Inflation, Money Growth, and I(2) Analysis

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Abstract

The paper discusses the dynamics of inflation and money growth in a stochastic framework, allowing for double unit roots in the nominal variables. It gives some examples of typical I(2) 'symptoms' in empirical I(1) models and provides both a non-technical and a technical discussion of the basic differences between the I(1) and the I(2) model. The notion of long-run and medium-run price homogeneity is discussed in terms of testable restrictions on the I(2) model. The Brazilian high inflation period of 1977:1-1985:5 illustrates the applicability of the I(2) model and its usefulness to address questions related to inflation dynamics.

JEL classification: C32, E41, E31.

Keywords: Cointegrated VAR, Price Homogeneity, Cagan Model, Hyper Inflation

1 Introduction

The purpose of this paper is to give an intuitive account of the cointegrated VAR model for I(2) data and to demonstrate that the rich structure of the I(2) model is particularly relevant for the empirical analyses of economic data characterized by strongly persistent shocks to the growth rates. Such data are usually found in applications of economic models explaining the determination of nominal magnitudes. For example, the explicit assumption of a nonstationary error term in some models of money demand during periods of high or hyper inflation (Cagan, 1956, Sargent, 1977), implies that nominal money and prices are I(2). Thus, the empirical analysis of such models would only make sense in the I(2) model framework.

However, as argued in Juselius and Vuojesevic (2003), prices in hyper-inflationary episodes should not be modelled as an I(2) but rather as an explosive root process. Though such episodes are (almost by definition) short they are usually preceded by periods of high inflation rates for which the I(2) analysis is more adequate. Even though inflationary shocks in such periods are usually large, it is worth stressing that the (double) unit root property, as such, is not related to the *magnitude* but the *permanence* of shocks. Therefore, we may equally well find double unit roots in prices during typical periods of low inflation rates, like the present one, and not just in periods of high inflation rates like the seventies. But, while the persistence of shocks determine whether price inflation is I(1) or I(0), the magnitude of inflationary shocks is probably much more indicative of a risk for hyper inflation. High inflation periods are, therefore, particularly interesting as they are likely to contain valuable information about the mechanisms which subsequently might lead to hyper-inflation.

The empirical application to the Brazilian high-inflation period of 1977-1985 offers a good illustration of the potential advantages of using the I(2) model and demonstrates how it can be used to study important aspects of the inflationary mechanism in periods preceding hyper inflation.

The Cagan hyper inflation model is first translated into set of testable empirical hypotheses on the pulling and pushing forces described by the cointegrated I(2) model in AR and MA form. The paper finds strong empirical support for one of the hypothetical pulling forces, the Cagan money demand relation with the opportunity cost of holding money measured by a combination of CPI inflation and currency depreciation in the black market. The Cagan's α coefficient, defining the average inflation rate at which government can gain maximum seignorage, is estimated to be approximately 40-50% which is usually considered to describe hyper inflation. Thus, it seems likely that the seed to the subsequent Brazilian hyper inflation episode can be found in the present data. This is further supported by the finding that (1) there is a small explosive root in the VAR model, (2) the condition for long-run price homogeneity was strongly violated, and (3) the CPI price inflation showed lack of equilibrium correction behavior. The latter is associated with the widespread use of wage and price indexation, which prohibited market forces to adjust back to equilibrium after a price distortion. As a consequence domestic price inflation gained momentum as a result of increasing inflationary expectations in the foreign exchange market.

The organization of the paper is as follows: Section 2 discusses money growth and inflation in a Cagan type of high / hyper inflation model framework. Section 3 reformulates the high inflation problem in a stochastic framework allowing for double unit roots in the nominal variables. Section 4 discusses typical 'symptoms' in the VAR analysis when incorrectly assuming that the data are I(1) instead of I(2) and gives a first intuitive account of the basic difference between the I(1) and the I(2) analysis. Section 5 defines formally the I(2) model in the AR and the MA form, discusses the role of deterministic components in the I(2) model and introduces the two-step procedure for determining the two cointegration rank indices. Section 6 gives an interpretation of the various components in the I(2) model and illustrates with the Brazilian data. Section 7 discusses long-run and medium-run price homogeneity and how these can formulated as testable restrictions on the I(2) model. Section 8 presents the empirical model for money growth, currency depreciation and price inflation in Brazil. Section 9 concludes.

2 Money growth and inflation

It is widely believed that the growth in money supply in excess of real productive growth is the cause of inflation, at least in the long run. The economic intuition behind this is that other factors are limited in scope, whereas money in principle is unlimited in supply (Romer, 1996). Generally, the reasoning is based on equilibrium in the money market so that money supply equals money demand:

$$M/P = L(R, Y^r), \tag{1}$$

where M is the money stock, P the price level, Y^r real income, R an interest rate, and $L(\cdot)$ the demand for real money balances. In a high (and accelerating) inflation period, the Cagan model for hyper inflation predicts that aggregate money demand is more appropriately described by :

$$M/P = L(\pi^e, Y^r), \quad L_{\pi^e} < 0, \;\; L_{Y^r} > 0 \tag{2}$$

where π^e is expected inflation.

The latter model (2) is chosen as the baseline model in the subsequent empirical analysis of the Brazilian high inflation experience in the seventies until the mid eighties. The data consists of money stock measured as M3, the CPI price index, the black market spot exchange rate, and the real industrial production and covers the period 1977:1,...,1985:5.

The graphs of the data in levels and differences (after taking logs) gives a first indication of the order of integration. The growth rates of all three nominal variables in Figure 1 exhibit typical I(1) behavior, implying that the levels of the variables are I(2). In contrast the graphs of the log of the industrial production in levels and differences in Figure 2 do not suggest I(2) behavior: The smooth behavior typical of I(2) variables is not present in the level of industrial production and the differenced process looks significantly mean-reverting.

The middle part of Figure 2 demonstrates how real money stock (*lnM3 - lnCPI*) and real exchange rates have evolved in a nonstationary manner and increasingly so after 1981. Figure 2, lower panel compares the levels and the differences of the official and black market rate exchange rate. While the official rate seems to have stayed below the black market rate for some periods the graphs show that the two major devaluations brought the two series back to the same level. Thus, it seems likely that the black market exchange rate is a good proxy for the 'true' value of the Brazilian currency in this period.

When data are nonstationary, the Cagan model can be formulated as a cointegrating relation, i.e.:

$$(M/P)_t - L(\pi_t^e, Y_t) = v_t \tag{3}$$

where v_t is a stationary process measuring the deviation from the steady-state position at time t.

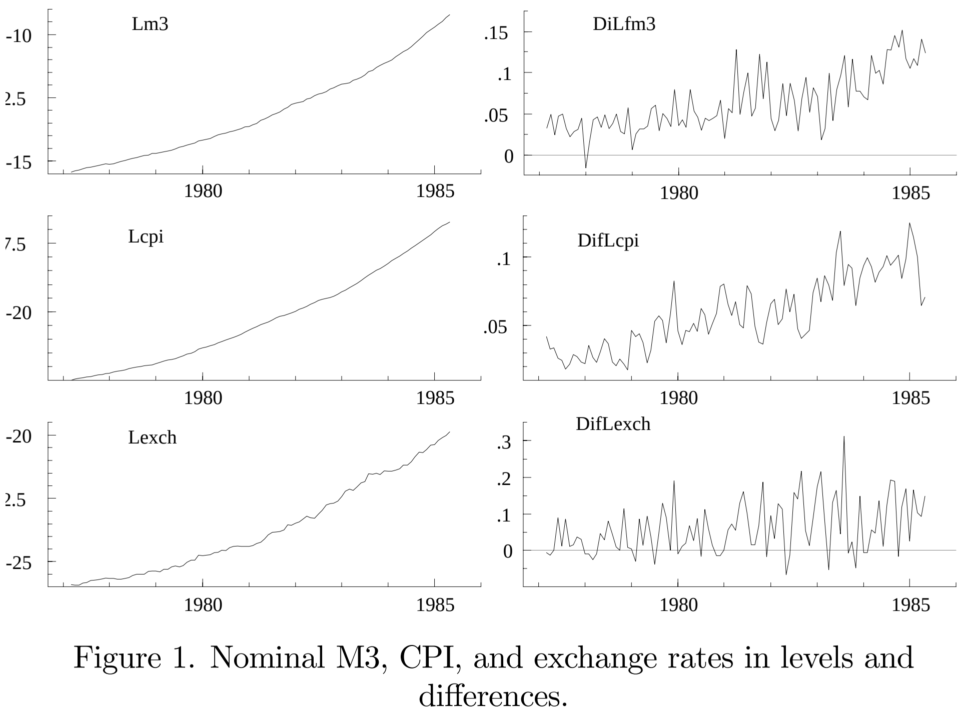

Figure 1. Nominal M3, CPI, and exchange rates in levels and differences.

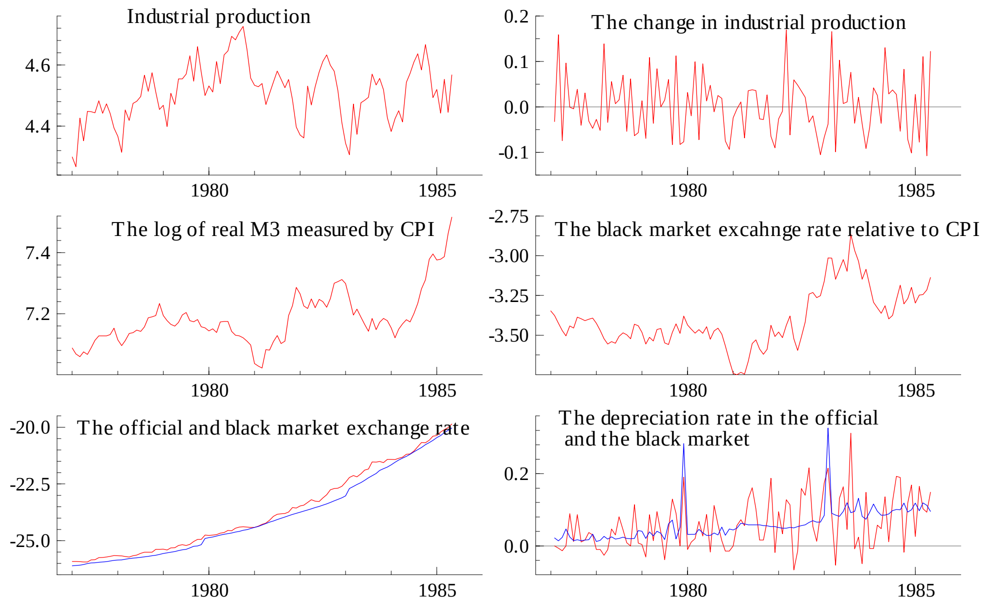

Figure 2. The graphs of industrial production in levels and differences (upper part), M3 and exchange rate both deflated with CPI (middle panel), and the black and white market exchange rate in levels and differences (lower panel).

The stationarity of v_t implies that whenever the system has been shocked it will adjust back to equilibrium and is, therefore, essential for the interpretation of (3) as a steady-state relation. If v_t is nonstationary as explicitly assumed in Sargent (1977) money supply has deviated from the steady-state value of money demand. As this case generally implies a double unit root in the data, the choice of the I(2) model for the econometric analysis seems natural. Therefore, when addressing empirical questions related to the mechanisms behind inflation and money growth in a high or hyper inflation regime we need to understand and interpret the I(2) model .

3 Formulating the economic problem in a stochastic framework

Cointegration and stochastic trends are two sides of the same coin: if there is cointegration there are also common stochastic trends. Therefore, to be able to address the transmission mechanism of monetary policy in a stochastic framework it is useful first to consider a conventional decomposition into trend, $\mathcal{T}$, cycle, $\mathcal{C}$, and irregular component, $\mathcal{I}$, of a typical macroeconomic variable.

$$X = \mathcal{T} \times \mathcal{C} \times \mathcal{I}$$

and allow the trend to be both deterministic, $\mathcal{T}_d$, and stochastic, $\mathcal{T}_s$, i.e. $\mathcal{T} = \mathcal{T}_s \times \mathcal{T}_d$, and the cyclical component to be of long duration, say 6-10 years, C_l, and of shorter duration, say 3-5 years, $\mathcal{C}_s$, i.e. $\mathcal{C} = \mathcal{C}_l \times \mathcal{C}_s$. The reason for distinguishing between short and long cycles is that a long/short cycle can either be treated as nonstationary or stationary depending on the time perspective of the study. For example, the graph of the trend-adjusted industrial production in Figure 5, lower panel, illustrates long cycles in the data that were found nonstationary by the statistical analysis.

An additive formulation is obtained by taking logarithms:

$$x = (t_s + t_d) + (c_l + c_s) + i \tag{4}$$

where lower case letters indicate a logarithmic transformation. Even if the stochastic trends are of primary interest for the subsequent analyses, a linear time trend is needed to account for average linear growth rates typical of most economic data.

3.1 Stochastic and deterministic trends

As an illustration of a trend-cycle decomposition we consider the following vector of variables $x_t = [m, p, s^b, y^r]_t,\ t = $1977:1,...,1985:5, where m is the log of M3, p is the log CPI, s^b is the log of black market exchange rate, and y^r is the log of industrial production. All variables are treated as stochastic and will be modelled, independently of whether they are considered endogenous or exogenous in the economic model.

A stochastic trend describes the cumulated impact of all previous *permanent* shocks on a variable, i.e. it summarizes all the shocks with a long lasting effect. This is contrary to a *transitory* shock, the effect of which cancels either during the next period or over the next few periods. For example, the income level of a household can be thought of as the cumulation of all previous permanent income changes (shocks), whereas the effect of temporary shocks, like lottery prizes, will not cumulate as it is only a temporary change in income.

If inflation rate is found to be I(1), then the present level of inflation can be thought of as the sum of all previous shocks to inflation, i.e.

$$\pi_t = \sum_{i=1}^{t} \varepsilon_i + \pi_0. \tag{5}$$

Because the effect of transitory shocks disappears in the cumulation a stochastic trend, t_s, is defined as the cumulative sum of previous permanent shocks, $t_{s,t} = \sum_{i=1}^{t} \varepsilon_i$. The difference between a linear stochastic and a linear deterministic trend is that the increments of a stochastic trend change randomly, whereas those of a deterministic trend are constant over time. Figure 3 illustrates three different stochastic trends measured as the once cumulated residuals from the money, price and exchange rate equations.

A representation of prices is obtained by integrating (5) once, i.e.

$$p_t = \sum_{s=1}^{t} \pi_s = \sum_{s=1}^{t} \sum_{i=1}^{s} \varepsilon_i + \pi_0 t + p_0. \tag{6}$$

Thus, if inflation is $I(1)$ with a nonzero mean (as most studies find), prices are $I(2)$ with a linear trend. Figure 4 illustrates the twice and once cumulated residuals from the CPI price equation of the VAR model defined in the next section.

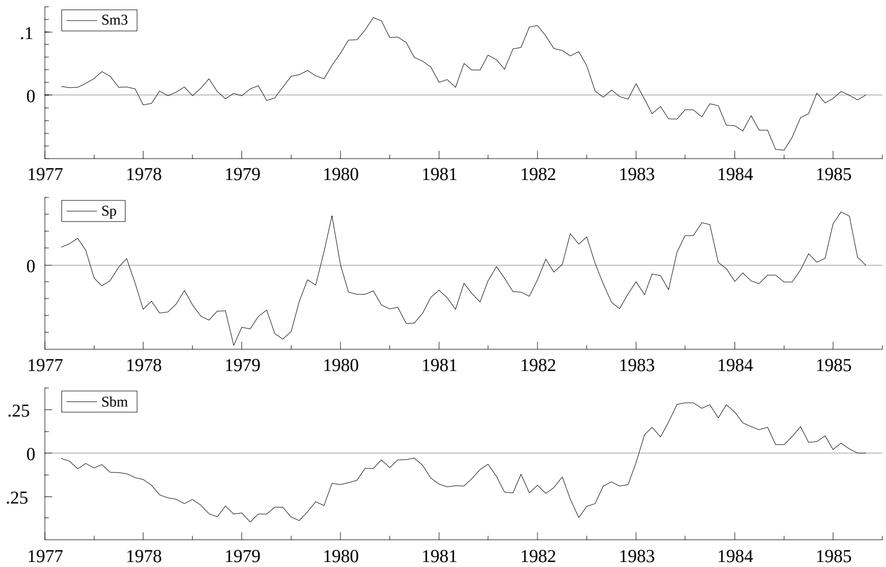

Figure 3. The graphs of the cumulated residuals from the money, price, and exchange rate equations of the estimated VAR.

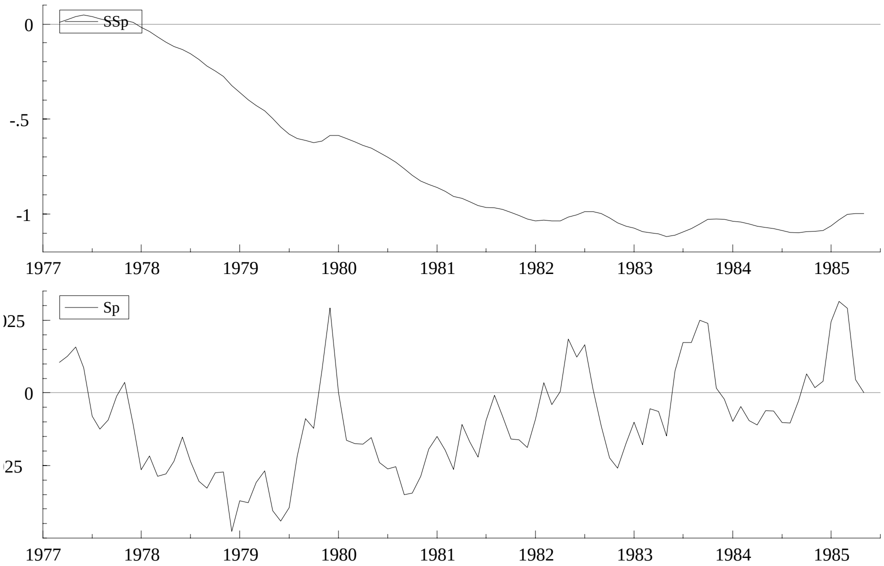

Figure 4. The graphs of the twice and once cumlated residuals from the price equation.

3.2 A trend-cycle scenario

Given the set of variables discussed above, one would expect (at least) two autonomous shocks $u_{1,t}$ and $u_{2,t}$, of which $u_{1,t}$ is a nominal shock and $u_{2,t}$ is a real shock. If there are second order stochastic trends in the data it seems plausible that they have been generated from the nominal shocks. We will, therefore, tentatively assume that the second order long-run stochastic trend t_s in (4) is described by the twice cumulated nominal shocks, $\sum_{s=1}^{t}\sum_{i=1}^{s}u_{1i}$. The long cyclical components c_l in the data will then be described by a combination of the once cumulated nominal shocks, $\sum_{i=1}^{t}u_{1i}$, and the once cumulated real shocks, $\sum_{i=1}^{t}u_{2i}$. This allows us to distinguish empirically between the long-run stochastic trend in nominal levels, $\sum_{s=1}^{t}\sum_{i=1}^{s}u_{1i}$, the medium-run stochastic trend in nominal growth rates, $\sum_{i=1}^{t}u_{1i}$, and the medium-run stochastic trend in real activity, $\sum_{i=1}^{t}u_{2i}$. Figure 5 illustrates.

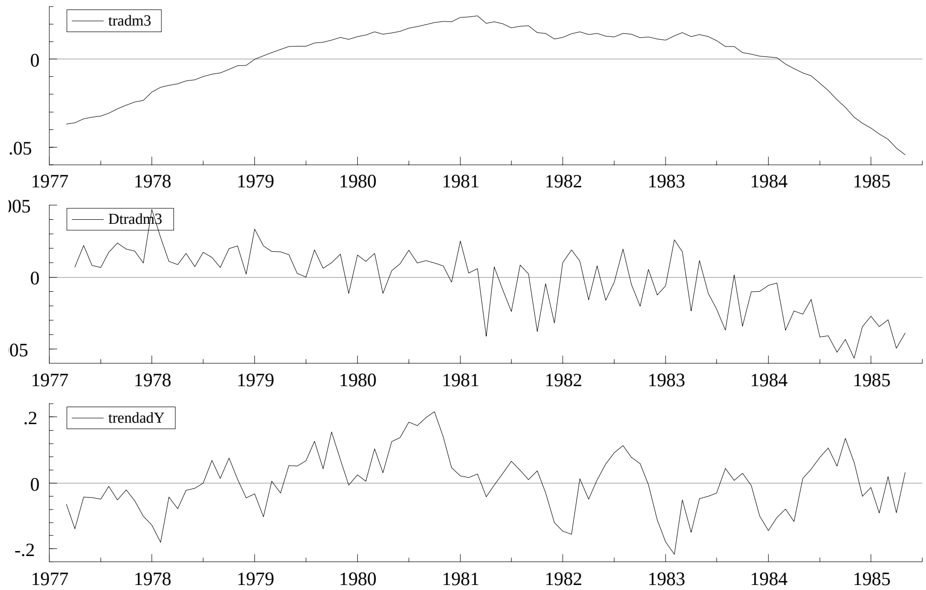

Figure 5. The graphs of trend-adjusted M3 in levels and differences (upper and lower panel) and trend-adjusted industrial production (lower panel).

The trend-cycle formulation below illustrates the ideas:

$$\begin{bmatrix} m_t \\ p_t \\ s_t^b \\ y_t^r \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ 0 \end{bmatrix} \left[\sum_{s=1}^{t} \sum_{i=1}^{s} u_{1i} \right] + \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \\ d_{31} & d_{32} \\ d_{41} & d_{42} \end{bmatrix} \begin{bmatrix} \sum_{i=1}^{t} u_{1i} \\ \sum_{i=1}^{t} u_{2i} \end{bmatrix} + \begin{bmatrix} g_1 \\ g_2 \\ g_3 \\ g_4 \end{bmatrix} [t] + \text{stat.comp.} \tag{7}$$

The deterministic trend component, $t_d = t$, is needed to account for linear growth trends present in the levels of the variables. If $g_4 = 0$ and $d_{41} = 0$ in (7), then $\sum_{i=1}^{t} u_{2,i}$ is likely to describe the long-run trend in industrial production. In this case it may be possible to interpret $\sum_{i=1}^{t} u_{2,i}$ as a "structural" unit root process (cf. the discussion in King, Plosser, Stock and Watson (1991) on stochastic versus deterministic real growth models).

If, on the other hand, $g_4 \neq 0$, then it seems plausible that the long-run real trend can be approximated by a linear deterministic time trend. In this case $\sum_{i=1}^{t} u_{2,i}$ is likely to describe medium-run deviations from the linear trend, i.e. the long business cycle. The graph of the trend-adjusted industrial production in the lower panel of Figure 5 illustrates such a long cycle starting from the long upturn from 1977-1980:6 and ending with the downturn 1980:6-1984. Note also the shorter cycles of approximately a year's duration imbedded in the long cycle.

Therefore, the possibility of interpreting the second stochastic trend, $\sum_{i=1}^{t} u_{2,i}$, as a long-run structural trend depends crucially on whether one includes a linear trend in (7) or not.

The trend components of m_t, p_t, s_t, and y_t in (7) can now be represented by:

$$\begin{array}{llllll} m_t = c_1 \sum\sum u_{1i} & +d_{11} \sum u_{1i} & +d_{12} \sum u_{2i} & +g_1 t + stat.\, comp. \\ p_t = c_2 \sum\sum u_{1i} & +d_{21} \sum u_{1i} & +d_{22} \sum u_{2i} & +g_2 t + stat.\, comp \\ s_t = c_3 \sum\sum u_{1i} & +d_{31} \sum u_{1i} & +d_{32} \sum u_{2i} & +g_3 t + stat.\, comp \\ y_t = & +d_{41} \sum u_{1i} & +d_{42} \sum u_{2i} & +g_4 t + stat.\, comp \end{array} \tag{8}$$

If $(c_1, c_2, c_3) \neq 0$, then $\{m_t,\ p_t, s_t\} \sim I(2)$. If, in addition, $c_1 = c_2 = c_3$ then

$$\begin{array}{rllll} m_t - p_t = & (d_{11} - d_{21}) \sum u_{1i} & +(d_{12} - d_{22}) \sum u_{2i} & +(g_1 - g_2)t & +stat.comp. \\ p_t - s_t = & (d_{21} - d_{31}) \sum u_{1i} & +(d_{22} - d_{32}) \sum u_{2i} & +(g_2 - g_3)t & +stat.comp. \\ m_t - s_t = & (d_{11} - d_{31}) \sum u_{1i} & +(d_{12} - d_{32}) \sum u_{2i} & +(g_1 - g_3)t & +stat.comp. \\ y_t = & +d_{41} \sum u_{1i} & +d_{42} \sum u_{2i} & +g_4 t & +stat.comp. \end{array} \tag{9}$$

The real variables are at most $I(1)$ but, unless $(g_1 = g_2)$, $(g_2 = g_3)$, and $(g_1 = g_3)$, they are I(1) around a linear trend. Figure 5 illustrates the trend-adjusted behavior of real M3 and industrial production.

Long-run price homogeneity among all the variables implies that both the long-run stochastic I(2) trends and the linear deterministic trends should cancel in (9). But, even if overall long-run homogeneity is rejected, some of the individual components of $\{m_t - p_t,\ p_t - s_t,\ m_t - s_t\}$ can, nevertheless, exhibit long-run price homogeneity. For example, the case $(m_t - p_t) \sim I(1)$ is a testable hypothesis which implies that money stock and prices are moving together in the long-run, though not necessarily in the medium-run (over the business cycle).

The condition for long-run and medium-run price homogeneity is $\{c_{11} = c_{21}$, and $d_{11} = d_{21}\}$, i.e. that the nominal shocks u_{1t} affect nominal money and prices in the same way both in the long run and in the medium run. Because the real stochastic trend $\sum u_{2i}$ is likely to enter m_t but not necessarily p_t, testing long-run and medium-run price homogeneity jointly is not equivalent to testing $(m_t - p_t) \sim I(0)$. Testing the composite hypothesis is more involved than the long-run price homogeneity alone.

It is important to note that $(m_t - p_t) \sim I(1)$ implies $(\Delta m_t - \Delta p_t) \sim I(0)$, i.e. long-run price homogeneity implies a stationary spread between price inflation and money growth. In this case the stochastic trend in inflation is the same as the stochastic trend in money growth. The econometric formulation of long-run and medium-run price-homogeneity in the I(2) model will be discussed in Section 7.

When overall long-run price homogeneity holds it is convenient to transform the nominal system (8) to a system consisting of real variables and a nominal growth rate, for example:

$$\begin{bmatrix} m_t - p_t \\ s_t - p_t \\ \Delta p_t \\ y_t \end{bmatrix} = \begin{bmatrix} d_{11} - d_{21} & d_{12} - d_{22} \\ d_{21} - d_{31} & d_{22} - d_{32} \\ c_{21} & 0 \\ d_{41} & d_{42} \end{bmatrix} \begin{bmatrix} \sum_{i=1}^{t} u_{1,i} \\ \sum_{i=1}^{t} u_{2,i} \end{bmatrix} + \begin{bmatrix} g_1 - g_2 \\ g_2 - g_3 \\ 0 \\ g_4 \end{bmatrix} [t] + ... \tag{10}$$

Given long-run price homogeneity all variables are at most $I(1)$ in (10). The nominal growth rate (measured by Δp_t, Δm_t, or Δs_t) is only affected by the once cumulated nominal trend, $\sum_{i=1}^{t} u_{1,i}$, but all the other variables can (but need not) be affected by both stochastic trends, $\sum_{i=1}^{t} u_{1,i}$ and $\sum_{i=1}^{t} u_{2,i}$.

The case $(m_t - p_t - y_t) \sim I(0)$, i.e. the inverse velocity of circulation is a stationary variable, requires that $d_{11} - d_{21} - d_{41} = 0$, $d_{12} - d_{22} - d_{42} = 0$ and $g_1 - g_2 - g_4 = 0$. If $d_{11} = d_{21}$ (i.e. medium run price homogeneity), $d_{22} = 0$ (real stochastic growth does not affect prices), $d_{41} = 0$ (medium-run inflationary movements do not affect real income), and $d_{12} = d_{42}$, then $m_t - p_t - y_t \sim I(0)$. In this case real money stock and real aggregate income share one common trend, the real stochastic trend $\sum u_{2i}$. The stationarity of money velocity, implying common movements in money, prices, and income, would then be consistent with the conventional monetarist assumption as stated by Friedman (1970) that "inflation always and everywhere is a monetary problem". This case would correspond to model (1) in Section 2.

The case $(m_t - p_t - y_t) \sim I(1)$, implies that the two common stochastic trends affect the level of real money stock and real income differently. Cagan's model of money demand in a high (hyper) inflation period suggests that the nonstationarity of the liquidity ratio is related to the expected rate of inflation $\mathcal{E}_t(\Delta p_{t+1})$. The latter is generally not observable, but as long as $\mathcal{E}_t(\Delta p_{t+1}) - \Delta p_t$ is a stationary disturbance, one can replace the unobserved expected inflation with actual inflation without loosing cointegration. The condition that $\{\mathcal{E}_t(\Delta p_{t+1}) - \Delta p_t\} \sim I(0)$ seems plausible considering that $\{\Delta p_{t+1} - \Delta p_t\} \sim I(0)$ when $p_t \sim I(2)$. It amounts to assuming that $\{\mathcal{E}_t(\Delta p_{t+1}) - \Delta p_{t+1}\} \sim I(0)$, i.e. agents' inflationary expectations do not systematically deviate from actual inflation. Therefore, from a cointegration point of view we can replace the expected inflation with the actual inflation:

$$m_t - p_t - y_t + a_1 \Delta p_t \sim I(0), \tag{11}$$

or, equivalently:

$$(m_t - p_t - y_t) + a_2 \Delta s_t \sim I(0).$$

where under the Cagan model $a_1 > 0$, $a_2 > 0$.

4 Diagnosing I(2)

VAR models are widely used in empirical macroeconomics based on the assumption that data are I(1) without first testing for I(2) or checking whether a near unit root remains in the model after the cointegration rank has been imposed. Unfortunately, when the data contains a double unit root essentially all inference in the I(1) model is affected. To avoid making wrong inference it is, therefore, important to be able to diagnose typical I(2) symptoms in the I(1) VAR model.

For the Brazilian data, the unrestricted VAR model was specified as:

$$\begin{gathered} \Delta x_t = \Gamma_1 \Delta x_{t-1} + \Pi x_{t-2} + \mu_1 t + \mu_0 + \Phi_p Dp83.8_t + \Phi_s Qs_t + \varepsilon_t, \\ \varepsilon_t \sim N_p(0, \Omega\ ),\ t = 1, ..., T \end{gathered} \tag{12}$$

where $x_t = [m_t, p_t, s_t^b, y_t^r]$, $t =$*1977:1,...,1985:5,* $\Pi = \alpha\beta'$, $\mu_1 = \alpha\beta_1$, $\mu_{01} = \alpha\beta_{01}$, and $(\Gamma_1, \mu_0, \Phi_p, \Phi_s, \Omega)$ are unrestricted. The estimates have been calculated using CATS for RATS, Hansen and Juselius (1994). Misspecification tests are reported in the Appendix.

The data are distinctly trending and we need to allow for linear trends both in the data and in the cointegration relations when testing for cointegration rank (Nielsen and Rahbek, 2000). The industrial production, y_t^r, exhibits strong seasonal variation and we include 11 seasonal dummies, Qs_t, and a constant, μ_0 in the VAR model. Finally, the graphs of the differenced black market exchange rate and nominal M3 money stock exhibited an extraordinary large shock at 1983:8, which was accounted for by an unrestricted impulse dummy $Dp83.8_t = 1$ for t = 1983:8 and 0 otherwise. A permanent shock to the changes corresponds to a level shift in the variables, which may or may not cancel in the cointegration relations. To account for the latter possibility the shift dummy, $Ds83.8_t = 0$ for t = 1983:8 and 1 otherwise, was restricted to be in the cointegration relations. It was found to be insignificant (p-value 0.88) and was left out.

The I(1) estimation procedure is based on the so called R-model in which the short-run effects have first been concentrated out:

$$R_{0t} = \alpha\beta' R_{1t} + \varepsilon_t. \tag{13}$$

where R_{0t} and R_{1t} are defined by:

$$\underbrace{\Delta x_t}_{I(1)} = \hat{B}_{11}\underbrace{\Delta x_{t-1}}_{I(1)} + const + B_{13}Dp_t + \underbrace{R_{0t}}_{I(0)} \tag{14}$$

and

$$\underbrace{\tilde{x}_{t-1}}_{I(2)} = \hat{B}_{21}\underbrace{\Delta x_{t-1}}_{I(1)} + const + B_{23}Dp_t + \underbrace{R_{1t}}_{I(2)}. \tag{15}$$

$\tilde{x}_t' = [m_t, p_t, s_t^b, y_t^r, t]$ and Dp_t is a catch-all for all the dummy variables. If $x_t \sim I(2)$ then $\Delta x_t \sim I(1)$ and (14) is a regression of an I(1) process on its own lag. Thus, the regressand and the regressor contain the same common trend which will cancel in regression. This implies that $R_{0t} \sim I(0)$, even if $x_t \sim I(2)$. On the other hand equation (15) is a regression of an I(2) variable, $\tilde{x}_{t-1}$, on an I(1) variable, Δx_{t-1}. Because an I(2) trend cannot be canceled by regressing on an I(1) trend, it follows that $R_{1t} \sim I(2)$.

Therefore, when $x_t \sim I(2)$ (13) is a regression of an $I(0)$ variable (R_{0t}) on an $I(2)$ variable (R_{1t}). Under the (testable) assumption that $\varepsilon_t \sim I(0)$, either $\beta' R_{1t} = 0$ or $\beta' R_{1t} \sim I(0)$ for the equation (13) to hold. Because the linear combination $\beta' R_{1t}$ transforms the process from I(2) to I(0), the estimate $\hat{\beta}$ is super-super consistent (Johansen, 1992). Even though β is precisely estimated in the I(1) model when data are I(2), the interpretation of $\beta' x_t$ as a stationary long-run relation has to be modified as will be demonstrated below.

It is easy to demonstrate the connection between $\beta' x_{t-2}$ and $\beta' R_{1t}$ by inserting (15) into (13) :

$$\begin{aligned} R_{0t} &= \alpha\beta' R_{1t} + \varepsilon_t \\ & \alpha\beta'(\tilde{x}_{t-1} - B_2\Delta x_{t-1}) + \varepsilon_t \\ &= \alpha(\beta'\tilde{x}_{t-1} - \beta' B_2 \Delta x_{t-1}) + \varepsilon_t \\ &= \alpha\underbrace{(\underbrace{\beta'\tilde{x}_{t-1}}_{I(1)} - \omega'\underbrace{\Delta x_{t-1}}_{I(1)})}_{I(0)} + \varepsilon_t \end{aligned} \tag{16}$$

where $\omega = \beta' B_2$. It appears that the stationary relations $\beta' R_{1t}$ consists of two components $\beta'\tilde{x}_{t-1}$ and $\omega'\Delta x_{t-1}$ both of which are generally I(1). The stationarity of $\beta' R_{1t}$ is, therefore, a consequence of cointegration between $\beta'\tilde{x}_{t-1} \sim I(1)$ and $\omega'\Delta x_{t-1} \sim I(1)$.

Thus, when data are I(2), $\beta_i'\tilde{x}_t \sim I(1)$, while $\beta_i' R_{1t} \sim I(0)$ for at least one i, $i = 1, ..., r$. It is, therefore, a clear sign of double unit roots (or, alternatively, a unit root and an explosive root) in the model when the graphs of $\beta_i'\tilde{x}_t$ exhibits nonstationary behavior whereas $\beta_i' R_{1t}$ looks stationary. As an illustration we have reported the graphs of all four cointegration relations (of which $\beta_1' R_{1t}$ and $\beta_2' R_{1t}$ are stationary) in <Figures 6-9>. The upper panels contain the relations, $\beta_i'\tilde{x}_t$, and the lower panels the cointegration relations corrected for short-run dynamics, $\beta_i' R_{1t}$.

Among the graphs in Figures 6 and 7 $\beta_1'\tilde{x}_t$ and $\beta_2'\tilde{x}_t$ exhibit distinctly nonstationary behavior whereas the graphs of the corresponding $\beta_i' R_{1,t}$ look reasonably stationary. This is strong evidence of double roots in the data. As all the remaining graphs seem definitely nonstationary, this suggests that $r = 2$ and that there is at least one $I(2)$ trend in the data.

Another way of diagnosing I(2) behavior is to calculate the characteristic roots of the VAR model for different choices of the cointegration rank r. When $x_t \sim I(2)$ the number of unit roots in the characteristic polynomial of the VAR model is $s_1 + 2s_2$, where s_1 and s_2 are the number of autonomous $I(1)$ and $I(2)$ trends respectively and $s_1 + s_2 = p - r$.

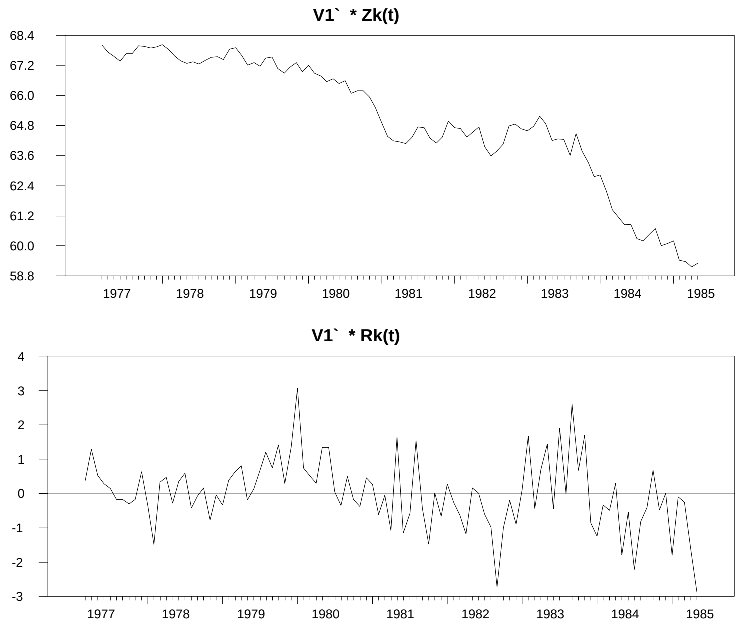

Figure 6. The graphs of $\beta_1' x_t$ (upper panel) and $\beta_1' R_{1t}$ (lower panel).

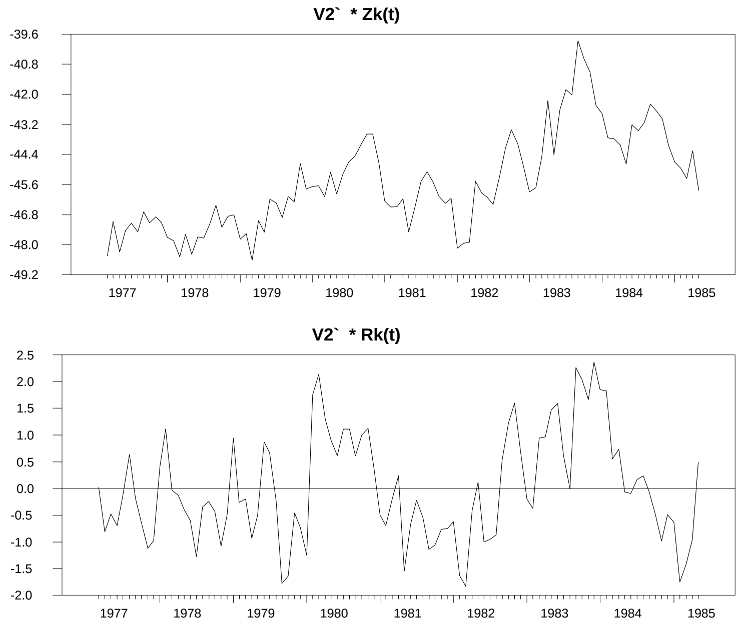

Figure 7. The graphs of $\beta_2' x_t$ (upper panel) and $\beta_2' R_{1t}$ (lower panel).

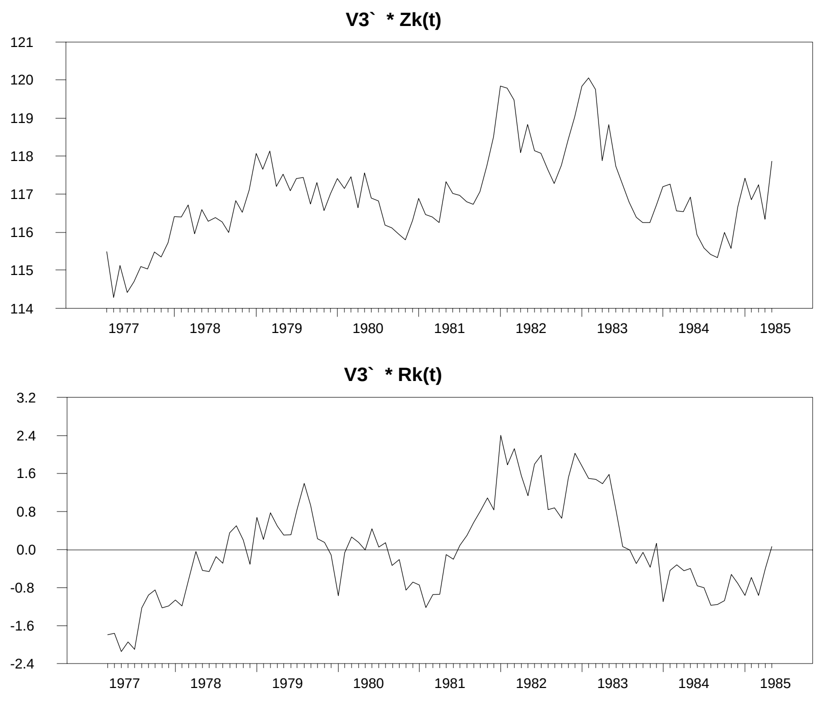

Figure 8. The graphs of $\beta_3' x_t$ (upper panel) and $\beta_3' R_{1t}$ (lower panel).

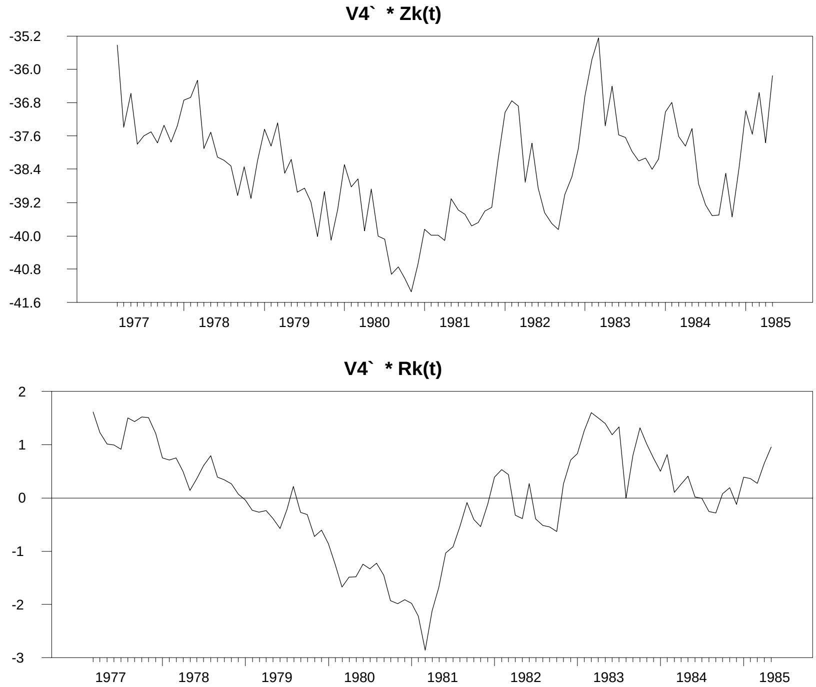

Figure 9. The graphs of $\beta_4' x_t$ (upper panel) and $\beta_4' R_{1t}$ (lower panel).

The characteristic roots contain information on unit roots associated with both Γ and Π, whereas the standard $I(1)$ trace test is only related to the number of unit roots in the Π matrix. If the data are $I(1)$ the number of unit roots (or near unit roots) should be $p - r$, otherwise $p - r + s_2$. Therefore, if for any reasonable choice of r there are still (near) unit roots in the model, it is a clear sign of I(2) behavior in at least some of the variables. Because the additional unit root(s) are related to Δx_{t-1}, i.e. belong to the matrix $\Gamma = I - \Gamma_1$, lowering the value of r does not remove the s_2 additional unit root associated with the I(2) behavior.

In the Brazilian nominal model there are altogether $p \times k = 4 \times 2 = 8$ eigenvalue roots in the characteristic polynomial which are reported below for $r = 1, ..., 4$. Unrestricted near unit roots are indicated with bold face.

$VAR(p)$	**1.002**	**0.97**	**0.90**	**0.90**	0.38	0.33	0.06	0.06
$r = 3$	1.0	**1.002**	**0.91**	**0.91**	0.38	0.33	0.06	0.06
$r = 2$	1.0	1.0	**0.99**	**0.86**	0.38	0.32	0.09	0.07
$r = 1$	1.0	1.0	1.0	**1.001**	0.61	0.33	0.09	0.00

In the unrestricted model two of the roots are very close to the unit circle, one is larger than unity possibly indicating explosive behavior, the other is a stable near unit root. In addition there is a complex pair of two fairly large roots. The presence of an unstable root can be seen in the graph of the first cointegration relation $\beta_1'\tilde{\mathbf{x}}_t$ in Figure 6: The equilibrium error in the 'steady-state' relation in levels grows in an unstable manner at the end of the period, but is 'compensated' by a similar increase in the inflation rate, so $\beta_1' R_{1,t}$ looks stationary. This suggests that the seed to the Brazilian hyper inflation in the subsequent period can already be found in the present data.

However, the explosive part of the root is very small and might not be statistically significant. In such a case we would expect the unstable root to disappear when restricting the rank. We notice that for $r = 3$ and $r = 1$ the explosive root is still left in the model, whereas for $r = 2$ it has disappeared. Independently of the choice of r, a near unit root remains in the model consistent with I(2), or moderately explosive, behavior. Therefore, we continue with $r = 2$ and disregard the possibility of an explosive root in the econometric analysis. Subsequently we will use the empirical results to demonstrate where in the model the seed to the subsequent hyper inflationary behavior can be found.

In most cases a graphical inspection of the data is sufficient to detect I(2) behavior and it might seem meaningless to estimate the I(1) model when x_t is in fact $I(2)$. However, a variety of hypotheses can be adequately tested using the I(1) procedure with the caveat that the interpretation of the cointegration results should be in terms of $CI(2,1)$ relations, i.e. relations which cointegrated from I(2) to I(1), and not from I(1) to I(0). One of the more important hypotheses which can be tested is the long-run price homogeneity of β to be discussed in Section 7.

5 Defining the I(2) model

It is useful to reformulate the VAR model defined in the previous section in acceleration rates, changes and levels:

$$\begin{gathered}\Delta^2 x_t = \Gamma\Delta x_{t-1} + \Pi x_{t-1} + \Phi_p D_{p,t} + \Phi_s Q_{s,t} + \mu_0 + \mu_1 t + \varepsilon_t, \\ \varepsilon_t \sim N_p(0, \Omega\ ),\ t = 1, ..., T\end{gathered} \tag{17}$$

where $\Gamma = -(I - \Gamma_1)$ and $\mu_1 = \alpha\mu_{1.0}$ is restricted to lie in $sp(\alpha)$ (cf. Section 5.3).

5.1 The AR formulation

The hypothesis that x_t is $I(2)$ is formulated in Johansen (1992) as two reduced rank hypotheses:

$$\Pi = \alpha\beta'\ ,\ \text{where}\ \alpha, \beta\ \text{are}\ p \times r \tag{18}$$

and

$$\alpha'_{\perp}\Gamma\beta_{\perp} = \zeta\eta', \text{ where } \zeta, \eta \text{ are } (p-r) \times s_1. \tag{19}$$

The first condition is the usual I(1) reduced rank condition associated with the variables in levels, whereas the second condition is associated with the variables in differences. The intuition is that the differenced process also contains unit roots when data are I(2). Note, however, that (19) is formulated as a reduced rank condition on the transformed Γ. The intuition behind this can be seen by pre-multiplying (17) with $\alpha_{\perp}$ (and post-multiplying by $\beta_{\perp}$). This makes the levels component $\alpha\beta' x_{t-2}$ disappear and reduces the model to a $((p-r) \times (p-r))$-dimensional system of equations in first- and second order differences. In this system the hypothesis of reduced rank of the matrix $\alpha'_{\perp}\Gamma\beta_{\perp}$ is tested in the usual way. Thus, the second reduced rank condition is similar to the first except that the reduced rank regression is on the $p-r$ common driving trends. Using (19) it is possible to decompose $\alpha_{\perp}$ and $\beta_{\perp}$ into the I(1) and I(2) directions:

$$\alpha_{\perp} = \{\alpha_{\perp 1}, \alpha_{\perp 2}\} \text{ and } \beta_{\perp} = \{\beta_{\perp 1}, \beta_{\perp 2}\}, \tag{20}$$

where $\alpha_{\perp 1} = \alpha_{\perp}(\alpha'_{\perp}\alpha_{\perp})^{-1}\zeta$ and $\beta_{\perp,1} = \beta_{\perp}(\beta'_{\perp}\beta_{\perp})^{-1}\eta$ is $p \times s_1$, $\alpha_{\perp 2} = \alpha_{\perp}\zeta_{\perp}$ and $\beta_{\perp 2} = \beta_{\perp}\eta_{\perp}$ is $p \times s_2$, and $\zeta_{\perp}, \eta_{\perp}$ are the orthogonal complements of ζ and η, respectively. Note that the matrices $\alpha_{\perp 1}, \alpha_{\perp 2}, \beta_{\perp 1}$, and $\beta_{\perp 2}$ are called $\alpha_1, \alpha_2, \beta_1$ and β_2 in the many papers on I(2) by Johansen and coauthors. The reason why we deviate here from the simpler notation is that we need to distinguish between different β and α vectors in the empirical analysis and, hence, use the latter notation for this purpose.

While the I(1) model is only based on the distinction between r cointegrating relations and $p-r$ non-cointegrating relations, the I(2) model makes an additional distinction between s_1 I(1) trends and s_2 I(2) trends. Furthermore, when $r > s_2$, the r cointegrating relations can be divided into $r_0 = r - s_2$ directly stationary $CI(2,2)$ relations (cointegrating from I(2) to I(0)) and s_2 polynomially cointegrating relations. This distinction will be illustrated in Section 6 based on the Brazilian data.

5.2 The moving average representation

The moving average representation of (17) describes the variables as a function of stochastic and deterministic trends, stationary components, initial values and deterministic dummy variables. It is given by:

$$
\begin{aligned}
x_t &= C_2 \sum_{s=1}^{t} \sum_{i=1}^{s} \varepsilon_i + C_2 \tfrac{1}{2}\mu_0 t^2 + C_2 \Phi_p \sum_{s=1}^{t} \sum_{i=1}^{s} Dp_i + C_2 \Phi_s \sum_{s=1}^{t} \sum_{i=1}^{s} Qs_i \\
&+ C_1 \sum_{s=1}^{t} \varepsilon_s + C_1 \Phi_p \sum_{s=1}^{t} Dp_s + C_2 \Phi_s \sum_{s=1}^{t} Qs_s + (C_1 + \tfrac{1}{2} C_2)\mu_0 t + \gamma_1 t \\
&+ Y_t + A + Bt, \;\; t = 1, ..., T
\end{aligned} \tag{21}
$$

where Y_t defines the stationary part of the process, A and B are functions of the initial values $x_0, x_{-1}, ..., x_{-k+1}$, and the coefficient matrices satisfy:

$$
C_2 = \beta_{\perp 2}(\alpha'_{\perp 2}\Psi\beta_{\perp 2})^{-1}\alpha'_{\perp 2}, \quad \beta' C_1 = -\overline{\alpha}'\Gamma C_2, \quad \beta'_{\perp 1} C_1 = -\overline{\alpha}'_{\perp 1}(I - \Psi C_2) \tag{22}
$$

where $\Psi = \Gamma\overline{\beta}\overline{\alpha}'\Gamma + I - \Gamma_1$ and the shorthand notation $\overline{\alpha} = \alpha(\alpha'\alpha)^{-1}$ is used. See Johansen (1992, 1995).

We denote $\widetilde{\beta}_{\perp 2} = \beta_{\perp 2}(\alpha'_{\perp 2}\Psi\beta_{\perp 2})^{-1}$ so that

$$
C_2 = \widetilde{\beta}_{\perp 2}\alpha'_{\perp 2} \tag{23}
$$

i.e. the C_2 matrix has a similar reduced rank representation as C_1 in the I(1) model. It is, therefore, natural to interpret $\alpha'_{\perp 2}\Sigma\Sigma\varepsilon_i$ as the second order stochastic trend that has affected the variables x_t with weights $\widetilde{\beta}_{\perp 2}$. However, the C_1 matrix cannot be decomposed similarly. It is a more complex function of the AR parameters of the model and the C_2 matrix and the interpretation of the parameters $\alpha_{\perp 1}$ and $\beta_{\perp 1}$ is less intuitive.

The MA representation (22) together with (23) can be used to obtain ML estimates of the stochastic and deterministic trends and cycles and their loadings in the intuitive scenario (8) of Section 3. This will be illustrated in Section 6.

5.3 Deterministic components in the I(2) model

It appears from (21) that an unrestricted constant in the model is consistent with linear and quadratic trends in the data. Johansen (1992) suggested the decomposition of the constant term μ_0 into the $\alpha, \alpha_{\perp 1}, \alpha_{\perp 2}$ projections:

$$
\mu_0 = \alpha\mu_0 + \gamma_0 + \gamma_1,
$$

where

- μ_0 is a constant term in the stationary cointegration relations,
- γ_0 is the slope coefficient of linear trends in the variables, and

- γ_1 is the slope coefficient of quadratic trends in the variables.

Quadratic trends in the levels of the variables is consistent with linear trends in the growth rates, i.e. in inflation rates, which generally does not seem plausible (not even as a local approximation). Therefore, the empirical model will be based on the assumption that the data contain linear but no quadratic trends, i.e. that $\gamma_1 = 0$.

Similar arguments can be given for the dummy variables. An unrestricted shift dummy, such as $Ds83.8_t$, in the model is consistent with a broken quadratic trend in the data, whereas an unrestricted blip dummy, such as $Dp83.8_t = \Delta Ds83_t$, is consistent with a broken linear trend in the data. Thus, a correct specification of dummies is important as they are likely to strongly affect both the model estimates and the asymptotic distribution of the rank test.

In many cases it is important to allow for trend-stationary relations in the I(2) model (Rahbek, Kongsted, and Jørgensen, 1999). In this case $\mu_1 t \neq 0$ and the vector μ_1 needs to be decomposed in a similar way as the constant term:

$$\mu_1 = \alpha' \mu_1 + \delta_0 + \delta_1,$$

where

- μ_1 is the slope coefficient of a linear trend in the cointegration relations,
- δ_0 is the slope coefficient of quadratic trends in the variables, and
- δ_1 is the slope coefficient of cubic trends in the variables.

Since the presence of deterministic quadratic or cubic trends are not very plausible we will assume that $\delta_0 = \delta_1 = 0$.

5.4 The determination of the two rank indices

The cointegration rank r can be determined either by the two-step estimation procedure in Johansen (1995) based on the polynomial cointegration property of $\beta' x_t$, or by the $FIML$ procedure in Johansen (1997) based on the $CI(2,1)$ property of $\beta' x_t$ and $\beta'_{\perp 1} x_t$. The idea of the two-step procedure is as follows: The first step determines $r = \overline{r}$ based on the trace test in the standard I(1) model and the estimates $\hat{\alpha}$ and $\hat{\beta}$. The second step determines $s_1 = \overline{s}_1$ by solving the reduced rank problem for the matrix $(\hat{\alpha}'_{\perp} \Gamma \hat{\beta}_{\perp})$. The practical procedure is to calculate the trace test for all possible combinations of r and s_1 so that the joint hypothesis (r, s_1) can be tested using the procedure in Paruolo (1996).

Table 1: Testing the two rank indices in the I(2) model

p-r	r	FIML test procedure: $Q(s_1, r)$				$Q(r)$	λ_i
4	0	323.87 [0.00]	220.09 [0.00]	149.68 [0.00]	99.01 [0.00]	95.91 [0.00]	0.43
3	1		141.95 [0.00]	73.89 [0.02]	51.69 [0.08]	44.16 [0.04]	0.24
2	2			48.92 [0.05]	24.14 [0.47]	19.67 [0.25]	0.12
1	3				13.40 [0.34]	7.29 [0.32]	0.05
s_2		4	3	2	1	0	

Based on a broad simulation study Nielsen and Rahbek (2003) show that the $FIML$ procedure has better size properties than the two-step procedure. The estimates here are, therefore, based on the $FIML$ procedure using a new version of CATS for RATS developed by Jonathan Dennis.

Table 1 reports the test of the joint hypothesis (r, s_1) with the 95% quantiles of the simulated distribution given in brackets. They are derived for a model with a linear trend restricted to be in the cointegration space. The test procedure starts with the most restricted model $(r = 0, s_1 = 0, s_2 = 4)$ in the upper left hand corner, continues to the end of the first row $(r = 0, s_1 = 4, s_2 = 0)$, and proceeds similarly rowwise from left to right until the first acceptance. The first acceptance is at $(r = 1, s_1 = 1, s_2 = 1)$ with a p-value of 0.08. However, the case $(r = 2, s_1 = 1, s_2 = 1)$ is accepted with a much higher p-value 0.47 and will be our preferred choice. As a matter of fact, the subsequent results will demonstrate that the second relation plays a crucial role in the prce mechanisms which led to hyper inflation.

To improve the small sample properties of the test procedures, a Bartlett correction can be employed (Johansen, 2000). Even though it significantly improves the size of the cointegration rank, the power of the tests is generally very low for I(2) or near I(2) data.

The Paruolo procedure delivers a correct size asymptotically, but does not solve the problem of low power. Because economic theory is often consistent with few rather than many common trends, a reversed order of testing might be preferable from an economic point of view. However, in that case the test will no longer deliver a correct asymptotic size.

Furthermore, when the $I(2)$ model contains intervention dummies that cumulate to trends in the DGP, standard asymptotic tables are no longer valid. For example, an unrestricted impulse dummy, like $Dp83.8_t$, will cumulate to a broken linear trend in the data. The asymptotic distributions for the $I(2)$ model do not account for this feature. Since the null of a unit root is not necessarily reasonable from an economic point of view, the low power and the impact of the dummies on the distributions can be a serious problem. This can sometimes be a strong argument for basing the choice of r and s_1 on prior information given by the economic insight as well as the statistical information in the data. As demonstrated in Section 4 such information can be a graphical inspection and the number of (near) unit roots in the characteristic polynomial of the VAR.

For the present choice of rank ($r = 2,\ s_1 = 1, s_2 = 1$) the characteristic roots of the VAR model became

$$1.0 \;\; 1.0 \;\; 1.0 \;\; 0.89 \;\; 0.39 \;\; 0.06 \;\; -0.09 \;\; -0.32$$

leaving a fairly large root in the model. Therefore, another possibility would have been to choose $r = 2\,, s_1 = 0, s_2 = 2$.

6 Interpreting the I(2) structure

It is no easy task to give the intuition for the different levels of integration and cointegration in the $I(2)$ model and how they can be translated into economically relevant relationships. Table 2 illustrates the I(2) decomposition of the Brazilian data, which is based on the following assumptions (anticipating the subsequent results):

$$m_t \sim I(2), p_t \sim I(2), s_t^b \sim I(2), y_t^r \sim I(1)$$

and

$$\underbrace{r = 2}_{r_0=1, r_1=1}\ , \text{ and } \underbrace{p - r}_{s_1=1, s_2=1} = 2$$

The left hand side of Table 2 illustrates the decomposition of x_t into two β and two $\beta_\perp$ directions corresponding to $r = 2$ and $p - r = 2$. This decomposition defines two stationary polynomially cointegrating relations, $\beta_1' x_t + \omega_1' \Delta x_t$ and $\beta_2' x_t + \omega_2' \Delta x_t$, and two nonstationary relations, $\beta_{\perp 1}' x_t \sim I(1)$ and $\beta_{\perp 2}' x_t \sim I(2)$. Note that $\beta_{\perp 1}' x_t$ is cointegrating from I(2) to I(1), and can become I(0) by differencing once, whereas $\beta_{\perp 2}' x_t$ is not cointegrating at all and, thus, can only become I(0) by differencing twice.

When $r > s_2$ the polynomially cointegrating relations can be further decomposed into $r_0 = r - s_2 = 1$ directly cointegrating relations, $\beta_0' x_t$, and $r_1 = r - r_0 = s_2 = 1$ polynomially cointegrating relations, $\beta_1' x_t + \kappa' \Delta x_t$, where κ is a $p \times s_2$ matrix proportional $\beta_{\perp 2}$.

Table 2: Decomposing the data vector using the I(2) model

The $\beta, \beta_\perp$ decomposition of x_t		The $\alpha, \alpha_\perp$ decomposition
$r = 2$	$[\underbrace{\beta'_{.1} x_t}_{I(1)} + \underbrace{\omega'_{.1} \Delta x_t}_{I(1)}] \sim I(0)$	α_1:short-run adjustment coefficients
	$[\underbrace{\beta'_{.2} x_t}_{I(1)} + \underbrace{\omega'_{.2} \Delta x_t}_{I(1)}] \sim I(0)$	α_2: short-run adjustment coefficients
$s_1 = 1$	$\beta'_{\perp 1} x_t \sim I(1)$	$\alpha'_{\perp 1} \sum_{i=1}^{t} \varepsilon_i$: I(1) stochastic trend
$s_2 = 1$	$\beta'_{\perp 2} x_t \sim I(2)$	$\alpha'_{\perp 2} \sum_{s=1}^{t} \sum_{i=1}^{s} \varepsilon_i$: I(2) stochastic trend

The right hand side of Table 2 illustrates the corresponding decomposition into the α and the $\alpha_\perp$ directions, where α_1 and α_2 measure the short-run adjustment coefficients associated with the polynomially cointegrating relations, whereas $\alpha_{\perp 1}$ and $\alpha_{\perp 2}$ measure the loadings to the first and second order stochastic trends.

Both $\beta' x_t$ and $\beta'_{\perp 1} x_t$ are $CI(2,1)$ but they differ in the sense that the former can become stationary by polynomial cointegration, whereas the latter can only become stationary by differencing. Thus, even in the $I(2)$ model the interpretation of the reduced rank of the matrix Π is that there are r relations that can become stationary either by cointegration or by multi-cointegration, and $p-r$ relations that only become stationary by differencing.

Thus, the $I(2)$ model can distinguish between the $CI(2,1)$ relations between levels $\{\beta' x_t, \beta'_{\perp 1} x_t\}$, the $CI(1,1)$ relations between levels and differences $\{\beta' x_{t-1} + \omega' \Delta x_t\}$, and finally the $CI(1,1)$ relations between differences $\{\beta'_{\perp 1} \Delta x_t\}$. As a consequence, when discussing the economic interpretation of these components, we need to modify the generic concept of "long-run" steady-state relations accordingly. We will here use the interpretation of

- $\beta'_0 x_t$ as a *static long-run equilibrium relation*,
- $\beta'_1 x_t + \kappa' \Delta x_t$ as a *dynamic long-run equilibrium relation*,
- $\beta'_{\perp 1} \Delta x_t$ as a *medium-run equilibrium relation*.

As mentioned above the parameters of Table 2 can be estimated either by the two-step procedure or by the $FIML$ procedure. Paruolo

Table 3: Unrestricted estimates of the $I(0)$, $I(1)$, and $I(2)$ directions of α and β

	m	p	s^b	y^r
	The stationary cointegrating relations			
$\hat{\beta}_0$	**1.00**	-0.07	**-0.91**	**-1.22**
$\hat{\beta}_1$	**-0.67**	**1.00**	-0.06	**0.37**
κ	**-4.87**	**-3.78**	**-5.48**	-0.30
	The adjustment coefficients			
$\hat{\alpha}_0$	**0.04**	**-0.03**	**0.17**	0.01
$\hat{\alpha}_1$	**0.10**	**0.04**	**0.11**	-0.01
	The nonstationary relations			
$\hat{\beta}_{\perp 1}$	**4.54**	0.46	**-3.99**	**6.69**
$\hat{\beta}_{\perp 2}$	**0.52**	**0.40**	**0.58**	-0.03
	The common stochastic trends			
$\hat{\alpha}_{\perp 1}$	0.038	-0.007	-0.012	**0.122**
$\hat{\alpha}_{\perp 2}$	**-0.053**	**-0.078**	**-0.016**	**-0.023**
$\hat{\sigma}_{\varepsilon}$	0.016	0.010	0.054	0.028

(2000) showed that the two-stage procedure gives asymptotically efficient ML estimates. The $FIML$ procedure solves just one reduced rank problem in which the eigenvectors determine the space spanned by $(\beta, \beta_{\perp 1})$, i.e. the $p - s_2$ $I(1)$ directions of the process. Independently of the estimation procedure, the crucial estimates are $\{\hat{\beta}, \hat{\beta}_{\perp 1}\}$, because for given values of these it is possible to derive the estimates of $\{\alpha, \alpha_{\perp 1}, \alpha_{\perp 2}, \beta_{\perp 2}\}$ and, if $r > s_2$, to further decompose β and α into $\beta = \{\beta_0, \beta_1\}$ and $\alpha = \{\alpha_0, \alpha_1\}$.

The parameter estimates in <Table 3> are based on the two-step procedure for $r = 2$, $s_1 = 1$, and $s_2 = 1$. We have imposed identifying restrictions on two cointegration relations by distinguishing between the directly stationary relation, $\beta_0' x_t$, and the polynomially cointegrated relation, $\beta_1' x_t + \kappa \Delta x_t$, where κ is proportional to $\beta_{\perp 2}$. Note, however, that this is just one of many identification schemes which happen to be possible because $r - s_2 = 1$. In Section 8 we will present another identified structure where both relations are polynomially cointegrating.

The $\widehat{\beta}'_{\perp 1} x_t$ relation is a $CI(2,1)$ cointegrating relation which only can become stationary by differencing. We interpret such a relation as a medium long-run steady-state relation. The estimated coefficients of $\widehat{\beta}_{\perp 1}$ suggest a first tentative interpretation:

$$\Delta y_t^r = 0.60 \Delta s_t^b - 0.68 \Delta m_t$$

i.e. real industrial production has increased in the medium run with the currency depreciation relative to the growth of money stock.

The estimate of $\alpha_{\perp 2}$ determines the stochastic $I(2)$ trend $\hat{\alpha}'_{\perp 2}\Sigma\Sigma\hat{\varepsilon}_i = \Sigma\Sigma\widehat{u}_{2i}$, where $\hat{\varepsilon}_i$ is the vector of estimated residuals from (17) and $\widehat{u}_{2t} = \hat{\alpha}'_{\perp 2}\hat{\varepsilon}_t$. Permanent shocks to money stock relative to price shocks, to black market exchange rates and to industrial production seem to have generated the $I(2)$ trend in this period. The standard deviations of the VAR residuals are reported in the bottom row of the table.

The estimate of $\alpha_{\perp 1}$ describes the second $I(1)$ stochastic trend, $\sum \widehat{u}_{2i} = \widehat{\alpha}'_{\perp 1} \sum \hat{\varepsilon}_i$. The coefficient to real industrial production has by far the largest weight in $\widehat{\alpha}_{\perp 1}$ suggesting that it measures an autonomous real shock. This is consistent with the hypothetical scenario (7) of Section 3.

Figure 10, upper panel, shows the graph of the I(2) stochastic trend, $\hat{\alpha}'_{\perp 2} \sum\sum \hat{\varepsilon}_i$, where $\hat{\alpha}_{\perp 2}$ is from Table 3. The graph in the middle panel is the differenced I(2) trend and the graph in the lower panel is the real stochastic trend given by $\sum \hat{\varepsilon}_{y,i}$.

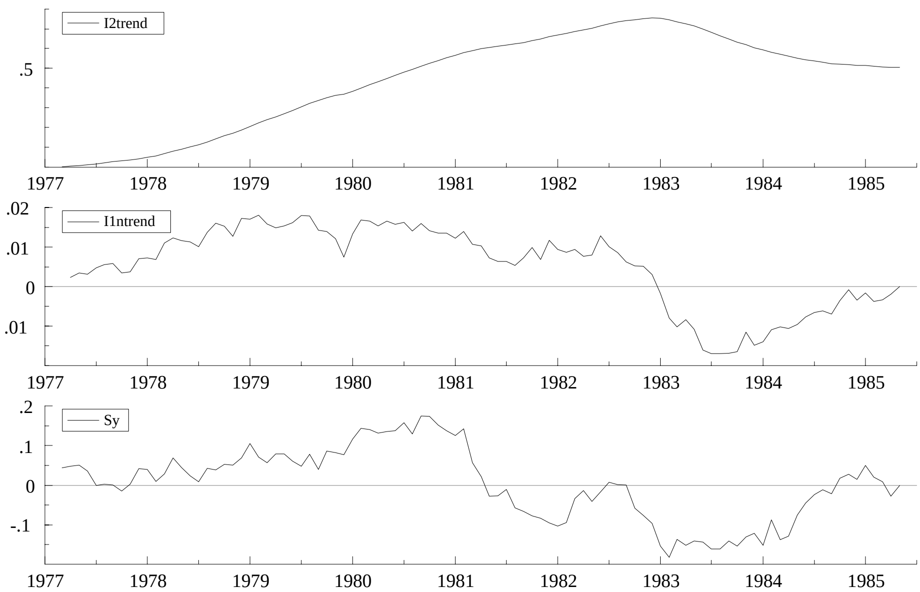

Figure 10. The graphs of the estimated I(2) trend in the upper panel, the nominal I(1) trend (i.e. the differenced I(2) trend) in the middel panel and the real I(1) trend in the lower panel.

The vector $\hat{\beta}_{\perp 2}$ describes the weights c_i, $i = 1, .., 4$ of the I(2) trend in the scenario (7) of Section 3 for the Brazilian variables. Nominal money, prices and exchange rates have large coefficients of approximately the same size, whereas the coefficient to real income is very small. This suggests that only the nominal variables are $I(2)$ consistent with the assumption behind the scenario in (7).

7 Nominal growth in the long run and the medium run

The notion of price homogeneity plays an important role for the analysis of price adjustment in the long run and the medium run. Both in the $I(1)$ and the $I(2)$ model, long-run price homogeneity can be defined as a zero sum restriction on β. Under the assumption that industrial production is not affected by the I(2) trend, long-run price homogeneity for the Brazilian data can be expressed as:

$$\begin{aligned} \beta_i' &= [a_i, -\omega_i a_i, -(1-\omega_i)a_i, *, *],\ i = 1, ..., 2, \\ \beta_{\perp 1}' &= [b, -\omega_3 b, -(1-\omega_3)b, *], \\ \beta_{\perp 2}' &= [c, c, c, 0]. \end{aligned} \tag{24}$$

where β and $\beta_{\perp 1}$ define $CI(2,1)$ relations and $\beta_{\perp 2}$ define the variables which are affected by the $I(2)$ trends. Overall price homogeneity is testable either as a joint hypothesis of the first two conditions or as a single hypothesis of the last condition in (24) (see, Kongsted, 2004). The first condition in (24) describes price homogeneity between the levels of the nominal variables. It can be easily tested in the standard I(1) model as a linear hypothesis on β either expressed as $R'\beta_i = 0$, $i = 1, 2, ..., r$, where for the Brazilian data $R' = [1, 1, 1, 0, 0]$ or, equivalently, as $\beta = H\varphi$ where φ is a $(p1-1) \times r$ matrix of free coefficients and

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

The hypothesis of price homogeneity was strongly rejected based on a LR test statistic of 41.9, asymptotically distributed as $\chi^2(2)$. We note that the first three coefficients of $\hat{\beta}_1$ in Table 3 do not even approximately sum to zero, whereas those of $\hat{\beta}_0$ are much closer to zero.

The $\hat{\beta}_{\perp 2}$ estimates in Table 3 suggest that nominal money stock and black market exchange rate have been similarly affected by the I(2) trend, whereas the CPI price index has a smaller weight. Furthermore, the coefficient to industrial production is close to zero, consistent with the hypothesis that the latter has not been affected by the I(2) trend. This can be formally tested based on the LR procedure (Johansen, 2004) as an hypothesis that industrial production is I(1). The test, distributed as $\chi^2(1)$, was accepted based a test statistic of 1.30 and a p-value of 0.25.

In the $I(2)$ model, there is the additional possibility of medium-run price homogeneity defined as homogeneity between nominal growth rates. This is, in general, associated with real variables being I(1). For example if $(m-p) \sim I(1)$ and $(s^b - p) \sim I(1)$, then $(\Delta m - \Delta p) \sim I(0)$ and $(\Delta s^b - \Delta p) \sim I(0)$ and there is medium-run price homogeneity in the sense of nominal growth rates being pairwise cointegrated (1, -1). Hence, a rejection of long-run price homogeneity implies a rejection of homogeneity between the nominal growth rates. We note that the first three coefficients of $\hat{\beta}_{\perp 1}$ in Table 3 do not even roughly sum to zero consistent with the rejection of long-run price homogeneity.

The previous section demonstrated that the levels component, Πx_{t-2} and the differences component, $\Gamma \Delta x_{t-1}$ in (17) are closely tied together by polynomial cointegration. In addition $\Gamma \Delta x_{t-1}$ contains information about $\beta'_{\perp} \Delta x_{t-1}$, i.e. about the medium long-run relation between growth rates. Relying on results in Johansen (1995) the levels and difference components of model (17) can be decomposed as:

$$\begin{aligned} \Gamma \Delta x_{t-1} + \Pi x_{t-1} = (\Gamma \overline{\beta}) & \underbrace{\boldsymbol{\beta}' \boldsymbol{\Delta} \mathbf{x}_{t-1}}_{I(0)} \\ & + (\alpha \overline{\alpha}' \Gamma \overline{\beta}_{\perp 1} + \alpha_{\perp 1}) \underbrace{\boldsymbol{\beta}'_{\perp 1} \boldsymbol{\Delta} \mathbf{x}_{t-1}}_{I(0)} \\ & + (\alpha \overline{\alpha}' \Gamma \overline{\beta}_{\perp 2}) \underbrace{\boldsymbol{\beta}'_{\perp 2} \boldsymbol{\Delta} \mathbf{x}_{t-1}}_{I(1)} \\ & + \alpha_1 \underbrace{\boldsymbol{\beta}'_1 \mathbf{x}_{t-1}}_{I(1)} \\ & + \alpha_0 \underbrace{\boldsymbol{\beta}'_0 \mathbf{x}_{t-1}}_{I(0)} \end{aligned} \tag{25}$$

where $\bar{\beta} = \beta(\beta'\beta)^{-1}$ and $\bar{\alpha}$ is similarly defined. The Γ matrix is decomposed into three parts describing different dynamic effects from the growth rates, and the Π matrix into two parts describing the effects from the stationary relation, $\beta'_0 x_{t-1}$, and the nonstationary relation, $\beta'_1 x_{t-1}$. The matrices in brackets correspond to the adjustment coefficients.

The interpretation of the first component in (25), $(\Gamma\overline{\beta})\beta'\Delta x_{t-1}$, is that prices are adjusting both to the equilibrium error between the price levels, $\beta' x_{t-2}$, and to the change in the equilibrium error, $\beta'\Delta x_{t-1}$. Under long-run price homogeneity it would have represented a homogeneous effect in inflation rates.

The second component, $(\alpha \overline{\alpha}' \Gamma \overline{\beta}_{\perp 1} + \alpha_{\perp 1}) \boldsymbol{\beta}'_{\perp 1} \boldsymbol{\Delta} \mathbf{x}_{t-1}$, corresponds to a stationary medium long-run relation between growth rates of nominal magnitudes. Because of the rejection of long-run price homogeneity, this represents a non-homogeneous effect in nominal growth rates.

The third component, $(\alpha\overline{\alpha}'\Gamma\overline{\beta}_{\perp 2})\,\boldsymbol{\beta}'_{\perp 2}\boldsymbol{\Delta}\mathbf{x}_{t-1}$, and the fourth component, $\alpha_1\beta_1'x_t$, are both I(1) relations which combine to a stationary polynomial cointegration relation, $\alpha_1(\beta_1'x_{t-1} + \kappa'\Delta x_{t-1}) \sim I(0)$, where $\alpha_1\kappa' = (\alpha\overline{\alpha}'\Gamma\overline{\beta}_{\perp 2})\beta'_{\perp 2}$.

The long-run matrix Π is the sum of the two levels components measured by:

$$\Pi = \alpha_0\beta_0' + \alpha_1\beta_1'.$$

Hypothetically, the Π matrix is likely to satisfy the condition for long-run price homogeneity in a regime where inflation is under control. Thus, the lack of price homogeneity is likely to be the first sign of inflation running out of control.

The growth-rates matrix Γ is the sum of the three different components measured by

$$\Gamma = (\Gamma\overline{\beta})\beta' + (\alpha\overline{\alpha}'\Gamma\overline{\beta}_{\perp 1} + \alpha_{\perp 1})\,\boldsymbol{\beta}'_{\perp 1} + (\alpha\overline{\alpha}'\Gamma\overline{\beta}_{\perp 2})\,\boldsymbol{\beta}'_{\perp 2}.$$

The Γ matrix is, however, not likely to exhibit medium-run price homogeneity, even under the case of long-run price homogeneity. This is because $R'\beta = 0$ implies $R'\beta_{\perp 2} \neq 0$. The intuition is as follows: When $\beta'x_t \sim I(0)$, a non-homogeneous reaction in nominal growth rates is needed to achieve an adjustment towards a stationary long-run equilibrium position. Therefore, medium-run price homogeneity interpreted as a zero sum restriction of rows of Γ would in general be inconsistent with overall long-run price homogeneity.

Table 4 reports the estimates of $\Gamma = -(I - \Gamma_1) = \alpha'_\perp\Gamma\beta_\perp$ and $\Pi = \alpha\beta'$. We notice that the coefficients of each row do not sum to zero. Next section will show that the difference is statistically significant. The diagonal elements of the Π matrix are particularly interesting as they provide information of equilibrium correction behavior, or the lack of it, of the variables in this system. We notice a significant positive coefficient in the diagonal element of the domestic prices, which in a single equation model would imply accelerating prices. In a VAR model absence of equilibrium correction in one variable can be compensated by a sufficiently strong counteracting reaction from the other variables in the system. It is noticeable that the only truely market determined variable, the black market exchange rate, is significantly equilibrium-correcting variable, whereas money stock is only borderline so.

Section 3 demonstrated that the unrestricted characteristic roots of the VAR model contained a small explosive root, which disappeared

Table 4: The unrestricted parameter estimates

	The estimated $\Gamma = \alpha'_{\perp}\Gamma\beta_{\perp}$ matrix				
	Δm_t	Δp_t	Δs_t^b	Δy_t^r	
$\Delta^2 m_t$:	-1.07	-0.06	0.00	0.02	
$\Delta^2 p_t$:	-0.02	-0.55	0.01	-0.01	
$\Delta^2 s_t^b$:	-0.42	0.42	-0.92	0.03	
$\Delta^2 y_t^r$:	-0.13	0.20	0.04	-1.32	
	The estimated $\Pi = \alpha\beta'$ matrix				
	m_{t-1}	p_{t-1}	s_{t-1}^b	y_{t-1}^r	trend
$\Delta^2 m_t$:	-0.03 (-1.7)	$\mathbf{0.11}$ (7.5)	$\mathbf{-0.05}$ (-3.3)	0.00 (0.4)	-0.001 (-6.6)
$\Delta^2 p_t$:	$\mathbf{-0.05}$ (-5.1)	$\mathbf{0.03}$ (3.3)	$\mathbf{0.03}$ (3.1)	$\mathbf{0.05}$ (4.8)	0.00 (0.1)
$\Delta^2 s_t^b$:	$\mathbf{0.11}$ (1.9)	$\mathbf{0.15}$ (2.8)	$\mathbf{-0.19}$ (-3.9)	$\mathbf{-0.15}$ (-2.6)	$\mathbf{0.002}$ (4.2)
$\Delta^2 y_t^r$:	0.02 0.7	-0.01 -0.3	-0.01 -0.5	-0.02 -0.6	-0.00 (-0.1)

when two unit roots were imposed. Nevertheless, the positive diagonal element of prices suggest that the spiral of price increases which subsequently became hyper inflation had already started at the end of this sample.

8 Money growth, currency depreciation, and price inflation in Brazil

Long-run price homogeneity is an important property of a nominal system and rejecting it is likely to have serious implications both for the interpretation of the results and for the validity of the nominal to real transformation. The empirical analysis of Durevall (1998) was based on a nominal to real transformation without first testing its validity. We will here use the I(2) model for the empirical investigation of the money-price spiral without having to impose invalid long-run price homogeneity.

8.1 Identifying the β relations

The estimates of β_0, β_1 and κ in Table 3 are uniquely identified by the $CI(2,2)$ property of $\beta_0' x_t$. However, other linear combinations of β_0 and β_1 may be more relevant from an economic point of view, but these will be $I(1)$ and will, therefore, have to be combined with the differenced I(2) variables to become stationary.

To obtain more interpretable results three overidentifying restrictions have been imposed on the two β relations. The LR test of overidentifying restrictions, distributed as $\chi^2(3)$ became 1.41 and the restrictions were accepted based on a p-value of 0.70. The estimates of the two identified relations became:

$$\begin{aligned} \beta_{1,t}^{c\prime} x_t &= m_{t-1} - s_{t-1}^{b} - y_{t-1}^{r} - \underset{(-2.5)}{0.005} trend \\ \beta_{2,t}^{c\prime} x_t &= p_{t-1} - \underset{(18.3)}{0.64} (m_{t-1} - y_{t-1}^{r}) - \underset{(-2.5)}{0.008} trend \end{aligned} \tag{26}$$

The first relation is essentially describing a trend-adjusted liquidity ratio, except that the black market exchange rate is used instead of the CPI as a measure of the price level. The liquidity ratio with CPI instead of the exchange rate was strongly rejected. This suggests that inflationary expectations were strongly affected by the expansion of money stock and that these expectations influenced the rise of the black market nominal exchange rate.

Both relations need a liner deterministic trend. The estimated trend coefficient of the first relation in suggests that 'the liquidity ratio' grew on average with 6% (0.005×12 × 100) per year in this period. The second relation shows that prices grew less than proportionally with the expansion of M3 money stock relative to industrial production after having accounted for an average price increase of approximately 9% (0.008×12 × 100) per year.

The graphs in Figure 11 of the liquidity ratio based on the nominal exchange rate and on the CPI index, respectively, may explain why nominal exchange rates instead of domestic prices were empirically more relevant in the first relation. It is interesting to note that the graphs are very similar until the end of 1980, whereafter the black market exchange rate started to grow faster than CPI prices. Thus, the results suggest that money stock grew faster than prices in the crucial years before the first hyper inflation episode, but also that the depreciation rate of the black market currency was more closely related to money stock expansion. This period coincided with the Mexican moratorium, the repercussions of which were strongly and painfully felt in the Brazilian economy. The recession and the major decline of Brazilian exports caused the government to abandon its previous more orthodox policy of fighting inflation by maintaining a revalued currency and, instead, engage in a much looser monetary policy. For a comprehensive review of the Brazilian exchange rate policy over the last four decades, see Bobomo and Terra (1999).

Under the assumption that the black market exchange rate is a fairly good proxy for the 'true' value of the Brazilian currency, the following scenario seems plausible: The expansion of money stock needed to finance the recession and devaluations in the first case increased inflationary expectations in the black market, which then gradually spread to the whole domestic economy. Because of the widespread use of wage and price indexation in this period there were no effective mechanisms to prevent the accelerating price inflation.

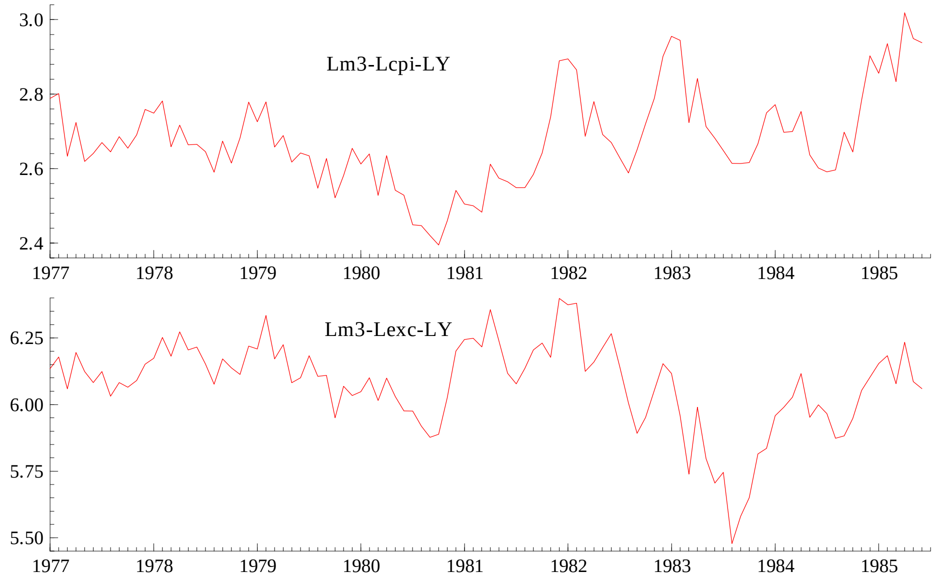

Figure 11. The graphs of inverse velocity with CPI as a price variable (upper panel) and with nominal exchange rate (lower panel).

8.2 Dynamic equilibrium relations

This scenario can be further investigated by polynomial cointegration. In the I(2) model $\beta' x_t \sim I(1)$ has to be combined with the nominal growth rates to yield a stationary dynamic equilibrium relation. The two identified relations, $\beta'_{1,1}x_t$ and $\beta'_{1,2}x_t$ in (26) need to be combined with nominal growth rates to become stationary. Table 5 reports various versions of the estimated dynamic equilibrium relations.

The first dynamic steady-state relation corresponds essentially to Cagan's money demand relation in periods of hyper inflation. However, the price level is measured by the black market nominal exchange rate and the opportunity cost of holding money is measured both by the CPI inflation and by the currency depreciation. The coefficient to inflation corresponds to Cagan's α coefficient which defines the average inflation rate $(1/\alpha)$ at which the government can obtain maximum seignorage. The present estimate suggests average inflation rates of an order of magnitude of 0.40-0.50 which corresponds to the usual definition of hyper inflation periods.

The second relation is more difficult to interpret from a theoretical

Table 5: Estimates of the polynomially cointegrated relations

The dynamic equilibrium relations $\beta' x_t + \omega' \Delta x_t$				
	$\hat{\beta}'_{1,1} x_t$	$\omega_{1,1} \Delta m_t$	$\omega_{1,2} \Delta p_t$	$\omega_{1,3} \Delta s^b_t$
(1)	1.0	-0.62 (1.1)	2.52 (3.4)	0.59 (2.7)
(2)	1.0	-	2.02 (3.4)	0.53 (2.5)
	$\hat{\beta}'_{1,2} x_t$	$\omega_{2,1} \Delta m_t$	$\omega_{2,2} \Delta p_t$	$\omega_{2,3} \Delta s^b_t$
(3)	1.0	-5.80 (67)	-11.32 (9.9)	-0.34 (1.0)
(4)	1.0	-6.02 (7.1)	-11.38 (10.0)	-
(5)	1.0	$-$	-16.57 (15.4)	-
(6)	1.0	-11.42 (12.4)	-	-

point of view but seems crucial for the mechanisms behind the increasingly high inflation of this period and the hyper inflation of the subsequent periods. Eq. (3) shows that the 'gap' between prices and 'excess' money as measured by $\beta'_{1,2} x_t$ is cointegrated with changes in money stock and prices, but not with currency depreciation. Eq. (4) combines $\beta'_{1,2} x_t$ with money growth, Δm, and price inflation, Δp, Eq. (5) with Δp and Eq. (6) with Δm. Although both nominal growth rates are individually cointegrating with $\beta'_{1,2} x_t$, there is an important difference between them: The relationship between money growth and the relation $\beta'_{1,2} x_t$ suggests error-correcting behavior in money stock, whereas the one between price inflation and $\beta'_{1,2} x_t$ indicates lack error-correcting behavior in prices. The latter would typically describe a price mechanism leading ultimately to hyper inflation unless counterbalanced by other compensating measures, such as currency control.

8.3 The short-run dynamic adjustment structure

The inflationary mechanisms will now be further investigated based on the estimated short-run dynamic adjustment structure. Current as well as lagged changes of industrial production were insignificant in the system and were, therefore, left out. Thus, real growth rates do not seem to have had any significant effect on the short-run adjustment of nominal growth rates which is usually assumed to be the case in a high inflation regime. Furthermore, based on a F-test the lagged depreciation rate was also found insignificant in the system and was similarly left out. Table 6 reports the estimated short-run structure of the simplified model. Most of the significant coefficients describe feed-back effects from the

Table 6: Dynamic adjustment and feed-back effects in the nominal system

Ref.	Regressors:	Eq.:	Δm_t	Δp_t	Δs_t^b
	Δm_{t-1}		0.33 (4.2)	0.11 (2.4)	0.91 (2.7)
	Δp_{t-1}		0.59 (5.6)	0.76 (12.1)	–
Table 5 (2)	$(\hat{\beta}_{1,1}x - \hat{w}_{1,1}\Delta x)_{t-1}$		-0.03 (-2.3)	-0.03 (-2.9)	0.08 (1.9)
Table 5 (4)	$(\hat{\beta}_{1,2}x - \hat{w}_{1,2}\Delta x)_{t-1}$		0.06 (6.4)	0.02 (4.3)	0.06 (2.0)
Table 3	$\hat{\beta}'_{\perp 1}\Delta x_t$		$+0.008$ (-2.2)	-0.005 (2.1)	-
			1.0		
	Residual correlations:		-0.02	1.0	
			0.08	-0.12	1.0

dynamic steady-state relations defined by Eq. (2) and Eq. (4) in Table 5 and the medium-run steady-state relation between growth rates, $\beta'_{\perp 1}\Delta x_t$ defined in Table 3. It is notable that the residual correlations are altogether very small, so that interpretation of the results should be robust to linear transformations of the system.

The short-run adjustment results generally confirm the previous findings. Price inflation has not been equilibrium correcting in the second steady-state relation, whereas the growth in money stock has been so in both of the two dynamic steady-state relations. The depreciation of the black market exchange rate has been equilibrium correcting to the first steady-state relation measuring the liquidity ratio relation and has been positively affected strongly affected by the second price 'gap' relation. Furthermore, it has reacted strongly to changes in money stock confirming the above interpretation of the important role of inflationary expectations (measured by changes in money stock) for the currency depreciation rate.

After the initial expansion of money stock at around 1981 (which might have been fatal in terms of the subsequent hyper inflation experience) money supply seems primarily to have accommodated the increasing price inflation. The lack of equilibrium correction behavior in the latter was probably related to the widespread use of wage and price indexation in this period. Thus, the lack of market mechanism to correct for excessive price changes allowed domestic price inflation to gain momentum as a result of high inflationary expectations in the foreign exchange market.

9 Concluding remarks

The purpose of this paper was partly to give an intuitive account of the cointegrated I(2) model and its rich (but also complicated) statistical structure, partly to illustrate how this model can be used to address important questions related to inflationary mechanisms in high inflation periods. The empirical analysis was based on data from the Brazilian high inflation period, 1977:1-1985:5. An additional advantage of this period was that it was succeeded by almost a decade of hyper-inflationary episodes. The paper demonstrates empirically that it is possible to uncover certain features in the data and the model which at an early stage may suggest a lack of control in the price mechanism. Thus, a violation of two distinct properties, price homogeneity and equilibrium correction, usually prevalent in periods of controlled inflation, seemed to have a high signal value as a means to detect an increasing risk for a full-blown hyper inflation. The paper demonstrates that:

1. prices started to grow in a non-homogeneous manner at the beginning of the eighties when the repercussions of the Mexican moratorium strongly and painfully hit the Brazilian economy. The expansion of money stock needed to finance the recession and devaluations increased inflationary expectations in the black market, which then spread to the whole domestic economy.

2. the widespread use of wage and price indexation in this period switched off the natural equilibrium correction behavior of the price mechanism. Without other compensating control measures which might have dampened inflationary expectations, it was not possible to prevent price inflation to accelerate.

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Table 7: Misspecification tests

Univariate misspecification tests				
	y^r	s^b	m	p
Normality,$\chi^2(2)$	0.66 (0.72)	1.71 (0.43)	0.36 (0.84)	1.67 (0.43)
AR(1)	0.19 (0.66)	0.00 (0.95)	0.03 (0.87)	1.27 (0.26)
Skewness	-0.13	0.21	0.09	-0.21
Kurtosis	3.06	3.29	2.62	3.27
Multivariate misspecification tests				
Normality, $\chi^2(8)$		4.43	(0.82)	
AR(1)		5.59	(0.99)	
AR(4)		62.21	(0.54)	

under rational expectations: I. International Economic Review 18, 1, 59-82.

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11 Appendix A: Misspecification diagnostics

The univariate normality test in Table A.1 is a Jarque-Bera test, distributed as $\chi^2(2)$. The multivariate normality test is described in Doornik and Hansen (1995) distributed as $\chi^2(8)$. The AR-test is the F-test described in Doornik (1996), page 4. P-values are in brackets.

Figure A.1 shows the residual auto-correlograms and cross-correlograms of order 10 for all four equations. Figure A.2 shows the residual histograms compared to the normal distribution for all equations.

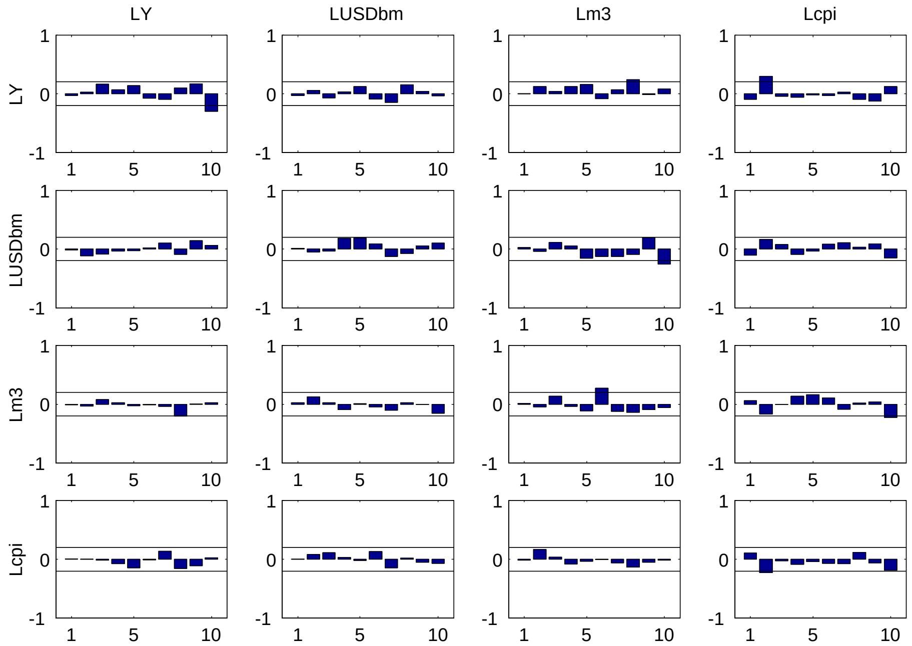

Figure A.1: Residual autocorrelograms and crosscorrelograms.

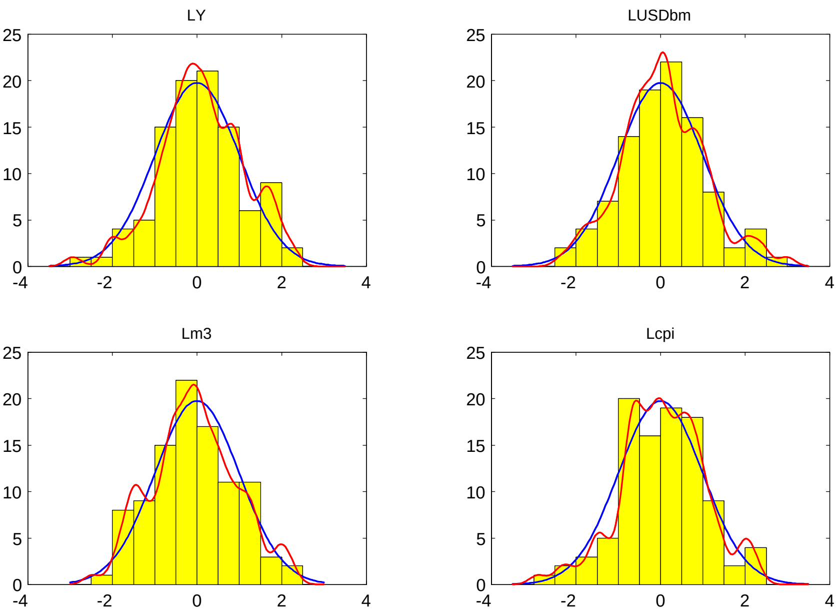

Figure A.2: Residual histograms for the four equations.