.
(ii) $(e_1, e_2(X_1))$ is a preferred personal equilibrium for the agent.
where $U_R = U_R^1 + U_R^2$ corresponds to the sum of the agent's per-period reservation utilities.

Part (i) corresponds to the standard individual rationality restriction: the agent accepts the offer if and only if the total utility he gets from the contract is not lower than his reservation utility. Part (ii) corresponds to the incentive compatibility restrictions implied by the agent's personal equilibrium. When agents have reference-dependent preferences and their reference point corresponds to their rational expectations about the effort path and the income beliefs induced by the contract, the personal equilibrium makes sure the agent will actually execute the desired effort path. However, this notion of incentive compatibility is more complex than that with classical reference-independent preferences. This is because the contract not only has to ensure that the agent actually executes the contingent desired actions but also that the agent plans those desired actions and does not deviate from his plan as information about the performance measure arrives. As a consequence, it is useful to consider the requirement of the effort path being a personal equilibrium as two sets of incentive compatibility restrictions: (1) those that, given the reference, make sure the agent actually implement the desired action and (2) those that make sure the agent does not deviate to other rational plans when reviewing his future plans as information arrives. I call the first set of incentive compatibility restrictions (part (i) in Definition 1) as "planning incentive compatibility restrictions", and the second set of restrictions as "executing incentive compatibility restrictions" (part (ii) and (iii) in Definition 1).

Finally, as usual since Grossman and Hart (1983), I consider the principal's problem as a two-step decision problem. First, find the contract that implements a given effort path $(e_1, e_2(X_1))$ at the minimum cost. Formally, for a given effort path $(e_1, e_2(X_1))$ the first step corresponds to the program (P)

$$\min_{\{s_n s_{nm}\}_{m=1}^{N}} \quad \sum_{n=1}^{N} \pi_n(e_1)[s_n + \sum_{m=1}^{N} \pi_m(e_2) s_{nm}]$$

$$\text{subject to} \qquad EU_0(e_1, e_2(X_1)|e_1, e_2(X_1)) \geqslant U_R$$
$$(e_1, e_2(X_1)) \quad \text{is a preferred personal equilibrium}$$

The second step in the Grossman and Hart (1983) approach consists in choosing the effort path (and its correspondent contract) that maximizes her profits. Though the statement of this problem is straightforward, I will skip it as I focus on the first step.

4 Basic Properties of the Optimal Contract

In this section I present a basic characterization of the optimal contract when agents are loss averse. I contrast the results under reference-dependent preferences with those under classical reference-independent preferences.

4.1 First-best Contracts

I start by considering the shape of the optimal contract when the state perfectly reveals the agent's action, i.e. the first-best contract. Lemma 1 presents a basic description of gain-loss utility under certainty.[26]

Lemma 1 *(Gain-loss Utility under Certainty is Zero)*
Assume A1-A-2 and that for every effort $e_j \in \mathcal{E}$ *there exists an* $n \in \{1, \ldots, N\}$ *such that* $\pi_n(e_j) = 1$ *and* $\pi_n(e_k) = 0$ *for all* $e_k \neq e_j$*. Then* $\int G(\widetilde{s}_{t,\tau}, \widetilde{e}_{t,\tau}|\widetilde{s}_{t-1,\tau}, \widetilde{e}_{t-1,\tau})d\Pi^S_\tau = 0$ *for* $t = 1$ *and* $\tau = 1, 2$ *and for* $t = 2$ *and* $\tau = 2$*,* $\forall \lambda \geqslant 1$*.*

Lemma 1 states that gain-loss utility is zero when there is no uncertainty even if the agent is loss averse. Intuitively, when the state perfectly reveals the action and the agent has rational expectations the agent cannot experience any feelings of gain or loss since he will never expect any other payment but the certain one given the effort he exerted.

With this lemma at hand, the following proposition describes the first-best contracts (S^{FB}) for classical reference-independent and reference-dependent preferences.

Proposition 1 *(First-Best Contracts)*
Assume A1-A2 and that for every effort $e_j \in \mathcal{E}$ *there exists an* $n \in \{1, \ldots, N\}$ *such that* $\pi_n(e_j) = 1$ *and* $\pi_n(e_k) = 0$ *for all* $e_k \neq e_j$*. Then the optimal contract* S *implementing* $(e_1, e_2(X_1))$ *does not depend on* η*. Moreover,* S *can be such that* $s_{nm} = s_{n'm}$ *for* $n \neq n'$ $\forall m$*.*

[26] This lemma replicates Proposition 3 in Kőszegi and Rabin (2006) in a moral hazard environment.

Proposition 1 presents two basic characteristics of the first best contracts. First, the contract with reference-dependent preferences is equal to that with classical reference-independent preferences. Herweg, Müller, and Weinschenk (2008) establish the same result for a static model of moral hazard with loss averse agents with preferences as in Kőszegi and Rabin (2006). The intuition behind this result comes straight from Lemma 1: under certainty gain-loss utility is zero and thus reference-dependent agents are behaviorally equivalent to consumption utility maximizers. Second, first-best contracts can be implemented without memory in wages, i.e. as a sequence of one-period contracts. Intuitively, when the contract can be written upon effort the principal does not face a risk-incentive trade-off and thus memory plays no role in alleviating it. As a result, a forcing contract in every period minimizes the principal's expected cost of implementing the desired action.[27]

4.2 Second-Best Contracts

I now study the shape of the optimal contract when the outcome measure does not reveal the agent's effort and the principal cannot observe the agent's action, i.e. the second-best contract. The aim is to prove that several standard features of this contract under classical reference-independent preferences still hold when agents have reference-dependent preferences. I start by proving the optimal contract exists when agents have reference-dependent preferences. Furthermore when consumption utility is concave such a contract is unique.

Assumption 3 *(Outcome does not Reveal the Action)*
For every $n \in \{1, \ldots, N\}$ there are at least two actions $e_j \neq e_k$ such that $\pi_n(e_j) \neq 0$ and $\pi_n(e_k) \neq 0$.

Proposition 2 *(Existence and Uniqueness of Second Best Contracts)*
Assume A1-A3 and fix $\eta > 0$ and an implementable effort path $(e_1, e_2(X_1))$. Then, there exists a contract S implementing $(e_1, e_2(X_1))$ at minimum expected cost for the principal. Moreover, if $u''(\cdot) < 0$, S is unique.

Just as in the reference-independent case, the assumption of consumption utility being strictly increasing allows us to uniquely define the principal's problem in a utility equivalent problem, which under the assumption of the value function being piece-wise linear (Assumption 2) corresponds to the minimization of well-behaved convex function with linear constraints.[28]

[27] Notice that without discounting, the first cost can also be implemented with a forcing long-term contract, i.e., a contract paying $\underline{s}$ in the second period if the desired first-period outcome is not observed and paying a fixed first-period wage of $u^{-1}(c(e_1) + U_{R_1})$. This contract, however, does not help highlighting the role of memory when the agent's action is not observable. See Proposition 4. For a proof of the optimality of first-best contracts without memory with classical preferences see Lambert (1983).

[28] The assumption of $\mathcal{E}$ being a discrete set is also important since it allows me to avoid the first order approach. See Herweg, Müller, and Weinschenk (2008) for a model of static moral hazard with Kőszegi and Rabin (2006)

From now on we assume that the outcome measure does not reveal the unobservable agent's action. Proposition 3 states that some of the basic characteristic of the second-best contract with classical reference-independent preferences also hold with reference-dependent preferences.

Proposition 3 *(Basic Properties of Second-Best Contracts)*
Assume A1-A3 and fix $\eta > 0$ and an implementable effort path $(e_1, e_2(X_1))$. Then, in the optimal contract S implementing the effort path
(i) the IR is binding.
(ii) the expected cost equals that of the first best contract if $e_1 = e_2(x_n) = e_{\min}$ $\forall n$ where e_{min} is such that $c(e_{min}) = \min\{c(\mathcal{E})\}$ or if there exists a subset $\mathcal{X}_j \subseteq \mathcal{X}$ such that $\sum_{x_n \in \mathcal{X}_j} \pi_n^j = 0 < \sum_{x_n \in \mathcal{X}_j} \pi_n^k$ $\forall e_k \in \mathcal{E} \backslash e_j$ where $c(e_k) < c(e_j)$ for $e_j = e_1$ and equivalently for $e_j = e_2(x_n)$ $\forall x_n$.

Part (i) states that any contract minimizing the principal's expected cost will leave no rents to the agent. Much like with reference-independent preferences, if the IR does not bind the principal can decrease by the same magnitude the consumption utility equivalents of all payments in a given period–without affecting gain-loss utility–decreasing thus her expected cost of implementing the action path.[29] Part (ii) states conditions under which the second-best contract achieves the same cost as a first-best contract. There are two cases here. First, if the principal wants to implement the least-cost effort path. The intuition is the same as with classical reference-independent preferences: if the interest of the agent and those of the principal are aligned there is no trade-off between risk-allocation and incentives. Second, if there is a (possibly different) shifting support for each of the desired actions in the effort path; that is, if for every desired action there is a certain range of the performance measure which cannot be achieved under the desired action but only under low cost ones. In this case the principal can write a sequence of forcing contracts with a sufficiently high penalization so that the agent will never take the risk of not executing the desired action path.[30]

It is interesting to note that with reference-dependent preferences, linear consumption utility does not imply that the second-best contract achieves the cost of the first-best contract as it is the case with reference-independent preferences. In fact, recall that with risk neutral classical preferences the principal can "sell the firm" at a fixed price to the agent (equal to her expected utility) and achieve first best. With reference-dependent preferences this is no longer true: even

preferences where the action is assumed to be continuous and the first order approach is used. As the authors point out, assumptions over the cost function must be made for the first order approach to be valid. The reason lies in the equilibrium gain-loss utility being negative (see Lemma 2 in the next section). Finally, the assumption of $\mathcal{E}$ being finite is not crucial. In fact, proving the existence of the contract when $\mathcal{E}$ is infinite only requires proving that the expected cost function is lower semicontinuous instead of simply continuous as I have done in the proof. I have opted to assume a finite set because there seem to be no additional interesting insights when $\mathcal{E}$ is assumed to be infinite.

[29] This result is not surprising: despite being reference-dependent, the agent's preferences over income lotteries are still independent of his effort choice due to the separability assumption (see Assumption 1).

[30] Notice though that this relaxes our initial assumption that $\pi_n^j > 0$ $\forall n, j$. However, just as in the case of classical reference-independent preferences, this may create some existence problem since proving that the set of IC and IR contracts is not bounded becomes harder. For details see Grossman and Hart (1983).

when consumption utility is linear, agents are still loss averse and thus they will not be willing to absorb the risk that such agreement involves.

5 Fixed-Wage Contracts

I now turn to the main prediction, that is, explore the intertemporal allocation of incentives when agents have dynamic reference-dependent preferences and the effort choice is private information. Proposition 4 presents our benchmark: the shape of optimal long-term contracts when agents have standard reference-independent preferences. From now on, unless otherwise stated, Assumption 1 is assumed to hold.

Proposition 4 *(Long-Term Contract with Reference-Independent Classical Preferences)*
Assume A1 and A3 and fix $\eta = 0$ and an effort path $(e_1, e_2(X_1))$. Then, the optimal contract S implementing $(e_1, e_2(X_1))$ is such that
(i) if $u^{''} < 0$, $s_n \neq s_{n'}$ for at least one $n \neq n'$, and $s_{nm} \neq s_{n'm}$ for $n \neq n'$ and at least one m.
(ii) if $u^{''} = 0$, $s_n \neq s_{n'}$ for at least one $n \neq n'$ and $s_{nm} = s_{n'm}$ for all m and for at least one $n \neq n'$.

Proposition 4 part (i) replicates the result in Rogerson (1985) (see also Lambert (1983), Murphy (1986), Malcomson and Spinnewyn (1988), Chiappori, Macho, Rey, and Salanié (1994)). It says that when the agent has standard risk-averse preferences optimal payments are contingent on the whole outcome history and the contract is said to display memory in wages. With risk-averse agents the intuition is as follows. By splitting incentives across time, the long-term contract increases the insurance provided to the agent relative to a sequence of one-period contracts, ameliorating the standard risk-sharing problem and allowing the principal to pay a smaller risk premium to implement the same action path. For present purposes, this prediction implies that when agents are risk-averse it is never optimal for the principal to cluster incentives in one period. Part (ii) corroborates this intuition by stating that if consumption utility function is linear the optimal contract displays no memory: since the agent is not bothered by risk there are no gains from intertemporal risk-sharing. In other words, the principal can write a sequence of forcing-short term contract and achieve first best.[31]

Before turning to our main proposition we present a lemma and add one final assumption. Lemma 2 generalizes the result in Kőszegi and Rabin (2009) by describing the value of the gain-loss utility in the equilibrium.

[31] From Proposition 2 we know that the optimal contract is not unique when the agent has a linear consumption utility function. As discussed in footnote 27, the first-best cost can also be achieved with a contract using memory. However, such contract does not highlight the use of memory as alleviating the risk-incentive trade-off. Thus, we opt to emphasize the first-best contract without memory.

Lemma 2 *(Equilibrium Gain-loss Utility is Non-Positive)*
Fix $\eta > 0$ and an an effort path $(e_1, e_2(X_1))$. Then, for the optimal contract S implementing $(e_1, e_2(X_1))$

$$\int G(\widetilde{s}_{t,\tau}(\widetilde{e}_{t,\tau})|\widetilde{s}_{t-1,\tau}(\widetilde{e}_{t-1,\tau}))d\Pi_\tau^S \leqslant 0$$

for $t = 1$ and $\tau = 1, 2$ and for $t = 2$ and $\tau = 2$ with equality if $\lambda = 1$.

Lemma 2 says that if the action path is a personal equilibrium, expected gain-loss utility is negative for loss-averse agents. Intuitively, the agent is hurt by the prospect of fluctuations in his income beliefs: he expects the payment realization to disappoint him with the same probability that he expects it to pleasantly surprise him, so due to loss aversion the expected utility from changes in income beliefs is unpleasant. By noticing that beliefs can only change in risky environments, this feature of the preference structure implies that the agent dislikes risk from the reference-dependent component of his utility function.

To understand why Lemma 2 holds, consider the following simple example illustrating contemporaneous gain-loss utility. An agent who works every Friday is paid the same day 100 dollars with probability $\pi(e_h)$ if he works hard (if he exerts e_h), and 80 dollars otherwise. On Thursday, the agent forms an effort plan $\widetilde{e}$ to be executed on Friday. Such an effort plan implies that the agent believes that on Friday he will earn 100 dollars with probability $\pi(\widetilde{e})$ and 80 dollars otherwise. To compute the expected utility the agent must consider the utility he will get in each possible future scenario. He knows that if he is successful on Friday he will get a gain equal to $(1-\pi(\widetilde{e_h}))\mu(100-80)$ by comparing the percentiles of the reference distribution with the degenerate distribution putting mass one into 100 (see the example in Section 3). Furthermore, he knows that if he fails on Friday he will experience a loss equal to $\pi(\widetilde{e})\mu(80-100)$. As a consequence, the total expected contemporaneous gain-loss utility corresponds to

$$\pi(e_h)(1-\pi(\widetilde{e_h}))\mu(100-80) + (1-\pi(e_h))\pi(\widetilde{e})\mu(80-100) \tag{3}$$

Equation (3) shows that the probability of the gain scenario corresponds to $\pi(e_h)(1-\pi(\widetilde{e_h}))$ while that of the loss scenario is $(1-\pi(e_h))\pi(\widetilde{e})$. When the agent has rational expectations, however, he will only implement actions he planned and he only plans actions he is willing to implement. Thus, in equilibrium $\pi(e_h)(1-\pi(\widetilde{e_h})) = (1-\pi(e_h))\pi(\widetilde{e})$. Then, using the piece-wise linearity of the value function (see Assumption 2), equation (3) corresponds to $(1-\lambda)\pi(e_h)(1-\pi(e_h))20$, which is negative whenever $\lambda > 1$.

Assumption 4 presents one final assumption over the outcome probability distribution.

Assumption 4 *(MLRP)*
For any e_k, $e_j \in \mathcal{E}$ such that $e_k \leqslant e_j$, the ratio $\pi_n(e_k)/\pi_n(e_j)$ is non-increasing in $n \in \{1, \ldots, N\}$.

Intuitively, Assumption 4 states that exerting more costly actions implies that higher outcome levels are more likely. This is a very reasonable assumption for a model where the actions correspond to effort level and the performance measure are outcomes positively related to the principal's utility: the higher the effort the agent exerts, the more likely higher outcomes will be observed. Whenever Assumption 4 holds for actions $e_k \leqslant e_j$, I say that the income distribution $\Pi^S(e_j)$ "likelihood ratio dominates" $\Pi^S(e_k)$ for $\tau = 1, 2$ (denoted $\Pi^S(e_j) \succsim_{LRD} \Pi^S(e_k)$).

I now present the main result. Proposition 5 states sufficient conditions for the long-term contract to allocate all incentives into future payments.

Proposition 5 *(Long-Term Contracts with Reference-Dependent Preferences)*
Assume A1-A4 and fix $\eta > 0$ and an effort path $(e_1, e_2(X_1))$. Then there is a $\bar{\gamma} > 0$ and a $M_1 > 0$ such that if $\gamma < \bar{\gamma}$ and $|u^{''}(s)| \leqslant M_1$ for all $s \geqslant \underline{s}$, the optimal contract implementing $(e_1, e_2(X_1))$ sets $s_n = s_{n'}$ for all $n, n^{'}$ and a first-period fixed wage, $s_{nm} \neq s_{n'm}$ for all $n \neq n^{'}$ and at least one $n, n^{'}$.

To illustrate the mechanism behind the main proposition, consider a two-action, two-effort model where for simplicity I assume the second-period action is verifiable and that the agent does not experience prospective gain-loss utility ($\gamma = 0$). Assume further that consumption utility is linear and that the principal wants to implement high effort in both periods. The high effort consumption disutility is c and that of low effort is zero. Define a contract S as follows. The first-period contract pays s_h after a success, and $s_\ell < s_h$ after a failure, meanwhile the second-period contract pays $\bar{s}$ after a second-period success and $\underline{s} < \bar{s}$–where $\underline{s}$ is defined in Assumption 1–after a second-period failure, independently of the first-period performance. Consider now an alternative contract $\hat{S}$ paying a fixed first-period payment $s = \pi(e_h)s_h + (1 - \pi(e_h))s_\ell$, where $\pi(e_h)$ is the probability of success given high effort and (s_h, s_ℓ) correspond to the first-period payments under contract S. Contrary to contract S, assume that $\hat{S}$ sets second-period contracts contingent on the first-period outcome realization: if a high outcome is observed in the second period, the contract pays $\bar{s} + \rho_h$ after a first-period success and $\bar{s} - \rho_\ell$ after a first-period failure with $\rho_h, \rho_\ell \in \mathbb{R}_+$. Moreover, if a low outcome is observed in the second period, the contract pays $\underline{s}$.

I show that for the right choices of ρ_h and ρ_ℓ, $\hat{S}$ is a profitable deviation from S, i.e., $\hat{S}$ implements the same effort path at cheaper expected cost. Notice that since the fixed-first-period payment under contract $\hat{S}$–s–equals the first-period expected consumption utility of contract S, the total expected cost of the two contracts will be equal if $\pi_h\rho_h - (1 - \pi_h)\rho_\ell = 0$. As a consequence, the principal will be indifferent between the two contracts and thus it suffices to show that under the alternative contract $\hat{S}$, high effort in both periods is a PPE and that the agent is better-off relative to contract S.

I start by exploring the conditions on ρ_h and ρ_ℓ such that high effort in every period is a PPE under $\hat{S}$. Notice that because the action is observable in the second period, and thus second-period contracts are forcing contracts, the only credible action for period two is high effort (otherwise the agent gets infinite disutility from $\underline{s}$). Thus, the second-period implementing IC is trivially satisfied under both S and $\hat{S}$.

Consider now the first-period implementing IC. To understand this IC restriction, we show how the two contracts S and $\hat{S}$ set incentives. Under contract S, the increase in total period one total expected utility the agent gets from exerting high effort instead of low effort, having planned to exert high effort, corresponds to:[32]

$$EU_1(e_h|e_h;S) - EU_1(e_\ell|e_h;S) = [\pi(e_h) - \pi(e_\ell)]\big\{\underbrace{(s_h - s_\ell)} + \underbrace{\phi(s_h - s_\ell)}\big\} - c - \mu(c) \geqslant 0$$

where $\phi \equiv \eta[(1-\pi_h) + \lambda\pi_h]$.[33] The first bracket shows incentives coming from first-period consumption utility: implementing high effort increases the probability of getting the high payment and decreases the probability of a low payment. The second bracket shows incentives coming from first-period contemporaneous gain-loss utility: implementing high effort increases the probability of a gain and decreases the probability of a loss. The difference in costs corresponds to the increase in costs of exerting high instead of low effort, having planned high effort: the consumption disutility of high effort plus the forgone gain of a pleasant surprise of working less than expected. Equivalently, under contract $\hat{S}$, the implementing first-period IC corresponds to:[34]

$$EU_1(e_h|e_h;\hat{S}) - EU_1(e_\ell|e_h;\hat{S}) = [\pi(e_h) - \pi(e_\ell)]\big\{0 + 0 + (\rho_h + \rho_\ell)\big\} - c - \mu(c) \geqslant 0$$

[32] To derive this equation, first build contemporaneous gain-loss utility. For this, recall that if a success is observed in the first period, the agent will experience a gain equal to $\eta(1-\pi(e_h))(s_h - s_\ell)$. If a failure is observed, the agent gets a loss equal to $-\lambda\eta(1-\pi(e_h))(s_h - s_\ell)$ (see example in Section 3). Since in equilibrium the implemented action equals the planned one, the gain scenario happens with probability $\pi(e_h)$ and the loss scenario happens with probability $(1-\pi(e_h))$. Then, notice that for a given effort plan made in period zero to be executed in period one, $\widetilde{e}_{0,1}$, the agent's total period-one expected utility of exerting e_h corresponds to:

$$EU(e_h|\widetilde{e}_{0,1}, S) = \pi(e_h)[s_h + \eta(1-\pi(\widetilde{e}_{0,1}))(s_h - s_\ell) + \bar{s}] + (1-\pi(e_h))[s_\ell - \lambda\eta\pi(\widetilde{e}_{0,1})(s_h - s_\ell) + \bar{s}] - c + \mu(-c - c(\widetilde{e}_{0,1}))$$

This equation follows simply by first setting $\widetilde{e}_{0,1} = e_h$ in the equation above, constructing the same equation for e_ℓ and then taking differences between the two. Notice, thus, that ϕ in fact corresponds to $\eta[(1-\pi(\widetilde{e}_{0,1})) + \lambda\pi(\widetilde{e}_{0,1})]$. In equilibrium, however, planned actions equal implemented actions.

[33] Because the second-period action is observable, the notation of all the utility functions in this example suppress the second-period action.

[34] To derive this equation notice that for a given effort plan made in period zero to be executed in period one, $\widetilde{e}_{0,1}$, the agent's total period-one expected utility of exerting e_h corresponds to:

$$EU_1(e_h|\widetilde{e}_{0,1}, \hat{S}) = \pi(e_h)[s + 0 + \bar{s} + \rho_h] + (1-\pi(e_h))[s + 0 + \bar{s} - \rho_\ell] - c + \mu(-c - c(\widetilde{e}_{0,1}))$$

The equation follows simply by first setting $\widetilde{e}_{0,1} = e_h$ in the equation above, constructing the same equation for e_ℓ and then taking differences between the two.

Since contract $\hat{S}$ uses a fixed wage in the first period, there is no change in consumption utility nor in gain-loss utility (the first two zeros above). However, under this alternative contract, second-period payments depend on the first-period outcome, and thus a new source of incentive arises: second-period consumption utility.[35] This is because if the agent executes high effort, he increases the probability of getting the high payment $\bar{s}+\rho_h$ relative to the low one $\bar{s}-\rho_\ell$.

From the two equations above is straightforward to see that by setting

$$(\rho_h+\rho_\ell)=(1+\phi)(s_h-s_\ell) \tag{4}$$

the period-one implementing IC under contract $\hat{S}$ holds, i.e. having planned to execute high effort, the agent will actually work hard under contract $\hat{S}$. In fact, using equation (4) together with $\pi_h\rho_h-(1-\pi_h)\rho_\ell=0$, the optimal quantities ρ_h^* and ρ_ℓ^* such that the principal is indifferent between the two contract and the agent will execute the desired action in period one are uniquely defined.[36]

The final step for high effort to be a PPE under $\hat{S}$, is to show that the period-zero planning IC holds. For this notice that if the disutility the agent gets in period zero from planning low effort but then deviating to high effort is big enough, an effort path with low effort in the first period may also be credible. Thus, it must be the case that under the deviation contract $\hat{S}$, the total period-zero expected utility of planning and exerting high effort is greater than that of planning and exerting low effort in the first-period. In fact, under the alternative contract $\hat{S}$ this condition is satisfied by the first-period implementing IC since this contract only relies on consumption utility–and thus on the implemented action–to put incentives.[37] Having found the optimal amount of second-period incentives for $\hat{S}$ to implement high effort, the question then is whether the agent is better or worse-off under this alternative contract. To find the answer, I compare the total period-zero expected utility the agent gets under both contracts. Under the original contract S the total period-zero equilibrium expected utility corresponds to:

$$EU_0(e_h,|e_h;S)=\underbrace{\pi(e_h)s_h+(1-\pi(e_h))s_\ell}-\underbrace{\eta(\lambda-1)\pi(e_h)(1-\pi(e_h))(s_h-s_\ell)}+\bar{s}-2c$$

The first bracket corresponds to first-period expected consumption utility. The second bracket corresponds to period-one expected contemporaneous gain-loss utility, which because of loss aversion is

[35] If $\gamma>0$ then prospective gain-loss utility also helps to set incentives.

[36] The optimal difference between the second-period continuation contracts therefore corresponds to $\rho_h^*=(1-\pi_h)(1+\phi)(s_h-s_\ell)$ and $\rho_\ell^*=\pi_h(1+\phi)(s_h-s_\ell)$.

[37] To see this notice that, by adding and subtracting conveniently and by noticing that equilibrium period-total utility equals that in period zero, period-zero planing IC can be written as

$$EU_0(e_h|e_h;\hat{S})-EU_0(e_\ell|e_\ell;\hat{S})=[EU_1(e_h|e_h;\hat{S})-EU_1(e_\ell|e_h;\hat{S})]-[EU_1(e_\ell|e_\ell;\hat{S})-EU_1(e_\ell|e_h;\hat{S})]$$

Furthermore, with very little work one can see that under this contract $EU_1(e_\ell|e_\ell;\hat{S})-EU_1(e_\ell|e_h;\hat{S})=-\mu(c)$ and thus $EU_0(e_h|e_h;\hat{S})-EU_0(e_\ell|e_\ell;\hat{S})=[EU_1(e_h|e_h;\hat{S})-EU_1(e_\ell|e_h;\hat{S})]+\mu(c)$.

negative. Notice further that there is no prospective gain-loss utility nor second-period contemporaneous gain-loss utility because $\gamma = 0$ and second-period contracts do not depend upon first-period performance. Equivalently, under contract $\hat{S}$, the total period-zero expected utility corresponds to

$$EU_0(e_h, |e_h; \hat{S}; \rho_h^*, \rho_\ell^*) = \underbrace{s} + \underbrace{\bar{s} + \pi(e_h)\rho_h - (1-\pi(e_h))\rho_\ell} - 2c = \pi(e_h)s_h + (1-\pi(e_h))s_\ell + \bar{s} - 2c$$

The first bracket corresponds to period-one expected utility, which consists only of first-period expected consumption utility. The second bracket corresponds to second-period expected utility which consists, again, uniquely of second-period expected consumption utility.

Therefore, the difference between the total period-zero expected utility under $\hat{S}$ and S corresponds to:

$$EU_0(e_h|e_h; \hat{S}, \rho_h^*, \rho_\ell^*) - EU_0(e_h|e_h; S) = \eta(\lambda - 1)\pi(e_h)(1-\pi(e_h))(s_h - s_\ell) > 0 \qquad (5)$$

which is positive because the agent is loss averse ($\lambda > 1$).

Intuitively, when $\gamma = 0$, the agent is not hurt by fluctuations in his beliefs about future income. As a consequence, the principal can defer all the risk–and thus all the incentives–into the future to minimize the disutility the agent gets from bearing risk. In doing so, the optimal contract relies heavily on future consumption utility to restore the incentives lost from fixing the first-period wage. In other words, a payment structure exploiting future consumption utility to set incentives is cheaper than one using gain-loss utility to motivate the agent, since it minimizes the disutility the agent gets from fluctuations in his income beliefs and thus from bearing risk.

When the agent is risk-averse, an intertemporal risk-sharing argument from consumption utility arises. Just as in the case of classical reference-independent preferences case, the concavity of the consumption utility function pushes towards spreading incentives across time. In such case, all the calculations above hold but for $\{\rho_h^*, \rho_\ell^*\}$ defined in utility terms. As a consequence, an extra assumption is needed to ensure that the expected cost of the alternative contract does not outweigh that of the original contract. Such a restriction corresponds to a not-too-concave restriction over the consumption utility function so that the cost of deferring payments to the second period do not outweigh the benefits of the fixed first-period wage.

When $\gamma > 0$, increasing the risk from future payments triggers prospective gain-loss disutility, which hurts the agent but helps the principal to set first-period incentives. However, if γ is small enough, the principal can defer incentives by increasing those coming from future consumption utility–without altering second-period incentives by moving all payments within a continuation contract by the same amount–and still increase the agent's total period-zero expected utility. A sufficiently small γ ensures that the increase in the total expected utility from shutting down first-period contemporaneous gain-loss utility outweighs the increase in the disutility from prospective gain-loss utility.[38]

Whether γ is big or small is an empirical matter, despite it seems reasonable that agents have a greater concern about present payments relative to future payments. I now further explore the importance of γ in the shape of the optimal contract by considering the case where changes in beliefs about future consumption resonate more strongly than changes in beliefs about present consumption. In such a case the increase in the continuation contract distance needed to preserve incentives would be small since prospective gain-loss utility is strong and helps to set incentives. However, this also implies a strong decrease in total period-zero total expected utility.

Proposition 6 *(No Memory)*
Fix $\eta > 0$ and an effort path $(e_1, e_2(X_1))$. Then there is a $\underline{\gamma} > 0$ such that if $\gamma > \underline{\gamma}$, the optimal contract implementing $(e_1, e_2(X_1))$ sets $s_n \neq s_{n'}$ for at least one $n \neq n^{'}$ and $s_{nm} = s_{n'm}$ for all $n \neq n^{'}$, $\forall m$.

Proposition 6 states that if prospective gain-loss utility is strong enough, then the optimal contract will display two basic characteristics. First, it will use contingent payments in the first period. The intuition here is the reverse of that in Proposition 5: if news about future consumption resonates more than news about current consumption, then it would be too expensive to set a first-period fixed wage since the savings from shutting down first-period contemporaneous gain-loss utility do not compensate the costs in increasing prospective gain-loss utility, as mentioned before. Second, wages will not display memory. The intuition is that using memory necessarily implies the agent will experience prospective gain-loss utility, which is negative (see Lemma 2). If γ is big enough, this creates a big disutility from the period-zero perspective, so big in fact that is it cheaper to use only first-period payments to motivate the agent.

6 Random Contracts

This section explores the principal's decision of which performances measures to use depending on their informativeness when writing the fixed-first-period wage contract. In particular, we explore

[38] This result is more likely to hold if the agent experiences diminishing sensitivity. Since the deviation contract does not rely on second-period contemporaneous gain-loss utility to replace incentives and since a lower prospective gain-loss disutility can be replaced with (cheaper) incentives coming from second-period consumption utility, a non-piece-wise-linear value function should increase the range of γ for which the result holds. To the contrary, the assumption of the principal being risk neutral is relevant since a risk averse principal is not willing to absorb the risk of the fixed-present wage for free. As a consequence, an assumption would have to be made about the principal being non too risk averse for the result to hold. This is no different from the classical model, however, where the optimal contract also depends on the principal's risk aversion assumption. This result also depends on the separability assumption made over the gain-loss utility function. In particular, if the gain-loss disutility in the second period depends negatively upon that in the first period, then fixing the first-period wage may trigger a higher prospective gain-loss disutility that would make the fixed-wage contract too expensive. Finally, the assumption of the agent consuming all his period income is also important. If the agent receives income news but he is forbidden to spend it, then the Kőszegi and Rabin (2009) model predicts the agent should not experience contemporaneous gain-loss disutility, as such news brings no information about present consumption. As a consequence, the principal would have no incentives to set a present-fixed wage.

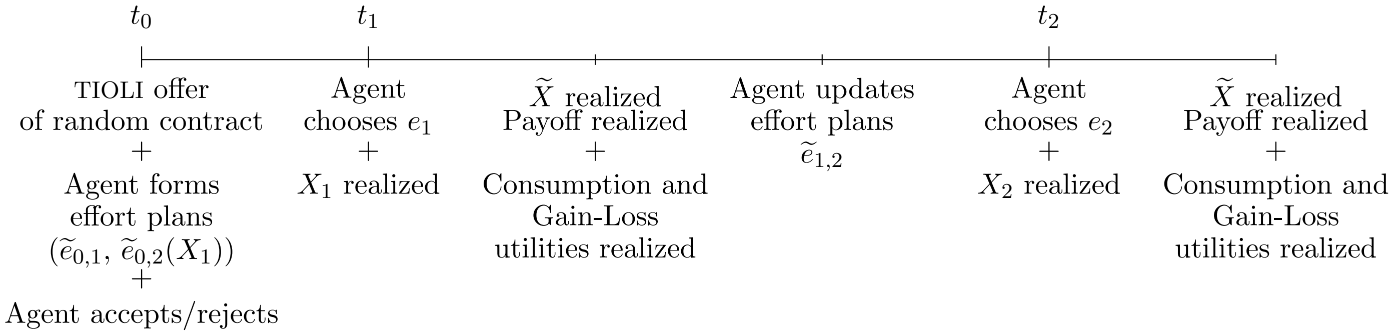


Figure 4: Timing of the Principal-Agent Interaction with Random Contracts

the convenience of paying the agent using a random contract, i.e., a contract that makes payments contingent on outcome measures uncorrelated with the agent's action. In this case, the principal offers the agent a menu of contracts for each period. The each-period menu consists of one contract for each possible realization of the outcome measure. Once the outcome is realized a pure randomization device uncorrelated with the agent's action–such as a wheel spun–is used to determine the actual payment from the contract previously chosen by the outcome realization.[39] Formally, let $\widetilde{X}$ drawn from the finite set $\widetilde{\mathcal{X}}$ correspond to the random device uncorrelated with the agent's action, which is assumed to be used in both period one and two. Let the random contract be denoted by $S(X_1, X_2, \widetilde{X})$ with a first-period contact $S_1(X_1, \widetilde{X})$ and a second period contract corresponding to $S_2(X_1, X_2, \widetilde{X})$.

The timing of the principal-agent interaction is presented in Figure 4. In period zero the principal makes a TIOLI offer of $S(X_1, X_2, \widetilde{X})$ to the agent. Given the contract, the agent forms an effort plan $\widetilde{e}_{0,1}$ and $\widetilde{e}_{0,2}$ for the upcoming periods. With these effort plans and the associated income beliefs he accepts or rejects the long-term contract. In the first period the agent takes and effort decision e_1 and the performance measure X_1 is realized according to which one of the contracts in the first-period menu is chosen. Next, $\widetilde{X}$ is realized for the fist time and the first-period payment–independent of the agent's performance–is realized and thus consumption and gain-loss utility in payments are experienced. At the end of period one the agent updates his effort plans for the second period given the realization of X_1. At the beginning of the second-period the agent takes a second effort decision e_2 and the performance measure X_2 is realized according to which one of the contracts in the second-period menu is chosen. Next, $\widetilde{X}$ is realized for the second time and the agent receives his second-period payments.

Proposition 7 shows that it is not optimal for the principal to use a contract using ex-post

[39]This type of random contract is usually denoted as "ex-post random" contract since the wheel is spun *after* the agent has executed the action. A second type of randomness can be included if the wheel is spun *before* the agent executes the action. Such a contracts are called "ex-ante" random contracts, and under specific assumptions over the third derivative of the consumption utility function, they can be optimal when the agent has classical preferences. See Fellingham, Kwon, and Newman (1984) and Arnott and Stiglitz (1988)

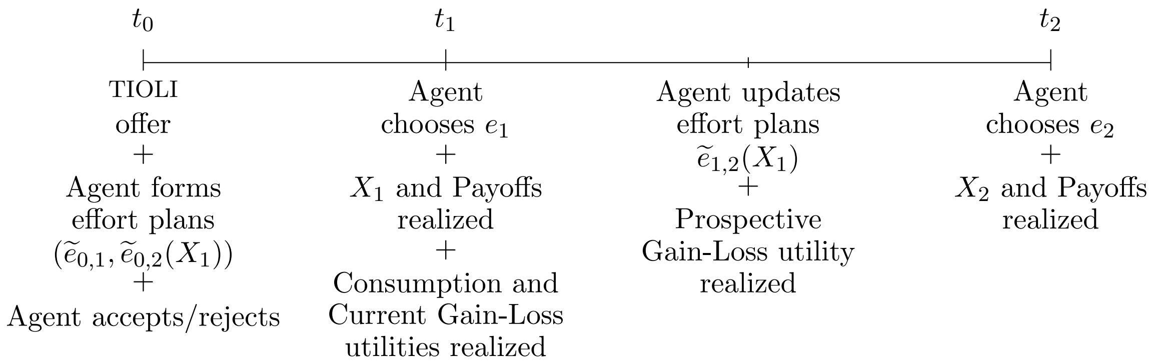


Figure 5: Timing of the Principal-Agent Interaction

randomness to implement the desired effort path when the optimal contract uses a fixed-first-period wage.

Proposition 7 *(Non-Optimality of Ex-Post Random Contracts)*
Assume A1-A4 and fix $\eta > 0$ and an effort path $(e_1, e_2(X_1))$. Then, there is a $M_1 > 0$ such that if $\gamma = 0$ and $|u''(s)| \leqslant M_1$ for all $s \geqslant \underline{s}$, $EC(S(X_1, X_2, \widetilde{X})) > EC(S(X_1, X_2))$.

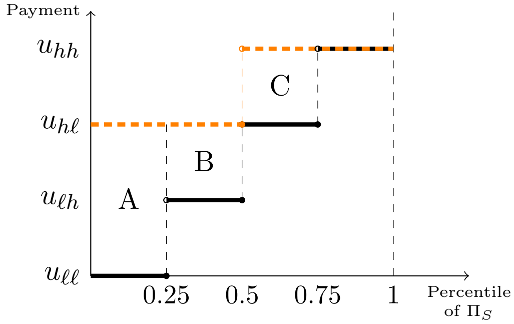
Intuitively, and just as with classical preferences, the principal can increase the agent's utility by offering him the certainty equivalent of the random payment. This is because the random device is not correlated with the agent's action–and thus it does not help to provide incentives–and the agent dislike risk. Then, because the principal's objective function is convex, the result follows by Jensen's inequality.[40] Hence, just as with classical reference-independent preferences, the principal will never use performance measures that are uncorrelated with the agent's unobserved effort.

Finally, it is interesting to notice that the certainty equivalent associated with the random device of an agent with reference-dependent preferences is smaller than that of an agent with classical preferences because he not only dislikes risk from the concavity of the consumption utility function, but also from the reference-dependent component of the utility function (see Lemma 2). As a consequence he is willing to accept even less money than a classical agent with the same consumption utility function to give up on the ex-post randomness of the contract.

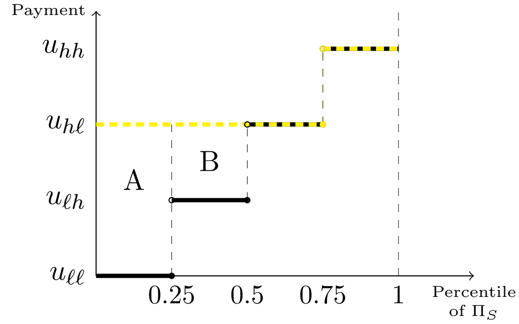
7 Timing of the Prospective Gain-Loss Utility Realization

This section examines a modification in the timing of the prospective gain-loss utility realization. To capture the intuition that experiencing a loss or gain from news about second-period payments

[40]Notice that Proposition 7 assumes $\gamma = 0$. This assumption is made only for simplicity and should be straightforwardly generalizable to small values of γ as in the main proposition. Moreover, I conjecture that the non-optimality of random contracts should also hold if γ is big so that a fixed-first-period wage is not optimal.



(a) $\widetilde{e}_{1,2} = e_h$



(b) $\widetilde{e}_{1,2} = e_\ell$

Figure 6: Prospective gain-loss utility realization when $X_1 = x_h$ as function of $\widetilde{e}_{1,2}$.

may affect second-period effort plans, I now assume prospective gain-loss utility is experienced after the agent has reviewed his second-period effort plans.[41] To keep the structure of incentives tractable, I further reduce the analysis to a two-effort $e_t \in \{e_\ell, e_h\}$, two-outcomes $x_t \in \{x_\ell, x_h\}$ model in which the principal wants to implement high effort in both periods. Let $S(X_1) = (s_h, s_\ell)$ and $S(X_1, X_2) = (s_{nh}, s_{n\ell})$ for $n = h, \ell$ correspond to the first and second period contracts. The rest of the interaction equals that in the original set up. Figure 5 shows the new timeline and Proposition 8 formalizes how second period incentives are affected by the timing of the prospective gain-loss utility realization.

Proposition 8 *(Slacking After Success)*
Fix $\eta > 0$ and assume the principal wants to implement $e_1 = e_2(x_\ell) = e_2(x_h) = e_h$. Then, the optimal contract implementing (e_h, e_h) is such that $s_{hh} - s_{h\ell} \geqslant s_{\ell h} - s_{\ell\ell}$.

Proposition 8 says that the optimal contract has to provide the agent with a bigger payment increase after a first-period success relative to a first-period failure. The intuition behind this result is as follows. When a high outcome is observed in the first period the agent experiences a gain from knowing that in the second period he will be paid with the high continuation contract. Since prospective gain-loss utility is realized after the agent reviews his plans for the second period, its size depends on the agent's effort plans for the second period. To understand how is the relationship between the planned effort and the prospective gain-loss utility gain, panel (a) in Figure 6 computes the the prospective gain-loss utility gain when the agent plans to exert high effort and the probability of success is 0.5. The solid line represents the reference distribution of second-period payments from period zero perspective, meanwhile the dashed line represents the expected second-period payment distribution. Since the agent compares the percentiles in the reference distribution with those in

[41]Notice that the equilibrium concept must be extended when changing the timing of prospective gain-loss utility realization. This is because now period-zero planning IC does not necessarily imply that the agent will form the correct plans. As consequence, a period-two planning restriction must be added. See the proof in Section 10.

the expected one (see Assumption 2), the prospective gain-loss utility gain corresponds to the area A+B+C. Panel (b) shows the size of the realized prospective gain if the agent plans to exert low effort in period two when the probability of success under low effort corresponds to 0.25. As before, the solid line represents the reference distribution–the equilibrium distribution of second-period payments from period zero perspective–meanwhile the dashed line corresponds to the expected second-period payment distribution. In this case the prospective gain-loss utility gain corresponds to the area A+B because planning to exert low effort in period two decreases the probability of pleasantly being surprised by u_{hh} given that $u_{h\ell}$ was planned. In other words, planning high effort maximizes the distance between the reference and the actual distribution of second-period payments.

Equivalently, if a failure is observed in the first period, the agent experiences a loss from knowing that in the second period he will be paid with the low continuation contract. The size of such loss is decreasing in the agent's second-period effort plans since planning high effort minimizes the distance between the actual second-period payments distribution and the reference distribution. In particular, planning high effort decreases the probability of being unpleasantly surprised by $u_{\ell\ell}$ given that $u_{h\ell}$ was planned. As consequence, the prospective gain-loss utility realization makes it easier to plan high effort for any realization of the first-period outcome. However, because of loss aversion, the effect of prospective gain-loss utility in planning is stronger in the loss scenario and thus is harder to motivate the agent after a first-period success.

Three important things to notice. First, even though this result mimics that with classical preferences where is easier to motivate the agent after a failure because the marginal utility is high, the rationale is different. Under reference-dependent preferences high effort is easier to motivate irrespective of the first-period realization but it is by loss aversion that the result follows. Moreover, this result holds even if consumption utility is linear, whereas with classical preferences payments are independent of first-period performance. Second, with reference-dependent preferences the temporal distance between the planning and the action execution is relevant. In fact, if the principal allows enough time after X_1 as been observed and the action execution, the agent may get used to his effort beliefs, rendering the realization of prospective gain-loss utility irrelevant for planning. Third, this result holds given that the agent planned in period zero to exert high effort in period two.[42]

8 Limitations and Future Research

This paper discusses the intertemporal allocation of incentives when agents have dynamic reference-dependent preferences. I show that in a completely classical environment and under reasonable assumptions over the utility function, the optimal contract defers all incentives into future payments.

[42]Changing the time of the prospective gain-loss utility would not affect our main result. To see this in pour Section 5 example, notice that at the end of period one, when the agent is reviewing his the second-period effort plan given the first-period outcome realization, there will be no other credible plans to deviate to and thus the first-period planning IC is also trivially satisfied under both S and $\hat{S}$.

This prediction reconciles the apparent dissociation between the models based on the risk-incentive trade-off and the shape of real contracts using present fixed wages. I further prove that, despite difference in the timing of incentive allocation, this optimal contract shares many features of the optimal contract with classical preferences: it does not leave rents to the agent, achieves first best when actions are least cost and there is a shifting support and ex-post random contracts are never optimal.

I emphasize that this model is about the principal's decision to defer incentives into the future. As a consequence, as it stands, it does not shed light about (1) how far incentives should be deferred or (2) how those deferred incentives should be spread among future periods. In fact, a probably pathological prediction of this model is that, when the sensitivity to income news is decreasing in the income period to which the news refers, the principal will always want to defer risk as far into the future as possible if the time structure of the problem allows her to do so. Extending the model to T periods would allow me to tackle this issue. In fact, I believe this pathological prediction may be overcome if the role of a standard exponential discount factor δ is considered. If $\delta < 1/r$– where r is the discount rate at which the principal can borrow or save–using future consumption utility to provide present incentives gets more and more expensive since the agent is less sensitive to incentives coming from future consumption utility the further in the future incentives are deferred. As a consequence, the principal won't be able to kick incentives into the future indefinitely because the amount of incentives needed would increase so much that no matter how small γ is (as long as is different form zero) the utility increase from shutting down present gain-loss utility will not outweigh the increase in prospect gain-loss disutility.

A second interesting question that can be explored in a T-period model relates to the timing of payments and feedback through the outcome realization. Until now, the model assumes that the principal pays the agent at the end of each period, once the performance measure has been realized. An extension of the model may deal with the case when there is a unique payment for first and second-period effort to be paid at the end of the principal-agent relationship. In such a framework, the principal may face the decision of whether to disclose the performance measure at the end of period one or disclose it at the end of period two together with the second-period outcome. Notice that in the classical model the preference structure is robust to the disclosure policy: only the informational structure of the problem changes. This is no longer true for dynamic reference-dependent preferences. In fact, the disclosure policy is a relevant choice variable for the principal because it determines whether the agent experiences gain-loss utility: if the principal decides not to disclosure x_n, then the agent only experiences second-period consumption utility and second-period current gain-loss utility. To the contrary, if the principal disclosures x_n in period one, then the agent will experience prospective gain-loss utility at the end of period one.

Extending the results to a T-period model may also give us some insights into why there is less contract renegotiation than what classical static moral hazard theory predicts. In fact, if the T-period-optimal contract with reference-dependent agents sets in every period a wage that does not depend on present performance but only on past performance–and thus can be considered fixed from the same period perspective–, then the model would predict that renegotiation is actually not necessary. The agent is perfectly insured in the present and cannot renegotiate future dates since the performance measure has not yet been observed.

9 Appendix A

9.1 Gain-loss Utility in Effort

In this section we present gain-loss utility in the effort domain. Making a similar definition of p relative to that for the payments domain but for the effort distribution (see Assumption 2), we have that gain-loss utility in the effort domain corresponds to

$$G(\widetilde{e}_{t,\tau}|\widetilde{e}_{t-1,\tau}) = \int_0^1 \mu\big(- c(e_{t,\tau}(p)) + c(\widetilde{e}_{t-1,\tau}(p))\big) dp$$

for $t = 0$ and $\tau = 1, 2$ and for $t = 1$ and $\tau = 2$ where we have suppressed the tilde notation because $e_{t,\tau}$ represents the actually implemented action. Consider first contemporaneous gain-loss utility for $t = 1$. Then, we have that if the agent implements action e_1 having planned $\widetilde{e}_{0,1}$, contemporaneous gain-loss utility corresponds to

$$G(e_1|\widetilde{e}_{0,1}) = \mu(c(\widetilde{e}_{0,1}) - c(e_1))$$

To see this, recall that $\widetilde{e} \in \mathcal{X}$ and thus for the first period the actual and the reference effort distributions are degenerate. Second-period contemporaneous gain-loss utility is defined analogously. For $(t, \tau) = (1, 2)$, we have that if the agent implements effort e_1 having planned $(\widetilde{e}_{0,1}, \widetilde{e}_{0,2})$, prospective gain-loss utility corresponds to

$$G(\widetilde{e}_{t,\tau}|\widetilde{e}_{t-1,\tau}) = \int_0^1 \mu\big(- c(e_{t,\tau}(p)) + c(\widetilde{e}_{t-1,\tau}(p))\big) dp$$

9.2 Description of the Total Utilities

From now on we rewrite gain-loss utility in effort and income in its discrete equivalent. Such conversion is done using the quantile comparison described in Section 3. Start by considering total period-two expected utility. Having observed $X_1 = x_n$ and thus having planned $\widetilde{e}_{1,2}$ at the end of period one, the total expected utility of implementing effort e_2 corresponds to

$$EU_2(e_2(x_n)|\widetilde{e}_{1,2}(x_n); x_n) = \sum_m \pi_m^2 u_{nm} + \sum_m \sum_\ell \pi_m^2 \pi_\ell^{1,2} \mu(u_{nm} - u_{n\ell}) - c(e_2) + \mu(c(\widetilde{e}_{1,2}) - c(e_2)) \quad (6)$$

where $\pi_m^2 \equiv \pi_m(e_2)$, $\pi_\ell^{1,2} \equiv \pi_\ell(\widetilde{e}_{1,2})$, $u_{nm} \equiv u(s_{nm})$ and s_{nm} are second-period payments given $X_1 = x_n$ and $X_2 = x_m$. The first term corresponds to the period-two expected consumption utility given the agent executes effort e_2. The second term corresponds to contemporaneous gain-loss utility as presented in Assumption 2. The last two terms represent the total cost. The term $c(e_2)$ corresponds to the consumption utility cost of exerting effort e_2, meanwhile the last term is the gain-loss utility of deviating from the planned effort made at the end of period one, $\widetilde{e}_{1,2}$ to e_2.

At the end of period one, having exerted e_1 in period one and having observed $X_1 = x_n$, the total expected utility of planning the credible action $\widetilde{e}_{1,2}$ corresponds to

$$EU_2(\widetilde{e}_{1,2}(x_n)|\widetilde{e}_{1,2}(x_n); x_n) = \sum_m \pi_m^{1,2} u_{nm} + \sum_m \sum_\ell \pi_m^{1,2} \pi_\ell^{1,2} \mu(u_{nm} - u_{n\ell}) - c(e_2) \tag{7}$$

The first term corresponds to second-period expected consumption utility of planning to exert $\widetilde{e}_{1,2}$. The second term is second-period contemporaneous gain-loss utility, knowing that $\widetilde{e}_{1,2}$ will be the the next period reference and knowing that he will execute the plan. Finally, notice there is no contemporaneous gain-loss utility in effort since the agent has rational expectations.

At the beginning of period one, before observing X_1 and having planned $(\widetilde{e}_{0,1}, \widetilde{e}_{0,2}(X_1))$ in period zero, the total expected utility of executing e_1 corresponds to

$$\begin{aligned} EU_1(e_1\widetilde{e}_{0,2}(X_1)|\widetilde{e}_{0,1}, \widetilde{e}_{0,2}(X_1)) = & \sum_n \pi_n^1 u_n + \sum_n \sum_\ell \pi_n^1 \pi_\ell^{0,1} \mu(u_n - u_\ell) + \gamma \sum_n \sum_\ell \sum_m \pi_n^1 \pi_\ell^{0,1} \pi_m^{0,2} \mu(u_{n\ell} - u_{\ell m}) \\ & + \sum_n \pi_n^1 EU_2(\widetilde{e}_{0,2}(x_n)|\widetilde{e}_{0,2}(x_n); x_n) \\ & - c(e_1) + \mu(c(\widetilde{e}_{0,1}) - c(e_1)) + \gamma \sum_n \sum_\ell \pi_n^1 \pi_\ell^{0,1} \mu(c(\widetilde{e}_2(x_\ell)) - c(\widetilde{e}_2(x_n))) \end{aligned} \tag{8}$$

where $\pi_n^1 \equiv \pi_n(e_1)$ and $\pi_n^{0,1} \equiv \pi_n(\widetilde{e}_{0,1})$, $u_n \equiv u(s_n)$ and s_n are first-period payments and others defined as before. The first line represents period-one total expected utility from exerting e_1 having planned to exert $\widetilde{e}_{0,1}$. As before, the first term is expected consumption utility and the second term is contemporaneous gain-loss utility. The third term corresponds to prospective gain-loss utility as defined in Assumption 2. The second line represents period-two total expected utility. The third and final line represents total period-one expected utility from effort. The first and second term correspond to consumption and gain-loss utilities from implementing action e_1 having planned $\widetilde{e}_{0,1}$ and the last term corresponds to prospective gain-loss utility in effort from second period effort being a contingent strategy.

Finally, in period zero, the agent's total expected utility corresponds to

$$\begin{aligned} EU_0(\widetilde{e}_{0,1}\widetilde{e}_{0,2}(X_1)|\widetilde{e}_{0,1}, \widetilde{e}_{0,2}(X_1)) = & \sum_n \pi_n^{0,1} u_n + \sum_n \sum_\ell \pi_n^{0,1} \pi_\ell^{0,1} \mu(u_n - u_\ell) + \gamma \sum_n \sum_\ell \sum_m \pi_n^{0,1} \pi_\ell^{0,1} \pi_m^{0,2} \mu(u_{n\ell} - u_{\ell m}) \\ & + \sum_n \pi_n^{0,1} EU_2(\widetilde{e}_{0,2}(x_n)|\widetilde{e}_{0,2}(x_n); x_n) \\ & - c(\widetilde{e}_{0,1}) + \gamma \sum_n \sum_\ell \pi_n^{0,1} \pi_\ell^{0,1} \mu(c(\widetilde{e}_2(x_\ell)) - c(\widetilde{e}_2(x_n))) \end{aligned} \tag{9}$$

The description of period-zero total expected utility follows that of period one. The only difference is that in this period the agent does not execute an action but forms plans. Since he has rational expectations, those plans must coincide with the actual action he plans to implement, thus he does not experience contemporaneous gain-loss utility in effort and the probabilities are defined in term of the plans uniquely. As a consequence $EU_0(\widetilde{e}_{0,1}\widetilde{e}_{0,2}(X_1)|\widetilde{e}_{0,1}, \widetilde{e}_{0,2}(X_1)) =$

$EU_1(\widetilde{e}_{0,1}\widetilde{e}_{0,2}(X_1)|\widetilde{e}_{0,1},\widetilde{e}_{0,2}(X_1))$.

10 Appendix B: Proofs

Proof of Lemma 1

Consider first-period contemporaneous gain-loss utility. Start noticing that if for $e_j \in \mathcal{E}$ $\pi_n^j = 1$, then $\pi_{n'}^j = 0$ $\forall n' \neq n$. Consider first the equilibrium path. In this case $e_1 = \widetilde{e}_{0,1}$ and thus $\forall e_1 \in \mathcal{E}$ $\exists!$ $n(e_1) \in \{1, \ldots, N\}$ such that $\pi_{n(e_1)}^1 \pi_{n(e_1)}^{0,1} \neq 0$ and $\pi_{n(e_1)}^1 \pi_\ell^{0,1} = 0$ $\forall \ell \neq n(e_1)$. Therefore,

$$\sum_{n=1}^{N}\sum_{\ell=1}^{N} \pi_n^1 \pi_\ell^{0,1} \mu(u_n - u_\ell) = \pi_{n(e_1)}^1 \pi_{n(e_1)}^{0,1} \mu(u_{n(e_1)} - u_{n(e_1)}) = 0$$

Second, consider the outside equilibrium path. In this case $e_1 \neq \widetilde{e}_{0,1}$, and thus $\pi_n^1 \pi_\ell^{0,1} = 0$ $\forall n, \ell$ and thus $\sum_{n=1}^N \sum_{\ell=1}^N \pi_n^1 \pi_\ell^{0,1} \mu(u_n - u_\ell) = 0$. The proof for second-period contemporaneous gain-loss utility is analogous $\forall x_n$. Consider now prospective gain-loss utility from period one perspective. Consider first the equilibrium path. If $e_1 = \widetilde{e}_{0,1} = \widetilde{e}_{0,2}$, then $\forall e_1 \in \mathcal{E}$ $\exists!$ $n(e_1) \in \{1, \ldots, N\}$ such that $\pi_{n(e_1)}^1 \pi_{n(e_1)}^{0,1} \pi_{n(e_1)}^{0,2} \neq 0$ and $\pi_{n(e_1)}^1 \pi_\ell^{0,1} \pi_m^{0,2} = 0$ $\forall \ell \neq n(e_1)$, $m \neq n(e_1)$ or $\ell \neq m$. Therefore,

$$\sum_{n=1}^{N}\sum_{\ell=1}^{N}\sum_{m=1}^{N} \pi_n^1 \pi_\ell^{0,1} \pi_m^{0,2} \mu(u_{n\ell} - u_{\ell m}) = \pi_{n(e_1)}^1 \pi_{n(e_1)}^{0,1} \pi_{n(e_1)}^{0,2} \mu(u_{n(e_1)n(e_1)} - u_{n(e_1)n(e_1)}) = 0$$

Second, if $e_1 = \widetilde{e}_{0,1} \neq \widetilde{e}_{0,2}$, then $\pi_n^1 \pi_\ell^{0,1} \pi_m(\widetilde{e}_{0,2}) = 0$ $\forall n, \ell, m$. Consider now the outside equilibrium path. In such case $e_1 \neq \widetilde{e}_{0,1}$ and thus $\pi_n^1 \pi_\ell^{0,1} = 0$ $\forall n, \ell$ and thus $\sum_{n=1}^N \sum_{\ell=1}^N \sum_{m=1}^N \pi_n^1 \pi_\ell^{0,1} \pi_m^{0,2} \mu(u_{n\ell} - u_{\ell m}) = 0$.. From period zero perspective, the proof is analogous.

■

Proof of Proposition 1

From Lemma 1 we have that equation (1) becomes $v_t = V(s_t, e_t)$ $\forall \eta$. Thus the optimal contract does not depend on η. To see that the optimal contract implementing $(e_1, e_2(X_1))$ does not necessarily displays memory, notice that when the action is observable for e_1 $\exists!$ $n(e_1) \in \{1, \ldots, N\}$ such that $\pi_n^1 = 1$ and thus $\pi_{n'}^1 = 0$ $\forall n' \neq n$. Defining $m(e_2) \in \{1, \ldots, N\}$ in analogous manner, we have that the IR restriction to implement $(e_1, e_2(x_n))$ corresponds to

$$\sum_{n=1}^{N} \pi_n(e_1)[u_n + \sum_{m=1}^{N} \pi_m(e_2) u_{nm}] - c(e_1) - c(e_n) = u_{n(e_1)} + u_{m(e_2)} - c(e_1) - c(e_n) \geqslant U_{R_1} + U_{R_2}$$

Thus, the principal can write a long-term-forcing contract as

$$S_1 = \begin{cases} u^{-1}(U_R^1 + c(e_1)) & \text{if } X_1 = x_{n(e_1)} \\ \underline{s} & \text{otherwise.} \end{cases} \quad \text{and} \quad S_2 = \begin{cases} u^{-1}(U_R^2 + c(e_2)) & \text{if } X_2 = x_{m(e_2)} \\ \underline{s} & \text{otherwise.} \end{cases}$$

where $\underline{s}$ is defined in Assumption 1. It is straight to see that such contract implements the action path and is least cost since the principal's objective function is decreasing in the agent's payments. Notice that in fact, there is a family of forcing contracts implementing the first best, as any payment $u^{-1}(\underline{s}) \leqslant s < u^{-1}(U_R^1 + c(e_1))$ implements e_1 at first best cost. Same for the second-period contract.

■

Proof of Proposition 2
Let $\mathcal{U} = \big(\mathcal{U}_1(X_1), \mathcal{U}_2(X_1, X_2)\big)$ be the utility equivalent of the contract S. Because $u^{'}(\cdot) > 0$ (see Assumption 1) such representation is unique. Thus, we can write the principal problem as

$$\min_{\big\{\{u_n\}_{n=1}^N, \{u_{nm}\}_{m=1}^N\big\}} \qquad EC_1(u_n) + EC_2(u_{nm}) = \sum_n \pi_n^1 u^{-1}(u_n) + \sum_n \sum_m \pi_m^2 u^{-1}(u_{nm})$$

$$\text{subject to} \qquad EU_0(e_1, e_2(X_1)|e_1, e_2(X_1)) \geqslant U_R$$

$$(e_1, e_2(X_1)) \quad \text{is a Preferred Personal Equilibrium}$$

where the objective function is convex. Moreover, because the function $\mu(\cdot)$ is piece-wise linear (Assumption 2), then is straightforward to see that all the expected utilities in the restriction set are linear in u_n and u_{nm} $\forall n, m$ (see Definition 1 and Appendix A). Thus the principal's problem is a convex problem with linear constraints. Consider the set of all contracts $\mathcal{U}$ implementing the action path

$$\mathcal{C}(e_1, e_2(X_1)) \equiv \Big\{ \sum_{n=1}^N \pi_n(e_1)[u^{-1}(u_n) + \sum_{m=1}^N \pi_m(e_2) u^{-1}(u_{nm})\big] | \mathcal{U} \text{ implements } (e_1, e_2(X_1) \Big\}$$

We are done if show that $\mathcal{C}(e_1, e_2(X_1))$ has an infimum that is achieved. We prove that the set is bounded below. Start by noticing that, because U_R is bounded, there must exist a finite $\widetilde{U}_{R_2}(x_n)$ $\forall n$ so that $EU_2(e_2(x_n)|e_2(x_n); x_n) = \widetilde{U}_{R_2}(x_n)$ (recall the agent is fully committed in period zero to stay in the relationship and thus the principal is not committed to $\text{EU}_2(e_2(x_n)|e_2(x_n); x_n) \geqslant U_{R_2}$ $\forall n$). Thus, using equation (6) in equilibrium, we have

$$\sum_m \pi_m^2 u_{nm} - c(e_2) \geqslant \sum_m \pi_m^2 u_{nm} + \sum_m \sum_\ell \pi_m^2 \pi_\ell^2 \mu(u_{nm} - u_{n\ell}) + c(e_2) \geqslant \widetilde{U}_{R_2}(x_n) \geqslant \widetilde{U}_{R_2}^{\max}$$

where $\widetilde{U}_{R_2}^{\max} \equiv max\{\widetilde{U}_{R_2}(x_n)\}_{n=1}^N$ and the second inequality comes from the fact that contemporaneous gain-loss utility is negative in equilibrium (see Lemma 2). Applying the definition of convexity twice, we have that a lower bound for the second-period contract corresponds to:

$$\sum_n \sum_m \pi_n^1 \pi_m^2 u^{-1}(u_{nm}) \geqslant \sum_n \pi_n(e_1) u^{-1}\Big(\sum_m u_{nm}\Big) \geqslant u^{-1}\Big(\widetilde{U}_{R_2}^{\max} + c(e_2)\Big)$$

Doing a similar analysis for period-one payments, we have that

$$\sum_n \pi_n^1 u^{-1}(u_n) \geqslant u^{-1}\Big(\sum_n \pi_n^1 u_n\Big) \geqslant u^{-1}\Big(U_R + c(e_1)\Big)$$

Thus the set $\mathcal{C}(e_1, e_2(X_1))$ is bounded below. Since the set is closed by construction, it as an infimum. The rest of the proof follows exactly that in Grossman and Hart (1983) by proving that this lower bound is achieved and then applying Weierstrass' theorem.

I now prove that the optimal contract is unique. By contradiction, suppose not. Let $U' = (U_1^{'}, U_2^{'})$ be also an optimal contract implementing $(e_1, e_2(x_n))$ where $u_n^{'} \neq u_n$ and $u_{nm}^{'} \neq u_{nm}$

for at least one n and one m. Define a new contract U^{α} as $u^{\alpha}_{n} = \alpha u_n + (1-\alpha)u^{'}_{n}$ and $u^{\alpha}_{nm} = \alpha u_{nm} + (1-\alpha)u^{'}_{nm}$, $\alpha \in (0,1)$ (where without loss of generality we have assumed α is the same for fist and second- period contracts). I start proving that such contract still implements the same effort path. Consider first the IR restriction under U^{α}. First period consumption utility can be written as $\sum_{n=1}^{N}\pi_n^{0,1}u_n^{\alpha} = \sum_{n=1}^{N}\pi_n^{0,1}(\alpha u_n + (1-\alpha)u'_n) = \alpha\sum_{n=1}^{N}\pi_n^{0,1}u_n + (1-\alpha)\sum_{n=1}^{N}\pi_n^{0,1}u^{'}_n$, and equivalently for second-period consumption utility. From equation (10) in the proof of Lemma 2, because the value function is assumed to be piece-wise linear, first period gain-loss utility can be written in linear form as $(1-\lambda)\sum_{n=1}^{N}\sum_{\ell<n}\pi_n^{0,1}(u_n^{\alpha} - u_{\ell}^{\alpha}) = \alpha(1-\lambda)\sum_{n=1}^{N}\sum_{\ell<n}u_n + (1-\alpha)(1-\lambda)\sum_{n=1}^{N}\sum_{\ell<n}u^{'}_n$, and equivalently for second-period contemporaneous gain-loss utility. Also from the proof of Lemma 2, prospective gain-loss utility can be separated into its positive and negative components so find a shape for prospective gain-loss utility under U^{α} as function of U and $U^{'}$ payments. Consider first from equation (12) its positive component,

$$\begin{aligned}
&\sum_{n=1}^{N}\sum_{\ell<n}^{N}\sum_{m=1}^{N}\pi_n^1\pi_\ell^{0,1}\pi_m^{0,2}(u_{n\ell}^{\alpha} - u_{\ell m}^{\alpha}) + \sum_{n=1}^{N}\sum_{\ell=n}^{N}\sum_{m<\ell}^{N}\pi_n^1\pi_\ell^{0,1}\pi_m^{0,2}(u_{n\ell}^{\alpha} - u_{\ell m}^{\alpha}) \\
&= \alpha\Big[\sum_{n=1}^{N}\sum_{\ell<n}^{N}\sum_{m=1}^{N}\pi_n^1\pi_\ell^{0,1}\pi_m^{0,2}(u_{n\ell} - u_{\ell m}) + \sum_{n=1}^{N}\sum_{\ell=n}^{N}\sum_{m<\ell}^{N}\pi_n^1\pi_\ell^{0,1}\pi_m^{0,2}(u_{n\ell} - u_{\ell m})\Big] \\
&+(1-\alpha)\Big[\sum_{n=1}^{N}\sum_{\ell<n}^{N}\sum_{m=1}^{N}\pi_n^1\pi_\ell^{0,1}\pi_m^{0,2}(u^{'}_{n\ell} - u^{'}_{\ell m}) + \sum_{n=1}^{N}\sum_{\ell=n}^{N}\sum_{m<\ell}^{N}\pi_n^1\pi_\ell^{0,1}\pi_m^{0,2}(u^{'}_{n\ell} - u^{'}_{\ell m})\Big]
\end{aligned}$$

Working the negative terms from equation (13) in the same fashion, and putting together first and second-period consumption utility with first and second-period contemporaneous gain-loss utility, is easy to see that

$$EU_0(e_1, e_2(x_n)|e_1, e_2(x_n); U^{\alpha}) = \alpha EU_0(e_1, e_2(x_n)e_1, e_2(x_n); U) + (1-\alpha)EU_0(e_1, e_2(x_n)|e_1, e_2(x_n); U^{'})$$

Thus, since the IR holds for U and $U^{'}$, it must also hold for U^{α}. Exactly the same logic shows that the IC restrictions are all satisfied for U^{α} as they are satisfied for U and $U^{'}$. As a consequence, U^{α} implements the same effort path as U and U'. Finally, notice that because of Jensen's inequality,

$$EC_t(u_n^{\alpha}) \leqslant \alpha EC_t(u_n) + (1-\alpha)EC_t(u'_n) = EC_t(u_n) \qquad t = 1, 2$$

with at least one inequality strict if $u^{''}(\cdot) < 0$ which contradicts the optimality of U.

■

Proof of Proposition 3

I start proving that the IR binds in the principal's problem. By contradiction, assume the contract S is the least cost contract implementing the effort path $(e_1, e_2(X_1))$ where $EU_0(e_1, e_2(X_1)|e_1, e_2(X_1); S) > U_R$. Consider an alternative contract $\hat{S} = (s_n - \varepsilon, s_{nm})$ $\varepsilon > 0$. Notice that because $\hat{S}$ does not modify second-period payments, second-period implementing IC is satisfied. Consider first-period implementing IC. By equation (8), first-period contemporaneous utility under contract $\hat{S}$ corresponds to $\sum_{n=1}^{N}\sum_{\ell=1}^{N}\pi_n^1\pi_\ell^{0,1}\mu((u_n-\varepsilon)-(u_\ell-\varepsilon)) = \sum_{n=1}^{N}\sum_{\ell=1}^{N}\pi_n^1\pi_\ell^{0,1}\mu(u_n - u_\ell)$, which is equal to contemporaneous gain-loss utility under S. Thus, first-period implementing IC holds under $\hat{S}$ with the same argument as in the classical model. The same analysis is valid for period-zero planning IC. The result follows from $\mathrm{EC}_1(S)$ being decreasing in first period payments and thus S cannot be a least-cost contract.

Consider now part (*ii*). Consider first the case when the principal wants to implement $e_1 = e_2(x_n) = e_{\min}$ $\forall n$ where e_{min} is such that $c(e_{min}) = \min\{c(\mathcal{E})\}$. The proof follows straight from noticing that a contract S^{min}, $s_n^{min} = u^{-1}(U_R^1 + c(e_{min}))$ $\forall n$ and $s_{nm}^{min} = u^{-1}(U_R^2 + c(e_{min}))$ $\forall n, m$ implements the action path and at first best cost (for a description of the first-best cost, see proof of Proposition 1). To see that the contract is IR, notice from equation (2) that gain-loss utility (contemporaneous and prospective) is zero with fixed wages, thus

$$EU_0(e_{\min}, e_{\min}|e_{\min}, e_{\min}; S^{min}) = \sum_n \pi_n^{min}(U_R^1 + c(e_{min})) + \sum_n \sum_m \pi_m^{min}(U_R^2 + c(e_{min})) - 2c(e_{min}) = U_R$$

where $\pi_n^{min} \equiv \pi_n(e_{min})$. An equivalent analysis show that under S^{min} all the IC constraints are met.

Consider now the case where there exists a subset $\mathcal{X}_j \subseteq \mathcal{X}$ such that $\sum_{x_n \in \mathcal{X}_j} \pi_n^j = 0 < \sum_{x_n \in \mathcal{X}_j} \pi_n^k$ $\forall e_k \in \mathcal{E} \backslash e_j$ such that $c(e_k) < c(e_j)$ for $e_j = e_1$ and equivalently for $e_j = e_2(x_n)$ $\forall x_n$ where we use the notation $\mathcal{X}_2(x_n)$ to make clear that the set $\mathcal{X}_2$ may depend on x_n. The proof follows straight from noticing that contract S consisting of the following two spot implementable forcing contracts

$$S_1 = \begin{cases} u^{-1}(U_R^1 + c(e_1)) & \text{if } x_n \notin \mathcal{X}_1 \\ \underline{s} & \text{otherwise.} \end{cases} \quad \text{and} \quad S_2(x_n) = \begin{cases} u^{-1}(U_R^2 + c(e_2)) & \text{if } x_n \notin \mathcal{X}_2(x_n) \\ \underline{s} & \text{otherwise.} \end{cases} \quad \forall n$$

implements the desired action path and achieves first best contract. To do so, is straight to check the IR and IC constraints hold.

■

Proof of Proposition 4

We start proving part (*i*), following the proof in Rogerson (1985). To see that first-period payments are contingent, let $U = (u_n, u_{nm})$ be the unique utility equivalent of the optimal contract implementing the effort path $(e_1, e_2(x_n))$. Consider the following deviation contract $\hat{U} = (\hat{u}_n, \hat{u}_{nm})$ where $\hat{u}_n = u_n$ and $\hat{u}_{nm} = u_{nm}$ and $\hat{u}_{n'} = u_{n'} - \rho$ and $\hat{u}_{n'm} = u_{n'm} + \rho$ for one $n' \neq n$ for all $m \in \{1, \ldots, N\}$. Notice that the agent's expected utility is the same under both contracts since $\pi_{n'}^1[(u_{n'} - \rho) + \sum_m \pi_m^2(u_{n'm} + \rho)] = \pi_{n'}^1[u_{n'} + \sum_m \pi_m^2 u_{n'm}]$. Moreover, since ρ does not depend on the action, $\hat{U}$ also implements $(e_1, e_2(x_n))$ and the agent is indifferent between the two contracts. Then, for U to be the optimal contract, the value $\rho = 0$ must minimize $u^{-1}(u_n - \rho) + \sum_m \pi_m(e_2(x_n))u^{-1}(u_{nm} + \rho)$ Taking first derivatives and using the fact $\rho = 0$ in the optimal contract, I have that the following condition must hold:

$$\frac{1}{u'(u^{-1}(u_n))} = \sum_{m=1}^{N} \frac{\pi_m(e_2(x_n))}{u'(u^{-1}(u_{nm}))}$$

Because the right hand side depends on x_n (as the notation emphasizes), then it must be the case that $u_n \neq u_{n'}$ for $n \neq n'$. To see that the optimal contract will display memory, suppose not. Then, it must be the case that $u_{nm} = u_{km}$ $\forall n, k$ and thus $e_2(x_n) = e_2(x_k)$. Therefore, from the equation above,

$$\frac{1}{u'(u^{-1}(u_n))} = \sum_{m=1}^{N} \frac{\pi_m(e_2(x_n))}{u'(u^{-1}(u_{nm}))} = \sum_{m=1}^{N} \frac{\pi_m(e_2(x_k))}{u'(u^{-1}(u_{km}))} = \frac{1}{u'(u^{-1}(u_k))}$$

implying that $u_n = u_k$, which is a contradiction.

To prove part (ii) it suffices to show that when $u''(\cdot) = 0$, the optimal long term contract without memory equals the first-best contract. Let $u_{nm} = u_{n'm} \equiv u_m$ be the second period payments in the contract without memory. The expected cost of implementing $(e_1, e_2(x_n))$ under such contract corresponds to:

$$\begin{aligned}
\sum_n \pi_n^1 u^{-1}(u_n) + \sum_m \pi_m^2 u^{-1}(u_m) &= u^{-1}\Big(\sum_n \pi_n^1 u_n + \sum_m \pi_m^2 u_m\Big) \\
&= u^{-1}\big(U_R + c(e_1) + c(e_2)\big) \\
&= u^{-1}\big(U_R^1 + c(e_1)\big) + u^{-1}\big(U_R^2 + c(e_2)\big)
\end{aligned}$$

where the first line follows from the fact that the IR constraint binds because of the usual arguments. Then, from Proposition 1 we the principal can implement the action path at the same cost of the first-best contract. As a consequence, memory cannot decrease the principal's expected cost .

■

Proof of Lemma 2

Let $u_n \equiv u(s_n)$, $u_{nm} \equiv u(s_{nm})$ be the utility equivalent of contract S. Because of Assumption 1, such representation is unique. Order this utility contract in ascending order with respect to n. Consider first expected contemporaneous gain-loss utility, that is $\int G(\widetilde{s}_{t\tau}|\widetilde{s}_{t-1,\tau})d\Pi_\tau^S$ for $(t,\tau) = (1,1)$ and $(t,\tau) = (2,2)$. Consider $(t,\tau) = (1,1)$,

$$\begin{aligned}
\int G(s_1|\widetilde{s}_{0,1})d\Pi_1^S &= \int\int_0^1 \mu\big(u(s_1(p)) - u(\widetilde{s}_{0,1}(p))\big)dpd\Pi_1^S \\
&= \sum_{n=1}^N \sum_{\ell=1}^N \pi_n(e_1)\pi_\ell(\widetilde{e}_{0,1})\mu(u_n - u_\ell) \\
&= \sum_{n=1}^N \sum_{\ell\leqslant n} \pi_n(e_1)\pi_\ell(\widetilde{e}_{0,1})\mu(u_n - u_\ell) + \sum_{n=1}^N \sum_{\ell\geqslant n} \pi_n(e_1)\pi_\ell(\widetilde{e}_{0,1})\mu(u_n - u_\ell) \\
&= \sum_{n=1}^N \sum_{\ell\leqslant n} \pi_n(e_1)\pi_\ell(\widetilde{e}_{0,1})\mu(u_n - u_\ell) + \sum_{n=1}^N \sum_{\ell\leqslant n} \pi_n(\widetilde{e}_{0,1})\pi_\ell(e_1)\mu(u_\ell - u_n) \\
&= \sum_{n=1}^N \sum_{\ell\leqslant n} \pi_n(e_1)\pi_\ell(e_1)[\mu(u_n - u_\ell) + \mu(u_\ell - u_n)] \leqslant 0 \qquad (10)
\end{aligned}$$

where in the last line I have used the fact that in equilibrium $\widetilde{e}_{0,1} = e_1$. Notice further that if $\lambda = 1$, then $\mu(u_n - u_\ell) = |\mu(u_\ell - u_n)|$ and thus $\int G(s_\tau|\widetilde{s}_{t-1,\tau})d\Pi_\tau^S = 0$. For $(t,\tau) = (2,2)$ the proof is analogous by considering $\int G(\widetilde{s}_{2,2}|\widetilde{s}_{1,2})d\Pi_2^S = \int\int_0^1 \mu\big(u(s_2(p)) - u(\widetilde{s}_{1,2}(p))\big)dpd\Pi_2^S = \sum_{m=1}^N \sum_{\ell=1}^N \pi_m(e_2)\pi_\ell(\widetilde{e}_{1,2})\mu(u_{nm} - u_{n\ell})$ for any given n. Finally, notice that the latter proof does not rely on the assumption of the value function $\mu(\cdot)$ being piece-wise linear.

Consider now expected prospective gain-loss utility (gain-loss utility for $(t,\tau) = (1,2)$). Notice that the difference between this utility source and contemporaneous gain-loss utility for period two is that, for the latter, and given the realization of x_n, second-period payments are still a non-degenerate distribution (a continuation contract $S_2(x_n)$), whereas in second-period contemporaneous gain-loss utility the second-period payment is a degenerate distribution since x_n has already been observed. Thus, using the fact that prospective gain-loss utility is realized before plans are updated and thus the only relevant case is $e_2(x_n) = \widetilde{e}_2(x_n)$, then

$$\gamma \int G(s_2|\widetilde{s}_{0,2}) d\Pi_2^S \propto \int G(s_2|\widetilde{s}_{0,2}) d\Pi_2^S = \int\int_0^1 \mu\big(u(s_2(p)) - u(\widetilde{s}_{0,2}(p))\big) dp d\Pi_2^S$$
$$= \sum_{n=1}^{N}\sum_{\ell=1}^{N}\sum_{m=1}^{N} \pi_n(e_1)\pi_\ell(\widetilde{e}_{0,1})\pi_m(\widetilde{e}_{0,2})\mu(u_{n\ell} - u_{\ell m}) \quad (11)$$

Ordering the payments as $s_{nm} \geqslant s_{nm'}$ for $m \geqslant m'$ $\forall n$ and that $s_{nN} \geqslant s_{(n-1)1}$ for $\forall n$, I can use the same strategy as in contemporaneous gain-loss utility of splitting the positive terms from the negative ones in equation (11) as

$$\sum_{n=1}^{N}\sum_{\ell<n}^{N}\sum_{m=1}^{N} \pi_n^1\pi_\ell^{0,1}\pi_m^{0,2}\mu(u_{n\ell} - u_{\ell m}) + \sum_{n=1}^{N}\sum_{\ell=n}^{N}\sum_{m<\ell}^{N} \pi_n^1\pi_\ell^{0,1}\pi_m^{0,2}\mu(u_{n\ell} - u_{\ell m}) \quad (12)$$
$$+\sum_{n=1}^{N}\sum_{\ell>n}^{N}\sum_{m=1}^{N} \pi_n^1\pi_\ell^{0,1}\pi_m^{0,2}\mu(u_{n\ell} - u_{\ell m}) + \sum_{n=1}^{N}\sum_{\ell=n}^{N}\sum_{m>\ell}^{N} \pi_n^1\pi_\ell^{0,1}\pi_m^{0,2}\mu(u_{n\ell} - u_{\ell m}) \quad (13)$$

where I have shortened the notation as $\pi_n(e_1) \equiv \pi_n^1$, $\pi_\ell(\widetilde{e}_{0,1}) = \pi_\ell^{0,1}$ and $\pi_m(\widetilde{e}_{0,2}) \equiv \pi_m^{0,2}$, and those terms in (12) are positive whereas those in equation (13) are negative. Contrarily to the case of contemporaneous gain-loss utility, the terms in (12) are not the same as those in (13). To make them comparable, I add an subtract so to create neighboring payments and use the value function specification $\mu(x) = x$ if $x > 0$ and $\mu(x) = \lambda x$ if $x < 0$. Consider first equation (12). For $n > \ell$, $s_{n\ell} - s_{\ell m} = s_{n\ell} \pm s_{n(\ell-1)} \pm \dots \pm s_{n1} \pm s_{(n-1)N} \pm \dots s_{(n-1)1} \pm \dots \pm s_{\ell N} \pm \dots \pm s_{\ell(m+1)} - s_{\ell m}$; for $n = \ell > m$, $s_{nn} - s_{nm} = s_{nn} \pm s_{n(m+1)} \pm \dots \pm s_{n(n-1)} - s_{nm}$, and for a given x_n, with a lot of tedious work I can rewrite (12) as

$$= \sum_{n=1}^{N-1}\sum_{\ell\neq 1}^{N}\Big[\sum_{i=1}^{n-1}\pi_i^{0,1} + \pi_n^{0,1}\sum_{j=1}^{\ell-1}\pi_j^{0,2}\Big](s_{n\ell} - s_{n(\ell-1)}) + \sum_{n=2}^{N}\sum_{i=1}^{n-1}\pi_i^{0,1}(s_{n1} - s_{(n-1)N})$$
$$+ \sum_{\ell=1}^{N-1}\Big[\sum_{i=\ell+1}^{N-1}\pi_i^{0,1} + \pi_N^{0,1}\sum_{j=1}^{\ell}\pi_j^{0,2}\Big](s_{N(\ell+1)} - s_{N\ell})$$

Adding and subtracting in a similar fashion, I can rewrite (13) as:

$$= -\lambda\sum_{n=2}^{N}\sum_{\ell\neq 1}^{N}\Big[\sum_{i=n+1}^{N}\pi_i^{0,1} + \pi_n^{0,1}\sum_{j=\ell}^{N}\pi_j^{0,2}\Big](s_{n\ell} - s_{n(\ell-1)}) - \lambda\sum_{n=2}^{N}\sum_{i=n}^{N}\pi_i^{0,1}(s_{n1} - s_{(n-1)N})$$
$$- \lambda\sum_{\ell=1}^{N-1}\Big[\sum_{i=2}^{\ell}\pi_i^{0,1} + \pi_1^{0,1}\sum_{j=\ell+1}^{N}\pi_j^{0,2}\Big](s_{1(\ell+1)} - s_{1\ell})$$

Then, changing the notation from n to k so to sum over the realizations of x_n and rearranging, I have that expected prospective gain-loss utility can be written in terms of neighboring payments as:

$$
\begin{aligned}
&\Big\{ \sum_{k=1}^{N-1} \sum_{n=k+1}^{N} \sum_{\ell>1}^{N} \pi_n^1 \Big[\sum_{i=1}^{k-1} \pi_i^{0,1} + \pi_k^{0,1} \sum_{j=1}^{\ell-1} \pi_j^{0,2} \Big] (s_{k\ell} - s_{k(\ell-1)}) + \sum_{n=1}^{N} \sum_{k=2}^{n} \sum_{i=1}^{k-1} \pi_n^1 \pi_i^{0,1} (s_{k1} - s_{(k-1)N}) \qquad (14)\\
&+ \sum_{k=2}^{N} \sum_{\ell>1}^{k} \pi_k^1 \Big[\sum_{i=\ell}^{k-1} \pi_i^{0,1} + \pi_k^{0,1} \sum_{j=1}^{\ell-1} \pi_j^{0,2} \Big] (s_{k\ell} - s_{k(\ell-1)}) \Big\} - \\
\lambda \quad &\Big\{ \sum_{k=2}^{N} \sum_{n=1}^{k-1} \sum_{\ell>1}^{N} \pi_n^1 \Big[\sum_{i=k+1}^{N} \pi_i^{0,1} + \pi_k^{0,1} \sum_{j=\ell}^{N} \pi_j^{0,2} \Big] (s_{k\ell} - s_{k(\ell-1)}) + \sum_{n=1}^{N} \sum_{k=n+1}^{N} \sum_{i=k}^{N} \pi_n^1 \pi_i^{0,1} (s_{k1} - s_{(k-1)N})\\
&+ \sum_{k=1}^{N-1} \sum_{\ell=k+1}^{N} \pi_k^1 \Big[\sum_{i=k+1}^{\ell-1} \pi_i^{0,1} + \pi_k^{0,1} \sum_{j=\ell}^{N} \pi_j^{0,2} \Big] (s_{k\ell} - s_{k(\ell-1)}) \Big\}
\end{aligned}
$$

Now I can prove that the latter expression is non-positive. In order to do so, I will conveniently sum up the terms. Consider first the sum of the second and the fifth terms in equation (14):

$$
\begin{aligned}
& \sum_{n=1}^{N} \sum_{k=2}^{n} \sum_{i=1}^{k-1} \pi_n^1 \pi_i^{0,1} (s_{k1} - s_{(k-1)N}) - \lambda \sum_{n=1}^{N} \sum_{k=n+1}^{N} \sum_{i=k}^{N} \pi_n^1 \pi_i^{0,1} (s_{k1} - s_{(k-1)N})\\
= \quad & \sum_{n=1}^{N} \Big[\sum_{k=2}^{n} \sum_{i=1}^{k-1} \pi_n^1 \pi_i^{0,1} - \lambda \sum_{k=n+1}^{N} \sum_{i=k}^{N} \pi_n^1 \pi_i^{0,1} \Big] (s_{k1} - s_{(k-1)N})\\
= \quad & (1-\lambda) \sum_{k=2}^{N} \sum_{n=k}^{N} \sum_{i=1}^{k-1} \pi_n \pi_i (s_{k1} - s_{(k-1)N}) \leqslant 0
\end{aligned}
$$

where in the last line I have used the fact that in equilibrium $\widetilde{e}_{0,1} = e_1$. Consider now the first and last terms in equation (14) for $k = 1$:

$$
\begin{aligned}
& \sum_{n=2}^{N} \sum_{\ell>1}^{N} \pi_n^1 \Big[\sum_{i=1}^{0} \pi_i^{0,1} + \pi_1^{0,1} \sum_{j=1}^{\ell-1} \pi_j^{0,2} \Big] (s_{1\ell} - s_{1(\ell-1)}) - \lambda \sum_{\ell=2}^{N} \pi_1^1 \Big[\sum_{i=2}^{\ell-1} \pi_i^{0,1} + \pi_1^{0,1} \sum_{j=\ell}^{N} \pi_j^{0,2} \Big] (s_{1\ell} - s_{1(\ell-1)}) \Big\} \qquad (15)\\
= \quad & \sum_{n=2}^{N} \sum_{\ell>1}^{N} \pi_n^1 \pi_k^{0,1} \sum_{j=1}^{\ell-1} \pi_j^{0,2} (s_{1\ell} - s_{1(\ell-1)}) - \lambda \sum_{\ell=2}^{N} \pi_1^1 \Big[\sum_{i=2}^{\ell-1} \pi_i^{0,1} + \pi_1^{0,1} \sum_{j=\ell}^{N} \pi_j^{0,2} \Big] (s_{1\ell} - s_{1(\ell-1)}) \Big\}\\
= \quad & \pi_1^1 \sum_{\ell>1}^{N} \Big\{ \sum_{n=2}^{N} \sum_{j=1}^{\ell-1} \pi_n^1 \pi_j^{0,2} - \lambda \Big[\sum_{i=2}^{\ell-1} \pi_i^{0,1} + \pi_1^{0,1} \sum_{j=\ell}^{N} \pi_j^{0,2} \Big] \Big\} (s_{1\ell} - s_{1(\ell-1)}) \qquad (16)
\end{aligned}
$$

where in the last line I have used the fact that in equilibrium $\pi_1^1 = \pi_1^{0,1}$. Consider now the curly bracket in equation (16). Changing the notation for convenience, adding and subtracting conveniently, and then rearranging, I have

$$\begin{aligned}
&\sum_{n=2}^{N}\sum_{j=1}^{\ell-1}\pi_n^1\pi_j^{0,2}-\lambda\Big[\sum_{i=2}^{\ell-1}\pi_i^{0,1}+\pi_1^{0,1}\sum_{j=\ell}^{N}\pi_j^{0,2}\Big]\\
=\;&\pi_1^{0,2}\sum_{n=2}^{N}\pi_n^1+\sum_{n=2}^{N}\sum_{j=2}^{\ell-1}\pi_n^1\pi_j^{0,2}\pm\pi_1^{0,2}\sum_{n=2}^{\ell-1}\pi_n^1-\lambda\Big[\sum_{j=1}^{N}\sum_{n=2}^{\ell-1}\pi_n^{0,1}\pi_j^{0,2}+\pi_1^{0,1}\sum_{j=\ell}^{N}\pi_j^{0,2}\Big]\\
=\;&(1-\lambda)\Big[\pi_1\sum_{n=\ell}^{N}\pi_n+\sum_{n=1}^{N}\sum_{j=2}^{\ell-1}\pi_n\pi_j\Big]
\end{aligned}$$

where in the last line I have used $e^1=\widehat{e}^{0,1}$. Thus equation (15) corresponds to

$$(1-\lambda)\pi_1\sum_{\ell>1}^{N}\Big\{\sum_{n=\ell}^{N}\pi_n+\sum_{n=1}^{N}\sum_{j=2}^{\ell-1}\pi_n\pi_j\Big\}(s_{1\ell}-s_{1(\ell-1)})\leqslant 0$$

Second, consider the sum of the third and fourth terms on (14) for $k=N$.

$$\sum_{\ell>1}^{N}\pi_N^1\Big[\sum_{i=\ell}^{N-1}\pi_i^{0,1}+\pi_N^{0,1}\sum_{j=1}^{\ell-1}\pi_j^{0,2}\Big](s_{N\ell}-s_{N(\ell-1)})-\lambda\sum_{n=1}^{N-1}\sum_{\ell>1}^{N}\pi_n^1\Big[\sum_{i=N+1}^{N}\pi_i^{0,1}+\pi_N^{0,1}\sum_{j=\ell}^{N}\pi_j^{0,2}\Big](s_{k\ell}-s_{k(\ell-1)}) \quad (17)$$

$$=\sum_{\ell>1}^{N}\pi_N^1\Big[\sum_{i=\ell}^{N-1}\pi_i^{0,1}+\pi_N^{0,1}\sum_{j=1}^{\ell-1}\pi_j^{0,2}\Big](s_{N\ell}-s_{N(\ell-1)})-\lambda\sum_{n=1}^{N-1}\sum_{\ell>1}^{N}\pi_n^1\pi_N^{0,1}\sum_{j=\ell}^{N}\pi_j^{0,2}(s_{k\ell}-s_{k(\ell-1)})$$

$$=\pi_N^1\sum_{\ell_1}^{N}\Big\{\sum_{i=\ell}^{N-1}\pi_i^{0,1}+\pi_N^{0,1}\sum_{j=1}^{\ell-1}\pi_j^{0,2}-\lambda\sum_{n=1}^{N-1}\pi_n^1\sum_{j=\ell}^{N}\pi_j^{0,2}\Big\}(s_{N\ell}-s_{N(\ell-1)}) \quad (18)$$

where in the last line I have used the fact that in equilibrium $\pi_N^1=\pi_N^{0,1}$. Consider now the curly bracket in equation (18). Slightly changing the notation for convenience, adding and subtracting conveniently, and then rearranging, I have

$$\begin{aligned}
\sum_{i=\ell}^{N-1}\pi_i^{0,1}+\pi_N^{0,1}\sum_{j=1}^{\ell-1}\pi_j^{0,2}-\lambda\sum_{n=1}^{N-1}\pi_n^1\sum_{j=\ell}^{N}\pi_j^{0,2}\;&=\;\sum_{j=\ell}^{N-1}\pi_j^{0,1}+\pi_N^{0,1}\sum_{j=1}^{\ell-1}\pi_j^{0,2}\pm\pi_N^{0,1}\sum_{j=\ell}^{N}\pi_m^{0,2}\\
&=\;(1-\lambda)(1-\pi_N)\sum_{j=\ell}^{N}\pi_j
\end{aligned}$$

where in the last lien I have used $\pi_1^1=\pi_1^{0,1}$. Thus equation (17) corresponds to

$$(1-\lambda)(1-\pi_N)\pi_N^1\sum_{\ell=1}^{N}\sum_{j=\ell}^{N}\pi_j(s_{N\ell}-s_{N(\ell-1)})\leqslant 0$$

Finally, consider all terms $2\leqslant k\leqslant N-1$ in the first, third , fourth and last term in equation (14).

For a given k and a given ℓ, I have

$$\Big\{ \sum_{n=k+1}^{N} \pi_n^1 \Big[\sum_{i=1}^{k-1} \pi_i^{0,1} + \pi_k^{0,1} \sum_{j=1}^{\ell-1} \pi_j^{0,2} \Big] + \pi_k^1 \Big[\sum_{i=\ell}^{k-1} \pi_i^{0,1} + \pi_k^{0,1} \sum_{j=1}^{\ell-1} \pi_j^{0,2} \Big] \Big\} (s_{k\ell} - s_{k(\ell-1)})$$
$$- \quad \lambda \Big\{ \sum_{n=1}^{k-1} \pi_n^1 \Big[\sum_{i=k+1}^{N} \pi_i^{0,1} + \pi_k^{0,1} \sum_{j=\ell}^{N} \pi_j^{0,2} \Big] + \pi_k^1 \Big[\sum_{i=k+1}^{\ell-1} \pi_i^{0,1} + \pi_k^{0,1} \sum_{j=\ell}^{N} \pi_j^{0,2} \Big] \Big\} (s_{k\ell} - s_{k(\ell-1)})$$

Five possible cases arise here depending on the relative magnitudes of ℓ, $k-1$ and $k+2$. With some further work, one can prove that all the terms above are negative in each of the five cases, which concludes the proof.

■

Before proving the main result, we present the following lemma.

Lemma 3 *(Outside Equilibrium Contemporaneous Gain-Loss Utility)*
For any actions $e_j \neq e_k \in \mathcal{E}$, if $\Pi_\tau^S(e_j) \succsim_{LRD} \Pi_\tau^S(e_k)$, then
(i) $\int G(\widetilde{s}_{t,\tau}(e_k)|\widetilde{s}_{t-1,\tau}(e_j))d\Pi_\tau^S \leqslant 0$
(ii) $\int G(\widetilde{s}_{t,\tau}(e_j)|\widetilde{s}_{t-1,\tau}(e_j))d\Pi_\tau^S - \int G(s_\tau(e_k)|\widetilde{s}_{t-1,\tau}(e_j))d\Pi_\tau^S \leqslant 0$
for $t=1$ and $\tau=1$ and $t=2$ and $\tau=2$.

Proof of Lemma 3
To keep notation simple, I will use first period contemporaneous gain-loss utility. To prove part (i), notice

$$\begin{aligned}
\sum_n \sum_\ell \pi_n^k \pi_\ell^j \mu(u_n - u_\ell) &= \eta \sum_n \sum_{\ell<n} \pi_n^k \pi_\ell^j (u_n - u_\ell) - \lambda\eta \sum_n \sum_{\ell>n} \pi_n^k \pi_\ell^j (u_\ell - u_n) \\
&= \eta \sum_n \sum_{\ell<n} \pi_n^k \pi_\ell^j (u_n - u_\ell) - \lambda\eta \sum_n \sum_{\ell<n} \pi_n^j \pi_\ell^k (u_n - u_\ell) \\
&= \eta \sum_n \sum_{\ell<n} [\pi_n^k \pi_\ell^j - \lambda \pi_n^j \pi_\ell^k](u_n - u_\ell)
\end{aligned}$$

Because $\Pi_\tau^S(e_j) \succsim_{LRD} \Pi_\tau^S(e_k)$,then for all n, $n > \ell$

$$\pi_n^k \pi_\ell^j \leqslant \pi_n^j \pi_\ell^k \leqslant \lambda \pi_n^j \pi_\ell^k \leqslant$$

where the last inequality holds because $\lambda \geqslant 1$. Then it must be the case that $[\pi_n^k \pi_\ell^j - \lambda \pi_n^j \pi_\ell^k] \leqslant 0$, which concludes the proof. Part (ii) is obvious from the fact that gains are increasing in n whereas losses are decreasing in n.

■

Proof of Proposition 5
Let S, $u_n \neq u_{n'}$ for at least one $n \neq n^{'}$ and $u_{nm} = u_{n'm}$, $n \neq n'$ for at least one m be the optimal contract implementing the effort path $(e_1, e_2(x_n))$. I prove that the alternative contract $\hat{S}$,

$\hat{u}_n = \hat{u}_{n'} = \sum_n \pi^1 u_n \equiv \bar{u}$ $\forall n \neq n'$ and $\hat{u}_{nm} = u_{nm} + \rho_n$, $\forall n, m$ $\rho_n \in \mathbb{R}$ implements $(e_1, e_2(x_n))$ at a lower expected cost than S. Consider first period-two implementing IC under contract S. Ordering the payments as in Lemma 2 and using equation (6) from Section 9.2 and Definition 1, these restrictions correspond to

$$\begin{aligned}EU_2(e_2(x_n)|e_2(x_n);x_n,S) - EU_2(e_j|e_2(x_n);x_n,S) &= \sum_m [\pi_m^2 - \pi_m^j]\big(u_{nm} + \sum_\ell \pi_\ell^2 \mu(u_{nm} - u_{n\ell})\big)\\ &\geqslant c(e_2(x_n)) - c(e_j) \qquad \forall e_j \in \mathcal{E}\backslash e_2(x_n) \qquad \forall n\end{aligned}$$

where recall $\pi_m^2 \equiv \pi_m(e_2(x_n))$ and $\pi_m^j \equiv \pi_m(e_j)$. The agent's expected utility under the alternative contract $\hat{S}$ corresponds to

$$\begin{aligned}EU_2(e_2(x_n)|e_2(x_n);x_n,\hat{S}) &= \sum_m \pi_m^2(u_{nm} + \rho_n) + \sum_m \sum_\ell \pi_m^2 \pi_\ell^{1,2} \mu(u_{nm} - u_{n\ell}) - c(e_2(x_n))\\ &= EU_2(e_2(x_n)|e_2(x_n);x_n,S) + \rho_n\end{aligned}$$

Thus, for all $e_j \in \mathcal{E}\backslash e_2(x_n)$ and for all n,

$$EU_2(e_2(x_n)|e_2(x_n);x_n,\hat{S}) - EU_2(e_j|e_2(x_n);x_n,\hat{S}) = EU_2(e_2(x_n)|e_2(x_n)|e_2(x_n);x_n,S) - EU_2(e_j|e_2(x_n);x_n,S)$$

so $\hat{S}$ does not distort second period incentives and thus period-two implementing IC is satisfied under $\hat{S}$. Consider now first-period implementing IC. Using equation (8) from Section 9.2 and Definition 1, under S these restrictions correspond to

$$\begin{aligned}&EU_1(e_1, e_2(X_1)|e_1{,}e_2(X_1);S) - EU_1(e_j, e_2(X_1)|e_1, e_2(X_1);S) =\\ &\sum_n [\pi_n^1 - \pi_n^j]\Big\{u_n + \sum_\ell \pi_\ell^1 \mu(u_n - u_\ell) + \gamma \sum_\ell \sum_m \pi_\ell^1 \pi_m^2 \mu(u_{n\ell} - u_{\ell m})\\ &\qquad + EU_2(e_2(x_n)|e_2(x_n);x_n,S) + \gamma \sum_\ell \pi_\ell^1 \mu\big(c(e_2(x_\ell) - c(e_2(x_n)))\big)\Big\} \geqslant c(e_1) - c(e_j) \quad \forall e_j \in \mathcal{E}\backslash e_1\end{aligned}$$

where notice that the difference between the utilities is taken for the implemented action given the reference. Under the alternative contract, the agent's first-period utility corresponds to:

$$\begin{aligned}EU_1(e_1, e_2(X_1)|e_1, e_2(X_1);\hat{S}) = \sum_n \pi_n^1 \Big\{&\bar{u} + \gamma \sum_\ell \sum_m \pi_\ell^{0,1} \pi_m^{0,2} \mu((u_{n\ell} + \rho_n) - (u_{\ell m} + \rho_\ell))\\ &+ EU_2(e_2(X_1)|e_2(x_n);S) + \rho_n\\ &+ \gamma \sum_\ell \pi_\ell^1 \mu\big(c(e_2(x_\ell) - c(e_2(x_n)))\big)\Big\} - c(e_1) - c(\widetilde{e}_{0,2})\end{aligned}$$

Working for the period-one implementing IC under this alternative contract and operating we have

$$\begin{aligned}&EU_1(e_1, e_2(X_1)|e_1, e_2(X_1);\hat{S}) - EU_1(e_j, e_2(X_1)|e_1, e_2(X_1);\hat{S}) =\\ &EU_1(e_1, e_2(X_1)|e_1, e_2(X_1);S) - EU_1(e_j, e_2(X_1)|e_1, e_2(X_1);S) - \sum_n [\pi_n^1 - \pi_n^j]\Big\{u_n + \sum_\ell \pi_\ell^1 \mu(u_n - u_\ell)\Big\}\\ &+ \gamma \sum_n [\pi_n^1 - \pi_n^j]\Big\{\sum_\ell \sum_m \pi_\ell^1 \pi_m^2 [\mu(u_{n\ell} - u_{\ell m} + \rho_n - \rho_\ell) - \mu(u_{n\ell} - u_{\ell m})]\Big\} + \sum_n [\pi_n^1 - \pi_n^j]\rho_n \geqslant 0 \quad \forall e_j \in \mathcal{E}\backslash e_1\end{aligned}$$

where the first curly bracket corresponds to the incentive lost from fixing the first-period wage, the second one–weighted by γ–corresponds to the incentives gained from prospective gain-loss utility whereas the last corresponds to the incentives gained from second-period consumption utility. To simplify the notation, define $\phi(\rho_n, \rho_\ell) \equiv \mu(u_{n\ell} - u_{\ell m} + \rho_n - \rho_\ell) - \mu(u_{n\ell} - u_{\ell m})$. Then, incentives are preserved if the sequence $\{\rho_n\}_{n=1}^N$ satisfies

$$\sum_n [\pi_n^1 - \pi_n^j]\Big\{u_n + \sum_\ell \pi_\ell^1 \mu(u_n - u_\ell)\Big\} = \gamma \sum_n [\pi_n^1 - \pi_n^j]\Big\{\sum_\ell \sum_m \pi_\ell^1 \pi_m^2 \phi(\rho_n, \rho_\ell)\Big\} + \sum_n [\pi_n^1 - \pi_n^j]\rho_n \quad \forall e_j \in \mathcal{E}\backslash e_1 \tag{19}$$

Existence of such sequence can be straightly proved by showing setting the system of $J-1$ equations as $A\rho = B$–where A is a $(J-1) \times N$ matrix and B is a $(J-1)$ vector–and showing that B can be written as a linear combination of the cols of A.

Consider first the set $\widetilde{\mathcal{E}} = \{e \in \mathcal{E} | c(e) \leqslant c(e_1)\} \subseteq \mathcal{E}$ and consider period-zero planning IC. Adding and subtracting, and noticing that the equilibrium total period one and period zero coincide, is straightforward to see that the period-zero planning IC can be written as

$$\begin{aligned}
&EU_0(e_1, e_2(X_1)|e_1, e_2(X_1); \hat{S}) - EU_0(e_j, e_k(X_1)|e_j, e_k(X_1); \hat{S}) \\
=&\Big(EU_1(e_1, e_2(X_1)|e_1, e_2(X_1); \hat{S}) - EU_1(e_j, e_2(X_1)|e_1, e_2(X_1); \hat{S})\Big) \quad -\Big(EU_1(e_j, e_k(X_1)|e_j, e_k(X_1); \hat{S}) \\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad -EU_1(e_j, e_2(X_1)|e_1, e_2(X_1); \hat{S})\Big)
\end{aligned}$$

Because under the optimal sequence $\{\rho^*\}$ the implementing IC holds, it is sufficient to find conditions for the second line to be negative. For this, notice that this planning IC is necessary iff $(e_j, e_k(x_n))$ is a credible plan, i.e. if $EU_0(e_j, e_k(x_n)|e_j, e_k(x_n)) \geqslant EU_0(e_j, e_2|e_j, e_k(x_n))$. Thus,

$$\begin{aligned}
&EU_1(e_j, e_k(x_n)|e_j, e_k(x_n); \hat{S}) - EU_1(e_j, e_2|e_1, e_2(x_n); \hat{S}) \\
\leqslant &EU_1(e_j, e_2(x_n)|e_j, e_k(x_n); \hat{S}) - EU_1(e_j, e_2(x_n)|e_j, e_k(x_n); \hat{S}) \\
\leqslant &EU_0(e_j, e_2(x_n)|e_j, e_2(x_n)) - EU_1(e_j, e_2|e_1, e_2(x_n))
\end{aligned}$$

where the last line follows from the fact that second-period IC holds. Thus, if

$$\gamma \leqslant \frac{\min_j\{\mu(c(e_1) - c(e_j))\}}{\max_j\{\sum_n \sum_\ell \sum_m [\pi_n^j \pi_\ell^j \pi_m^2 - \pi_n^j \pi_\ell^1 \pi_m^2]\phi(\rho_n^*, \rho_\ell^*)\}} = \bar{\gamma}_1 \tag{20}$$

the period-zero planning IC is implied by the period-one implementing IC. Furthermore, notice that $\bar{\gamma}_1 > 0$ because the numerator is positive for all j since prospective gain-loss utility is decreasing in the reference distribution.

Consider now the difference between total period-zero utility between contracts for the optimal sequence $\{\rho_n^*\}$. Modifying equation (9) in Section 9.2 and using Definition 1, we have that under contract $\hat{S}$ the total period-zero expected utility corresponds to,

$$\begin{aligned} EU_0(e_1, e_2(X_1)|e_1, e_2(X_1); \hat{S}) = & \sum_n \pi_n^1 \bar{u} + \gamma \sum_n \sum_\ell \sum_m \pi_n^1 \pi_\ell^1 \pi_m^2 \mu(u_{n\ell} - u_{\ell m} + \rho_h - \rho_\ell) \\ & + \sum_n \pi_n^1 [EU_2(e_2(x_n)|e_2(x_n); x_n; S) + \rho_n] \\ & - c(\tilde{e}_{0,1}) + \gamma \sum_n \sum_\ell \pi_n^{0,1} \pi_\ell^{0,1} \mu(c(e_2(x_\ell)) - c(e_2(x_n))) \end{aligned} \tag{21}$$

Taking differences between equation (9) in Section 9.2 and equation (21), we have that the deviation contract gives a higher expected utility if

$$\gamma \sum_n \sum_\ell \sum_m \pi_n^1 \pi_\ell^1 \pi_m^2 \phi(\rho_n^*, \rho_\ell^*) + \sum_n \pi_n^1 \rho_n \leqslant - \sum_n \sum_\ell \pi_n^1 \pi_n^1 \sum_\ell^1 \mu(u_n - u_\ell) \tag{22}$$

where the RHS and LHS are positive because of Lemma 2 and because $\{\rho_n^*\}$ is increasing, respectively. Using equations (19) for the optimal sequence, we can rewrite equation (22) as

$$\sum_n [\pi_n^1 - \pi_n^j](u_n - \rho_n^*) - \sum_n \sum_\ell^1 \mu(u_n - u_\ell) \geqslant \gamma[- \sum_n \sum_\ell \sum_m \pi_n^j \pi_\ell^1 \pi_m^2 \phi(\rho_n^*, \rho_\ell^*)] \tag{23}$$

Thus, if $\gamma \leqslant \min\{\bar{\gamma}_1, \bar{\gamma}_2\}$ where

$$\bar{\gamma}_2 = \frac{\min_j \{\sum_n [\pi_n^1 - \pi_n^j](u_n - \rho_n^*) - \sum_n \sum_\ell^1 \mu(u_n - u_\ell)\}}{\max_j \{- \sum_n \sum_\ell \sum_m \pi_n^j \pi_\ell^1 \pi_m^2 \phi(\rho_n^*, \rho_\ell^*)\}} \tag{24}$$

where $\bar{\gamma}_2 > 0$ because of equation (19) and Lemma 3. Then $\hat{S}$ implements the action path at the same expected cost for the principal and increases the agent's total period-zero utility in the restricted set $\widetilde{\mathcal{E}}$.

We are done in the linear consumption utility case if can show that adding actions does not decrease the principal's expected cost under the contract $\hat{S}$. To see this, first notice that working for the period-zero planning IC for both contracts and operating, we have that period-zero planning IC holds under contract $\hat{S}$ can be written as

$$\begin{aligned} & EU_0(e_1, e_2(X_1)|e_1, e_2(X_1); \hat{S}) - EU_0(e_j, e_k(X_1)|e_j, e_k(X_1); \hat{S}) = \\ & EU_0(e_1, e_2(X_1)|e_1, e_2(X_1); S) - EU_0(e_j, e_k(X_1)|e_j, e_k(X_1); S) - \sum_n [\pi_n^1 - \pi_n^j]\{u_n - \rho_n^*\} \\ & - \sum_n \sum_\ell [\pi_n^1 \pi_\ell^1 - \pi_n^j \pi_\ell^j] \mu(u_n - u_\ell) + \gamma \sum_n \sum_\ell \sum_m [\pi_n^1 \pi_\ell^1 \pi_m^2 - \pi_n^j \pi_\ell^j \pi_m^k] \phi(\rho_n^*, \rho_\ell^*) \geqslant 0 \quad \forall (e_j, e_k(X_1)) \in \mathcal{E}^{PE} \end{aligned} \tag{25}$$

where $\mathcal{E}^{PE}$ is as in Definition 1. By contradiction, assume that there exists a $e_j \in \mathcal{E}\backslash\widetilde{\mathcal{E}}$ such that

$$
\begin{aligned}
&EU_1(e_j, e_2(X_1)|e_1, e_2(X_1); \hat{S}) - EU_1(e_1, e_2(X_1)|e_1, e_2(X_1); \hat{S}) = \\
&EU_1(e_j, e_2(X_1)|e_1, e_2(X_1); S) - EU_1(e_1, e_2(X_1)|e_1, e_2(X_1); S) - \sum_n [\pi_n^j - \pi_n^1]\Big\{u_n + \sum_\ell \pi_\ell^1 \mu(u_n - u_\ell)\Big\} \\
&+ \gamma \sum_n [\pi_n^j - \pi_n^1]\Big\{\sum_\ell \sum_m \pi_\ell^1 \pi_m^2 \phi(\rho_n^*, \rho_\ell^*) + \sum_n [\pi_n^j - \pi_n^1]\rho_n^* \geqslant 0
\end{aligned}
$$

however, since S implements the action path we know that

$$EU_1(e_j, e_2(X_1)|e_1, e_2(X_1); S) - EU_1(e_1, e_2(X_1)|e_1, e_2(X_1); S) < 0$$

which contradicts equation (19). An analogous proof holds for period-zero planning IC. This completes the proof for the linear case.

Consider now the case where $u^{''} < 0$. Let $EC(S)$ and $EC(\hat{S})$ correspond to the expected cost of each contract. First, notice that if $\hat{S}$ is the optimal contract, given $\{\rho_n^*\}$, the fixed-wage u^* that maximizes the principal's profits corresponds to

$$u^* = U_R + c(e_1) - \gamma \sum_n \sum_\ell \sum_m \pi_n^1 \pi_\ell^1 \pi_m^2 \mu(u_{n\ell} - u_{\ell m} + \rho_h^* - \rho_\ell^*) - \sum_n \pi_n^1 EU_2(e_2(x_n)|e_2(x_n); S) < \sum_n \pi_n^1 u_n \tag{26}$$

where the last inequality follows from the IR under contract S. Thus the difference between the first-period expected cost between contract S and $\hat{S}$

$$EC_1(S) - EC_1(\hat{S}) = \sum_n \pi_n^1 u^{-1}(u_n) - u^{-1}(u^*) > \sum_n \pi_n^1 u^{-1}(u_n) - u^{-1}\Big(\sum_n \pi_n^1 u_n\Big) > 0$$

To consider the difference between the second-period expected cost, recall that the optimal deviation contract sets $\sum_n \pi_n \rho_n^* = 0$. Defining the sets $N_+ = \{n|\rho_n^* \geqslant 0\}$ and $N_- = \{n|\rho_n^* < 0\}$,

$$
\begin{aligned}
EC_2(S) - EC_2(\hat{S}) &= \sum_n \sum_m \pi_n^1 \pi_m^2 \Big(u^{-1}(u_{nm}) - u^{-1}(u_{nm} + \rho_n^*)\Big] \\
&= \sum_m \pi_m^2 \Bigg(\sum_{n \in N_-} \pi_n^1 \Big[u^{-1}(u_{nm}) - u^{-1}(u_{nm} + \rho_n^*)\Big] - \sum_{n \in N_+} \pi_n^1 \Big[u^{-1}(u_{nm} + \rho_n^*) - u^{-1}(u_{nm})\Big]\Bigg)
\end{aligned}
$$

Noticing that the first term is positive, for the contract $\hat{S}$ to have a smaller expected cost than S it suffices to find a condition for the following to hold

$$
\begin{aligned}
\sum_{n \in N_+} \sum_m \pi_n^1 \pi_m^2 \Big[u^{-1}(u_{nm} + \rho_n^*) - u^{-1}(u_{nm})\Big] &\leqslant \sum_n \pi_n^1 u^{-1}(u_n) - u^{-1}(u^*) \\
&+ \sum_{n \in N_-} \sum_m \pi_n^1 \pi_m^2 \Big[u^{-1}(u_{nm}) - u^{-1}(u_{nm} + \rho_n^*)\Big]
\end{aligned}
$$

where we have already showed that the right hand side is positive. For that notice that for all m, $n \in N_+$ there exists an $u_{nm}^+ \in (u_{nm}, u_{nm} + \rho_n^*)$ such that $u^{-1^{'}}(u_{nm}^+)\rho_n^* = u^{-1}(u_{nm} + \rho_n^*) - u^{-1}(u_{nm})$ where $u^{-1^{'}}(\cdot)$ is the inverse's derivative. Defining $u_{\max}^+ = \max\{\{u_{nm}^+\}_{n \in N_+, m}\}$, we can find an upper bound for the increase in the second-period expected cost,

$$\sum_{n\in N_+}\sum_m \pi_n^1\pi_m^2\Big[u^{-1}(u_{nm}+\rho_n^*)-u^{-1}(u_{nm})\Big]=\sum_{n\in N_+}\sum_m \pi_n^1\pi_m^2 u^{-1'}(u_{nm}^+)\rho_n^*\leqslant u^{-1'}(u_{\max}^+)\sum_{n\in N_+}\pi_n^1\rho_n^*$$

Analogously, we can define a lower bound for the second-period expected cost gain by noticing that for all m, $n\in N_-$ there exists an $u_{nm}^-\in(u_{nm}+\rho_n^*,u_{nm})$ such that $u^{-1'}(u_{nm}^-)(-\rho_n^*)=u^{-1}(u_{nm})-u^{-1}(u_{nm}+\rho_n^*)$ so that

$$u^{-1'}(u_{\min}^-)\sum_{n\in N_-}\pi_n^1(-\rho_n^*)\leqslant\sum_{n\in N_-}\sum_m \pi_n^1\pi_m^2\Big[u^{-1}(u_{nm})-u^{-1}(u_{nm}+\rho_n^*)\Big]$$

where $u_{\min}^-=\min\{\{u_{nm}^-\}_{n\in N_-,m}\}<u_{\max}^+$ because $\{\rho^*\}$ is nondecreasing. Now, by assumption, there exists a M_1 such that $u^{-1''}(s)\leqslant M_1\ \forall s$. Thus, using the mean value theorem again, we now that $u^{-1'}(u_{\max}^+)-u^{-1'}(u_{\min}^-)\leqslant M_1(u_{\max}^+-u_{\min}^-)$. Thus, the expected cost of $\hat S$ is no greater than that of S if M_1 is such that

$$\begin{aligned}M_1&\leqslant\frac{\sum_n\pi_n^1u^{-1}(u_n)-u^{-1}(u^*)+u^{-1'}(u_{\min}^-)\big(\sum_{n\in N_-}\pi_n^1(-\rho_n^*)-\sum_{n\in N_+}\pi_n^1\rho_n^*\big)}{(u_{\max}^+-u_{\min}^-)}\\&=\frac{\sum_n\pi_n^1u^{-1}(u_n)-u^{-1}(u^*)}{(u_{\max}^+-u_{\min}^-)}\end{aligned}$$

where this upper bound is positive by Jensen's inequality and equation (26). This concludes the proof.

■

Proof of Proposition 6

The fact the contract uses contingent payments in the first period follows straight from the proof of Proposition 5 by defining $\gamma>\min\{\bar\gamma_1,\bar\gamma_2\}$. To see that the contract does not use memory if γ is big enough, suppose the contract S is such that $u_n\neq u_n^{'}$ for at least one $n\neq n^{'}$ and $u_{nm}=u_{n'm}$ for $u_n\neq u_n^{'}$ $\forall m$. Consider now a deviation contract using memory in wages, $\hat S$, $\hat u_n=u_n+\rho_n$ $\rho_n\in\mathbb{R}$, $\rho_n\neq 0$ for at least one n and $\hat u_{nm}\neq\hat u_{nm}^{'}$, $n\neq n^{'}$ for at least one m.

Consider the case of linear consumption utility. Building the first-period implementing IC and the period-zero planning IC, using a proof equivalent to that in the proof of Proposition 5, one can show that there exists a $\{\rho_n^*\}$ such that $\hat S$ implements the effort path. Consider now the difference between total period-zero expected utility under contract $\hat S$ relative to S

$$\begin{aligned}EU_0(\hat S)-EU_0(S)=&\sum_n\pi_n^1\rho_n^*+\sum_n\sum_\ell\pi_n^1\pi_\ell^1\left[\mu(u_n-u_\ell+(\rho_n^*-\rho_\ell^*))-\mu(u_n-u_\ell)\right]\\&+\gamma\Big(\sum_n\sum_\ell\sum_m\pi_n^1\pi_\ell^1\pi_m^2\mu(\hat u_{n\ell}-\hat u_{\ell m})+\sum_n\sum_\ell\pi_n^1\pi_\ell^1\mu(c(e_2(x_\ell))-c_2(x_n))\Big)\\&+\sum_n\pi_n^1\big(EU_2(e_s(x_n)|e_2(x_n);x_n,\hat S)-EU_2(e_s(x_n)|e_2(x_n);x_n,S)\big)\end{aligned}$$

where $EU_0(\hat S)$ stands for $EU_0(e_1,e_(x_n)|e_1,e_(x_n);\hat S)$ and the same for s. Noticing that from Lemma 2 the second line is negative, then is straight to see that for any $\{\rho_n^*\}$ and any $EC(\hat S)-EC(S)$

there is always a γ such that $EU_0(e_1|e_1;\hat{S}) < EU_0(e_1|e_1;S)$ so that $\hat{S}$ is not a profitable deviation.

■

Proof of Proposition 7

Let $\widetilde{\pi}_m = \mathbb{P}\{\widetilde{X} = \widetilde{x}_m\} > 0$ for all $\widetilde{x}_m \in \mathcal{X}$ represent the density of the random device. Consider first the utilities the agent gets under the random contract S. Let $EU_2^2(e_2(x_n)|e_2(x_n);S)$ represent the total expected utility at the end of period-two once the agent has exerted e_2, which corresponds to

$$EU_2^2(e_2(x_n)|e_2(x_n);S) = \sum_m \widetilde{\pi}_m u_{nm} + \sum_m \sum_\ell \widetilde{\pi}_m \widetilde{\pi}_\ell \mu(u_{nm} - u_{n\ell})$$

Thus, the total expected utility at the beginning of period two having planned to exert $\widetilde{e}_{1,2}(x_n)$ at the end of period one

$$EU_2^1(e_2(x_n)|\widetilde{e}_{1,2}(x_n);S) = \sum_n \pi_n^2 \Big[\sum_m \widetilde{\pi}_m u_{nm} + \sum_m \sum_\ell \widetilde{\pi}_m \widetilde{\pi}_\ell \mu(u_{nm} - u_{n\ell})\Big] - c(e_2(x_n)) + \mu\big(c(e) - c(\widetilde{e}_{1,2}(x_n))\big)$$

Thus, the second-period implementing IC for any n corresponds to

$$\sum_n [\pi_n^2 - \pi_n^j]\Big[\sum_m \widetilde{\pi}_m u_{nm} + \sum_m \sum_\ell \widetilde{\pi}_m \widetilde{\pi}_\ell \mu(u_{nm} - u_{n\ell})\Big] - \big(c(e_2(x_n)) - c(e_j)\big) - \mu(c(e_2(x_n)) - c(e_j))$$

(notice that a period-one planning IC equal the latter except for the fact that $\mu\big(c(e_2(x_n)) - c(e_j)\big) = 0\big)$. Because Proposition 5 assumptions hold, let $\bar{u}_m$ be the first-period payment depending on the first period $\widetilde{X}$ but not on X_1. As before, the expected utility flow of the agent once the agent has executed the first-period action corresponds to $\sum_m \widetilde{\pi}_m \bar{u}_m + \sum_m \sum_\ell \widetilde{\pi}_m \widetilde{\pi}_\ell \mu(\bar{u}_m - \bar{u}_\ell)$. Thus, the total period-one expected utility of implementing action e_1 having planned corresponds to

$$\begin{aligned} EU_1^1(e_1, \widetilde{e}_{0,2}(x_n)|\widetilde{e}_{0,1}, \widetilde{e}_{0,2}(x_n);S) = &\sum_m \widetilde{\pi}_m \bar{u}_m + \sum_m \sum_\ell \widetilde{\pi}_m \widetilde{\pi}_\ell \mu(\bar{u}_m - \bar{u}_\ell) \\ &+ \sum_n \pi_n^1 EU_2^1(\widetilde{e}_{0,2}(x_n)|\widetilde{e}_{0,2}(x_n); x_n, S) - c(e_1) - \mu\big(c(\widetilde{e}_{0,1}) - c(e_1)\big) \end{aligned}$$

Thus for all $e_j \in \mathcal{E}\backslash e_1$ the period-one implementing IC corresponds to

$$\begin{aligned} &EU_1^1(e_1, \widetilde{e}_{0,2}(x_n)|\widetilde{e}_{0,1}, \widetilde{e}_{0,2}(x_n);S) - EU_1^1(e_j, \widetilde{e}_{0,2}(x_n)|\widetilde{e}_{0,1}, \widetilde{e}_{0,2}(x_n);S) \\ &= \sum_m \widetilde{\pi}_m \bar{u}_m + \sum_m \sum_\ell \widetilde{\pi}_m \widetilde{\pi}_\ell \mu(\bar{u}_m - \bar{u}_\ell) + \sum_n [\pi_n^1 - \pi_n^j] EU_2^1(\widetilde{e}_{0,2}(x_n)|\widetilde{e}_{0,2}(x_n); x_n, S) - c(e_1) - \mu\big(c(\widetilde{e}_{0,1}) - c(e_1)\big) \end{aligned}$$

where the period-zero planning IC equals this for $\mu\big(c(\widetilde{e}_{0,1}) - c(e_1)\big) = 0$. Then, by noticing that paying $u_{nm} = \sum_m \widetilde{\pi}_m u_{nm} + \sum_m \sum_\ell \widetilde{\pi}_m \widetilde{\pi}_\ell \mu(u_{nm} - u_{n\ell})$ and $\bar{u}_m = \sum_m \widetilde{\pi}_m \bar{u}_m + \sum_m \sum_\ell \widetilde{\pi}_m \widetilde{\pi}_\ell \mu(\bar{u}_m - \bar{u}_\ell)$ all the IC are satisfied and the agent is indifferent between the two contracts, the result follows straight from Jensen's inequality.

■

Proof of Proposition 8

We first restate the equilibrium concept. Now, period-zero planning IC does not ensure that the agent chooses the right plans at the end of period one and thus a period-one planning IC must be explicitly stated.

Definition 3 *(Modified PPE)*
Given a contract S, the effort path $(e_1^{PE}, e_2^{PE}(X_1))$ *is a "personal equilibrium" (PE) for the agent if*

$$\begin{array}{llcll}
(i) & EU_0(e_1^{PE}, e_2^{PE}(X_1)|e_1^{PE}, e_2^{PE}(X_1)) & \geqslant & EU_0(e_1, e_2(X_1)|e, e_2(X_1)) & \forall (e_1, e_2(X_1)) \in \mathcal{E}^{PE} \\
(ii) & EU_1(e_1^{PE}, e_2^{PE}(X_1)|e_1^{PE}, e_2^{PE}(X_1)) & \geqslant & EU_1(e_1, e_2^{PE}(X_1)|e_1^{PE}, e_2^{PE}(X_1)) & \forall e_1 \in \mathcal{E} \\
(iii) & EU_1(e_2^{PE}(x_n)|e_2^{PE}(x_n); x_n) & \geqslant & EU_1(e_2|e_2; x_n) & \forall e_2 \in \mathcal{E}_n^{PE}, \forall x_n \\
(iv) & EU_2(e_2^{PE}(x_n)|e_2^{PE}(x_n); x_n) & \geqslant & EU_2(e_2|e_2^{PE}(x_n); x_n) & \forall e_2 \in \mathcal{E}, \forall x_n
\end{array}$$

where $\mathcal{E}^{PE} \equiv \{e \in \mathcal{E}, e_2(X_1) \in \mathcal{E} \times \cdots \times \mathcal{E} | (ii), (iii)$ *and* (iv) *hold for* $e = e_1^{PE}$ *and* $e_2 = e_2(X_1)^{PE}\}$ *and* $\mathcal{E}_n^{PE} \equiv \{e \in \mathcal{E} | (iv)$ *holds for* $e = e_2^{PE}$ *given* $X_1 = x_n\}$.

First consider period-two implementing IC for any given realization of the first-period outcome $x_n \in \{x_h, x_\ell\}$,

$$\begin{aligned}
EU_2(e_h|e_h; x_n) - EU_2(e_\ell|e_h; x_n) = {} & [\pi_h - \pi_\ell](u_{nh} - u_{n\ell}) \\
& + [\pi_h(1-\pi_h) - \lambda\pi_h(1-\pi_h)](u_{nh} - u_{n\ell}) \\
& - [\pi_\ell(1-\pi_h) - \lambda\pi_h(1-\pi_\ell)](u_{nh} - u_{n\ell}) - c - \mu(c) \geqslant 0
\end{aligned}$$

I now build period one planning IC. From the analysis in Section 7, I have that after a first period failure the change in the prospective gain-loss utility loss from planning high effort for the second period instead of low, corresponds to $\lambda\eta(\pi_h - \pi_\ell)(u_{\ell h} - u_{\ell\ell}) \geqslant 0$. Thus, the period-one planning IC after a first-period failure corresponds to

$$\begin{aligned}
EU_2(e_h|e_h; x_\ell) - EU_2(e_\ell|e_\ell; x_\ell) = {} & [\pi_h - \pi_\ell](u_{\ell h} - u_{\ell\ell}) \\
& + [\pi_h(1-\pi_h) - \lambda\pi_h(1-\pi_h)](u_{\ell h} - u_{\ell\ell}) \\
& - [\pi_\ell(1-\pi_h) - \lambda\pi_h(1-\pi_\ell)](u_{\ell h} - u_{\ell\ell}) \\
& + \lambda\gamma(\pi_h - \pi_\ell)(u_{\ell h} - u_{\ell\ell}) - c \geqslant 0
\end{aligned}$$

Comparing the planning and implementing ICs, one can see that if

$$c(1-\gamma) > (\pi_h - \pi_\ell)[\lambda(1 - \pi_\ell - \gamma) - \pi_\ell] \tag{27}$$

then the planning IC implies the implementing IC and thus it must bind (in the opposite case the implementing IC binds). Equivalently, after a first first-period success, the change in prospective gain-loss utility from planning high effort for the second period instead of low, corresponds to $\eta(\pi_h - \pi_\ell)(u_{hh} - u_{h\ell}) \geqslant 0$. Thus, period-one planning IC after a first-period success corresponds to

$$\begin{aligned}
EU_2(e_h|e_h; x_h) - EU_2(e_\ell|e_\ell; x_h) = {} & [\pi_h - \pi_\ell](u_{hh} - u_{h\ell}) \\
& + [\pi_h(1-\pi_h) - \lambda\pi_h(1-\pi_h)](u_{hh} - u_{h\ell}) \\
& - [\pi_\ell(1-\pi_h) - \lambda\pi_h(1-\pi_\ell)](u_{hh} - u_{h\ell}) \\
& + \gamma\eta(\pi_h - \pi_\ell)(u_{hh} - u_{h\ell}) - c \geqslant 0
\end{aligned}$$

where have used that the principal wants to implement high effort in both periods. Comparing the planning and implementing ICs, one can see that if

$$c(1-\gamma) > (\pi_h - \pi_\ell)[(1 - \pi_\ell - \gamma) - \pi_\ell] \tag{28}$$

then the planning IC implies the implementing IC and thus it must bind (in the opposite case the implementing IC binds). Thus, four cases arise here. If both implementing IC's bind, because their RHS are equal, then by making them equal is straight to see that $(u_{\ell h} - u_{\ell\ell}) = (u_{hh} - u_{h\ell})$. If both planning IC's bind, doing the same, one can see that $(u_{\ell h} - u_{\ell\ell}) < (u_{hh} - u_{h\ell})$. The other two cases follow using the same logic and using the conditions in equations (27) and (28).

■

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