# Weak reactions on <sup>12</sup>C within the Continuum Random Phase Approximation with partial occupancies

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## Abstract

We extend our previous studies of the neutrino-induced reactions on <sup>12</sup>C and muon capture to include partial occupation of nuclear subshells in the framework of the continuum random phase approximation. We find, in contrast to the work by Auerbach et al., that a partial occupation of the  $p_{1/2}$ subshell reduces the inclusive cross sections only slightly. The extended model describes the muon capture rate and the  ${}^{12}C(\nu_e, e^-){}^{12}N$  cross section very well. The recently updated flux and the improved model bring the calculated <sup>12</sup>C( $\nu_{\mu}, \mu^{-}$ )<sup>12</sup>N cross section ( $\approx 17.5 \cdot 10^{-40}$  cm<sup>2</sup>) and the data  $((12.4 \pm 0.3 \text{(stat.}) \pm 1.8 \text{(syst.}) \cdot 10^{-40} \text{ cm}^2)$  closer together, but does not remove the discrepancy fully.

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#### I. INTRODUCTION

In recent years the study of neutrino-nucleus reactions has attracted continuously growing interest. Besides astrophysical motivations this interest has particularly been inspired by the use of neutrino-nucleus reactions as a laboratory to study fundamental properties of neutrinos. The best studied nucleus by  $\nu$ -induced reactions is <sup>12</sup>C, which is an ingredient of liquid scintillator and plays an important role in the oscillation search experiments performed by the KARMEN[[1](#page-10-0)] and LSND[[2\]](#page-10-0) collaborations.

The exclusive and inclusive charged-current reactions  ${}^{12}C(\nu_e, e^-){}^{12}N$  have been measured by the KARMEN and LSND collaborations for neutrinos generated by muon decay at rest [\[1](#page-10-0)–[4\]](#page-10-0). The KARMEN collaboration has supplemented this by a measurement of the neutral current<sup>12</sup>C( $\nu, \nu'$ )<sup>12</sup>C\*(15.11 MeV) cross section [[5\]](#page-10-0). Using the LAMPF pion-decay-in-flight neutrino source, the LSND collaboration has also measured the <sup>12</sup>C( $\nu_\mu$ ,  $\mu$ <sup>-</sup>)<sup>12</sup>N cross section [\[6](#page-10-0)]. Together with the precisely known inclusive and exclusive muon-capture rates on  ${}^{12}$ C [\[7](#page-10-0)], these measurements represent a challenge to the theorists.

Various data on the exclusive reactions (also including electro-magnetic transitions and beta decays), leading to the triad of  $T = 1, J^{\pi} = 1^{+}$  analog states in the  $A = 12$  nuclei (these are the <sup>12</sup>B and <sup>12</sup>N ground states and the excited state in <sup>12</sup>C at 15.11 MeV), appear to be consistent and are theoretically well described, validating the total neutrino flux determination[[8\]](#page-10-0). However, the situation is somewhat different for the inclusive reactions; the term is adopted here for the reactions leading to final states in <sup>12</sup>B and <sup>12</sup>N other than the ground state (in the following denoted by  ${}^{12}B^*$  and  ${}^{12}N^*$ , respectively) and involving states in the continuum. The data on inclusive weak processes  $({}^{12}C(\nu_e, e^-)^{12}N^*, {}^{12}C(\nu_\mu, \mu^-)^{12}N^*$ and muon capture) are usually not simultaneously reproduced by calculations [\[9](#page-10-0),[10](#page-10-0)], with a notable exception [\[11](#page-10-0)]. For example, a reasonable model to calculate these cross sections is the continuum random phase approximation (RPA). In fact, one finds that this model describes the inclusive muon capture rate and the <sup>12</sup>C( $\nu_e$ ,  $e^-$ )<sup>12</sup>N<sup>\*</sup> cross section very well, but it overestimates the <sup>12</sup>C( $\nu_{\mu}, \mu^{-}$ )<sup>12</sup>N<sup>\*</sup> data noticeably. Of course, these three weak processes probe the nuclear response at different momentum  $(q)$  and energy  $(\omega)$  transfers, and thus the good description of the capture rate (with average values  $\overline{\omega} \approx 25$  MeV,  $\overline{q} \approx 80$  MeV/c) and the  $\nu_e$ -induced cross section ( $\overline{\omega} \approx 23$  MeV,  $\overline{q} \approx 50$  MeV/c) might simply indicate that the model does well for low values of q and  $\omega$ , but it is less reliable at larger momentum and energy transfers which are important for the <sup>12</sup>C( $\nu_\mu$ ,  $\mu^-$ )<sup>12</sup>N<sup>\*</sup> experiment ( $\overline{\omega} \approx 37$  MeV,  $\overline{q} \approx 200$  MeV). Such hypothesis has, however, been invalidated by the continuum RPA study of the inelastic electron scattering on  ${}^{12}$ C where the model has been demonstrated to describe well the data for momentum and energy transfer regions relevant for the LSND experiment [\[12\]](#page-10-0).

Could the origin of the discrepancy be nuclear correlations missing in the continuum RPA model? This is in fact what the authors of Ref. [\[11](#page-10-0)] suggest. First, they point out that, due to pairing effects, there are about 1.6 nucleons occupying the  $p_{1/2}$  subshell in the <sup>12</sup>C ground state, rather than the closed  $p_{3/2}$  configuration (and empty  $p_{1/2}$  subshell) assumed in the previous continuum RPA calculations. Second, they argue on the basis of a standard RPA calculation that this partial occupation in fact allows one to simultaneously describe all three inclusive weak processes.

Obviously the method of choice to test this conjecture is the interacting shell model.

But a reliable calculation of the <sup>12</sup>C( $\nu_{\mu}, \mu^{-}$ )<sup>12</sup>N<sup>\*</sup> cross section requires inclusion of at least the complete  $3\hbar\omega$  model space, which is a formidable task even on modern computers. Short of such a large-scale shell model calculation, we have performed a set of continuum (standard) RPA calculations for the various inclusive (and exclusive) cross sections extending our previous model to allow for partial occupancies of the subshells. The second motivation for repeating our previous calculations within an improved model is that recently the decayin-flight neutrino spectrum for the LSND experiment has been revised. With the updated flux, the experimental cross section slightly increased to  $(12.4 \pm 0.3(\text{stat.}) \pm 1.8(\text{syst.}) \cdot 10^{-40}$ cm<sup>2</sup> [[13\]](#page-10-0). As the LSND experiment is such an important benchmark we feel obliged to also consistently update our calculated cross section allowing for a meaningful comparison.

The continuum random phase approximation for nuclei with closed-shell configurations is well documented in the literature[[14](#page-10-0)]. A generalization of this formalism to partial occupancies is readily achieved by multiplying the relevant matrix elements in Eq. (17) of Ref.[[14\]](#page-10-0) with the partial occupation numbers,  $n_h$ , for the hole states, thus

$$
\langle \mathbf{p} \mathbf{h}^{-1} | V | \mathbf{p}' \mathbf{h}'^{-1} \rangle \to n_h n_{h'} \langle \mathbf{p} \mathbf{h}^{-1} | V | \mathbf{p}' \mathbf{h}'^{-1} \rangle. \tag{1}
$$

After this replacement, the numerical solution of the Continuum RPA equations follow the lines as outlined in Ref. [\[14](#page-10-0)]. The orthogonalization of the particle states on the hole states is performed by Schmidt's procedure.

With the exception of the  $T = 1$  triad of bound states in the  $A = 12$  nuclei, all other final states (which contribute to the inclusive cross section) lie above the particle emission threshold and therefore are described as resonances within the continuum RPA model. As residual interactions we used as before the finite-range G-matrix [\[15\]](#page-10-0) derived from the Bonn NN potential (BP) and, in order to estimate the theoretical uncertainties, also an empirical zero-range Landau-Migdal force (LM).

To derive an estimate of the partial occupancies of the  $p_{1/2}$  subshell, we have performed a shell model calculation for <sup>12</sup>C with the OXBASH code[[16\]](#page-10-0) and the Cohen-Kurath inter-action[[17\]](#page-10-0). This calculation suggests a partial occupation for the  $p_{1/2}$  subshell of 0.75 for protons and neutron, respectively. This result is rather close to the value 0.8 used in Ref. [\[11](#page-10-0)].

First we recalculated the various exclusive processes leading to the  $T = 1$  triad in the  $A = 12$  nuclei. Since these states are discrete we used the standard RPA in this calculation, again including partial occupancies as outlined above. As single particle states we considered the complete  $(1s)(1p)(2s1d)(2p1f)$  model space. For the proton and neutron  $p_{3/2}$  and  $p_{1/2}$ single particle energies we used the experimental values; the others have been determined from a Woods-Saxon potential which describes the  ${}^{12}$ C charge distribution well [\[18](#page-10-0)]. We have checked that an enlargement of the model space does not affect our results. Our RPA results are compared to data in Tables 1 and 2.

In Table 1 the rescaling factors N needed to bring calculation and data in agreement are shown. Note, that allowing the partial occupancy of the  $p_{1/2}$  subshell reduces the value of N substantially, from about 4 to less than 2. At the same time the rescaling factors are essentially identical for the three processes considered, showing consistency. The necessity of using such renormalization reflects again the well known fact that random phase approximation calculations do not describe all (proton-neutron) correlations necessary to reproduce the quenching of the Gamow-Teller strength[[26](#page-10-0),[27](#page-10-0)]. This aim can be only achieved in

shell-model calculations within a complete major shell, introducing additionally the renormalization of the spin operator by the universal factor 0.77 [\[28–30](#page-10-0)]. Deviating from this universal finding, the Cohen-Kurath interaction had been designed to describe the weak interaction transition rates in  ${}^{12}$ C without incorporating the renormalization of the spin operator. Thus the fact that  $N > 1$  in our calculation again shows the wellknown result that the reproduction of the Gamow-Teller strength requires the consideration of genuine correlations beyond a simple mean-field approach.

In Table 2 we compare the results on the exclusive neutrino induced reactions with data. The entries are based on  $N^{new} = 1.5$  and  $N^{old} = 4.0$ . The agreement remains good, and at the same time, as mentioned above, the required renormalization is substantially smaller than in our previous RPA calculation. The reduction is caused by the destructive interference of the  $p_{3/2}$  and  $p_{1/2}$  configurations in the RPA description of the Gamow-Teller transition, as discussed by Auerbach et al. [\[11\]](#page-10-0).

We then studied the three inclusive processes within our improved continuum RPA model. The noticeable mixture of  $p_{1/2}$  configuration into the <sup>12</sup>C ground state allows for the excitation of a larger number of resonances than in the previous calculation. This is demon-strated in Fig. [1,](#page-9-0) where the <sup>12</sup>C( $\nu_{\mu}, \mu^{-}$ )<sup>12</sup>N<sup>\*</sup> cross section (calculated with the BP force) is shown as a function of the excitation energy  $\omega$  of the nucleus. However, this figure also demonstrates that the strength going into these additional resonances is taken away from those transitions already present for the pure  $p_{3/2}$  ground state. In fact, Table 3 shows that the total inclusive rate and cross sections are only slightly reduced by the mixing of a  $p_{1/2}$ component into the ground state. The inclusive muon capture rate and the <sup>12</sup>C( $\nu_e, e^-$ )<sup>12</sup>N<sup>∗</sup> cross section agree very well with data: For muon capture the improved continuum RPA yields for both residual interactions rates close to the experimental value  $(32.8 \cdot 10^3 \text{ s}^{-1})$ , and the calculated <sup>12</sup>C( $\nu_e$ ,  $e^{-}$ )<sup>12</sup>N<sup>\*</sup> cross sections are also close to the most recent data  $(5.1 \pm 0.6 \pm 0.5) \cdot 10^{-42}$  cm<sup>2</sup> [\[3](#page-10-0)] and  $(5.7 \pm 0.6 \pm 0.6) \cdot 10^{-42}$  cm<sup>2</sup> [[4\]](#page-10-0). Our calculation of the inclusive <sup>12</sup>C( $\nu_{\mu}, \mu^{-}$ )<sup>12</sup>N<sup>\*</sup> cross section results also in a reduced value compared to our previous study, where the reduction is about half  $(0.8 \cdot 10^{-40} \text{ cm}^2 \text{ for BP, } 1.9 \cdot 10^{-40} \text{ cm}^2$ for LM) due to the revised flux and half  $(0.7 \cdot 10^{-40} \text{ cm}^2 \text{ for BP}, 0.9 \cdot 10^{-40} \text{ cm}^2 \text{ for LM})$ due to the partial occupation of the  $p_{1/2}$  subshell in the ground state wave function. Our new total <sup>12</sup>C( $\nu_\mu$ ,  $\mu$ <sup>-</sup>)<sup>12</sup>N cross section (inclusive + normalized exclusive) is then 17.8 · 10<sup>-40</sup>  $\text{cm}^2$  and  $17.5 \cdot 10^{-40}$  cm<sup>2</sup> for the BP- and LM-force, respectively, to be compared with the last experimental value of  $((12.4 \pm 0.3(\text{stat.}) \pm 1.8(\text{syst.}) \cdot 10^{-40} \text{ cm}^2)$ . Thus the disturbing discrepancy is less severe than before, but it not entirely removed.

Our result does not support in detail the argumentation in Ref. [\[11\]](#page-10-0) that the discrepancy between RPA calculations and data can be essentially fully removed if  $p_{1/2}$  partial occupancy is considered (Ref. [\[11](#page-10-0)] gives  $(13.5-15.2)\cdot 10^{-40}$  cm<sup>2</sup>) and all inclusive and exclusive processes can be simultaneously and consistently described. We also mention that Singh et al. [\[32](#page-10-0)] have studied the various processes based on the local density approximation and find a total (exclusive + inclusive) muon capture rate of  $(3.6 \pm 0.22) \cdot 10^4$  s<sup>-1</sup> [[33](#page-11-0)] (the experimental value for the total rate is  $(3.8 \pm 0.10) \cdot 10^4 \text{ s}^{-1})$  and a <sup>12</sup>C( $\nu_\mu$ ,  $\mu$ <sup>-</sup>)<sup>12</sup>N cross section of (16.5 ± 1.3)10<sup>−</sup><sup>40</sup> cm<sup>2</sup> [\[32](#page-10-0)], slightly lower, but consistent with our new result. Notably, Mintz and collaborators predicted the <sup>12</sup>C( $\nu_{\mu}, \mu^{-}$ )<sup>12</sup>N cross section [\[34](#page-11-0)], but the model assumptions within the elementary particle model used in the approach of these authors are questionable and not justified for this reaction [\[12\]](#page-10-0).

In summary, we have found that the consideration of a partial  $p_{1/2}$  occupancy lowers the exclusive cross sections and muon capture rate noticeably, but it reduces the results for the inclusive processes only slightly. The reason for the reduction of the exclusive cross sections has been discussed above. But can one understand why the continuum RPA is apparently able to describe the inclusive processes without the need for parameter adjustment, unlike the exclusive Gamow-Teller transitions? Another way of thinking about this problem is the question to what extent the correlations of nucleons within the p-shell influence the results. To shed the light on this, one can evaluate the total strength of various operators  $O$ , i.e. the norm of the vector  $\hat{O}(q.s.)$ , with and without the effect of correlations. Note that the total strength depends only on the ground state wave function and is therefore relatively easy to evaluate. (Here we follow the points made in Ref.[[35](#page-11-0)]).

For the positive parity operators this is done in Table 4. Table 4 illustrates the well known fact that the p shell correlations are very important for the Gamow-Teller operator  $\sigma\tau$ . (Note that the RPA gives the total Gamow-Teller strength of 5.5, only slightly reduced when compared to the 'naive' estimate of the independent particle model. Also, the exact shell model predicts that a strength of  $0.12$  goes to excited  $1^+$  states, which are absent in the naive model.)

But the situation with the quadrupole operators is rather different. The total  $p$  shell strength of the spin-independent operator, and the strength summed over the multipoles of the spin-dependent operators is affected by the correlations only on the  $30 - 40\%$  level, even though the individual spin-dependent multipoles are affected more. Moreover, the p shell strength represents only a small fraction of the total quadrupole strength, which is concentrated in the  $2\hbar\omega$  excitations, unaffected by correlations as long as we assume that the ground state has only p shell nucleons.

The lowest  $2^+$  resonance has a dominating  $(0p)$  configuration and thus its contribution to the various cross sections might be affected by the present more flexible description of the <sup>12</sup>C ground state. We find that the contribution of the  $2^+$  multipole to the  $\nu_e$ -induced cross section and to the muon capture rate increases slightly (by  $0.02 \cdot 10^{-42}$  cm<sup>2</sup> and  $0.17 \cdot 10^3$ s<sup>-1</sup>, respectively), but is still negligible compared to the total cross section and rate  $(0.8\%$ and 2.8%, respectively). For the  $\nu_{\mu}$ -induced reaction, the effects of the partial occupancy formalism and of the improved experimental neutrino flux nearly cancel each other so that the partial cross section of the  $2^+$  multipole slightly decreases (by  $0.08 \cdot 10^{-40}$  cm<sup>2</sup>). As the total cross section is also reduced, the contribution of the  $2^+$  multipole, of which the  $(0p)$ configuration is only a small fraction, to the total cross section is increased to 12% in our present calculation.

However, the inclusive reactions we are considering are dominated by the excitations of the negative parity states. To what extent does the strength depend on the occupation of the  $p_{1/2}$  subshell for these operators? According to Ref. [\[11](#page-10-0)] the cross sections should be noticeably reduced when there are about 1.6 nucleons in the  $p_{1/2}$  subshell. In order to test this point, we plot in Fig. [2](#page-9-0) the dipole and quadrupole strength as a function of the  $p_{1/2}$  subshell occupation. We find that when summed over multipoles the strength is totally independent of this occupation number. But even the individual multipoles depend on the occupation numbers only mildly. We find that the  $p$  shell correlations are relatively unimportant for the full strength.

Of course, the partial occupation of the  $p_{1/2}$  subshell leads to the occurrence of new

states, reshifting the strength. As the  $p_{1/2}$  orbital reflects an excited state compared to the  $p_{3/2}$  orbital, the strength is shifted slightly to higher energies. Thus, even if the total strength is unchanged, this redistribution of strength changes the cross section due to the related change of kinematics. As the strength is shifted to higher energies, its weight in the cross section is reduced, as apparent from our results. The relative importance of this effect decreases with increasing momentum and energy transfer, explaining why we find a reduction of about 20% for the  $\nu_e$  induced experiment, while it is only 4% for the  $\nu_\mu$ -induced reaction.

The LSND collaboration has recently conducted a search for  $\nu_{\mu} \rightarrow \nu_{e}$  oscillations for high energetic  $\nu_{\mu}$  ( $E_{\nu_{\mu}} \leq 300 \text{ MeV}$ ) stemming from  $\pi^{+}$  decay in flight. The oscillation signal consists of isolated, high-energy electrons (60 MeV $\leq E_{e^-} \leq 200$  MeV) in the detector coming from charged current <sup>12</sup>C( $\nu_e$ ,  $e^-$ )<sup>12</sup>N reactions. Such events cannot be caused by  $\nu_e$ -neutrinos from pion decay at rest, which are also produced in the beam stop at LAMPF, because their energies is lower than 52.8 MeV. In the experiment an excess in the number of these events was found, which, if interpreted as an oscillation signal, leads to an oscillation probability of  $(2.6 \pm 1.0 \pm 0.5) \times 10^{-3}$  [\[36\]](#page-11-0). This value and its systematic error depends dominantly on the knowledge of the inclusive <sup>12</sup>C( $\nu_e$ ,  $e^-$ )<sup>12</sup>N cross section and the absolute neutrino flux through the detector. To aid the analysis, we present in Table 5 this cross section calculated withinthe improved RPA model. Comparing with our previous results in Ref. [[10](#page-10-0)], we find for most of the neutrino energies a reduction of the cross section between 5–10%. As this reduction is smaller than the 10%-error attributed to the uncertainty of <sup>12</sup>C( $\nu_e$ ,  $e^{-}$ )<sup>12</sup>N cross section in the LSND-analysis[[36](#page-11-0)], it does not require a re-analysis of the experiment.

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#### TABLES

TABLE I. The rescaling factors N of the matrix elements for the exclusive transitions were determined for both of the applied residual interactions by comparison of the calculated rates  $(\omega_{\rm BP}^{\rm new}, \omega_{\rm LM}^{\rm new})$  with the experimental beta-decay rates and partial muon capture rate to the <sup>12</sup>B ground state (in units of  $s^{-1}$ ). Also shown are the previous results ( $\omega_{\rm BP}^{\rm old}$ ,  $\omega_{\rm LM}^{\rm old}$ ) obtained for a completely filled  $p_{3/2}$  subshell.

| process   | data |  |  | Ref. $\omega_{\rm BP}^{\rm new}$ $\omega_{\rm LM}^{\rm new}$ $N_{\rm BP}^2$ $N_{\rm LM}^2$ $\omega_{\rm BP}^{\rm old}$ $\omega_{\rm LM}^{\rm old}$ $N_{\rm BP}^2$ $N_{\rm LM}^2$ |  |  |
|---|------|--|--|--|--|--|
| $\left  {}^{12}C(\mu^-,\nu_\mu) {}^{12}B_{g.s.} \right  6.050 \pm 300(*)$ |      |  |  | $[19-22]$ 9 330 8 538 1.54 1.41 22 780 25 400 3.77 4.20  |  |  |
| $ ^{12}B_{g.s.}(\beta^-)^{12}C_{g.s.} $ 33.36 ± 0.13                      |      |  |  | $[23]$ 49.6 44.3 1.49 1.33 123.6 128.8 3.71 3.86   |  |  |
| $12N_{\rm g.s.}(\beta^+)^{12}\rm{C}_{\rm g.s.}$ 59.58 ± 0.46              |      |  |  | $[23]$ 98.7 88.2 1.66 1.48 247.1 257.4 4.15 4.32   |  |  |

TABLE II. The exclusive cross sections for charged and neutral current neutrino scattering (in units of  $10^{-42}$  cm<sup>2</sup>). The results of our improved RPA calculation for both of the applied residual interactions ( $\sigma_{\rm BP}^{\rm new}$ ,  $\sigma_{\rm LM}^{\rm new}$ ) are compared to the data and the previous RPA calculation ( $\sigma_{\rm BP}^{\rm old}$ ,  $\sigma_{\rm LM}^{\rm old}$ ) without partial  $p_{1/2}$  subshell occupation.

| process   | data.   | Ref.                         | $\sigma_{\rm BP}^{\rm new}$ | $\sigma_{\text{LM}}^{\text{new}}$ | $\sigma_{\rm BP}^{\rm old}$ | $\sigma_{\text{LM}}^{\text{old}}$ |
|---|---|------------------------------|-----------------------------|-----------------------------------|-----------------------------|-----------------------------------|
| ${}^{12}C(\nu_e, e^{-}){}^{12}N_{\rm g.s.}$               | $10.5 \pm 1.0(stat.) \pm 1.0(syst.)$            | $[24]$                       |                             |                                   |                             |                                   |
|   | $8.9 \pm 0.6$ (stat.) $\pm 0.75$ (syst.)        |                              | 8.9                         | 8.9                               | 9.3                         | 9.3                               |
|   | $9.1 \pm 0.4$ (stat.) $\pm 0.9$ (syst.)         | $\left\lceil 6 \right\rceil$ |                             |                                   |                             |                                   |
| ${}^{12}C(\nu_{\mu}, \mu^{-}){}^{12}N_{\rm g.s.}$         | $66 \pm 10(\text{stat.}) \pm 10(\text{syst.})$  | $\lceil 6 \rceil$            | 68                          | 73                                | 63                          | 63                                |
| ${}^{12}C(\nu,\nu'){}^{12}C^*(15.11)$                     | $10.4 \pm 1.0(stat.) \pm 0.9(syst.)$            | [5]                          | 10.5                        | 10.5                              | 10.5                        | 10.6                              |
| $\left[{}^{12}C(\nu_\mu, \nu'_\mu)^{12}C^*(15.11)\right]$ | $3.2 \pm 0.5({\rm stat.}) \pm 0.4({\rm syst.})$ | [25]                         | 2.8                         | 2.7                               | 2.8                         | 2.8                               |

TABLE III. The inclusive muon capture rate  $\omega$ , (in 10<sup>3</sup> s<sup>-1</sup>) and the cross sections  $\sigma$  for the <sup>12</sup>C( $\nu_e, e^{-}$ )<sup>12</sup>N<sup>\*</sup> (in units of 10<sup>-42</sup> cm<sup>2</sup>) and the total (inclusive + exclusive) cross section for the <sup>12</sup>C( $\nu_{\mu}, \mu^{-}$ )<sup>12</sup>N (in 10<sup>-40</sup> cm<sup>2</sup>) reactions.. The results of our improved continuum RPA calculation  $((\omega/\sigma)^{new})$  are compared to the data and the previous  $((\omega/\sigma)^{old})$  continuum RPA calculation without partial  $p_{1/2}$  subshell occupation.

| process                           | data                   | Ref.  | $(\omega/\sigma)_{\rm BP}^{\rm new}$ | $(\sigma)^{\rm new}_{\rm LM}$<br>$\omega/$ | $(\omega/\sigma)_{\rm BP}^{\rm old}$ | $)_{\rm LM}^{\rm old}$<br>$(\omega/\sigma)$ |
|-----------------------------------|------------------------|---|--------------------------------------|--|--------------------------------------|---|
| $12C(\mu^-,\nu_\mu)^{12}B^*$      | $32.8 \pm 0.8$         | $\left[31\right]$   | 32.7                                 | 31.3                                       | 34.2                                 | 33.3  |
| $ {}^{12}C(\nu_e,e^-){}^{12}N^* $ | $5.1 \pm 0.6 \pm 0.5$  | $\overline{3}$  | 5.4                                  | 5.6  | $6.3\,$                              | 5.9   |
|                                   | $5.7 \pm 0.6 \pm 0.6$  | $[4] % \includegraphics[width=1\textwidth]{images/TrDiM-Architecture.png} \caption{The figure shows the results of the estimators in the left hand side.} \label{TrDiM-Architecture}$ |                                      |  |                                      |   |
| $12C(\nu_{\mu}, \mu^{-})^{12}N$   | $12.4 \pm 0.3 \pm 1.8$ | $[13]$  | 17.8                                 | 17.5                                       | $19.3\,$                             | 20.3  |

TABLE IV. The full strength within the nuclear  $p$  shell evaluated for the operators in column 1. The 'SM' column is the exact shell model result calculated with the Cohen-Kurath interaction. The column named 'naive' corresponds to the  $(p_{3/2})^8$  configuration. In the last column the strength for transitions with  $2\hbar\omega$  is shown for comparison.

| Operator                                    | SM   | naive | $2\hbar\omega$ |
|---|------|-------|----------------|
| Gamow-Teller $(\sigma\tau)$                 | 1.51 | 8.00  | 0.00           |
| $r^2Y_2$                                    | 1.37 | 1.98  | 9.95           |
| $r^2(Y_2\sigma)^{I=1}$                      | 0.11 | 0.08  |                |
| $r^{2}(Y_{2}\sigma)^{I=2}$                  | 0.33 | 0.75  |                |
| $r^{2}(Y_{2}\sigma)^{I=3}$                  | 0.20 | 0.00  |                |
| $\sum_{\lambda} r^2 (Y_2 \sigma)^{\lambda}$ | 0.64 | 0.83  | 7.47           |

TABLE V. Total <sup>12</sup>C( $\nu_e$ ,  $e^-$ )<sup>12</sup>N, <sup>12</sup>C( $\bar{\nu}_e$ ,  $e^+$ )<sup>12</sup>B, and exclusive cross sections to the <sup>12</sup>N and <sup>12</sup>B ground states for a mesh of neutrino energies  $E_{\nu_e}$ . The cross sections have been calculated with the BP-interaction and are given in units of  $10^{-42}$  cm<sup>2</sup>. Energies are in MeV, exponents are given in parentheses.

| $\overline{E_{\nu_e}}$ | ${}^{12}C(\nu_e, e^-){}^{12}N$ | ${}^{12}C(\nu_e, e^-){}^{12}N_{gs}$ | ${}^{12}C(\bar{\nu}_e,e^+){}^{12}B$ | ${}^{12}C(\bar{\nu}_e,e^+){}^{12}{\rm B}_{gs}$ |
|------------------------|--------------------------------|-------------------------------------|-------------------------------------|--|
| 20                     | $2.85(-1)$                     | $2.84(-1)$                          | $8.00(-1)$                          | $7.90(-1)$                                     |
| $30\,$                 | $5.65\,$                       | 4.90                                | $6.05\,$                            | $5.01\,$                                       |
| 40                     | $2.23 (+1)$                    | $1.46 (+1)$                         | $1.84 (+1)$                         | $1.17 (+1)$                                    |
| $50\,$                 | $5.99 (+1)$                    | $2.79 (+1)$                         | $4.14 (+1)$                         | $1.95 (+1)$                                    |
| 60                     | $1.31 (+2)$                    | $4.31 (+1)$                         | 7.80 $(+1)$                         | $2.72 (+1)$                                    |
| 70                     | $2.47 (+2)$                    | $5.82 (+1)$                         | $1.30 (+2)$                         | $3.41 (+1)$                                    |
| 80                     | 4.21 $(+2)$                    | 7.16 $(+1)$                         | $1.99 (+2)$                         | $3.97 (+1)$                                    |
| 90                     | 6.63 $(+2)$                    | $8.25 (+1)$                         | $2.84 (+2)$                         | 4.39 $(+1)$                                    |
| 100                    | $9.79 (+2)$                    | $9.03 (+1)$                         | 3.86 $(+2)$                         | 4.69 $(+1)$                                    |
| 110                    | $1.37 (+3)$                    | $9.49 (+1)$                         | $5.01 (+2)$                         | 4.90 $(+1)$                                    |
| 120                    | $1.85 (+3)$                    | $9.70 (+1)$                         | 6.29 $(+2)$                         | $5.04 (+1)$                                    |
| 130                    | $2.41 (+3)$                    | $9.70 (+1)$                         | 7.68 $(+2)$                         | $5.14 (+1)$                                    |
| 140                    | $3.06 (+3)$                    | $9.58 (+1)$                         | $9.17 (+2)$                         | $5.22 (+1)$                                    |
| 150                    | 3.79 $(+3)$                    | $9.37 (+1)$                         | $1.08 (+3)$                         | $5.29 (+1)$                                    |
| 160                    | 4.61 $(+3)$                    | $9.15 (+1)$                         | $1.24 (+3)$                         | $5.35 (+1)$                                    |
| 170                    | $5.51 (+3)$                    | $8.94 (+1)$                         | $1.42 (+3)$                         | $5.41 (+1)$                                    |
| 180                    | 6.49 $(+3)$                    | $8.77 (+1)$                         | $1.59 (+3)$                         | $5.47 (+1)$                                    |
| 190                    | 7.54 $(+3)$                    | $8.63 (+1)$                         | $1.78 (+3)$                         | $5.52 (+1)$                                    |
| 200                    | $8.66 (+3)$                    | $8.51 (+1)$                         | $1.96 (+3)$                         | $5.58(+1)$                                     |
| $210\,$                | $9.84 (+3)$                    | $8.42 (+1)$                         | $2.15 (+3)$                         | $5.63 (+1)$                                    |
| 220                    | $1.11 (+4)$                    | $8.35 (+1)$                         | $2.34 (+3)$                         | $5.68 (+1)$                                    |
| $230\,$                | $1.23 (+4)$                    | $8.28(+1)$                          | $2.53 (+3)$                         | $5.72 (+1)$                                    |
| $240\,$                | $1.36 (+4)$                    | $8.22 (+1)$                         | $2.72 (+3)$                         | $5.77 (+1)$                                    |
| $250\,$                | $1.50 (+4)$                    | $8.17 (+1)$                         | $2.91 (+3)$                         | $5.81 (+1)$                                    |
| 260                    | $1.63 (+4)$                    | $8.11 (+1)$                         | $3.11 (+3)$                         | $5.85 (+1)$                                    |
| $270\,$                | $1.76 (+4)$                    | $8.06 (+1)$                         | $3.30 (+3)$                         | $5.89 (+1)$                                    |
| $280\,$                | $1.90 (+4)$                    | $8.01 (+1)$                         | $3.50 (+3)$                         | $5.93 (+1)$                                    |
| 290                    | $2.03 (+4)$                    | 7.96 $(+1)$                         | 3.70 $(+3)$                         | $5.97 (+1)$                                    |
| 300                    | $2.16 (+4)$                    | 7.91 $(+1)$                         | 3.90 $(+3)$                         | $6.00 (+1)$                                    |

#### FIGURES

<span id="page-9-0"></span>FIG. 1. The <sup>12</sup>C( $\nu_{\mu}, \mu^{-}$ )<sup>12</sup>N<sup>\*</sup> cross sections as a function of the excitation energy  $\omega$  of the nucleus. The present results obtained with partial occupancy of the  $p_{1/2}$  orbital (labelled 'new') are compared to those assuming a pure  $(p_{3/2})^8$  closed-shell configuration for the <sup>12</sup>C ground state (labelled 'old').

FIG. 2. Total strength of the dipole (upper part) and quadrupole (lower part) operators versus the occupation of the  $p_{1/2}$  subshell. The curves are labeled by the corresponding angular momentum of the  $r^{l}(Y_{l}\sigma)^{I=\lambda}$  operators for  $l=1$  and  $l=2$ .

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