

# Inter-Jurisdictional Competition for Firms\*

by

**Robin Boadway**  
Queen's University  
boadwayr@qed.econ.queensu.ca

**Katherine Cuff**  
Queen's University  
cuffk@qed.econ.queensu.ca

**Nicolas Marceau**  
Université du Québec à Montréal  
marceau.nicolas@uqam.ca

October, 1999

**Abstract:** Regions inhabited with an immobile population of disabled and able individuals compete to attract mobile firms that provide jobs. The redistributive goal of regional governments is to support the disabled, who cannot work. Able individuals may work, be involuntary unemployed because of frictions in the labour market, or choose to be voluntary unemployed. Labour force participation decisions depend on regional redistributive policies. Both the size of workforce and tax on firms affect profits and therefore, firms' location decisions. Allowing regions to engage in tax competition may be efficient. If regions cannot tax firms, they compete by implementing inefficient redistributive policies.

**Key Words:** Inter-Jurisdictional Competition, Redistributive Policies, Unemployment

**JEL Classification:** H2, H7, J6

---

\* Presented at the Fourth Nordic Conference on the Economics of Local Government, Lund, Sweden, June 1999. We are grateful for comments by Lars-Erik Borge, Martin Browning, and Knud Munk. Financial support from the Social Sciences and Humanities Research Council of Canada is acknowledged.

# INTER-JURISDICTIONAL COMPETITION FOR FIRMS

by

**Robin Boadway, Queen's University**

**Katherine Cuff, Queen's University**

**Nicolas Marceau, Université du Québec à Montréal**

## 1. INTRODUCTION

Jurisdictions are typically constrained in achieving their policies by various sorts of competition from neighbouring jurisdictions, whether they be countries within an economic union, provinces within a federation or local and regional authorities within a unitary nation. The competition may arise from the mobility of capital, the mobility of labour, or cross-border trade in goods and services. There is a substantial literature on each of these form of inter-jurisdictional competition.<sup>1</sup> Considerable emphasis has been placed on the manner in which competition affects redistribution polices in a world where economies are becoming much more integrated, and labour and capital are increasingly mobile, while redistribution remains very much in the domain of lower-level jurisdictions.<sup>2</sup> This is also our concern in this paper.

Much of the literature on inter-jurisdictional competition in redistribution relies on relatively simplistic models in which the tax-transfer system is used to redistribute income from better-off to worse-off persons subject to some efficiency costs (perhaps arising from informational asymmetries) and subject to the constraints imposed by mobility. Though this is in keeping with the literature on optimal redistributive policy,<sup>3</sup> it belies much of what

---

<sup>1</sup> A recent survey of tax competition emphasizing capital and labour taxes may be found in Wilson (1999) and of commodity tax competition in Lockwood (1999). There is also a literature on competition in public expenditures (Keen and Marchand, 1997).

<sup>2</sup> This is a longstanding concern going back to Boskin (1973) and Pauly (1973), and its more recent incarnation in Wildasin (1993) and Hindriks (1998). The issues are thoroughly surveyed in Cremer *et al.* (1996).

<sup>3</sup> This literature is surveyed in Boadway and Keen (1999).

takes place in the real world. Actual redistributive policies are designed to a substantial extent to address problems arising from unemployment and the perceived shortage of jobs. Moreover, many of these policies are delivered outside of the progressive income tax system, including such policies as unemployment insurance, disability assistance and welfare.<sup>4</sup> These policies are typically administered by decentralized agencies which are often under the control of lower-level jurisdictions. It is natural to consider how the redistributive policies of these jurisdictions might be compromised by competition from their neighbours.

In this paper, we construct a simple model of redistribution in which those who receive transfers are unemployed, and unemployment is affected both by government policies and by frictions in the labour market. Jurisdictions, referred to as regions, implement the redistributive policies, but must compete with each other for firms which create jobs and reduce labour market frictions. In our model, persons may be working or unemployed. To focus on the importance of jobs, we assume there is only one type of worker, but there may be three types of unemployed persons — the voluntary unemployed, the involuntary unemployed, and those unable to work because of disability.<sup>5</sup> Regions want to redistribute towards the disabled, but their redistributive policies end up affecting the decision of workers to become voluntarily unemployed. As well, they can influence the number of jobs available. Job vacancies are created by firms, and we take the number of firms in the economy as a whole as given, reflecting, say, the number of entrepreneurs. Firms are mobile across regions, but workers are not. The filling of job vacancies within a region is governed by a simple matching technology which reflects the frictions associated with

---

<sup>4</sup> Recent literature has begun to focus on the importance of unemployment both as a source of need and as a constraint on redistribution: see Stiglitz (1999), Jacobsen and Sørensen (1999), and Boadway and Cuff (1999). There has also been some literature focusing on the consequences of unemployment for competition among governments (Fuest and Huber, 1999).

<sup>5</sup> Throughout this paper, we refer to an involuntarily unemployed individual as one who was unsuccessful at finding work. This is in contrast to a voluntarily unemployed individual who is one that did not look for work.

matching workers to jobs. As is standard, this matching process generates involuntary unemployment. Voluntary unemployment is obtained by supposing that workers, although equally productive, have differing utilities of leisure. Regional policies can influence the proportions of workers who are voluntarily and involuntarily unemployed. By increasing the number of workers who wish to participate in the job market, a region directly affects the wage rate and the number of jobs created and filled in its existing firms, but also indirectly affects the number of firms that locate in its region.

We investigate how the possibility of attracting firms affects a region's redistributive policies when it acts as a Nash competitor with respect to other regions. We show how the sub-game perfect Nash equilibrium (SPNE) depends upon which instruments are available to the regions. A key finding is that enabling regions to tax mobile firms and thereby to engage in self-defeating tax competition for firms might be a good policy. Without that instrument, they will compete for firms by adjusting their redistribution policies thereby rendering these policies inefficient.

To present our arguments in as simple a way as possible, we assume that regional governments are very well-informed. They can distinguish between the disabled and those able to work, and they can perfectly monitor whether or not those unemployed are actively searching for employment in the labour market.<sup>6</sup> The only individual characteristic they cannot observe is the utility that unemployed individuals derive from leisure. The redistributive policy instruments available to the regions include transfers to both the disabled and the voluntarily unemployed, and unemployment insurance payments to those involuntarily unemployed. We also consider the use of a minimum wage as a policy for redistribution.<sup>7</sup> The implications of firm mobility for the use of a minimum wage turn out

---

<sup>6</sup> The consequences for redistributive policy of being unable to distinguish the able from the disabled and from imperfectly monitoring the job search activities of the unemployed are examined in Boadway and Cuff (1999).

<sup>7</sup> The usefulness of a minimum wage in the presence of involuntary unemployment was shown in Boadway and Cuff (1999) in a model similar to this one. Minimum wages which create unemployment or underemployment may also be welfare-improving as in Drazen (1986),

to be similar to those for redistributive transfers.

As in any analysis of redistributive policy, the objective function of the government is important. We adopt a fairly simple policy objective for the regions, that of maximizing the welfare of the disabled subject to a given minimum level of expected utility for the workers. This has the implication that any gains from policy coordination go entirely to the disabled. A more general treatment (such as maximizing a weighted sum of utilities) would allow both the workers and the disabled to share any efficiency gains. But this would complicate our analysis (which already involves a three-stage process) considerably without affecting the qualitative results. As well, our formulation avoids the need to take a stand on the issue of what weight the policy-maker should give to the differing values attached to leisure by workers who are not employed.

We present the model in the next section and characterize the sub-game perfect equilibrium (SPNE) in Section 3. Since the model involves three stages, backward induction is used to solve for the SPNE. Formally, a sub-game perfect Nash equilibrium is a strategy profile containing strategies for the firms, the workers and the governments, which results in a Nash equilibrium in all sub-games. A comparison between this non-cooperative equilibrium (SPNE) and the cooperative equilibrium is made in Section 4. In Section 5, we allow for heterogenous regions and characterize and compare both the non-cooperative, the cooperative, and the unitary state equilibria. The use of a minimum wage as a redistributive policy instrument is then examined in Section 6. Finally in Section 7, we summarize our results and outline possible extensions of the model.

---

Guesnerie and Roberts (1987), and Marceau and Boadway (1994).

## 2. THE MODEL

We consider an economy with two identical regions, each with its own regional government. The regions could be part of a federation or an economic union. Since the regions are identical, we conduct our analysis for one of the two regions, that is for the representative region. Variables for the other region are denoted by a superscript ‘-’. The economy is populated with  $\bar{n}$  footloose entrepreneurs, each of which owns a single firm. Entrepreneurs locate in the region in which they can earn the largest profits. The redistributive policies chosen by each regional government, and the impact of these policies on regional labour markets, determines the allocation of entrepreneurs (firms) between regions. We denote by  $n$  the number of entrepreneurs that locate in the representative region, with  $n + n^- = \bar{n}$ . Initially, there are equal number of entrepreneurs in each region.

Each region is also inhabited by a number of immobile individuals. There are two types of such individuals:  $D$  disabled individuals, who are unable to work, and  $A$  able individuals, who can work. The population of each type of individual is normalized to unity. The utility of the disabled individuals is indexed by their consumption of a composite good,  $c$  (with a price normalized to one). Since disabled individuals cannot work, government transfers are the only source of income they have to finance their consumption.

In a given region,  $L$  able individuals enter the labour market while the remaining  $V = (1 - L)$  individuals opt for voluntary unemployment. There exist frictions in the labour market for able individuals, so when they enter the labour market in search of work, only a proportion  $p \in (0, 1)$  find a job. The  $E (= pL)$  successful individuals earn a wage  $w$  and supply, inelastically, one unit of labour. Those  $I = L - E$  individuals who are unsuccessful at finding work are involuntarily unemployed. A simple matching model is adopted (and described in the next section) which determines for a given number of able individuals in a region’s labour market the probability of employment  $p$ , the wage rate  $w$ , and each firm’s profits  $\pi$ .

Working individuals have utility  $u(c)$ , where  $c$  can differ from  $w$  because of taxes or

transfers, and where  $u'(c) > 0, u''(c) < 0$ : thus able individuals are risk averse. If they are not working because of either voluntary or involuntary unemployment their utility is given by  $u(c) + \delta$ , where  $\delta$  is distributed on the interval  $[\underline{\delta}, \bar{\delta}]$ , with  $\underline{\delta} > 0$ , according to the cumulative distribution  $F(\delta)$  and its corresponding density  $f(\delta) > 0$ . Since unemployed individuals earn no income, their consumption equals the transfer given to them by the government. These transfers may differ for voluntarily and involuntarily unemployed individuals since we assume the government can distinguish between them. The parameter  $\delta$  can be interpreted as the non-pecuniary utility differential between leisure and working. It could include a combination of the utility of leisure and the (dis)utility (stigma) received from collecting unemployment transfers. In any case, although the government knows  $F(\delta)$ , it cannot observe each individual's  $\delta$ . This implies that all those involuntarily unemployed must be treated alike, as must all those voluntarily unemployed.

The objective of a regional government is to maximize the consumption of its disabled population given a minimum level of expected utility of the able,  $\bar{U}$ . Since  $\delta$  cannot be observed, the expected utility constraint of the able will not be binding for all able individuals; indeed, as we shall see, it will only be binding for those with the lowest  $\delta$  ( $\delta = \underline{\delta}$ ). The welfare of the entrepreneurs is not taken explicitly into account, but a non-negative constraint on their after-tax profits implicitly serves that purpose. These assumptions simplify the government's problem and avoid the issue of assigning welfare weights to households of different tastes. The government must also balance its budget.

The government chooses taxes and transfers, which is equivalent to choosing the consumption bundles for the various types of individuals, making sure that these bundles satisfy appropriate incentive constraints. We denote by  $c_D$ ,  $c_V$ ,  $c_I$ , and  $c_E$  the consumption levels of the disabled, the voluntary unemployed, the involuntary unemployed, and the employed. Since working individuals in a region earn a wage  $w$ , those individuals pay taxes of  $w - c_E$ . As will become clear below, a regional government that cannot observe  $\delta$  will let individuals with a relatively large  $\delta$  be voluntary unemployed, while those with a low  $\delta$  will join the labour force and search for work. Without being able to observe  $\delta$ ,

it simply becomes too costly to induce all households to enter the labour force.<sup>8</sup> Each regional government therefore selects its consumption allocations such that there is some cut-off level  $\hat{\delta}$ , which separates those that remain voluntary unemployed ( $\delta > \hat{\delta}$ ) from those that join the labour force ( $\delta < \hat{\delta}$ ). In that sense, a regional government controls the size of its labour force  $L = F(\hat{\delta})$ , which in turn affects the equilibrium values of  $w$  and  $p$  and the profits of firm  $\pi$ . Thus,  $\hat{\delta}$  can be used as a strategic variable to attract firms from the other region. Regional governments may also impose a tax  $t$  on the firms operating within their jurisdictions.<sup>9</sup> When  $t$  is used, it also serves as a strategic variable to compete for firms. Each region also has a given amount of revenue from elsewhere in the economy, denoted by  $\bar{R}$ .

The problem we consider can be viewed as taking place in the following three consecutive stages.<sup>10</sup>

- *Stage 1: Choice of a Tax/Transfer Policy.* Each regional government chooses its tax/transfer policy  $\mathcal{P} = (c_D, c_V, c_E, c_I, t)$ , given the tax/transfer policy chosen by the other region,  $\mathcal{P}^-$ , and anticipating the consequences of its choices on the location and labour market equilibria. It is assumed that the regional government can perfectly monitor the job search activities of able individuals and can distinguish the voluntary from the involuntary unemployed. Given  $\mathcal{P}$ ,  $\hat{\delta}$  and therefore the number of individuals  $L = F(\hat{\delta})$  joining the labour force in the region is determined. The size of the labour

---

<sup>8</sup> This distinguishes our model from standard models of redistribution which rely on the revelation principle to induce all households to report their true types in an optimum.

<sup>9</sup> Alternatively, we could allow regional government's to impose an ad valorem tax on profits. A unit tax on the firm simplifies the analysis and does not affect the qualitative results.

<sup>10</sup> The timing of decisions is important in these types of models. If firms choose their locations first, many of the problems arising from regional competition for firms, or competition for capital more generally, would not arise. The importance of the timing of private versus public sector decisions for the efficiency of equilibria in federations is considered by Mitsui and Sato (1999) in the context of labour migration. Our assumption is that while labour migration decisions might be of a long-run nature, firms can move from one region to another much more readily.



force in the two regions  $(L, L^-)$ , together with the tax rates  $(t, t^-)$ , turn out to be all that is required to solve the next two stages.

- *Stage 2: Location Equilibrium.* Given  $(L, t, L^-, t^-)$ , firms' profits in the two regions are determined as a function of the number of firms in each region. The  $\bar{n}$  entrepreneurs allocate themselves between regions such that profits per firm are equalized. This results in an equilibrium allocation of entrepreneurs  $(n, n^-)$ .
- *Stage 3: Regional Labour Market Equilibrium.* At this stage, the tax/transfer policies, as summarized by  $(L, t, L^-, t^-)$ , and the firms' locations  $(n, n^-)$  are given. Firms post vacancies and offer to pay some wage to fill them. Equilibrium in each regional labour market yields a wage rate  $w$ , a level of profits per firm  $\pi$ , and a probability of employment for an individual searching for work  $p$ .

The following section analyzes these three stages in detail when regions act as Nash competitors. We then consider in Section 4 the case when regions behave cooperatively in choosing their policies, perhaps reflecting the inducement of a central government. This serves as the basis for a comparison of the SPNE and the cooperative equilibria.

### 3. THE SUB-GAME PERFECT NASH EQUILIBRIUM

To characterize the SPNE, we solve the problem recursively. Given that both regions are identical, it is natural to focus on symmetric equilibria.

#### Stage 3: Regional Labour Market Equilibrium

At this stage, the policies of the regional government which determine  $L$  are given as is the number of firms in the region  $n$ . To determine the regional labour market equilibrium, we adopt a simplified version of the search model of equilibrium unemployment from Johnson and Layard (1986).<sup>11</sup> In this model, both the wage rate and the level of employment are

---

<sup>11</sup> The main simplification is that we assume individual separations from firms are exogenous rather than endogenous. This model is similar to that used in Boadway and Cuff (1999).

determined by the firms bidding for the available labour force by offering a wage rate and posting a number of vacancies. The number of job vacancies filled in each firm depends on the wage it offers relative to that of other firms and a matching function whose argument is the ratio of involuntary unemployment to total vacancies. In equilibrium, all firms offer the same wage rate and post the same number of vacancies. Since the total labour force  $L$  has been determined at this stage, we can concentrate on the way in which bidding for those workers by the given number of firms yields an equilibrium. We begin by analyzing the profit-maximizing choice of a wage rate and a number of vacancies by the firms, and then turn to the labour market equilibrium that these entail.

### Firm Level Equilibrium

Let  $v$  be the number of vacancies offered by the representative firm and  $r$  be its relative wage rate, so its actual wage rate is  $rw$ , where  $w$  is the wage rate in the market at large. The proportion of vacancies that are filled is given by a matching function  $rm(u)$  where  $u$  is the ratio of unemployment to vacancies in the regional market as a whole ( $u = I/nv$ ) and  $m'(u) > 0$ ,  $m''(u) < 0$ . Let  $e$  be the number of workers the firm ends up employing (the number of its vacancies that are filled). In each period a proportion  $b$  of those employed will exogenously separate from the firm. In equilibrium, each firm's new hires must equal its separations,  $rm(u)v = be$ .

Suppose each firm has a strictly concave production function  $f(e)$ , and incurs a given cost for each vacancy created and job filled of  $\psi$ . The tax  $t$  levied by regions on firms has no effect on firm behaviour in a given region; it only affects their choice of location (Stage 2). Assuming after-tax profits are non-negative, the representative firm's problem is given by:

$$\max_{\{r,e\}} \pi = f(e) - rwe - \psi \left[ e + \frac{be}{rm(u)} \right]$$

The first-order conditions with respect to  $r$  and  $e$  are:

$$-we + \frac{\psi be}{r^2 m(u)} = 0 \tag{r}$$

$$f'(e) - rw - \psi \left[ 1 + \frac{b}{rm(u)} \right] = 0 \quad (e)$$

In equilibrium, all firms behave alike, so  $r = 1$ . Equations (r) and (e) can then be used to determine  $w(u)$  and  $e(u)$ . It is straightforward to show that  $w'(u) < 0$  and  $e'(u) > 0$ .<sup>12</sup> These in turn yield the profit function  $\pi(u)$ , where  $\pi'(u) > 0$ .<sup>13</sup>

### Regional Labour Market Equilibrium

In equilibrium, total employment must equal the number of firms times the number of workers each employs and total separations must equal the total number of new hires:

$$E = ne(u); \quad bE = m(u)nv = m(u)(L - E)/u$$

The solution to these gives  $E(n, L)$  and  $u(n, L)$ , where  $E_n(n, L) > 0$ ,  $E_L(n, L) > 0$ ,  $u_n(n, L) < 0$ ,  $u_L(n, L) > 0$ , all of which conform with intuition.<sup>14</sup> Both the number of firms in the given region and the size of the regional labour force positively affect the level of total employment. The number of firms negatively affects the ratio of involuntary unemployment to total vacancies, while the size of the labour force negatively affects it. As mentioned, individuals in the labour force face some probability  $p$  of finding work. In equilibrium, this probability is given by the ratio of total employment to the size of the labour force:  $p = E/L$ .

From the above expressions, we can solve for the equilibrium values of the wage rate, profits, and the probability of finding a job (where we abuse notation slightly):<sup>15</sup>

---

<sup>12</sup> From the first-order condition on  $r$ ,  $w(u) = \psi b/m(u)$  and  $w'(u) = -\psi b m'(u)/m(u)^2 < 0$ . Comparative statics on (e) yields:  $e'(u) = -2\psi b m'(u)/(m(u)^2 f''(e)) > 0$ .

<sup>13</sup> Totally differentiate  $\pi$  and use the envelope theorem to find  $\pi'(u) = 2\psi b e(u) m'(u)/m(u)^2 > 0$ .

<sup>14</sup> Comparative statics on the two equations gives:  $E_n = \Delta^{-1} b e(u) E (1 - \eta) > 0$ ,  $E_L = \Delta^{-1} m(u) n e'(u) > 0$ ,  $u_n = -\Delta^{-1} e(u) (b u + m(u)) < 0$ ,  $u_L = \Delta^{-1} m(u) > 0$ , where  $\Delta = b E (1 - \eta) + n e'(u) [b u + m(u)] > 0$  and  $\eta = [m'(u) u]/m(u) < 1$ .

<sup>15</sup> Comparative statics of the three expressions yields:  $w_n = w'(u) u_n > 0$ ,  $w_L = w'(u) u_L < 0$ ,  $\pi_n = \pi'(u) u_n < 0$ ,  $\pi_L = \pi'(u) u_L > 0$ ,  $p_n = E_n/L > 0$ , and  $p_L = -\Delta^{-1} p^2 b (1 - \eta) < 0$ , where  $\Delta$  and  $\eta$  are defined in the previous footnote.

- $w(n, L) = w(u(n, L))$ ,  $w_n > 0$  and  $w_L < 0$
- $\pi(n, L) = \pi(u(n, L))$ ,  $\pi_n < 0$  and  $\pi_L > 0$
- $p(n, L) = E(n, L)/L$ ,  $p_n > 0$  and  $p_L < 0$

An increase in the number of firms in the region, increases both the wage rate and the probability of finding work and decreases the profits per firm.

## Stage 2: Location Equilibrium

In Stage 2, the policies chosen in Stage 1 by the two regional governments, as summarized by their consequences for firms  $(L, t, L^-, t^-)$ , are given. Entrepreneurs are aware that when  $n$  entrepreneurs have already located in a region, net-of-tax profits will be  $\pi(n, L) - t$ . Since  $\pi_n < 0$ , a stable location equilibrium is the number of entrepreneurs locating in the representative region that solves  $\pi(n, L) - t = \pi^-(\bar{n} - n, L^-) - t^-$ . The solution to this equilibrium condition is denoted by  $n(L, t, L^-, t^-)$ . Total differentiation of this equilibrium condition yields:

$$n_L = -\nabla^{-1}\pi_L > 0, \quad n_t = \nabla^{-1} < 0, \quad n_{L^-} = \nabla^{-1}\pi_{L^-}^- < 0, \quad n_{t^-} = -\nabla^{-1} > 0$$

where  $\nabla = \pi_n + \pi_{n^-}^- < 0$ .

For later use, it is useful to rewrite the  $w(\cdot)$ ,  $\pi(\cdot)$ , and  $p(\cdot)$  functions explicitly as  $w(n(L, t, L^-, t^-), L)$ ,  $\pi(n(L, t, L^-, t^-), L)$ , and  $p(n(L, t, L^-, t^-), L)$ . It is then possible to obtain the following:<sup>16</sup>

$$\begin{aligned} \frac{dw}{dL} &= w_n n_L + w_L < 0, & \frac{dw}{dt} &= w_n n_t < 0, & \frac{dw}{dL^-} &= w_n n_{L^-} < 0, & \frac{dw}{dt^-} &= w_n n_{t^-} > 0 \\ \frac{d\pi}{dL} &= \pi_n n_L + \pi_L > 0, & \frac{d\pi}{dt} &= \pi_n n_t > 0, & \frac{d\pi}{dL^-} &= \pi_n n_{L^-} > 0, & \frac{d\pi}{dt^-} &= \pi_n n_{t^-} < 0 \end{aligned}$$

---

<sup>16</sup> By substitution,  $dw/dL = \nabla^{-1}w'(u)\pi'(u)u_L u_{n^-}^- < 0$ ,  $d\pi/dL = \nabla^{-1}\pi'(u)\pi'(u^-)u_L u_{n^-}^- > 0$ , and  $dp/dL = [\pi'(u)e(bm + u) - \pi_{n^-}^- \Delta]^{-1}[p^2 b(1 - \eta)\pi_{n^-}^-] < 0$ , where  $\nabla$  is defined in the text. The signs of the remaining total derivatives follow directly from the properties of  $w(\cdot)$ ,  $\pi(\cdot)$ ,  $p(\cdot)$ , and  $n(\cdot)$ .

$$\frac{dp}{dL} = p_n n_L + p_L < 0, \quad \frac{dp}{dt} = p_n n_t < 0, \quad \frac{dp}{dL^-} = p_n n_{L^-} < 0, \quad \frac{dp}{dt^-} = p_n n_{t^-} > 0$$

In the location equilibrium, an increase in either the size of the regional labour force or the tax on firms negatively affects both the region's wage rate and the probability of finding work and positively affects gross profits per firm. It is straightforward to show that net profits,  $\pi - t$ , is decreasing in  $t$ .<sup>17</sup> Therefore, to induce firms to locate in its region the regional government could either increase the size of its labour force or reduce its tax on firms. The efficacy of using these instruments to attract firms will be determined in the next stage.

### Stage 1: Choice of Tax/Transfer Policy

In this initial stage, a regional government selects its policy vector  $\mathcal{P} = (c_D, c_V, c_E, c_I, t)$ , taking as given that chosen by the other region,  $\mathcal{P}^-$ . The regional government understands that its policies will result in location and labour market equilibria that are completely described in the functions  $w(n(\cdot), L)$ ,  $\pi(n(\cdot), L)$ , and  $p(n(\cdot), L)$  from Stage 2.

Individuals' abilities and whether or not able individuals are actively searching for work are observable. Thus the government by monitoring the job search activities of the able can perfectly distinguish the voluntary from the involuntary unemployed. As mentioned, since the government cannot observe individuals' tastes for leisure, it is costly to induce individuals with very high utilities of leisure  $\delta$  into the labour force. So we assume that the regional government allows some individuals to be voluntarily unemployed ( $V > 0$ ). Let  $\hat{\delta}$  be the cut-off taste parameter in the optimum. This level of  $\hat{\delta}$  is a consequence of government tax-transfer policies, but for analytical convenience, we allow it to be chosen by the government as an artificial control variable. In choosing  $\hat{\delta}$ , the government is constrained by the fact that an individual with taste  $\hat{\delta}$  is indifferent between going into the labour force and being voluntarily unemployed. If the appropriate incentive constraint is satisfied, then all individuals with  $\delta \in [\underline{\delta}, \hat{\delta})$  are induced into the labour force,

---

<sup>17</sup> This follows directly from the fact that  $d\pi/dt = \pi_n / (\pi_n + \pi_{n^-}) \in (0, 1)$ , so  $d(\pi - t)/dt < 0$ .

and all individuals with  $\delta \in (\hat{\delta}, \bar{\delta}]$  choose to be voluntarily unemployed. This implies that  $L = F(\hat{\delta})$  and  $dL/d\hat{\delta} = f(\hat{\delta}) > 0$ : an increase in  $\hat{\delta}$  increases the number of individuals in the regional labour force. In what follows we use  $\hat{\delta}$  rather than  $L$  as a choice variable for the government, but obviously the effect of changes in  $\hat{\delta}$  translate directly into the effect of changes in  $L$  (e.g.,  $n_{\hat{\delta}} = n_L f(\hat{\delta})$ ).

The government is also constrained to ensure that firms that locate in its region earn non-negative net profits,  $\pi(\cdot) - t \geq 0$ . Otherwise, they will not operate. Using the above functions, the regional government's problem is given by

$$\max_{\{c_D, c_E, c_V, c_I, \hat{\delta}, t\}} c_D \quad (P)$$

subject to

$$u(c_V) + \delta \geq \bar{U}, \quad \delta \geq \hat{\delta} \quad (\mu_V)$$

$$EU_L(\delta) = p(\cdot)u(c_E) + (1 - p(\cdot)) [u(c_I) + \delta] \geq \bar{U}, \quad \delta \leq \hat{\delta} \quad (\mu_L)$$

$$c_D + [1 - F(\hat{\delta})]c_V + F(\hat{\delta}) [p(\cdot)(c_E - w(\cdot)) + (1 - p(\cdot))c_I] - tn(\cdot) \leq \bar{R} \quad (\lambda)$$

$$p(\cdot)u(c_E) + (1 - p(\cdot))[u(c_I) + \hat{\delta}] = u(c_V) + \hat{\delta} \quad (\phi)$$

$$\pi(\cdot) - t \geq 0 \quad (\alpha)$$

where equation labels refer to the Lagrange multipliers used in the problem. It is clear that the minimum expected utility constraint of individuals in the labour force will only bind at the lowest  $\delta$ ,  $\delta = \underline{\delta}$ , since their expected utility  $EU_L(\delta)$  is increasing in  $\delta$ . It is also the case that the left-hand side of  $(\mu_V)$  is increasing in  $\delta$ . Given that the incentive constraint  $\phi$  is satisfied, this implies that  $\mu_V = 0$  — the minimum utility constraint for the voluntary unemployed is always slack so we can neglect it in what follows.<sup>18</sup>

---

<sup>18</sup> In the full information optimum, the government would equate the utility of all voluntarily unemployed individuals to  $\bar{U}$ . When it can no longer observe the  $\delta$ 's, these individuals necessarily receive a utility greater than  $\bar{U}$ . Therefore,  $c_V$  is too high relative to the full information case. Rewriting the incentive constraint as  $pu(c_E) + (1-p)u(c_I) - p\hat{\delta} - u(c_V) = \varepsilon$ , we can see that an increase in  $\varepsilon$  allows the government to reduce  $c_V$ , and thereby increase social welfare. Thus,  $\phi > 0$ .

The Lagrangian expression for the government's problem is:

$$\begin{aligned} \mathcal{L} = & c_D + \mu_L [p(\cdot)u(c_E) + (1 - p(\cdot))[u(c_I) + \underline{\delta}] - \overline{U}] \\ & - \lambda [c_D + [1 - F(\hat{\delta})]c_V + F(\hat{\delta})(p(\cdot)(c_E - w(\cdot)) + (1 - p(\cdot))c_I) - tn(\cdot) - \overline{R}] \\ & - \phi [p(\cdot)u(c_E) + (1 - p(\cdot))[u(c_I) + \hat{\delta}] - u(c_V) - \hat{\delta}] + \alpha[\pi(\cdot) - t] \end{aligned}$$

and the first-order conditions are listed in the Appendix.<sup>19</sup> The first-order condition on  $c_D$  reduces to  $\lambda = 1$ : the shadow price of public funds is simply unity. Then, from the first-order conditions on  $c_E$  and  $c_I$ , we obtain  $u'(c_E) = u'(c_I) = F(\hat{\delta})/(\mu_L - \phi)$ . This implies that  $c_E = c_I$ , so there is full insurance in the optimum.<sup>20</sup> Given these results, the incentive constraint  $(\phi)$  reduces to

$$u(c_E) - u(c_V) = p(\cdot)\hat{\delta} \quad (\phi)$$

which implies that  $c_E > c_V$ , since  $\hat{\delta} > 0$ . Similarly, the constraint  $(\mu_L)$  reduces to

$$u(c_E) + (1 - p(\cdot))\underline{\delta} = \overline{U} \quad (\mu_L)$$

As stated previously, the cut-off taste parameter,  $\hat{\delta}$ , is chosen as an artificial variable. These two constraints indicate how  $c_E$  and  $c_V$  must change to induce an increase in  $\hat{\delta}$ . In order to keep  $(\mu_L)$  binding, an increase in  $\hat{\delta}$  must be accompanied by a decrease in  $c_E$  since  $p(\cdot)$  is strictly decreasing in  $\hat{\delta}$ . Then, from  $(\phi)$ , a sufficient condition for  $c_V$  also to decrease is that the elasticity of  $p(\cdot)$  with respect to  $\hat{\delta}$  be less than one:  $-(dp/d\hat{\delta})(\hat{\delta}/p) < 1$ . Assuming this to be satisfied, the larger the labour force in a given region, the lower the consumption transfers to all able individuals.

---

<sup>19</sup> Throughout the paper, we assume that the second-order conditions on the governments' problems are satisfied.

<sup>20</sup> The full insurance result depends critically on the government's ability to observe the job search activities of those unemployed so as to distinguish the voluntary from the involuntary unemployed. In the absence of perfect monitoring, it must be that  $c_E > c_I$  to induce enough of the unemployed to search for work (Boadway and Cuff, 1999). This would make our analysis much more complicated and would obscure the phenomenon of competition for firms we are concerned with here.

The first-order conditions of importance to us are those involving the choice of  $\hat{\delta}$  and  $t$ . Using  $c_E = c_I$ ,  $\lambda = 1$ , and the properties of  $w(n(\cdot), L)$ ,  $p(n(\cdot), L)$ , and  $\pi(n(\cdot), L)$  from the Stage 2 equilibrium, they reduce to:

$$n_{\hat{\delta}}\mathcal{L}_n + [\phi\hat{\delta} - \mu_L\underline{\delta} + F(\hat{\delta})w]p_{\hat{\delta}} + F(\hat{\delta})pw_{\hat{\delta}} + \alpha\pi_{\hat{\delta}} - f(\hat{\delta})[c_E - pw - c_V] + p\phi = 0 \quad (1)$$

$$n_t\mathcal{L}_n + n - \alpha = 0 \quad (2)$$

where  $\mathcal{L}_n = \partial\mathcal{L}/\partial n = [\phi\hat{\delta} - \mu_L\underline{\delta} + F(\hat{\delta})w]p_n + F(\hat{\delta})pw_n + \alpha\pi_n + t$  is the derivative of the Lagrangian with respect to the number of firms. Increasing the number of firms in the region increases both  $p$  and  $w$  and decreases  $\pi$ . So, the first two terms of  $\mathcal{L}_n$  are positive for sufficiently small  $\underline{\delta}$  and the third term is negative. The last term in the expression will be positive if the regional government imposes a tax in the SPNE and will be negative if it subsidizes firms in the SPNE. Overall, the sign of  $\mathcal{L}_n$  is ambiguous. It may be welfare-improving to increase or decrease the number of firms in the region and it is possible that there exists some optimal level of firms. The above first-order conditions, (1) and (2), along with those on  $c_D$ ,  $c_E$ , and  $c_V$  determine the equilibrium policies for the region,  $\mathcal{P}$ , as functions of those chosen in the other region. In a SPNE, each region's policies are optimal given the other region adopts its optimal policies. As regions are identical, there will be a symmetric SPNE in which both regional governments adopt the same policies and none of the firms move from their initial location. We now turn to characterize the cooperative equilibrium and compare it to the symmetric SPNE.

#### 4. COMPARING THE SPNE AND THE COOPERATIVE EQUILIBRIUM

In the symmetric SPNE, the two regions engage in self-defeating and possibly distorting competition for firms. It is self-defeating in the sense that in the SPNE, neither region succeeds in attracting firms from the other. It is possibly inefficient because the competition for firms may cause regions to induce inefficient values of  $\hat{\delta}$  and  $t$ . In a cooperative equilibrium, regions coordinate their policies so as to avoid this competition for firms, thereby generating socially efficient policies in each region. The gains from cooperation are shared



equally between the two regions since they are identical. A simple way to characterize this cooperative equilibrium is for a single regional government to maximize the consumption of its disabled population subject to the constraints of problem (P) above, but assuming that firms are immobile.<sup>21</sup> We then compare the choice of redistributive policies and tax rates in the cooperative equilibrium to those in the non-cooperative equilibrium (SPNE).

### The Cooperative Equilibrium

The government's problem is identical to problem (P) in Stage 1 of the previous section, but treating  $n$  as fixed and  $p$ ,  $w$ , and  $\pi$  as functions of  $\hat{\delta}$  only. The first-order conditions on the consumption allocations are identical: able individuals in the labour force receive full unemployment insurance ( $c_E = c_I$ ) and the shadow price of public funds is unity ( $\lambda = 1$ ). Given these, the first-order conditions on  $\hat{\delta}$  and  $t$  become:

$$[\phi\hat{\delta} - \mu_L\underline{\delta} + F(\hat{\delta})w]p_{\hat{\delta}} + F(\hat{\delta})pw_{\hat{\delta}} + \alpha\pi_{\hat{\delta}} - f(\hat{\delta})[c_E - pw - c_V] + p\phi = 0 \quad (3)$$

$$n - \alpha = 0 \quad (4)$$

From (4), all profits are taxed away in a cooperative equilibrium ( $t = \pi$ ,  $\alpha = n > 0$ ). The optimal choice of  $\hat{\delta}$  in (3) is such that the marginal cost of an increase in  $\hat{\delta}$ , or equivalently of  $L$ , just offsets the marginal benefit. The first two terms represent the effect of a change in  $\hat{\delta}$  on  $p$  and  $w$ . For a sufficiently small  $\underline{\delta}$ , the coefficient on  $p_{\hat{\delta}}$  will be positive, so these first two terms will both be negative: the cost of a marginal increase in  $L$  is that it reduces the wage rate and the probability of finding work. The third term is positive and represents the additional tax revenue the government receives when it increases the size of its labour force and thereby increases profits. The last term represents the weakening of the incentive constraint and is positive. As mentioned above, a marginal increase in  $\hat{\delta}$  reduces the amount of resources that must be transferred to the able individuals in order to

---

<sup>21</sup> The cooperative allocation is equivalent to the one arising from the maximization of the consumption of the disabled nationwide subject to the location and labour market equilibria, the expected utility constraint for the able individuals, the non-negativity constraint on net profits, and an economy-wide resource constraint.

keep the incentive constraint satisfied.<sup>22</sup> Finally, the second last term in (3) represents the direct effect of a change in the size of the labour force on resources. The cost in transfers from an additional involuntary unemployed Type 1 is given by  $c_E - pw$  and the saving in transfers from having one less Type 1 individual in the labour force is  $c_V$ . Whether or not there is a net savings in resources from a marginal increase in  $\hat{\delta}$  depends on the relative magnitude of the consumption transfers, the wage rate, and the probability of finding work.

The first-order conditions on  $\hat{\delta}$  and  $t$  in the SPNE, expressions (1) and (2), respectively, differ from the corresponding first-order conditions in the cooperative equilibrium by their first terms involving  $\mathcal{L}_n$ . Using (1) and (2) to substitute out the term  $\mathcal{L}_n$  and noting that  $n_{\hat{\delta}}/n_t = -\pi_{\hat{\delta}}$ , we obtain in the SPNE:

$$[\phi\hat{\delta} - \mu_L\underline{\delta} + F(\hat{\delta})w]p_{\hat{\delta}} + F(\hat{\delta})pw_{\hat{\delta}} + n\pi_{\hat{\delta}} - f(\hat{\delta})[c_E - pw - c_V] + p\phi = 0 \quad (5)$$

Equation (5), which applies for any  $\alpha \geq 0$ , is identical to the condition on  $\hat{\delta}$  in the cooperative equilibrium (3) when  $\alpha$  has been substituted out using (4). This implies that *the regional governments' choices of  $\hat{\delta}$  are not distorted from the cooperative equilibrium when they act as Nash competitors if they are free to choose  $t$* . Moreover, the actual level of  $\hat{\delta}$  in the cooperative and non-cooperative equilibria are *identical*, even though the values of  $t$  may differ. This is because changes in  $t$  which are common to the two regions do not affect the choice of consumption bundles for the able: any revenue changes they generate are transferred in their entirety to the disabled and are therefore non-distorting.<sup>23</sup> This result is independent of the equilibrium level of  $t$  in the non-cooperative equilibrium. It is conceivable that regional governments tax all profits in the SPNE so  $\alpha > 0$ , in which case it replicates the cooperative optimum. But the more likely case is that regional governments compete down the tax rate so that  $\alpha = 0$ . In this case, social welfare is necessarily lower

---

<sup>22</sup> Equivalently, as mentioned in footnote 18, the multiplier  $\phi > 0$ .

<sup>23</sup> This follows from the fact that changes in  $\bar{R}$  in the regions budget constraint cause a one-for-one change in  $c_D$ , which in turn leads to the shadow price of public funds  $\lambda$  being unity.

than in the cooperative optimum since the reduction in regional tax revenue results in decreased transfers to the disabled. It is even possible that regions will offer subsidies to firms in the SPNE.<sup>24</sup>

### **Achieving the Cooperative Equilibrium in a Federation**

The above comparison assumes that the tax on firms is levied solely by the competing regional governments. If there were a central authority which could tax firms, the adverse consequences of regional tax competition might be overcome. Suppose that redistribution policy remains in the hands of the regional governments, but that a central authority can tax firms and transfer the proceeds to the regional governments. The power to tax firms is a shared power — both the regions and the central authority can use it independently. This would be a realistic scenario in the case of a federation. Suppose also that the central authority is the first mover (Stackelberg leader). At Stage 0, it chooses a tax rate  $\tau$  to apply to firms and transfers the revenues equally to the regions.

It is straightforward to see that the cooperative equilibrium can be achieved if the central authority chooses the correct value of  $\tau$ . Since it can predict the outcome of the non-cooperative behaviour of the regions, it knows that the regions will choose the optimal level of  $\hat{\delta}$  if they are allowed to compete using  $t$ . It can also predict the level of after-regional-tax profits of the firms,  $\pi - t$ . It will choose its tax rate such that the after-tax profits of the firms is just zero,  $\pi - t - \tau = 0$ . The revenues it raises are then transferred to the regions, who use it entirely to enhance the transfer to the disabled. The full cooperative equilibrium is achieved, regardless of the value of  $t$ , including the case in

---

<sup>24</sup> To see this, we differentiate the Lagrangian with respect to  $t$  and evaluate the resulting expression at  $t = 0$ , to obtain:  $\partial\mathcal{L}/\partial t|_{t=0} = n_t\mathcal{L}_n + n$ , where  $\mathcal{L}_n$  is positive for sufficiently small  $\underline{\delta}$ , given  $t = 0$  and  $\alpha = 0$ . An increase in  $t$  from zero induces out-movement of firms as represented by the first negative term in the expression and increases tax revenue as represented by the second positive term. Overall, this expression can be positive or negative. If it is negative, the regional governments have an incentive to subsidize firms in their regions and  $t < 0$  in the SPNE, while if it is positive they have incentive to tax firms and  $t > 0$  in the SPNE. In the cooperative equilibrium, the first term is zero so it is always optimal for the regions to cooperatively impose a tax on firms.

which  $t < 0$  in the SPNE.

### **Do Competing Regions Improve Welfare By Taxing Firms?**

In the SPNE, regions compete for firms through their choice of  $t$  and  $\hat{\delta}$ . We showed above that the regional government's choice of  $\hat{\delta}$  will be undistorted from the cooperative value, but its choice of  $t$  may be inefficient. Suppose competing regions could not use the policy instrument  $t$ . This case may come about in a federation by constitutional restriction, or it may come about by agreement to transfer the power to tax firms to a central authority. In this case the non-cooperative choice of  $\hat{\delta}$  will be inefficient.

To see this, we compare the choice of  $\hat{\delta}$  in the SPNE to its value in the cooperative equilibrium assuming in both cases that  $t = 0$ , so  $\alpha = 0$ . In either case, full insurance still applies ( $c_E = c_I$ ) and  $\lambda = 1$ . When regions act cooperatively, the first-order condition determining  $\hat{\delta}$  is given by (3) with  $\alpha = 0$  and when regions act non-cooperatively, the first-order condition determining  $\hat{\delta}$  is given by (1) with  $\alpha = 0$ . These expressions differ only by the first term in (1),  $n_{\hat{\delta}}\mathcal{L}_n$ . Since  $t = \alpha = 0$ ,  $\mathcal{L}_n$  is positive for sufficiently small  $\underline{\delta}$  and this term is positive. This implies that from the symmetric SPNE, the two regions acting cooperatively will want to reduce  $\hat{\delta}$ , and from the cooperative equilibrium, competing regions will want to increase  $\hat{\delta}$ . Regions acting as Nash competitors will increase the size of their labour forces through their redistributive policies in order to attract firms even though in equilibrium no firms relocate. Relative to the cooperative equilibrium, regional labour forces will be too large.

Suppose instead regions compete for firms using  $t$ . Then we know from the above analysis that the optimal choice of  $\hat{\delta}$  in the SPNE is efficient and the cost of providing  $\bar{U}$  to the able individuals minimized. Relative to the SPNE when regions do not compete using  $t$ , there are more resources which can now be transferred to the disabled. Whether or not social welfare is higher in this case depends on the optimal value of  $t$  in the SPNE. *If regions tax firms in the SPNE, then social welfare is unambiguously higher than if regions did not have access to the policy instrument  $t$ .* In addition to the resources resulting from

choosing the efficient value of  $\hat{\delta}$ , the regions also raise revenue from taxing firms which can also be transferred to the disabled. If, on the other hand, the SPNE level of  $t$  is negative, then resources are diverted from the disabled individuals in order to subsidize firms. In this case, it is unclear whether or not social welfare will be increased if competing regions have access to  $t$ .

To summarize our results:

**Proposition 1:** *Comparing the SPNE and the cooperative equilibrium, we obtain that:*

- a. *Regional choices of  $c_E, c_I, c_V$  and  $L$  are the same in the two equilibria when regions can choose  $t$ . However,  $t$  will generally be lower in the SPNE than in the cooperative equilibrium, in which case so will social welfare ( $c_D$ ).*
- b. *A central authority can ensure that the SPNE is identical to the cooperative equilibrium — in terms of policies and welfare — by:*
  - (i) *Allowing regional governments to choose  $t$ ; and*
  - (ii) *Taxing all firms at the rate  $\tau$  such that  $\pi - t - \tau = 0$ , and transferring the revenues equally to each region.*
- c. *Regional redistributive policies differ in the two equilibria if  $t$  is not a regional choice variable. The SPNE choice of  $L$  is larger than in the cooperative equilibrium and social welfare ( $c_D$ ) is lower.*
- d. *In the SPNE, a sufficient condition for social welfare to increase if regions compete using  $t$  is for  $t > 0$  in the optimum.*

## 5. ASYMMETRIC REGIONS

An extension to the model considered in Section 3 is to allow for heterogenous regions. There are several ways regions can differ. For example, firms in the two regions may have different production functions reflecting regional differences in the availability of some

fixed factor, or they may have different costs of creating a vacancy and maintaining a filled position reflecting regional institutional and administrative differences. Likewise, the matching function may differ across regions. A simpler way to model heterogeneous regions is to assume they have different populations of able workers. Previously, the population of the able workers was normalized to one ( $A = 1$ ). In this section, we allow  $A$  to differ across the two regions. The size of the regional labour force is now given by  $L = F(\hat{\delta})A$ , and analogously for the other region.

We begin by characterizing the SPNE when regions can choose both their redistributive policies and taxes on firms. We then consider equilibria where the two regions coordinate their actions and thereby, internalize the externalities arising from their competition for firms. In this asymmetric setting, various divisions of the surplus from coordination between the two regions are possible. We focus on two outcomes — a cooperative equilibrium where there are no interregional transfers and a unitary state equilibrium where there is redistribution between regions such that all disabled receive the same transfer. Redistribution policies and equilibrium values for  $w$ ,  $p$  and  $n$  will differ between the regions, but for each region they will be independent of the way in which the surplus from coordination is divided. Both equilibria (or any other coordinated equilibrium which divides the surplus differently) will generate the same aggregate consumption for the disabled across both regions; but its division between the two regions will differ. We then show that a given region's optimal choice of  $\hat{\delta}$  in the SPNE is undistorted from its value in these two equilibria when regions act competitively, provided they are free to choose the tax on firms. Finally, we discuss how a central authority capable of imposing taxes on firms and making interregional transfers could achieve both the cooperative and the unitary state allocations.

### **The Sub-Game Perfect Nash Equilibrium**

The problem we are now considering still occurs in three stages and as before, the SPNE is solved for recursively. In this case, the regional labour market equilibrium (Stage 3) is

identical to that in Section 3. Likewise, the location equilibrium (Stage 2) is also unaffected by heterogenous regions. Net-of-tax profits must be equalized across the two regions in equilibrium as characterized in Section 3. We proceed directly to the representative region's choice of policies.

The region chooses all consumption levels,  $\hat{\delta}$  (or equivalently  $L$  since  $L = F(\hat{\delta})A$ ,  $dL/d\hat{\delta} = f(\hat{\delta})A$ ), and  $t$ . The Lagrangian is:

$$\begin{aligned} \mathcal{L} = & c_D + \mu_L [p(\cdot)u(c_E) + (1 - p(\cdot))[u(c_I) + \underline{\delta}] - \overline{U}] \\ & - \lambda [c_D + [1 - F(\hat{\delta})]Ac_V + F(\hat{\delta})A[p(\cdot)(c_E - w(\cdot)) + (1 - p(\cdot))c_I] - tn(\cdot) - \overline{R}] \\ & - \phi [p(\cdot)u(c_E) + (1 - p(\cdot))[u(c_I) + \hat{\delta}] - u(c_V) - \hat{\delta}] + \alpha[\pi(\cdot) - t] \end{aligned}$$

The first-order conditions on consumption yield  $\lambda = 1$  and  $c_E = c_I > c_V$  as before.

Using the properties of  $\pi(n(\cdot), L)$ ,  $w(n(\cdot), L)$ ,  $p(n(\cdot), L)$ , and  $n(\cdot)$  derived in Stage 2 and Stage 3 in Section 3, the first-order conditions on  $\hat{\delta}$  and  $t$  can be written as:

$$\begin{aligned} n_{\hat{\delta}}\mathcal{L}_n + [\phi\hat{\delta} - \mu_L\underline{\delta} + F(\hat{\delta})Aw]p_{\hat{\delta}} \\ + F(\hat{\delta})Apw_{\hat{\delta}} + \alpha\pi_{\hat{\delta}} - f(\hat{\delta})A[c_E - pw - c_V] + p\phi = 0 \end{aligned} \quad (6)$$

$$n_t\mathcal{L}_n + n - \alpha = 0 \quad (7)$$

where  $\mathcal{L}_n = \partial\mathcal{L}/\partial n = [\phi\hat{\delta} - \mu_L\underline{\delta} + F(\hat{\delta})A]p_n + F(\hat{\delta})Apw_n + \alpha\pi_n + t$ . Eliminating  $\mathcal{L}_n$  from (6) and (7) and noting that  $n_{\hat{\delta}}/n_t = -\pi_{\hat{\delta}} < 0$ , we obtain:

$$[\phi\hat{\delta} - \mu_L\underline{\delta} + F(\hat{\delta})Aw]p_{\hat{\delta}} + F(\hat{\delta})Apw_{\hat{\delta}} + n\pi_{\hat{\delta}} - f(\hat{\delta})A[c_E - pw - c_V] + p\phi = 0 \quad (8)$$

As  $A$  increases, the negative terms in equation (8) become more negative,<sup>25</sup> while the positive terms become less positive (or remain constant). This implies that the optimal choice of  $\hat{\delta}$  decreases as  $A$  increases. When regions have the same sized able population, we know from Section 3 that in the symmetric SPNE both regions choose the same level

---

<sup>25</sup> It is assumed  $\underline{\delta}$  is small enough so the first term in the square brackets is positive.

of  $\hat{\delta}$ . In this case, the regional governments' optimal choices of  $\hat{\delta}$ , given the optimal policy choices of the other region, are not the same. This implies the equilibrium values of  $w$  and  $p$  will also be different in the two regions. However, since  $L = F(\hat{\delta})A$  we cannot determine whether or not in the non-cooperative equilibrium the region with the larger able population has the larger labour force: for  $A > A^-$ , we can have  $L \gtrless L^-$ . Nor can we determine unambiguously in which region the disabled will be better off.<sup>26</sup>

### The Cooperative and the Unitary State Equilibria

When regions are identical, it is natural to focus on the cooperative outcome where both regions obtain identical shares of the surplus from coordination as we did in Section 4. With asymmetric regions, various divisions of this surplus are possible depending on the policy coordination process. As mentioned, we consider two types of coordinated equilibria here.<sup>27</sup> In the first type, which we refer to as the cooperative equilibrium, the regions coordinate their policies to internalize the effects on each other of the movement of firms (which now occurs in equilibrium) while at the same time maintaining budget balance within each region. This setting without interregional transfers is consistent with interpreting regions as nations in an economic union. Given the equilibrium consumption levels to the able individuals, the size of the labour forces and taxes on firms in each region, the consumption of the disabled in each region is determined from the regional budget constraint and will differ across regions. Formally, the cooperative equilibrium maximizes the sum of the consumption of the disabled in the two regions subject to all of the constraints facing each region — the minimum utility constraint, the incentive constraint and the regional budget

---

<sup>26</sup> Differentiating the Lagrangian with respect to  $A$  at the optimum, we find:  $\partial\mathcal{L}/\partial A = -(1 - F(\hat{\delta}))c_V - F(\hat{\delta})(c_E - pw)$  which is necessarily negative if  $c_E > pw$  and ambiguous otherwise.

<sup>27</sup> Alternatively, we could also allow for a cooperative bargaining equilibrium in which the division of the resources available for transfer to the disabled is determined through regional bargaining. This coordination process would only affect the values of the consumption transfers to the disabled across the regions, but would not affect the redistributive and taxation policies of the regions.



constraint.<sup>28</sup>

On the other hand, if regions comprise a single country, then a national government may be interested in equalizing the consumption of the disabled in the various regions. We refer to this case as the unitary state optimum. In this case, the central government maximizes the per capita consumption of the disabled over both regions subject to a single economy-wide resource constraint as well as the usual minimum expected utility and incentive constraints. This problem is similar to the cooperative case except that by aggregating regional budget constraints, lump-sum transfers between the disabled in the two regions are effectively allowed. It is straightforward to confirm that the consumption bundles to the different categories of able individuals, the regional taxes on firms, and the size of the regional labour forces are the same in both the cooperative and unitary state equilibria. Therefore, the regional wage rates and probabilities of employment and the distribution of firms across regions will also be the same. However, it is important to note that the levels of  $\hat{\delta}$  and therefore  $p$ ,  $w$  and  $\pi$  will generally be different across the two regions. The key result is that the total resources available for transfer to the disabled in the economy as a whole ( $c_D + c_D^-$ ) is identical in both equilibria. The only difference between the two allocations is the distribution of these resources to the disabled across the two regions.

To compare a given regions' choice of  $\hat{\delta}$  in the SPNE to its value in the cooperative and unitary state equilibria, it is sufficient to characterize the cooperative equilibrium. The Lagrangian of this problem is  $(\mathcal{L} + \mathcal{L}^-)$ , where  $\mathcal{L}$  and  $\mathcal{L}^-$  are the Lagrangian expressions defined above for each region. To simplify the analysis, we substitute out  $n^-(\cdot)$  in the Lagrangian using the fact that the number of firms across the two regions is fixed, so  $n^-(L^-, t^-, L, t) = \bar{n} - n(L, t, L^-, t^-)$ . The first-order conditions for  $\hat{\delta}$  and  $t$  are:

$$n_{\hat{\delta}}(\mathcal{L}_n + \mathcal{L}_n^-) + [\phi\hat{\delta} - \mu_L\underline{\delta} + F(\hat{\delta})Aw]p_{\hat{\delta}}$$

---

<sup>28</sup> We assume that the minimum levels of expected utility for the able persons in the two regions are identical. In a more general analysis, that need not be the case, but it would not affect the results we obtain for either the SPNE or the cooperative equilibrium.

$$+ F(\hat{\delta})Apw_{\hat{\delta}} + \alpha\pi_{\hat{\delta}} - f(\hat{\delta})A[c_E - pw - c_V] + p\phi = 0 \quad (9)$$

$$n_t(\mathcal{L}_n + \mathcal{L}_n^-) + n - \alpha = 0 \quad (10)$$

where  $\mathcal{L}_n^- = \partial\mathcal{L}^-/\partial n = -[\phi^- \hat{\delta}^- - \mu_L^- \underline{\delta}^- - F(\hat{\delta}^-)A^- w^-]p_n^- - F(\hat{\delta}^-)A^- p^- w_n^- - t^- - \alpha^- \pi_n^-$ . Combining (9) and (10) to eliminate  $(\mathcal{L}_n + \mathcal{L}_n^-)$  and noting that  $n_{\hat{\delta}}/n_t = -\pi_{\hat{\delta}}$ , we obtain:

$$[\phi\hat{\delta} - \mu_L\underline{\delta} + F(\hat{\delta})Aw]p_{\hat{\delta}} + F(\hat{\delta})Apw_{\hat{\delta}} + n\pi_{\hat{\delta}} - f(\hat{\delta})A[c_E - pw - c_V] + p\phi = 0 \quad (11)$$

Equation (11) characterizes the choice of  $\hat{\delta}$  in the cooperative equilibrium. This equation is identical to equation (8) implying that *a given region's choice of  $\hat{\delta}$  in the non-cooperative equilibrium with asymmetric regions is not distorted if the regions can compete in taxes.* In fact, the region chooses exactly the same value of  $\hat{\delta}$  in the SPNE as in the cooperative equilibrium (and therefore, the unitary state equilibrium) as long as it can compete in taxes.

### **Achieving the Cooperative and the Unitary State Equilibria in a Federation**

In the case of the cooperative equilibrium, decentralization proceeds precisely as in the symmetric case. The regions are given the responsibility for redistribution, and are allowed to tax firms as well. A central authority acting as first mover will extract all the profits from the firms for transfer to the disabled by choosing tax rates  $\tau$  and  $\tau^-$  for the two regions such that after-tax profits for firms in each region are driven down to zero. The proceeds from the taxes are returned to the regions from which they were collected.

The cooperative allocation will inevitably leave the disabled with a higher level of consumption in one region than the other. Since the tax levied on firms is effectively a lump-sum tax in the cooperative equilibrium, the unitary state optimum can be achieved by transferring the central authority's tax collections to the two regions in such a way as to equate the consumption levels of the disabled across the regions. (In principle, this might entail negative transfers to one of the regions. If this is not permissible, then the unitary state optimum cannot be decentralized.)

We can summarize our results in the following proposition.

**Proposition 2:** *In the case where regions are asymmetric:*

- a. *In the SPNE, regions choose different redistributive policies  $(c_D, c_E, c_I, c_V, L)$ , and as a result have different equilibrium values of  $w, p$  and  $n$ .*
- b. *If  $t$  is a choice variable, a given region's redistributive policies  $(c_E, c_I, c_V, L)$  and its equilibrium values of  $w, p$  and  $n$  are identical in all regimes (SPNE, cooperative equilibrium and unitary state equilibrium). Transfers to the disabled however differ across equilibria:*
  - (i) *Assuming tax competition causes  $t$  to be lower in the SPNE than in the cooperative equilibrium,  $c_D$  will be lower in the SPNE; and*
  - (ii) *In the cooperative equilibrium,  $c_D$  will differ across regions, while in the unitary state equilibrium, they are equalized by inter-regional transfers.*
- c. *A central authority can ensure that the SPNE is equivalent to:*
  - (i) *The cooperative equilibrium by imposing taxes on firms in the two region  $\tau$  and  $\tau^-$  such that  $\pi - t - \tau = 0$  and  $\pi^- - t^- - \tau^- = 0$ , and transferring the revenues to the regions in which they were raised; and*
  - (ii) *The unitary state equilibrium by imposing the same taxes on firms in the two regions  $\tau$  and  $\tau^-$  and transferring the revenues to the regions such that  $c_D$  is equalized across regions.*

## 6. MINIMUM WAGES

An alternative extension of the analysis in Section 3 is to allow for other instruments of redistribution by the regional governments. A common tool used for redistribution is the minimum wage, an instrument which is meant to be of assistance to low-wage workers. For it to be relevant in our context, it is necessary to consider our able individuals to be those

with low productivities. Elsewhere in the economy are higher-wage individuals from whom transfers to low income persons are mainly financed. We do not model them explicitly, but we suppose the revenue obtained from them has been included in the exogenous government revenue  $\bar{R}$ . The efficiency of a minimum wage in a model of involuntary unemployment similar to that used here has been examined by Boadway and Cuff (1999). They show that if  $\bar{U}$  is large enough, a binding minimum wage is a welfare-improving instrument.

Starting from this premise, we now allow the regional governments to choose a minimum wage.<sup>29</sup> As with tax-transfer policies, changes in the minimum wage will affect firms' profits and thereby change the incentive to migrate. The effect will be more direct now, since a binding minimum wage will mean that firms are price-takers in the labour market. The question we investigate is the extent to which regions will compete for firms through their choice of a minimum wage. It turns out that the answer depends upon what other instruments are available, in particular, whether regions are able to tax firms. The analysis is thus parallel to the above case.

Proceeding in a similar fashion, we first outline the SPNE when regions can select their redistributive policies, which now include both consumption transfers and a minimum wage for working individuals as well as a tax on firms. Next, we determine the cooperative equilibrium and compare the efficiency of the three different policy instruments in the two equilibria. We show that if regions can compete for firms with a tax, then the optimal decision choices of regional redistributive policies are the same as in the cooperative equilibrium.

### **The Sub-Game Perfect Nash Equilibrium**

The problem we are now considering still occurs in three stages. As before, the SPNE is solved for recursively.

---

<sup>29</sup> Again, the regions can be interpreted as sub-national levels of government in a federation, or nations in an economic union. In some federations, for example Canada, provinces do in fact set minimum wages within their jurisdictions.

### Stage 3: Regional Labour Market Equilibrium

Our matching model must be amended to account for the fact that firms are no longer free to choose their relative wage rate. Once a firm locates (and produces) in a given region, it must pay a minimum wage  $\bar{w}$  to all the workers it employs. If the minimum wage is binding, the firm's profits are  $f(e) - \bar{w}e - \psi[e + be/m(u)]$ . Price-taking behaviour on the part of the firm implies that its only choice variable is the number of workers to employ. The firm maximizes its profits with respect to  $e$ . The first-order condition is  $f'(e) - \bar{w} - \psi[1 + b/m(u)] = 0$ , which yields  $e(\bar{w}, u)$ , where  $e_{\bar{w}}(\bar{w}, u) < 0$ ,  $e_u(\bar{w}, u) > 0$  and  $\pi(\bar{w}, u)$ , where  $\pi_{\bar{w}}(\bar{w}, u) < 0$ ,  $\pi_u(\bar{w}, u) > 0$ .<sup>30</sup> Thus, increases in the minimum wage reduce both employment and profits, as expected.

In this stage, the size of the labour force  $L$  and the number of firms in the region  $n$  have been determined by Stages 1 and 2 respectively. Given  $\bar{w}$ , total employment  $E(n, L, \bar{w})$  and the unemployment to vacancy ratio  $u(n, L, \bar{w})$  are jointly determined by the regional labour market equilibrium conditions  $E = ne(\bar{w}, u)$  and  $bE = (m/u)[L - E]$ . A routine comparative static analysis yields  $E_n > 0$ ,  $E_L > 0$ ,  $E_{\bar{w}} < 0$  and  $u_n < 0$ ,  $u_L > 0$ ,  $u_{\bar{w}} > 0$ .<sup>31</sup> Changes in  $n$  and  $L$  affect  $E$  and  $u$  as before. The minimum wage decreases total employment and increases the ratio of total involuntary unemployment relative to total vacancies. Using the expressions  $u(n, L, \bar{w})$  and  $E(n, L, \bar{w})$ , we obtain:<sup>32</sup>

- $\pi(\bar{w}, n, L) = \pi(\bar{w}, u(n, L, \bar{w}))$ ,       $\pi_{\bar{w}} < 0$ ,  $\pi_n < 0$ ,  $\pi_L > 0$
- $p(\bar{w}, n, L) = E(n, L, \bar{w})/L$ ,       $p_{\bar{w}} < 0$ ,  $p_n > 0$ ,  $p_L < 0$

<sup>30</sup> Comparative statics on the firm's first-order condition yields  $e_{\bar{w}}(\bar{w}, u) = f''(e)^{-1} < 0$ ,  $e_u(\bar{w}, u) = -\psi bm'/(m^2 f''(e)) > 0$ . The envelope theorem gives  $\pi_{\bar{w}}(\bar{w}, u) = -e < 0$ ,  $\pi_u(\bar{w}, u) = \psi bem'/m^2 > 0$ .

<sup>31</sup> Specifically,  $E_n = \Delta^{-1} ebE(1 - \eta) > 0$ ,  $E_L = \Delta^{-1} mne_u > 0$ ,  $E_{\bar{w}} = \Delta^{-1} ne_{\bar{w}} bE(1 - \eta) < 0$ ,  $u_n = -\Delta^{-1} e(bu + m) < 0$ ,  $u_L = \Delta^{-1} m > 0$ ,  $u_{\bar{w}} = -\Delta^{-1} ne_{\bar{w}}(bu + m) > 0$  where  $\Delta = bE(1 - \eta) + ne_u[bu + m] > 0$  and  $\eta = [m'(u)u]/m(u)$ .

<sup>32</sup> Comparative statics of the expressions yields:  $\pi_{\bar{w}} = -\Delta^{-1} ebE(1 - \eta) < 0$ ,  $\pi_n = \pi_u u_n < 0$ ,  $\pi_L = \pi_u u_L > 0$ ,  $p_{\bar{w}} = E_{\bar{w}}/L < 0$ ,  $p_n = E_n/L > 0$ , and  $p_L = \Delta^{-1} p^2 b(1 - \eta) < 0$ , where  $\Delta$  and  $\eta$  are defined in the previous footnote.

## Stage 2: Location Equilibrium

For the entrepreneurs, the relevant policies chosen in Stage 1 by the two regional governments are  $(\bar{w}, L, t, \bar{w}^-, L^-, t^-)$ . Firms will locate such that net profits are equalized across the regions:

$$\pi(\bar{w}, n, L) - t = \pi^-(\bar{w}^-, \bar{n} - n, L^-) - t^-$$

This determines the equilibrium allocation of firms  $n(\bar{w}, L, t, \bar{w}^-, L^-, t^-)$ , where  $n_{\bar{w}} < 0$ ,  $n_L > 0$ ,  $n_t < 0$ ,  $n_{\bar{w}^-} > 0$ ,  $n_{L^-} < 0$ , and  $n_{t^-} > 0$ .<sup>33</sup> The regional minimum wage decreases the number of firms in the region and increases the number of firms in the other region. The effects of the regional labour force and tax on  $n$  are as in the previous case. Using the above solution for  $n(\cdot)$ , we can rewrite the  $\pi(\cdot)$  and  $p(\cdot)$  functions as  $\pi(\bar{w}, n(\bar{w}, L, t, \bar{w}^-, L^-, t^-), L)$  and  $p(\bar{w}, n(\bar{w}, L, t, \bar{w}^-, L^-, t^-), L)$ . From these, we obtain:<sup>34</sup>

$$\begin{aligned} \frac{d\pi}{d\bar{w}} &= \pi_n n_{\bar{w}} + \pi_{\bar{w}} < 0, & \frac{d\pi}{dL} &= \pi_n n_L + \pi_L > 0, & \frac{d\pi}{dt} &= \pi_n n_t > 0, \\ \frac{d\pi}{d\bar{w}^-} &= \pi_n n_{\bar{w}^-} < 0, & \frac{d\pi}{dL^-} &= \pi_n n_{L^-} > 0, & \frac{d\pi}{dt^-} &= \pi_n n_{t^-} < 0, \\ \frac{dp}{d\bar{w}} &= p_n n_{\bar{w}} + p_{\bar{w}} < 0, & \frac{dp}{dL} &= p_n n_L + p_L < 0, & \frac{dp}{dt} &= p_n n_t < 0, \\ \frac{dp}{d\bar{w}^-} &= p_n n_{\bar{w}^-} > 0, & \frac{dp}{dL^-} &= p_n n_{L^-} < 0, & \frac{dp}{dt^-} &= p_n n_{t^-} > 0. \end{aligned}$$

Any increase in the tax on firms increases gross profits per firm, but reduces net profits since the change in  $\pi$  with respect to  $t$  is less than one.<sup>35</sup> Therefore, it is clear from the above that if a regional government wants to induce firms to locate in its region, it will increase its labour force and decrease both its minimum wage and its tax on firms.

---

<sup>33</sup> The expressions for  $n_L$ ,  $n_t$ ,  $n_{L^-}$ , and  $n_{t^-}$  are identical to the case with no minimum wage, while  $n_{\bar{w}} = -\nabla^{-1}\pi_{\bar{w}} < 0$  and  $n_{\bar{w}^-} = \nabla^{-1}\pi_{\bar{w}^-} > 0$ , where  $\nabla = \pi_n + \pi_{n^-}$ .

<sup>34</sup> By substitution,  $d\pi/d\bar{w} = \nabla^{-1}\pi_{\bar{w}}\pi_{n^-}^- < 0$ ,  $d\pi/dL = \nabla^{-1}\pi_L\pi_{n^-}^- > 0$ , and  $dp/dL = [\pi'(u)e(bm+u) - \pi_{n^-}^- \Delta]^{-1}[p^2b(1-\eta)\pi_{n^-}^-] < 0$ , where  $\nabla$  is defined in the previous footnote and  $\Delta$  and  $\eta$  are defined in footnote 31. The signs on the remaining total derivatives follow directly from the properties of  $\pi(\cdot)$ ,  $p(\cdot)$ , and  $n(\cdot)$ .

<sup>35</sup> That is,  $d\pi/dt = \pi_n/(\pi_n + \pi_{n^-}^-) \in (0, 1)$ , so  $d(\pi - t)/dt < 0$ .

### Stage 1: Choice of Tax/Transfer Policy

The regional labour force is given by  $L = F(\hat{\delta})$ , where  $dL/d\hat{\delta} = f(\hat{\delta})$ . The government's problem is identical to problem (P) except that  $w = \bar{w}$  and is chosen by the government, given that  $n(\cdot)$ ,  $p(\cdot)$ , and  $\pi(\cdot)$  are all functions of  $\bar{w}$ . The first-order conditions on  $c_D$ ,  $c_E$ ,  $c_V$ ,  $c_I$  are the same as in the case without a minimum wage. These yield  $\lambda = 1$  and  $c_E = c_I > c_V$ .

The first-order conditions of interest are those on  $\bar{w}$ ,  $\hat{\delta}$ , and  $t$  and are listed in the Appendix. Using the results that  $c_E = c_I$  and  $\lambda = 1$ , and the properties of  $p(\bar{w}, n(\cdot), L)$  and  $\pi(\bar{w}, n(\cdot), L)$  from Stage 2, the first-order conditions on  $\bar{w}$ ,  $\hat{\delta}$ , and  $t$  reduce to:

$$n_{\bar{w}}\mathcal{L}_n + [\phi\hat{\delta} - \mu_L\hat{\delta} + \bar{w}F(\hat{\delta})]p_{\bar{w}} + \alpha\pi_{\bar{w}} + F(\hat{\delta})p = 0 \quad (12)$$

$$n_{\hat{\delta}}\mathcal{L}_n + [\phi\hat{\delta} - \mu_L\hat{\delta} + \bar{w}F(\hat{\delta})]p_{\hat{\delta}} + \alpha\pi_{\hat{\delta}} - f(\hat{\delta})[c_E - p\bar{w} - c_V] + p\phi = 0 \quad (13)$$

$$n_t\mathcal{L}_n + n - \alpha = 0 \quad (14)$$

where  $\mathcal{L}_n = \partial\mathcal{L}/\partial n = [\phi\hat{\delta} - \mu_L\hat{\delta} + \bar{w}F(\hat{\delta})]p_n + \alpha\pi_n + t$ . As in Section 3, the sign of this term is ambiguous.

### The Cooperative Equilibrium

In this case, no relocation occurs since both regions enact the same policies cooperatively. Thus, we can treat the number of entrepreneurs as fixed in each region so the choice of  $t$  does not affect  $p$  or  $\pi$ . However, government policy can affect both these variables through the choice of  $\bar{w}$  and  $\hat{\delta}$ . The problem is identical to the one given above, taking into account these considerations. The first-order conditions on  $\bar{w}$ ,  $\hat{\delta}$ , and  $t$ , assuming  $\bar{w} > w(n(\cdot), L)$  is welfare-improving,<sup>36</sup> become:

$$[\phi\hat{\delta} - \mu_L\hat{\delta} + F(\hat{\delta})\bar{w}]p_{\bar{w}} + \alpha\pi_{\bar{w}} + F(\hat{\delta})p = 0 \quad (15)$$

---

<sup>36</sup> To determine if a minimum wage is welfare-improving, we differentiate the Lagrangian with respect to  $\bar{w}$  and evaluate it at the market wage  $w(n(\cdot), L)$ . If this expression is positive, then increasing  $\bar{w}$  above  $w(n(\cdot), L)$  is welfare-improving. In the cooperative equilibrium, this expression is given by,  $\partial\mathcal{L}/\partial\bar{w}|_{\bar{w}=w(\cdot)} = [\phi\hat{\delta} - \mu_L\hat{\delta} + F(\hat{\delta})w(\cdot)]p_{\bar{w}} + \alpha\pi_{\bar{w}} + F(\hat{\delta})p$ . Substituting in for  $\pi_{\bar{w}}$ , the last two terms are positive. However, the first term is negative for small enough

$$[\phi\hat{\delta} - \mu_L\underline{\delta} + F(\hat{\delta})\bar{w}]p_{\hat{\delta}} + \alpha\pi_{\hat{\delta}} - f(\hat{\delta})[c_E - p\bar{w} - c_V] + p\phi = 0 \quad (16)$$

$$n - \alpha = 0 \quad (17)$$

By (12),  $\alpha = n > 0$ , implying as before that the governments acting cooperatively tax away all profits.

### Comparing the SPNE and the Cooperative Equilibrium

In the SPNE, combining the first-order condition on  $\bar{w}$  and  $t$ , expressions (12) and (14), to eliminate  $\mathcal{L}_n$  and noting that  $n_{\bar{w}}/n_t = -\pi_{\bar{w}}$ , we obtain:

$$[\phi\hat{\delta} - \mu_L\underline{\delta} + F(\hat{\delta})\bar{w}]p_{\bar{w}} + n\pi_{\bar{w}} + F(\hat{\delta})p = 0 \quad (18)$$

This expression is identical to (15), the first-order condition on  $\bar{w}$  in the cooperative equilibrium, when  $\alpha$  is substituted out using (17). Likewise, eliminating  $\mathcal{L}_n$  using the first-order conditions on  $\hat{\delta}$  and  $t$  in the SPNE, expressions (13) and (14), and noting that  $n_{\hat{\delta}}/n_t = -\pi_{\hat{\delta}}$ , we obtain:

$$[\phi\hat{\delta} - \mu_L\underline{\delta} + F(\hat{\delta})\bar{w}]p_{\hat{\delta}} + n\pi_{\hat{\delta}} - f(\hat{\delta})[c_E - p\bar{w} - c_V] + p\phi = 0 \quad (19)$$

Equation (19), which applies for any  $\alpha \geq 0$ , is identical to (16), the first-order condition on  $\hat{\delta}$  in the cooperative equilibrium, when  $\alpha$  is substituted out using (17). The implication of equations (18) and (19) is that *the choices of  $\hat{\delta}$  and  $\bar{w}$  are not distorted from the cooperative equilibrium when regions act as Nash competitors given that they are free to choose  $t$* . Moreover, analogously to before, the choices of  $\hat{\delta}$  and  $\bar{w}$  are *identical* to that of the cooperative solution. The fact that regions are competing in  $t$  implies that they may not be extracting the optimal amount of revenue from the firms. That simply reduces the level of  $c_D$  provided to the disabled without affecting redistributive policies applied to the able persons. Thus all of the results from the previous section, including the manner of

---

$\underline{\delta}$ , so the sign of the expression is ambiguous. In the non-cooperative equilibrium (assuming  $\alpha = 0$ ), this expression is given by  $\partial\mathcal{L}/\partial\bar{w}|_{\bar{w}=w(\cdot)} = n_{\bar{w}}\mathcal{L}_n + [\phi\hat{\delta} - \mu_L\underline{\delta} + w(\cdot)F(\hat{\delta})]p_{\bar{w}} + F(\hat{\delta})p$ . Both the first two terms are negative for small enough  $\underline{\delta}$  and the last term is positive, so the sign of this expression is also ambiguous.



decentralizing the cooperative solution, carry over to this case when there is a minimum wage in effect. *If a central authority could impose a tax on firms and transfer the proceeds to the regions in a lump-sum manner, the cooperative optimum could be achieved.* It is also straightforward to show that if the regions could not compete using  $t$ , their optimal choice of  $\bar{w}$  will be distorted downwards from the cooperative value. This follows directly from a comparison of the first-order conditions on  $\bar{w}$  in the SPNE, (12), and in the cooperative, (15), when  $t = 0$  and assuming the non-negative constraint on net profits is not binding.

These results are summarized in the following proposition.

**Proposition 3:** *When regional governments use a minimum wage for redistribution, the results obtained in Proposition 1 still apply. The SPNE value of  $\bar{w}$  is the same as in the cooperative equilibrium if regions can choose  $t$ ; if not,  $\bar{w}$  will generally be too low. A central authority can ensure that the SPNE is equivalent to the cooperative equilibrium by imposing a tax on firms and transferring the revenues to the regions.*

## 7. SUMMARY AND EXTENSIONS

We have shown that allowing regions to tax mobile firms and engage in tax competition may be efficient even in the absence of a central authority that internalizes the consequences of tax competition. If regions are allowed to use a tax to compete for firms then their redistributive policies will be identical to the cooperative allocation. This result is robust to allowing regions to use a minimum wage as a redistributive policy tool and allowing regions to differ in their populations of potential workers. Giving regional governments the right to tax firms unambiguously increases social welfare provided they levy positive taxes on firms in the SPNE. If they subsidize firms in equilibrium, then social welfare may or may not be higher compared to the case when they cannot engage in tax competition. One way to achieve the cooperative allocation is to allow a central authority to tax firms in both regions and redistribute the tax revenue among the two regions. The relative size of these transfers will depend on whether or not regions are identical. In the case of

heterogeneous regions, the unitary state optimum may only be decentralized if the central authority is allowed to redistribute between regions.

We have made several simplifying assumptions to derive these results. First, throughout this paper we have assumed that regional governments can perfectly monitor individuals for job search activities. However, most of the qualitative results derived under this assumption would be unchanged if we adopted the other extreme, that of regional governments unable to monitor individuals for job search activities. The one result that would change is that of full unemployment insurance. When the government cannot distinguish between the involuntary and voluntary unemployed, it will give less than full unemployment insurance to induce individuals to look for work. Alternatively, we could have also assumed that regional governments could imperfectly monitor individuals for job search activities. This would potentially give them one more policy instrument in which to affect  $\hat{\delta}$  in order to compete for firms — the intensity of monitoring. But in these cases of zero or imperfect monitoring, it would remain true that giving the regions the ability to tax firms would remove the incentive to compete via redistributive policies.

Assigning redistributive responsibilities to the regions accords with reality in many federations. Obviously, if the central authority could perfectly monitor individuals for job search activities then there would be no rationale to decentralize the power to tax firms to the regions.

Other possible extensions to the model include allowing the number of firms to be endogenous. If there is free-entry of identical entrepreneurs, then a fixed set-up or operating cost must be introduced to ensure an interior solution. A consequence of free-entry is that the government's redistributive policies which determine the size of the regional labour force will not affect the equilibrium wage rate or probability of finding work. The direct effect of any increase in the labour force on these variables is completely offset by the resulting change in the number of firms. However, if entrepreneurs are heterogeneous and differ either in their ability or set-up costs, then it is likely government redistributive

policies will affect the equilibrium wage rate and probability of finding work.

Another possible extension is to allow individuals to differ in productive ability. If the government could perfectly observe ability types, then the qualitative results of the analysis would not change. However, if ability types are unobservable to the government then we are back in the standard non-linear income taxation framework. Alternatively, extending the simple search model to allow for endogenous quits would be another fruitful exercise.

## 8. APPENDIX

### Section 3: Choice of Tax/Transfer Policy

The first-order conditions of the problem are:

$$1 - \lambda = 0 \tag{c_D}$$

$$[\mu_L - \phi]pu'(c_E) - \lambda F(\hat{\delta})p = 0 \tag{c_E}$$

$$[\mu_L - \phi](1 - p)u'(c_I) - \lambda F(\hat{\delta})(1 - p) = 0 \tag{c_I}$$

$$- \lambda[1 - F(\hat{\delta})] + \phi u'(c_V) = 0 \tag{c_V}$$

$$\begin{aligned} & \mu_L \frac{dp}{d\hat{\delta}} [u(c_E) - u(c_I) - \underline{\delta}] - \lambda \left[ F(\hat{\delta}) \frac{dp}{d\hat{\delta}} (c_E - w - c_I) - F(\hat{\delta})p \frac{dw}{d\hat{\delta}} - tn_{\hat{\delta}} \right] \\ & - \phi \frac{dp}{d\hat{\delta}} [u(c_E) - u(c_I) - \hat{\delta}] - \lambda f(\hat{\delta}) [p(c_E - w) + (1 - p)c_I - c_V] \\ & + p\phi + \alpha \frac{d\pi}{d\hat{\delta}} = 0 \end{aligned} \tag{\hat{\delta}}$$

$$\begin{aligned} & \mu_L \frac{dp}{dt} [u(c_E) - u(c_I) - \underline{\delta}] - \lambda \left[ F(\hat{\delta}) \frac{dp}{dt} (c_E - w - c_I) - F(\hat{\delta})p \frac{dw}{dt} - tn_t - n \right] \\ & - \phi \frac{dp}{dt} [u(c_E) - u(c_I) - \hat{\delta}] + \alpha \frac{d\pi}{dt} - \alpha = 0 \end{aligned} \tag{t}$$

### Section 6: Choice of Tax/Transfer Policy

The first-order conditions of the problem are  $(c_D)$ ,  $(c_E)$ ,  $(c_I)$ , and  $(c_V)$  given above and:

$$\begin{aligned} & \mu_L \frac{dp}{d\bar{w}} [u(c_E) - u(c_I) - \underline{\delta}] - \lambda \left[ F(\hat{\delta}) \frac{dp}{d\bar{w}} (c_E - \bar{w} - c_I) - F(\hat{\delta})p - tn_{\bar{w}} - n \right] \\ & - \phi \frac{dp}{d\bar{w}} [u(c_E) - u(c_I) - \hat{\delta}] + \alpha \frac{d\pi}{d\bar{w}} = 0 \end{aligned} \tag{\bar{w}}$$

$$\begin{aligned} & \mu_L \frac{dp}{d\hat{\delta}} [u(c_E) - u(c_I) - \underline{\delta}] - \lambda \left[ F(\hat{\delta}) \frac{dp}{d\hat{\delta}} (c_E - \bar{w} - c_I) - tn_{\hat{\delta}} \right] \\ & - \phi \frac{dp}{d\hat{\delta}} [u(c_E) - u(c_I) - \hat{\delta}] - \lambda f(\hat{\delta}) [p(c_E - \bar{w}) + (1 - p)c_I - c_V] \\ & + p\phi + \alpha \frac{d\pi}{d\hat{\delta}} = 0 \end{aligned} \tag{\hat{\delta}}$$

$$\begin{aligned} & \mu_L \frac{dp}{dt} [u(c_E) - u(c_I) - \underline{\delta}] - \lambda \left[ F(\hat{\delta}) \frac{dp}{dt} (c_E - \bar{w} - c_I) - tn_t - n \right] \\ & - \phi \frac{dp}{dt} [u(c_E) - u(c_I) - \hat{\delta}] + \alpha \frac{d\pi}{dt} - \alpha = 0 \end{aligned} \tag{t}$$

## 9. REFERENCES

- Boadway, R. and K. Cuff (1999), 'Monitoring Job Search as an Instrument for Targeting Transfers', *International Tax and Public Finance* **6**, 317-337.
- Boadway, R. and M. Keen (1999), 'Redistribution', in: A.B. Atkinson and F. Bourguignon, eds., *Handbook of Income Distribution* (North-Holland, Amsterdam), forthcoming.
- Boskin M.J. (1973), 'Local Government Tax and Product Competition and the Optimal Provision of Public Goods', *Journal of Political Economy* **81**, 203-210.
- Cremer, H., V. Fourgeaud, M. Leite-Monteiro, M. Marchand, and P. Pestieau (1996), 'Mobility and Redistribution: A Survey', *Public Finance* **51**, 325-352.
- Drazen, A. (1986), 'Optimal Minimum Wage Legislation', *Economic Journal* **96**, 774-784.
- Fuest, C. and B. Huber (1999), 'Tax Coordination and Unemployment', *International Tax and Public Finance* **6**, 7-26.
- Guesnerie, R. and K. Roberts (1987), 'Minimum Wage Legislation as a Second Best Policy', *European Economic Review* **31**, 490-498.
- Hindriks, J. (1998), 'Redistribution, Mobility and Political Failure', University of Exeter, *mimeo*.
- Jacobsen, H. and P.B. Sørensen (1999), 'Labour Tax Reform, the Goods Jobs and the Bad Jobs', University of Copenhagen: Economic Policy Research Unit, Working Paper 1999-1.
- Johnson, G.E. and P.R.G. Layard (1986), 'The Natural Rate of Unemployment: Explanation and Policy', in: O. Ashenfelter and P.R.G. Layard, eds., *Handbook of Labor Economics* (North-Holland, Amsterdam), 921-999.

- Keen, M. and M. Marchand (1997), 'Fiscal Competition and the Pattern of Public Spending', *Journal of Public Economics* **66**, 33-53.
- Lockwood, B. (1999), 'Inter-Regional Insurance', *Journal of Public Economics* **72**, 1-37.
- Marceau, N. and R. Boadway (1994), 'Minimum Wage Legislation and Unemployment Insurance as Instruments for Redistribution', *Scandinavian Journal of Economics* **96**, 67-81.
- Mitsui, K. and M. Sato (1999), 'Ex Ante Free Mobility, Ex Post Immobility, and Time-Consistent Policy in a Federal System', *Journal of Public Economics*, forthcoming.
- Pauly, M.V. (1973), 'Income Redistribution as a Local Public Good', *Journal of Public Economics* **2**, 35-58.
- Stiglitz, J. (1999), 'Taxation, Public Policy, and the Dynamics of Unemployment', *International Tax and Public Finance*, forthcoming.
- Wildasin, D.E. (1993), 'Income Distribution and Migration', *Canadian Journal of Economics* **27**, 637-656.
- Wilson, J. D. (1999), 'Theories of Tax Competition', *National Tax Journal*, forthcoming.