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Logical Rules and the Classification of Integral-Dimension Stimuli

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A classic distinction in perceptual information processing is whether stimuli are composed of separable dimensions, which are highly analyzable, or integral dimensions, which are processed holistically. Previous tests of a set of logical-rule models of classification have shown that separable-dimension stimuli are processed serially if the dimensions are spatially separated and as a mixture of serial and parallel processes if the dimensions are spatially overlapping (Fifić, Little, & Nosofsky, 2010; Little, Nosofsky, & Denton, 2011). In the current research, the logical-rule models are applied to predict response-time (RT) data from participants trained to classify integral-dimension color stimuli into rule-based categories. In dramatic contrast to the previous results for separable-dimension stimuli, analysis of the current data indicated that processing was best captured by a single-channel coactive model. The results converge with previous operations that suggest holistic processing of integral-dimension stimuli and demonstrate considerable generality for the application of the logical-rule models to predicting RT data from rule-based classification experiments.

Keywords: categorization, logical rules, response times, integral dimensions, coactive processing

A fundamental aspect of human cognition involves the manner in which people represent categories in memory and make decisions about category membership. There is growing agreement in the field that different cognitive systems may underlie the learning and representation of different kinds of categories. For example, a wide variety of ill-defined categories, such as those found in the natural world, may be represented in terms of prototypes or stored exemplars. Since the pioneering studies of Posner and Keele (1968) and Rosch (1973), most research has focused on these forms of ill-defined categorization.

By contrast, many other categories and concepts appear to be organized in terms of systems of logical rules. Examples include varieties of scientific and mathematical concepts, kinship terms, and linguistic systems. Indeed, recent research has seen greatly renewed interest in logical rule-based forms of category learning and representation. Some of this research has been aimed at predicting the difficulty of learning different concepts based on their structural complexity or the length of the rules required to represent them (e.g., Feldman, 2000; Goodman, Tenenbaum, Feldman, & Griffiths, 2008; Goodwin & Johnson-Laird, 2011; Vigo, 2009). Other approaches have focused on learning processes involved in the trial-by-trial construction of rules and exceptions to those rules (Erickson & Kruschke, 1998; Nosofsky, Palmeri, & McKinley, 1994). Still another major theme has been to identify the neural systems that mediate rule-based forms of category learning (e.g., Ashby & Maddox, 2005; Nomura et al., 2007) and rule-plus-exception learning (Davis, Love, & Preston, 2012).

Finally, modern research is also addressing the psychological mechanisms that underlie the *information-processing* of rule-based categorization (Bradmetz & Mathy, 2008; Fifić, Little, & Nosofsky, 2010; Lafond, Lacouture, & Cohen, 2009). One of the insights from this literature is that there is a wide variety of ways in which the cognitive system may process a stimulus when deciding whether a logical rule that defines a category has been satisfied. Furthermore, the time course of making rule-based categorization decisions will be strongly influenced by the nature of these underlying information-processing mechanisms.

In our view, because evaluating logical rules appears to be one of the major forms of categorization decision making, understanding the information processing mechanisms that underlie logical-rule use is of fundamental importance and can significantly advance our understanding of categorization behavior. As is well known, it is often extremely difficult to distinguish among predictions of alternative information-processing mechanisms based on analysis of choice-probability data alone; this limitation is certainly true in the domain of categorization. Instead, by also analyzing the *time course* of categorization decision making through

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the modeling of response-time (RT) data, one can achieve deeper insights into the psychological processes that implement rule-based categorization. To achieve this goal it is important to answer the questions: What should patterns of RT data look like if people are indeed using logical rules as a basis for making categorization decisions? And how will these patterns of RTs vary with the precise information-processing mechanisms that are used to implement the rules?

To address these questions, Fifić et al. (2010) and Little, Nosofsky, and Denton (2011) developed a set of logical-rule models of categorization RTs. As is true of an extremely diverse set of rule-based models in the field, the models from Fifić et al.'s (2010) framework assume that people categorize by making independent decisions about dimensional values and then combining these independent decisions using logical-rule connectives such as AND, OR, and NOT (Ashby & Gott, 1988; Bruner, Goodnow, & Austin, 1956). The main advance of the theory is that these logical-rule evaluations are implemented in an information-processing framework. As reviewed below, to achieve this implementation, the models synthesize decision-bound theory (Maddox & Ashby, 1996), random-walk RT models (Busemeyer, 1985), and mental architecture models of information processing (Schweickert, 1992; Sternberg, 1969; Townsend, 1984), enabling predictions of the time course of categorization for individual stimuli at the level of full RT distributions. As we demonstrate here, the rich RT data sets that are analyzed allow one to discriminate among a variety of distinct rule-based models, as well as to discriminate members of the class of rule-based models from other major alternatives in the field.

The logical-rule models address fundamental questions in the psychology of perception and categorization, such as whether multiple dimensions are processed sequentially in a *serial* fashion, simultaneously in *parallel*, or are pooled into a single *coactive* processing channel (Miller, 1982; Townsend & Wegner, 2004). The logical-rule framework has already been applied to investigate processing of a number of different stimulus types (see Figure 1), and distinct processing mechanisms are sometimes associated with the different types. For example, Fifić et al. (2010) and Little et al. (2011) used the models to analyze the speeded categorization of highly analyzable *separable*-dimension stimuli (Garner, 1974; Shepard, 1964), such as forms varying in shape and color. That research suggested that when the separable dimensions are also *spatially* separated (e.g., see the lamp stimuli in Figure 1), the processing of each dimension occurs in serial (Fifić et al., 2010; Little et al., 2011). By contrast, in cases involving spatially *overlapping* separable dimensions (e.g., see the rectangles with inset vertical line in Figure 1), rule-based categorization decision making involves a mixture of serial and parallel processing (Little et al., 2011).

An important contrast in the psychology of perception is drawn between stimuli composed of separable dimensions and those composed of *integral* dimensions, such as colors varying in brightness and saturation (Garner, 1974). A variety of converging operations suggests that, unlike separable-dimension stimuli, integral-dimension stimuli are processed holistically (Garner, 1974). For instance, people tend to sort integral stimuli based on overall similarity but separable stimuli based on individual dimensions (Garner, 1974); classification errors can be predicted from pairwise identification errors for integral stimuli (Shepard & Chang, 1963) but not for separable stimuli (Shepard, Hovland, & Jenkins, 1961); attention-learning in categorization tasks takes place efficiently for separable-dimension stimuli (Nosofsky, 1986) but not for integral-dimension stimuli (Nosofsky, 1987; Nosofsky & Palmeri, 1996); and stimulus similarities are better described by an Euclidean-distance metric if the stimulus dimensions are integral but by a city-block distance metric if they are separable (Shepard, 1987). Finally, for integral-dimension stimuli, RTs in a unidimensional categorization task are speeded or slowed by the inclusion of correlated or uncorrelated variation, respectively, on an irrelevant dimension. By contrast, RTs to separable-dimension stimuli

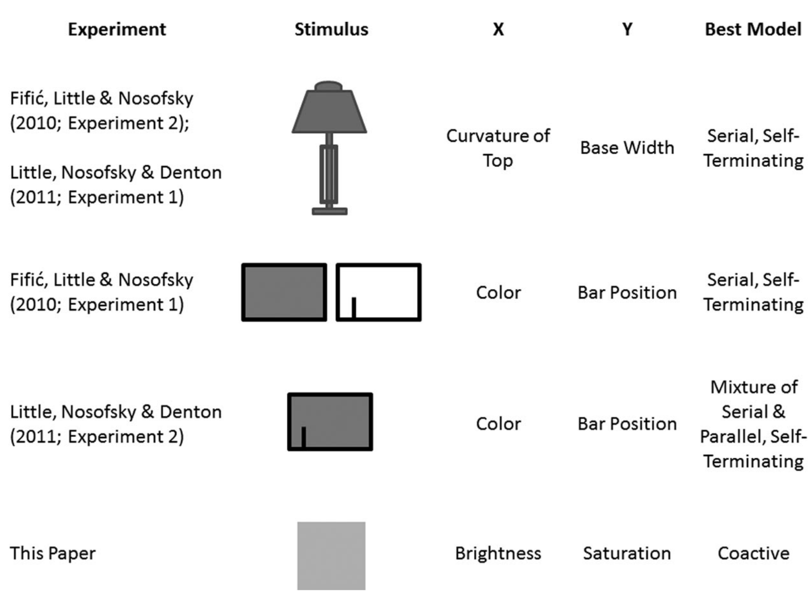

Figure 1. Examples of stimuli and dimensions previously tested using the logical-rules paradigm.

are largely unaffected by such irrelevant variation (Garner & Felfoldy, 1970). In summary, the distinction between integral- and separable-dimension stimuli is foundational, because the nature of the dimensions affects the manner in which stimuli are represented and how decision making takes place.

To achieve generality, a model of category-based information processing should be able to account for differences in performance based on perceptual separability versus integrality. As indicated in Figure 1, however, the logical-rule models framework has not yet been applied in categorization domains involving integral-dimension stimuli. The purpose of the present work was to fill that gap and to investigate the information-processing mechanisms by which people classify integral-dimension stimuli into rule-described categories. As we address in our General Discussion, the outcome of our experiments will yield a further source of converging operations that distinguish the processing of integral- versus separable-dimension stimuli.

We should note that Fifić, Nosofsky, and Townsend (2008) reported initial findings to suggest that classification of integral-dimension stimuli is accomplished by a coactive processing mechanism. The present research goes well beyond that initial work in several respects. First, we aim to use the logical-rule models to provide detailed quantitative accounts, at the level of individual subjects, of complete RT-distribution data associated with individual stimuli in tasks of rule-based classification of integral-dimension stimuli. Second, we conduct novel qualitative tests to contrast the manner in which subjects classify separable-dimension stimuli versus integral-dimension stimuli into the rule-based categories. Third, past research involving the logical-rule models focused on tests involving the performance of only small numbers of highly experienced observers. Here, we demonstrate in addition that these contrasts between separable-dimension versus integral-dimension stimuli are quite general effects, holding for large numbers of relatively unpracticed subjects as well. Before proceeding with these new tests and experiments, we first provide a review of the logical-rule models framework.

Logical-Rule Models

For efficiency, in this review we refer to the category space shown in Figure 2, which is used in the current experiment and which is highly diagnostic of which information-processing architecture best accounts for performance (cf. Townsend & Nozawa, 1995). The category space is populated by nine stimuli created by orthogonally combining three values of two continuous dimensions, *x* and *y*. The four stimuli in the upper right quadrant are members of the "target" category (A), and the remaining stimuli are members of the "contrast" category (B). Membership in the target category can be described by a conjunctive rule: A stimulus is a member of Category A if it has value greater than or equal to x_1 on dimension *x* AND greater than or equal to y_1 on dimension *y*. Membership in the contrast category can be described by a complementary disjunctive rule: A stimulus is a member of Category B if it has value less than x_1 on dimension *x* OR less than y_1 on dimension *y*. The logical-rule models presume that observers classify the objects by implementing these rules.

Following *general recognition theory* (Ashby & Townsend, 1986), the logical-rule models assume that the perception of each stimulus is represented by a bivariate normal distribution. Decision

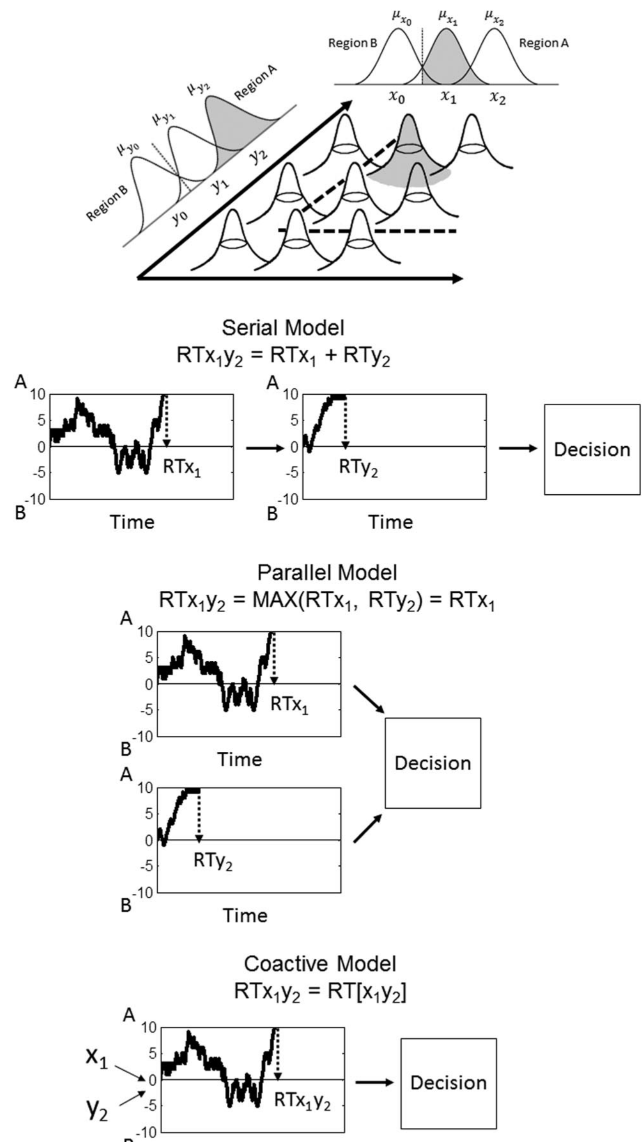

Figure 2. Top panel: Schematic illustration of the category structure used for testing the logical-rule models. Each stimulus is represented by a bivariate normal distribution over two dimensions, *x* and *y*. There are three values per dimension, combined orthogonally to produce the nine members of the stimulus set. The stimuli in the upper right quadrant of the space are the members of the "target" category (A), whereas the remaining stimuli are the members of the "contrast" category (B). Bottom panel: Three architectures are contrasted on how they process the target category stimulus, x_1y_2 (shaded in the top panel). The serial model processes both dimensions, one after the other, using the marginal *x* and *y* distributions to drive separate random walks; the final response time (RT) is given by the sum of the two individual RTs. The parallel model uses the marginal distributions to processes both dimensions simultaneously; because this stimulus is from the target category and requires exhaustive processing, the final RT for this stimulus is given by whichever dimension takes longer (i.e., the maximum RT for the two dimensions). The coactive model uses the joint bivariate normal distribution to drive a single random walk, which provides the final RT.

boundaries are established by the observer to separate Region A from Region B (the dashed lines in Figure 2). To make a category decision for a stimulus, the observer decides where the perceptual distribution falls along each dimension. In the present logical-rule models, these decisions are governed by random-walk processes driven by information that is sampled from the perceptual distributions.

First, the stimulus dimensions may be processed independently, with each dimension driving a separate random walk. That is, one random-walk process would determine to which side of the x decision boundary a stimulus falls, and a separate random-walk would do the same for dimension y. Furthermore, in this case, the independent dimensions may be processed in either *serial* or *parallel* fashion (see Figure 2, bottom panel). The final decision and RT are determined either by the random walk that finishes first, in the case of a *self-terminating* stopping rule, or by the output of both random walks, for cases involving an *exhaustive* stopping rule.

More specifically, suppose a stimulus is presented and the observer is making a decision regarding the stimulus's location on dimension x. A random-walk counter with an initial value zero accumulates information toward either the $+$A or $-$B criterion. At each time step, a sample is taken from the marginal perceptual distribution along dimension x. If that sample falls into the Category $-$A region, the counter steps toward $+$A; otherwise, the counter steps toward $-$B. The process continues until a criterion is reached. An analogous random-walk process takes place along dimension y. The decision time along each dimension is determined by the number of steps necessary to complete each random walk.

The final categorization response is determined by which of the logical-rules is satisfied. For the serial and parallel models, this decision is achieved by combining the results of the individual random walks according to whether the stopping rule is self-terminating or exhaustive. For example, if the first finishing random walk reaches the B criterion, then a self-terminating model would stop, because the disjunctive OR rule that defines Category B has already been satisfied. By contrast, exhaustive models assume that even if the first random walk reaches the B criterion, the second random walk must be completed prior to making the decision. Note that for the current category space, the self-terminating/exhaustive distinction applies only to stimuli from Category B, which instantiates an OR decision rule. By contrast, a Category *A* response can be successful only if the stimulus values are above the boundary on the y axis AND to the right of the boundary on the x axis; hence, exhaustive processing is required for Category *A* stimuli.

The preceding discussion pertained to cases involving independent decisions along each dimension. However, for cases involving integral-dimension stimuli, a different architecture based on coactive processing may be involved. According to the coactive model, instead of *separate* random walks operating along each dimension, there is a *single* random walk driven by samples from the *bivariate* perceptual distribution defined over dimensions x and y. On each step, a sample is drawn from the bivariate perceptual distribution associated with the stimulus. If the sampled percept falls in Region A (defined by the *combined* x and y decision bounds), then the single random walk steps toward Criterion $+$A. Otherwise, if the sampled percept falls in Region B (defined by the combined bounds), the random walk steps toward Criterion $-$B. The process continues until one of the criteria is reached. Note that the exhaustive/self-terminating distinction does not apply to the coactive model, because only a single random walk is taking place.

In their previous applications of the rule models, Fifić et al. (2010) and Little et al. (2011) assumed perceptual independence and perceptual separability of the dimensional distributions (Ashby & Townsend, 1986), because the stimuli varied along highly separable dimensions. Although the stimuli used in the current experiments were composed of integral dimensions, for simplicity we nevertheless maintained these assumptions in deriving quantitative fits from the models. More accurate predictions of the classification RTs could likely be achieved by first estimating more fine-grained representations for the stimuli (e.g., Maddox & Ashby, 1996; Nosofsky, 1986; Thomas, 1996). The crucial point, however, is that these simplifying assumptions do not a priori favor one of the processing architectures over the others. In our General Discussion, we report results from analyses that examine predictions from the logical-rule models in cases involving more complicated representational assumptions. As will be seen, the major qualitative contrasts for distinguishing among the alternative architectures remain intact.

Qualitative Contrasts (Target Category)

Fifić et al. (2010) provided a set of qualitative contrasts for differentiating the predictions from the alternative models. To help describe the predictions, we make reference to Figure 3 (left panel), which illustrates the brightness-saturation coordinates for the integral-dimension color stimuli used in our experiments and which provides a notation for the main stimulus types. The qualitative predictions from the models for the target-category members are illustrated schematically in the left panels of Figure 4. (Note: Figure 4 is presented in two separate 3 $\times$ 2 sections on consecutive pages.)

Following previous convention (Townsend & Nozawa, 1995), the four members of the target category are denoted LL, LH, HL, and HH, where "L" denotes a "low-salience" dimension value (close to the decision bound) and "H" denotes a "high-salience" value (see Figure 3). Naturally, H values lead to faster decisions than L values, because the H values are farther from the categorization decision bounds. The pattern of mean RTs for the target category can be quantified by using a measure known as the mean interaction contrast (MIC; Townsend & Nozawa, 1995), which expresses the type of interaction in mean RTs (in ms) between the factorially combined stimuli in the target category:

$$\text{MIC} = (RT_{LL} - RT_{LH}) - (RT_{HL} - RT_{HH}). \quad (1)$$

As shown in Figure 4 (left panels), the MIC distinguishes among the models based on whether MIC $= 0$, an *additive* pattern predicted by the serial models; MIC < 0, an *under-additive* pattern predicted by the parallel models; or MIC > 0, an *over-additive* pattern predicted by the coactive model (Townsend & Nozawa, 1995). The basis for these qualitative predictions for the target-category members has been described in numerous previous articles, including many that predate the logical-rules models framework (for a recent review, see Fifić et al., 2008). Therefore, we omit providing a detailed explanation in this section.

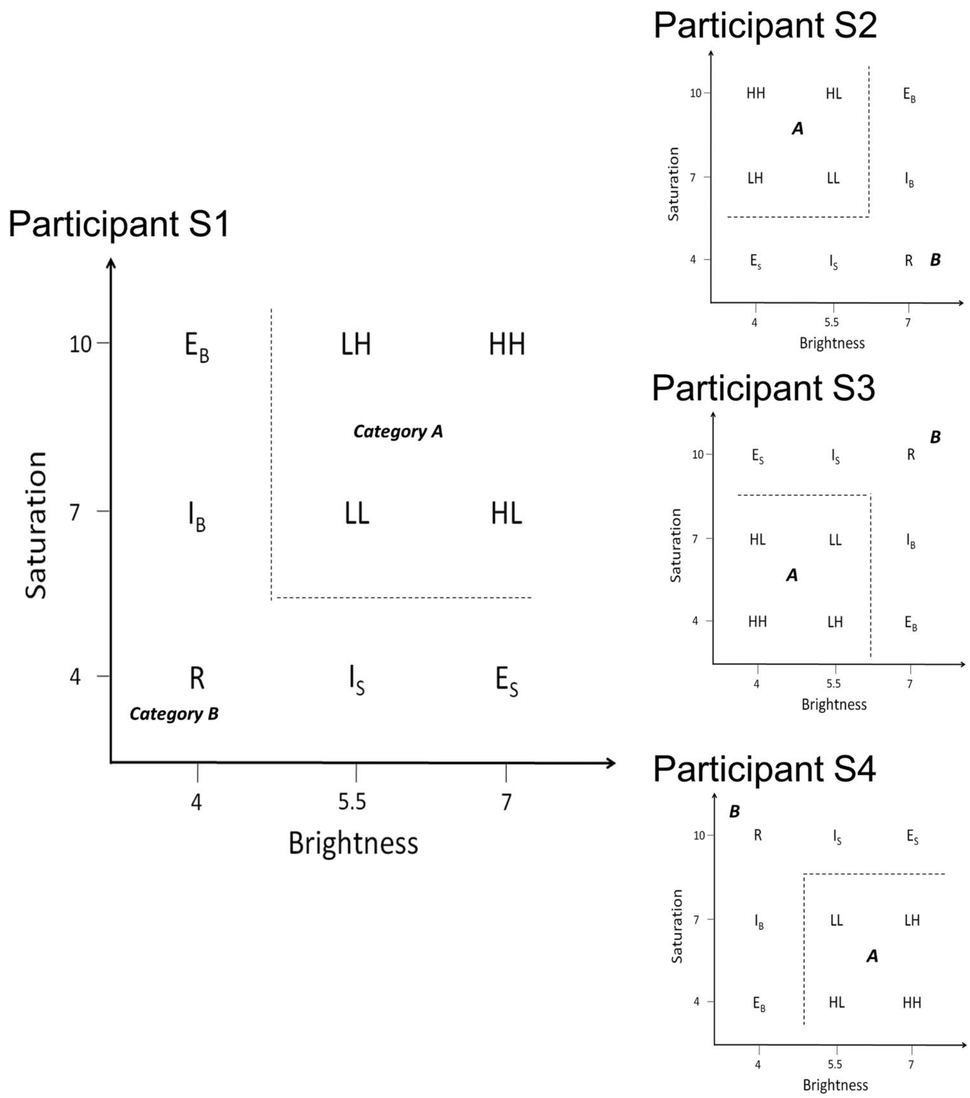

Figure 3. Schematic illustration of the category structure for each participant. A shorthand nomenclature is used to identify the main stimulus types in the category structure: For the target category (A), H and L refer to the high- and low-salience dimension values, respectively. The low-low (LL) stimulus is close to both dimensional boundaries and should therefore have the slowest response times (RTs), while the high-high (HH) stimulus is far from both boundaries and should have the fastest RTs. The low-high (LH) and high-low (HL) stimuli are close to only one of the boundaries and should have intermediate RTs. The Category B stimulus that satisfies the disjunctive OR rule on both dimensions is denoted as the redundant (R) stimulus. The Category B stimuli closest to the redundant stimulus are denoted the interior (I) stimuli, and the Category B stimuli furthest from the redundant stimulus are denoted the exterior (E) stimuli. Each participant received the same stimuli but with a different rotation of the category boundaries.

Qualitative Contrasts (Contrast Category)

The more novel qualitative contrasts to be examined in this work involve results from the contrast category. In particular, Fifić et al. (2010) showed that the patterns of mean RTs for the contrast category also differentiate the models (see Figure 4, right panels). Again, we make use of the notation in Figure 3 (left panel), where "R" denotes the "redundant stimulus" that satisfies the disjunctive rule on both dimensions; and where "I" and "E" denote the "interior" and "exterior" stimuli on each dimension. We focus our discussion on the models that assume self-terminating stopping rules, as those models are the most plausible models in this paradigm (see e.g., Fifić et al., 2010; Little et al., 2011).

Fixed-Order Serial Self-Terminating Model

Consider a fixed-order serial self-terminating model in which dimension x (brightness) is always processed before dimension y (saturation). Assuming accurate responding, the model predicts that the R, I_B and E_B stimuli will have the same RTs. The reason

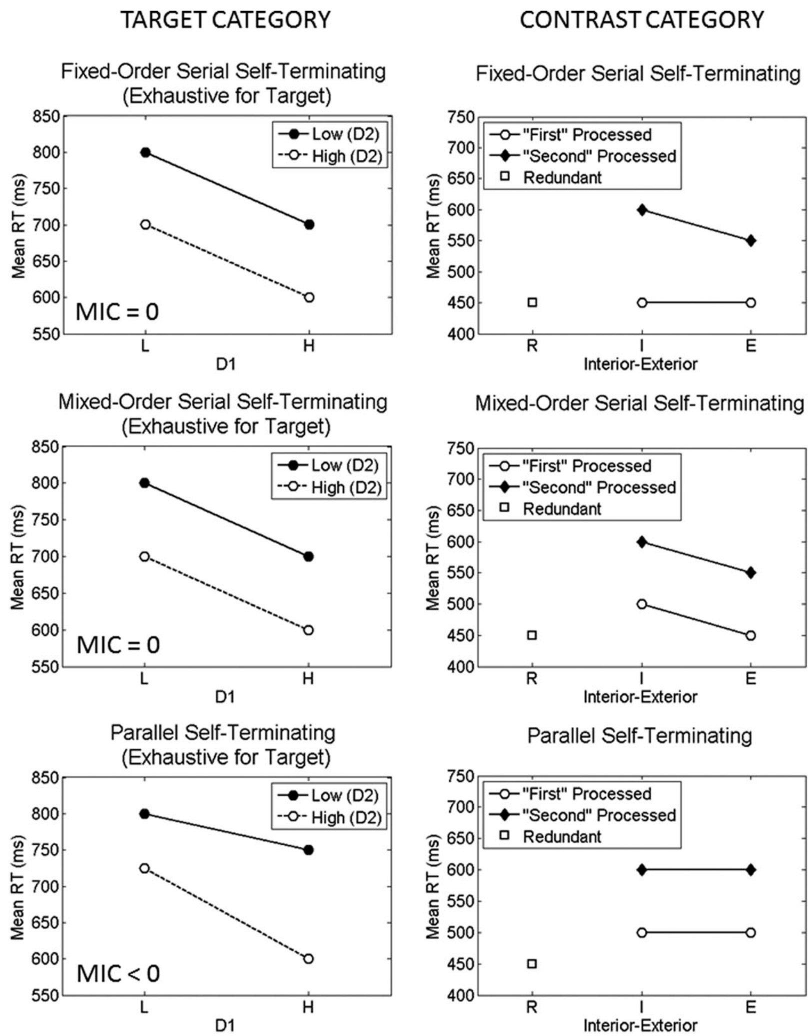

Figure 4. Summary predictions of mean response times (RTs) from the alternative logical-rule models of classification. Each row corresponds to one of the models. The left panels show the pattern of predictions for the target-category members, and the right panels show the pattern of predictions for the contrast-category members. As explained in the text, note that regardless of the stopping rule, correct classification of target-category members requires exhaustive processing. Left panels: MIC = mean interaction contrast; L = low-salience dimension value; H = high-salience dimension value; D1 = Dimension 1; D2 = Dimension 2. Right panels: R = redundant stimulus; I = interior stimulus; E = exterior stimulus. For the serial models, if *x* tends to be processed before *y*, then we refer to Dimension *x* as the first-processed dimension and to Dimension *y* as the second-processed dimension. This same terminology is also used for the parallel and coactive models if processing tends to be faster or more accurate on one dimension than the other. (*Figure 4 continues on the next page.*)

is that these stimuli are the same distance from the brightness bound and processing will self-terminate after the random walk on the brightness dimension is completed (the disjunctive rule is already satisfied). The model predicts that RTs for the interior and exterior stimuli on the saturation dimension (I_S and E_S) will be slower, because they require a second stage of processing. Furthermore, as shown in Figure 4 (top-right panel), the RT for I_S will be slower than for E_S; the reason is that, during the first stage in which brightness is processed, subjects will be slower to decide that I_S lies to the wrong side of the brightness bound (i.e., I_S is closer to the brightness bound than is E_S). The predictions of the mixed-order serial self-terminating model are explained in a similar fashion but with dimension *x* processed first on some trials and dimension *y* processed first on other trials.

Parallel Self-Terminating Model

The parallel self-terminating model assumes that both dimension *x* and dimension *y* are processed simultaneously and that processing ends as soon as a Category −B decision is made along either of these dimensions (i.e., the minimum processing time). This model predicts equal processing times for the I_B and E_B stimuli because the distance

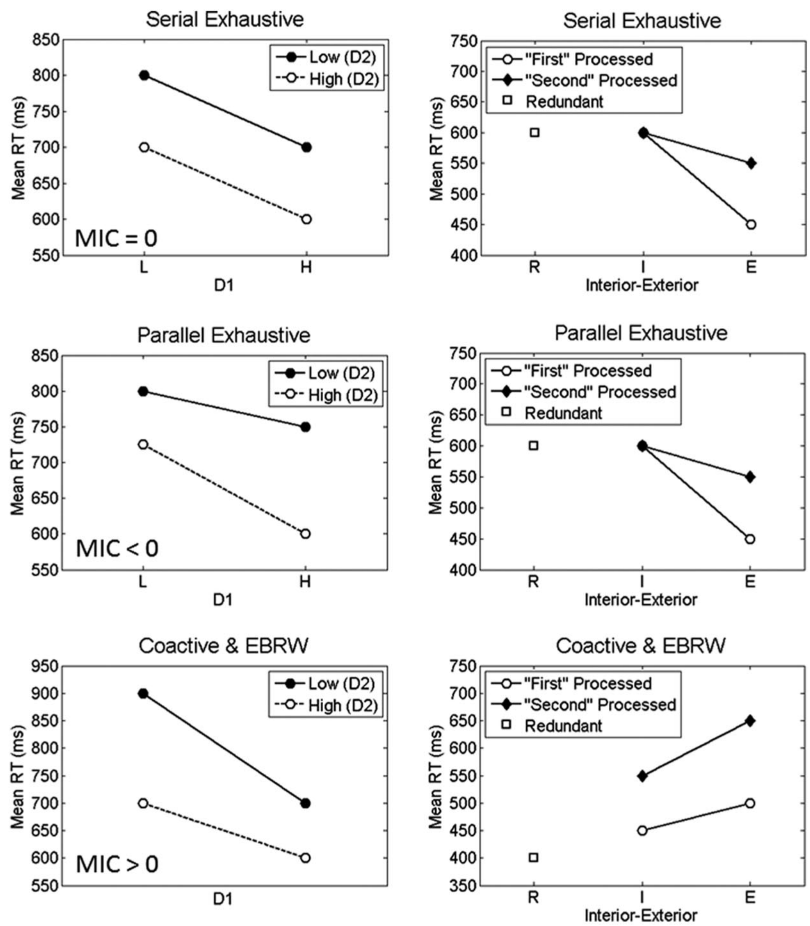

Figure 4 (continued).

of the brightness value from the brightness boundary is the same for both the I_B and the E_B stimulus. For an analogous reason, the parallel self-terminating model also predicts equal RTs for the I_S and E_S stimuli. This model predicts that the R stimulus should be processed faster on average than the I or the E stimulus because opportunities for self-termination arise on both the *x* and *y* dimensions: If there are more opportunities to self-terminate the minimum processing time will tend to be faster.

Coactive Model

Finally, the coactive model uniquely predicts that the interior stimuli will be processed *faster* than the exterior stimuli. This prediction may seem counterintuitive because the interior stimuli are closer to some of the category boundaries. In point of fact, however, according to the coactive model, the closer a contrast-category stimulus is to the redundant stimulus (see the lower left corner in Figure 2), the *faster* is its predicted RT. The reason is that, as one approaches the lower left corner, a greater proportion of a stimulus's *bivariate* perceptual distribution falls in the Category −B region (see illustration in Figure 2). Furthermore, as explained earlier, for the coactive model the step probabilities toward the −B criterion are determined solely by these bivariate proportions. That is, anytime a sampled bivariate percept falls in the B region that is defined by the combined *x* and *y* decision boundaries, the random walk steps toward the −B criterion. Thus, presentations of the interior stimuli lead to a more efficient random walk. This prediction of the coactive model has never been tested in previous research.

As we noted earlier, Fifić et al. (2008) conducted preliminary tests that classification of integral stimuli would involve coactive processing. However, that earlier study considered only a single contrast involving the over-additivity for the target category (MIC > 0). In the present work, we extend the analysis to the contrast category. The diagnosticity of the contrast-category RTs was not known when Fifić et al. conducted their study, and their design did not utilize all of the contrast-category items. As shown in Figure 4, each of the rule models yields its own unique signature of the pattern of mean RTs (considered collectively across the target and contrast categories), so the current paradigm is highly diagnostic. Second, Fifić et al. used only a nonparametric analysis that examined a single qualitative effect. By contrast, our approach also utilizes parametric model fitting with the goal of accounting quantitatively for the complete set of individual-stimulus RT distributions. In a nutshell, we examine the extent to which the coactive rule model can account for the complete set of data, which is an approach that provides far more rigorous tests than attempted previously.

Experiment 1

In Experiment 1, we tested four participants on their speeded classification of integral-dimension stimuli (colors varying in brightness and saturation), using the rule-based category structures in Figure 3. A novel property of our design was that a different rotation of the category space was used for each participant. We conducted this manipulation for generality and to reduce the dependence of the results on possible stimulus-specific properties associated with individual colors. Our central hypothesis is that, in contrast to previous results involving separable-dimension stimuli, our applications of the rule models will point toward coactive information processing.

Method

Participants. Four Indiana University students, with normal or corrected-to-normal vision, completed the experiment. They received $9 per session and a $3 bonus per session for accurate performance.

Stimuli. The stimuli were color squares (187 pixels × 187 pixels) varying in brightness and saturation. The stimuli were presented at a monitor resolution of 1,280 × 1,024, and subtended a visual angle of about 4.70 degrees.

Each color was selected from the Munsell hue 5R; the full set of nine stimuli was created by combining three levels of brightness (4.0, 5.5, and 7.0) and three levels of saturation (4.0, 7.0, and 10.0). The standard xyY coordinates corresponding to the Munsell brightness and saturations (available at http://www.cis.rit.edu/research/mcsl2/online/munsell_data/all.dat) were converted to RGB values by converting the xyY values first to CIE XYZ color space coordinates and then to RGB values using standard transformations (Rossel, Minasny, Roudier, & McBratney, 2006). The Munsell brightness-saturation levels and associated RGB coordinates for each of the stimuli are shown in Table 1.

The assignment of colors to categories was varied across participants, as illustrated in Figure 3.

Table 1
RGB Values for the Munsell (5R) Colors Used in Experiments 1 and 2

Brightness	Saturation	R	G	B
Integral stimulus condition				
4.0	4.0	130	86	84
4.0	7.0	150	75	75
4.0	10.0	167	62	66
5.5	4.0	170	125	122
5.5	7.0	192	114	111
5.5	10.0	212	102	101
7.0	4.0	208	163	160
7.0	7.0	233	153	148
7.0	10.0	255	142	137
Separable stimulus condition				
5.0	6.0	170	104	102
5.0	8.0	184	97	95
5.0	10.0	197	89	88

Note. R = red; G = green; B = blue.

Procedure. Participants completed five sessions on near consecutive days. In each session, participants first completed 27 practice trials, followed by 810 experimental trials grouped into six blocks of 15 presentations of each of the nine colors. During the experimental trials, each color was presented 90 times per session and 450 times over the entire experiment. The order of presentation was randomized anew within each block.

All responses were collected using the mouse. Participants rested their left and right index fingers on the mouse buttons throughout testing, with the left button indicating A and the right button B. Each trial began with a 1,770-ms fixation cross, and a warning tone initiated 1,070 ms after the onset of the cross and played for 700 ms.

The first session was a training session. Participants learned the categories through trial and error by responding to the individually presented members of the categories and receiving feedback. The feedback was provided for both correct and incorrect trials, and the stimulus and feedback remained on screen until the participant pressed a mouse button.

In the later test sessions, the stimulus was presented until either a response was made or 5,000 ms passed (a time out). Feedback was presented only after incorrect trials or time-out responses (e.g., ". . . TOO SLOW . . ."). RTs were recorded from the onset of a stimulus display up to the time of a response. A blank interval of 1,870 ms was inserted between each trial.

The contrasting qualitative model predictions all assume highly accurate responding; hence, the instructions emphasized accuracy. However, subjects were also informed that their RTs were being recorded and to execute their response as soon as they had made their decision.

Results and Discussion

For each participant/item combination, we excluded trials with RTs less than 150 ms or greater than two standard deviations above the mean.[1] Less than 4% of the data were excluded. Error rates were low, with the exception of the items with the slowest RTs for Participants 1 and 2 (see Table 2).

Mean-RT analyses. The mean correct RTs and error rates are reported in Table 2 and shown graphically in Figure 5. (The contrast category dimensions for each participant are labeled as either first-processed or second-processed depending on which of the two dimensions was processed faster or slower, respectively.) Comparison of Figures 4 and 5 suggests that the results are most consistent with the coactive model. First, replicating the previous results of Fifić et al. (2008), the mean RTs for the target category tend to show the over-additive pattern predicted by the coactive model (although the magnitude of over-additivity is small in some cases). The second and more novel result is that the mean RTs for the contrast category (right hand panels) get slower as one moves from the redundant (R) stimulus to the interior (I) stimuli to the exterior (E) stimuli, again supporting the predictions from the coactive model· We should note that the schematic predictions illustrated in Figure 4 (bottom-right panel) are intended to illus-

[1] We also analyzed the data after removing RTs greater than three standard deviations above the mean. The main analyses all showed that same qualitative pattern.

Table 2
Mean Correct RTs (ms) and Error Rates for the Individual Stimuli, Along With the Model Predictions From the Coactive Model

Variable	Item								
	HH	HL	LH	LL	E_S	I_S	E_B	I_B	R
Participant S1									
RT Observed	509.1	540.1	659.6	692.3	598.6	491.0	599.2	514.4	475.5
RT Model	504.5	533.8	651.6	705.3	580.1	518.6	568.5	550.4	464.1
SE	4.14	5.23	10.92	9.95	9.60	5.42	10.80	7.48	5.11
p(e) Observed	.01	.01	.06	.13	.06	.01	.03	.01	.00
p(e) Model	.00	.01	.10	.18	.05	.01	.03	.02	.00
Participant S2									
RT Observed	508.6	605.8	546.2	697.8	607.0	544.4	647.4	604.2	543.6
RT Model	490.3	637.7	552.8	723.6	596.6	563.7	651.5	618.1	523.6
SE	5.95	11.24	7.56	12.19	8.02	5.84	10.63	9.04	7.28
p(e) Observed	.00	.03	.02	.23	.02	.01	.05	.03	.00
p(e) Model	.00	.07	.01	.22	.03	.01	.07	.04	.00
Participant S3									
RT Observed	465.8	534.8	507.8	587.0	519.5	450.8	544.4	457.9	403.7
RT Model	461.0	519.4	501.2	620.0	494.7	464.1	528.1	471.3	424.8
SE	4.18	6.15	5.23	6.11	5.96	4.55	5.33	5.08	3.44
p(e) Observed	.00	.01	.03	.04	.01	.00	.02	.00	.00
p(e) Model	.00	.00	.00	.05	.00	.00	.01	.00	.00
Participant S4									
RT Observed	401.3	443.6	433.6	515.6	467.3	433.6	456.6	425.4	398.8
RT Model	401.0	439.7	436.8	499.7	461.9	443.3	447.6	435.5	399.2
SE	1.94	3.63	2.99	5.71	3.61	4.60	4.36	3.53	2.76
p(e) Observed	.00	.00	.00	.03	.01	.00	.01	.00	.01
p(e) Model	.00	.00	.00	.02	.01	.00	.00	.00	.00

Note. H and L refer to the high- and low-salience dimension values, respectively. RT = response time; HH = high-high; HL = high-low; LH = low-high; LL = low-low; E = exterior; I = interior; S = saturation; B = brightness; R = redundant; p(e) = proportion of errors; *SE* = the observed standard error of the mean for each item.

trate only a slowing of RTs as one moves from R to I to E. The relative degree of slowing across the first-processed and second-processed dimensions will depend on specific parameter settings in the model. As will be seen, the coactive model achieves accurate quantitative accounts of the observed results.

We conducted statistical tests to corroborate the descriptions of the data provided above. Table 3 presents the results of a 4 (session) × 2 (brightness level) × 2 (saturation level) analysis of variance conducted on the target-category RTs of each participant. In all cases, the main effects of brightness and saturation were highly significant, reflecting that subjects were faster to classify stimuli that were far from the category boundaries. (Because these main effects are obvious from inspection of Figure 5, the statistical tests are not shown in Table 3.) As in previous experiments (Little et al., 2011), there was also a significant main effect of session resulting from a slight speed-up effect. Most important, for two of the participants (S2 and S4), there was a significant interaction between brightness and saturation, confirming the over-additive pattern of mean RTs. For the remaining two participants (S1 and S3), however, the interaction was not significant (see Table 3).

Table 3 also presents the results of several paired *t* tests conducted on the mean RTs from the contrast category. For all participants, and on both the brightness and saturation dimensions, the external stimuli were always processed significantly more slowly than the internal stimuli. Furthermore, this effect was reasonably stable across sessions. For each subject, we conducted a three-way analysis of variance (ANOVA) using as factors dimension (saturation vs. brightness), item type (interior vs. exterior), and session. In none of the cases did session interact significantly with item type. Finally, as shown in Table 3, the internal stimuli were processed significantly more slowly than the redundant stimulus. (This effect was only marginal for stimulus I_S for participant S2.)

In sum, overall, the results point decidedly toward a coactive processing pattern. We now turn to detailed formal modeling of the RT-distribution data to provide rigorous tests of the coactive rule model and develop further contrasts with the other architectures.

Computational modeling. For simplicity, when modeling the perceptual distributions, we assumed that the mean location for each individual stimulus was given by its logical coordinates within the stimulus space. We also assumed equal variances within a given dimension; however, to model differences in overall discriminability between the dimensions, we estimated separate variance parameters for Dimension x (σ_x^2) and y (σ_y^2). To facilitate model fitting, we allowed continuous variation in the discrete $+A$ and $-B$ criterion parameters. Specifically, we assumed that the criterion locations were uniformly distributed and estimated *mean locations* for the $+A$ and $-B$ criteria and a *range*, *R,* for the uniform distribution. On each simulated trial, a criterion location was sampled randomly from the uniform distribution and then truncated to its integer-valued setting. We also assumed a log-normal distribution of residual times (encoding and motor-execution) with location-parameter μ_r and scale-parameter σ_r^2. A scaling parameter k translated the number of steps in each random walk into ms. In total, the logical-rule models have 10 free parameters: the perceptual-variance parameters σ_x^2 and σ_y^2; decision-

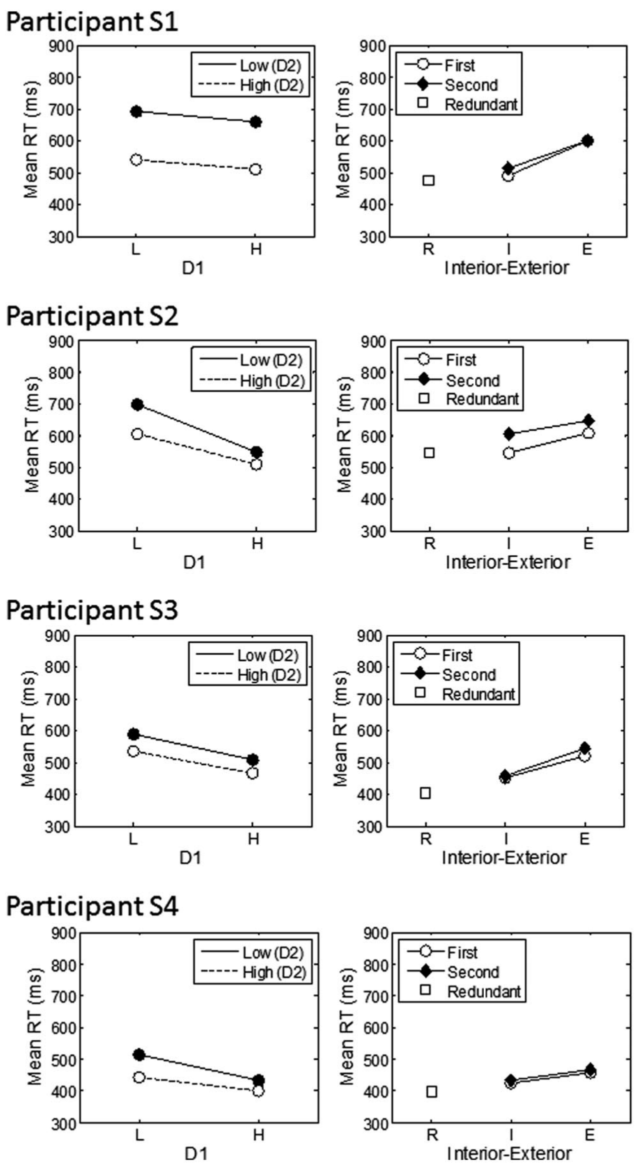

Figure 5. Observed mean response times (RTs) for the individual subjects and stimuli. Error bars represent ± 1 *SE*. Note that the standard error bars for the mean RTs are too small to be seen. The left panels show the results for the target-category stimuli, and the right panels show the results for the contrast-category stimuli. L = low-salience dimension value; H = high-salience dimension value; D1 = Dimension 1; D2 = Dimension 2; R = redundant stimulus; I = interior stimulus; E = exterior stimulus.

bound locations on Dimensions *x* and *y* (D_x and D_y); random-walk criterion locations $+A$ and $-B$; the uniform criterion range, *R*; residual-stage parameters μ_r and σ_r^2; and a scaling constant *k*. For the serial-self terminating model, an additional parameter, p_x, indicated the probability that dimension *x* was processed before dimension *y*.

Following Little et al. (2011), in addition to the serial, parallel, and coactive models, we also fitted a mixed serial-parallel model to the data. See Little et al. (2011) for a description of the additional free parameters used for fitting the mixed model to the data.

We fitted the models to the correct-RT distributions for each item separated into fixed width 50-ms bins (Heathcote, Brown, & Mewhort, 2002; Speckman & Rouder, 2004). We did not fit error-RT distributions because of the low overall error rates; however, the error rates strongly constrain the models, because the models are required to simultaneously fit the error rates and the correct-RT distributions. Specifically, we searched for the free parameters that maximized the multinomial log-likelihood fit function

$$\ln L = \sum_{i=1}^{n}\ln(N_i!) - \sum_{i=1}^{n}\sum_{j=1}^{m+1}\ln(f_{ij}!) + \sum_{i=1}^{n}\sum_{j=1}^{m+1}f_{ij}\cdot\ln(p_{ij}) \qquad (2)$$

where N_i the total number of times that item *i* ($i = 1, n$) was presented, f_{ij} is the frequency with which item *i* had a correct RT in the *j*th bin ($j = 1, m$) or was an error response ($m + 1$), and p_{ij} (which is a function of the model parameters) is the predicted probability that each item *i* had a correct RT in the *j*th bin or was an error. The log-likelihoods were converted to Bayesian information criteria (BIC; Schwarz, 1978) by penalizing the log-likelihood with an additional term based on the number of free parameters in the model, n_p, and the number of data observations, *M*:

$$BIC = -2\ln L + n_p\ln(M). \qquad (3)$$

The model that yields the smallest BIC is preferred.

Predictions of the RT distributions and error probabilities were generated using 10,000 simulations for each individual stimulus. Fifić et al. (2010, pp. 311–317) provided complete descriptions for the mechanics of each of the models. Numerical methods are also available for generating predictions for each of the models (Little, in press).

The model fits are shown in Table 4. It is important to reemphasize that in past cases involving separable-dimension stimuli (Fifić et al., 2010; Little et al., 2011), the serial self-terminating and mixed serial/parallel models fitted far better than did the coactive model. By contrast, with the present integral-dimension stimuli, we see a dramatic reversal. For all four subjects, the coactive model yields a much better fit than the serial self-terminating, serial-exhaustive, and parallel-exhaustive models. For three of the four subjects, the coactive model also fares considerably better than the parallel self-terminating model. (For S2, however, the parallel model yields a better fit, although not nearly as dramatic as the advantage of the coactive model for the other subjects.) Furthermore, making allowance for a mixture between parallel and serial processing fails to improve the BIC from the parallel model (except for S2, where the mixed serial-parallel model fits slightly better than the parallel model).

To understand the reason for these results, Figure 6 compares the mean correct-RT predictions from the serial self-terminating, parallel self-terminating, and coactive models. Clearly, the serial and parallel models fail to predict the pattern of RTs for the contrast category. By comparison, the coactive model achieves excellent accounts of the contrast-category results. In addition, the parallel model always incorrectly predicts an under-additive pattern of mean RTs for the target category, while the serial model fails to predict the over-additive pattern observed for S2 and S4. The coactive model predicts correctly the over-additivity for S2 and S4. Its main failing is that it over-predicts the degree of over-additivity for S1 and S3.

Table 3
Statistical Test Results for the Individual Subjects

Variable	Target category *df*	Target category *F*	Contrast category Paired comparison	Contrast category *M*	Contrast category *t*
Participant S1					
Session	3	29.35***	E_S–I_S	107.69	9.92***
Brightness × Saturation	**1**	**0.1**	E_B–I_B	84.89	6.46***
Session × Brightness × Saturation	3	0.47	E_S–R	123.16	11.58***
Error	1,339		I_S–R	15.47	2.08*
			E_B–R	123.75	10.53***
			I_B–R	123.16	11.58***
Participant S2					
Session	3	59.79**	E_S–I_S	62.67	6.31***
Brightness × Saturation	**1**	**12.87*****	E_B–I_B	43.19	3.11**
Session × Brightness × Saturation	3	7.33***	E_S–R	63.41	5.86***
Error	1,286		I_S–R	0.74	0.08†
			E_B–R	103.74	8.20***
			I_B–R	63.14	5.86***
Participant S3					
Session	3	39.98***	E_S–I_S	68.75	9.20***
Brightness × Saturation	**1**	**1.34**	E_B–I_B	86.51	11.76***
Session × Brightness × Saturation	3	0.73	E_S–R	115.85	16.89***
Error	1,378		I_S–R	47.11	8.23***
			E_B–R	140.77	22.21***
			I_B–R	115.85	16.89***
Participant S4					
Session	3	9.14***	E_S–I_S	33.69	5.72***
Brightness × Saturation	**1**	**28.99*****	E_B–I_B	31.19	5.55***
Session × Brightness × Saturation	3	0.32	E_S–R	68.54	15.12***
Error	1,389		I_S–R	34.85	6.43***
			E_B–R	57.80	11.13***
			I_B–R	68.54	15.12***

Note. E = exterior; I = interior; S = saturation; B = brightness; R = redundant. The mean-interaction-contrast test is highlighted in boldface.
$^{\dagger}p < .10$. $^{*}p < .05$. $^{**}p < .01$. $^{***}p < .001$.

In Figure 7, we present the fit of the coactive model to the full RT distributions observed for each of the individual stimuli of each of the participants. (The spatial layout for each participant has been rotated to match the category space shown in Figure 2.) The coactive model accounts well for not only the mean RTs but also for the detailed shapes of the individual-stimulus RT distributions. (Note also that the model accounts well for the error rates of the individual stimuli reported in Table 2, although the error rates are generally low.) In our view, these good quantitative accounts of the individual-stimulus RT-distribution data provide strong further support for the logical-rules framework of classification RTs (Fifić et al., 2010), while at the same time providing insights into the information-processing architecture that underlies the rule-based classification of integral-dimension stimuli.

Other model comparisons. The goal of our design was to develop sharp qualitative contrasts among only the members of the logical-rule family displayed in Figure 4. Nevertheless, it is useful to also consider prominent RT models from the field of perceptual categorization that are outside this family. In this section, we consider three such models: the distance-from-boundary model of Maddox and Ashby (1996), a prototype-based random-walk model (Nosofsky & Stanton, 2005), and an exemplar-based random-walk model (Nosofsky & Palmeri, 1997b).

According to Maddox and Ashby's (1996) distance-from-boundary model, the observer partitions a perceptual space into

Table 4
Negative Log-Likelihood and BIC Fits for Each Model

Model	S1	S2	S3	S4
Serial self-terminating	594 (1,277)	634 (1,358)	543 (1,175)	426 (941)
Parallel self-terminating	500 (1,082)	552 (1,184)	493 (1,066)	397 (874)
Mixed serial-parallel	498 (1,109)	**533 (1,179)**	492 (1,098)	393 (898)
Serial exhaustive	767 (1,615)	710 (1,500)	611 (1,303)	535 (1,151)
Parallel exhaustive	808 (1,697)	676 (1,433)	629 (1,338)	554 (1,188)
Coactive	**477 (1,035)**	559 (1,199)	**457 (995)**	**348 (776)**

Note. BIC = Bayesian information criterion; S = subject. Boldface type is used to indicate the best fit. BIC is shown in parentheses.

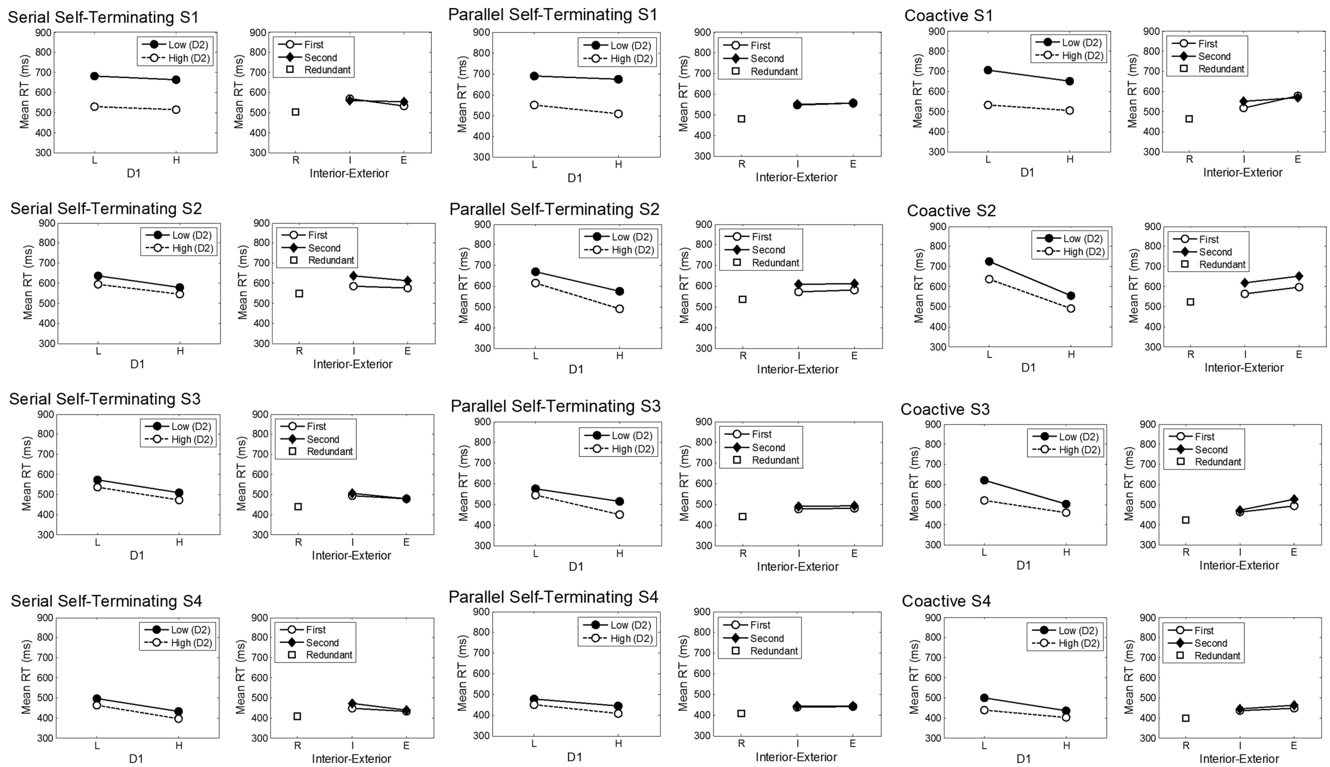

Figure 6. Mean response time (RT) predictions from the serial self-terminating, parallel self-terminating, and coactive models. L = low-salience dimension value; H = high-salience dimension value; D1 = Dimension 1; D2 = Dimension 2; R = redundant stimulus; I = interior stimulus; E = exterior stimulus.

category regions using decision boundaries. For the present design, we assume that the boundaries are the rule-based ones illustrated in our Figure 2. Presentation of a stimulus gives rise to a noisy perceptual representation in the space. The observer determines the region in which the perceptual representation falls and emits the appropriate categorization response. The RT for making the response is assumed to be some decreasing function of the distance of the perceptual representation from the decision boundary. For the present design, in which multiple decision boundaries are involved, additional assumptions are needed to instantiate this general hypothesis. One possibility is that the RT depends only on the distance of the percept from the closest *relevant* bound. For example, the speed of correct classification of I_s and E_s in our Figure 3 would depend only on their distance from the saturation boundary. An alternative possibility is that the RT depends on the distance of the percept from the closest bound, regardless of whether it is relevant or irrelevant. Regardless of which assumption is made, the distance-from-boundary model would fail to account for the main qualitative pattern of results in our experiment. For example, because I_s and E_s are equidistant from the relevant saturation bound, the first version of the model would predict incorrectly that those two stimuli should have identical RT distributions. And because I_s is closer to the brightness bound than is E_s, the second version of the model would predict that I_s would tend to have slower RTs than E_s (whereas our data show the opposite pattern). In our view, the present logical-rule models framework improves upon the descriptive RT-distance hypothesis by incorporating psychological processing assumptions. These include the sequential sampling of perceptual information to drive random-walk processes, mental architecture assumptions (e.g., serial vs. parallel vs. coactive processing), and stopping rules (self-terminating vs. exhaustive processing).

A second major theory of perceptual categorization is prototype theory (e.g., Reed, 1972), which holds that people classify objects according to their similarity to the central tendencies of the alternative categories. Nosofsky and Stanton (2005) developed an RT version of prototype theory, in which retrieved prototype information was presumed to drive a random-walk process (for details, see Nosofsky & Stanton, 2005, p. 610). Prototype models would be unable to account for the data in the present experiment. As is well known, accurate classification by means of a prototype strategy requires that the contrasting categories be linearly separable (see Reed, 1972). It is easy to see from inspection of Figure 2 that one cannot draw a straight line through the space that perfectly separates the members of Category A from Category B. Yet participants in our experiment performed with extremely high levels of accuracy on all stimuli.

An alternative model that *can* capture the general qualitative pattern of results in this experiment is Nosofsky and Palmeri's (1997b) exemplar-based random walk (EBRW) model. According to the EBRW, people store individual exemplars from each of previously studied categories in memory. When an item is presented for classification, the stored exemplars race to be retrieved. If the retrieved exemplar belongs to Category A then a random

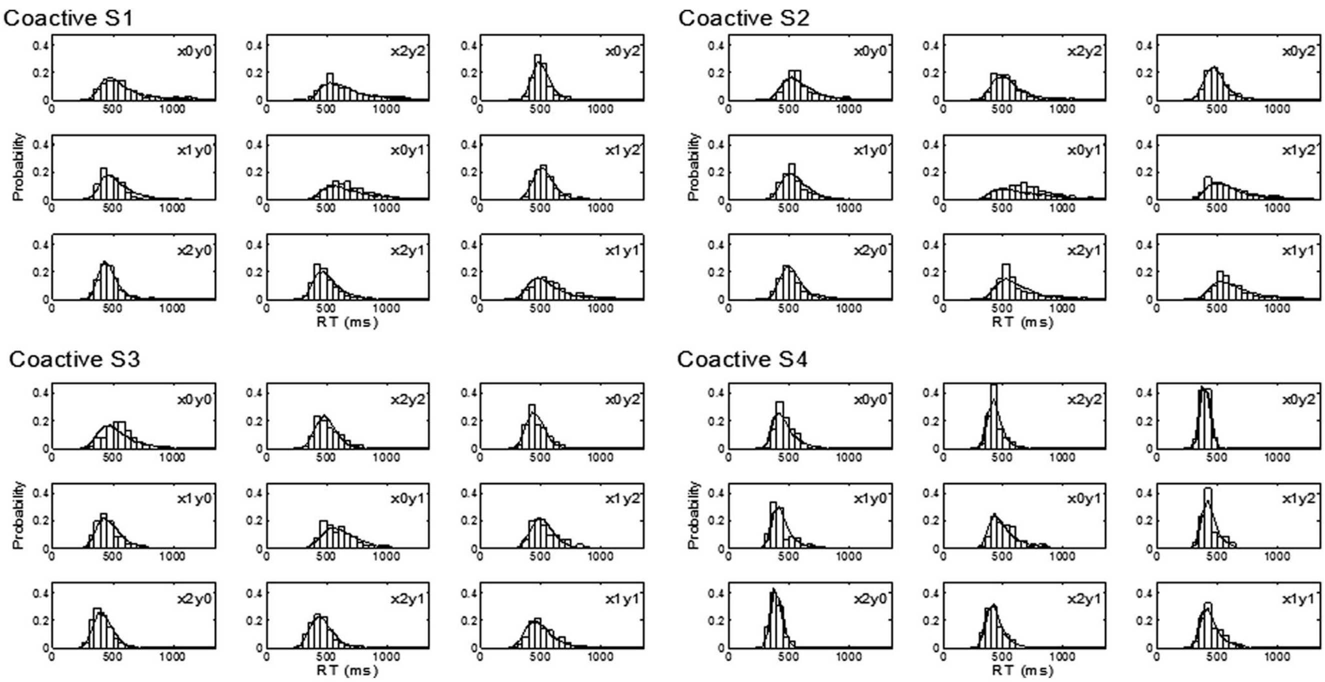

Figure 7. Fit (smooth curves) of the coactive model to the individual-stimulus response time (RT) distribution data (open bars) of the individual subjects. Each cell of each panel shows the RT distribution associated with an individual stimulus. Within each panel, the spatial layout of the stimuli has been rotated to match Figure 1 and Participant S1 in Figure 2.

walk counter takes a step toward the +A criterion. If a Category B exemplar is retrieved then the counter takes a step toward the −B criterion. The probability with which an exemplar is retrieved is determined by its similarity to the currently presented item. Nosofsky and Palmeri (1997b) derived formal predictions for the EBRW and showed that the RT could be predicted from the relative "summed similarity" between a presented item and the stored exemplars from the alternative categories. In general, the greater the summed similarity between an item and the exemplars of one category and the less the summed similarity to the other category, the faster the classification RT for that item.

Fifić et al. (2008, 2010) provided evidence that in the current paradigm the qualitative predictions of the EBRW are the same as those of the coactive rule model. The models make similar predictions because (a) both assume a single-channel random walk in the evidence-accumulation and decision process, and (b) the random-walk step probabilities from the exemplar model are strongly correlated with those from the coactive rule model. For example, as can be seen in Figure 3 (left panel), as one moves toward the lower left corner of the space, the summed similarity of items to the members of Category B increases, resulting in faster Category B RTs. Future research involving other experimental manipulations would be needed to sharply discriminate the coactive rule model from the EBRW model in this paradigm.[2]

In our view, although the coactive rule model and EBRW model yield similar predictions, our results still converge on a general conclusion: namely, that the type of information processing that underlies categorization performance for these integral-dimension stimuli is "holistic" in nature. That is, our results suggest that observers are *not* making separate cognitive decisions about the category region of the stimuli on each separate dimension and then combining those separate decisions to emit a final response. Instead, perceptual information from the individual dimensions is combined at an early stage, via, for example, coactive processing or retrieval of whole exemplars, and it is this holistic pooled information that is driving categorization decision making. For simplicity, in our ensuing discussions, we refer to this holistic form of perceptual-category decision making as *coactive processing*.

Summary and remaining issues. In sum, the overall qualitative pattern of results as well as the results from the computational modeling point strongly in the direction of a coactive information-processing architecture for classifying these integral-dimension stimuli into the rule-based categories. Nevertheless, there are a couple of issues that remain to be resolved. First, for

[2] We fitted the EBRW model to the RT-distribution data of the individual subjects by using the same methods as already described for the logical-rule models. The EBRW model provided slightly worse BIC fits than did the coactive-rule model for Subjects 1, 3, and 4, and a slightly better fit for Subject 2. Because our assumed stimulus configuration provides only an approximation to the true perceptual spaces of each of the individual observers, our view is that the small differences in quantitative fit are not very meaningful. We should note that in previous studies involving separable-dimension stimuli (Fifić et al., 2010; Little et al., 2011), the EBRW provided dramatically worse quantitative fits to the data than did the serial-processing and parallel-processing logical-rule models.

two of the participants, the predicted over-additive MIC for the target-category items failed to reach significance. Of course, we expect that there must be individual differences in the detailed perceptual spaces of the individual observers. Possibly, the lack of significant over-additivity for these two participants may be due to their perception of the individual colors diverging slightly from the positions assumed in Figure 3.[3] Second, the Experiment 1 results were limited to only four highly experienced observers. Our reason for collecting extensive data from only four observers was to obtain detailed RT-distribution data suitable for quantitative fitting at the individual-subject level. Nevertheless, questions may arise regarding the generality of our main qualitative effects. To address these issues, we decided to conduct a second experiment, using the same stimuli and category structure as before, but in which we collected data from a larger number of observers. Rather than fitting detailed RT distributions of individuals, the central goal was now to evaluate whether the fundamental qualitative predictions from the coactive model would be observed at the group level. In addition, as a source of comparison, we tested a separate group of subjects on the same rule-based category structure, except using separable-dimension stimuli instead of integral-dimension ones. As explained in our introduction and reviewed below, we expect to see dramatically different mean-RT signatures across the integral-dimension and separable-dimension conditions.

Experiment 2

In Experiment 2, we sought to replicate our main qualitative pattern of results from Experiment 1, namely, that for integral-dimension stimuli: (a) there should be an over-additive pattern of target-category mean RTs, and (b) the internal stimuli of the contrast category should have *faster* mean RTs, on average, than do the external stimuli. In direct contrast, categorization RTs for separable-dimension stimuli should show: (a) an additive or under-additive pattern of target-category mean RTs, and (b) a pattern in which mean RTs for the internal stimuli of the contrast category are either the same as or *slower* than for the external stimuli.

Method

Participants. Thirty-three participants from the University of Melbourne community were paid $10/session for participation. The qualitative predictions of RTs from the logical-rule models hold only under conditions in which observers are responding with high accuracy (for detailed simulation results, see Fifić et al., 2008). Therefore, participants were required to achieve a high level of accuracy (i.e., >90% correct categorization of each stimulus) to be included in the main data analysis. If a participant failed to achieve this level on the first attempt, he or she was invited to complete a second session. If the participant failed to achieve the required accuracy in the second session, the data were discarded. Nine participants were removed due to below-criterion accuracy. Five of the retained participants needed to complete the second session to meet the accuracy criterion.

Stimulus condition (integral vs. separable) and category rotation (see Figure 3) were balanced across participants. Participants removed for failure to achieve the accuracy criterion were replaced with new participants in the same condition and rotation. After deleting participants who failed to meet the accuracy criterion, there were 24 participants in total, three participants in each of the Stimulus-Condition × Rotation-Condition cells of the design.

Stimuli. There were two stimulus conditions: an integral-dimension condition and a separable-dimension condition. The stimuli in the integral-dimension condition were the same colors as used in Experiment 1.

The stimuli in the separable-dimension condition were colored rectangles with an inset vertical line (the same stimuli used in Experiment 2 of Little et al., 2011). The rectangles were 225 pixels wide and 150 pixels high, displayed in red (Munsell hue 5R, brightness-value 5) with a 10-pixel black border. The inset vertical line, which extended from the lower left corner of the rectangle, was 100 pixels tall × 10 pixels wide. The saturation of the rectangles varied in three levels (Munsell-chroma levels 6, 8, and 10), and the vertical line varied in the pixel-distance from the left-hand side of the rectangle (30, 40, 50 pixels). Munsell hue, chroma, and value coordinates were converted to RGB values using the same method described in Experiment 1 (see Table 1). The separable-dimension stimuli subtended a vertical visual angle of about 3.64° and a horizontal visual angle of about 5.79°.

The same rule-based category structure was used as in Experiment 1 (see Figure 3). Participants responded to each stimulus by pressing the left mouse button for Group A and the right mouse button for Group B. As in Experiment 1, the assignment of stimuli to categories was varied across participants. For the integral-dimension condition, the category boundaries were rotated as illustrated in Figure 3. An analogous set of rotations was created for the separable-dimension condition but with bar position replacing brightness as the horizontal dimension in Figure 3 and with the saturation values updated to chroma levels 6, 8, and 10. (We used slightly different chroma spacings across the integral-dimension and separable-dimension conditions to achieve roughly equal discriminability across the conditions.)

Procedure. Participants completed nine practice trials (one repetition of each item) followed by 90 training trials (10 repetitions of each item). Following the training trials, participants completed 540 test trials (60 reps of each item) with a break every 90 trials. The procedure during the training trials was identical to the procedure used in the first training session in Experiment 1. The procedure during the test trials was identical to the procedure used in the test sessions in Experiment 1 with the following exceptions: (a) to encourage highly accurate responding, at each break in the test trials, all of the stimuli were presented on screen with the percentage of correct responses for that stimulus in the previous block. (b) The duration of the warning tone was shortened to 100 ms.

Results and Discussion

The practice and training trials were excluded from the analysis. For the test trials, for each participant/item combination we excluded trials with RTs less than 150 ms or greater than three standard deviations above the mean. Error trials were also excluded. Less than 3% of the data were excluded. As shown in Table 5, average error rates were low were for all items in both conditions.

[3] This factor may also be relevant in assessing the meaningfulness of the better fit of the mixed serial-parallel model compared to the coactive model for Subject 2.

Table 5
Error Rates for Each Condition in Experiment 2 for Each Item

	Item								
Condition	HH	HL	LH	LL	E_1	I_1	E_2	I_2	R
Integral	.00	.01	.00	.06	.03	.02	.02	.02	.00
Separable	.01	.00	.01	.03	.03	.02	.03	.01	.00

Note. H and L refer to the high- and low-salience dimension values, respectively. HH = high-high; HL = high-low; LH = low-high; LL = low-low; E = exterior; I = interior; R = redundant.

In the main RT analyses that we report below, we collapsed across rotations and analyzed the data only at the level of stimulus condition (integral vs. separable) and item type. We collapsed across rotation for two reasons. First, there were only three subjects per individual rotation. Second, because the dimensions differed across the two stimulus conditions, aligning the rotation variable across the stimulus conditions is arbitrary.

Mean correct RTs for the main item types are shown graphically in Figure 8, with the results from the integral condition shown in the top panels, and the results from the separable condition shown in the bottom panels. First, consider the target-category results (left panels). For the integral condition, the target-category mean RTs show an over-additive pattern, consistent with the predictions of the coactive model (see Figure 4). By contrast, for the separable condition, the target-category mean RTs are additive or slightly under-additive, consistent with either a serial or parallel model.

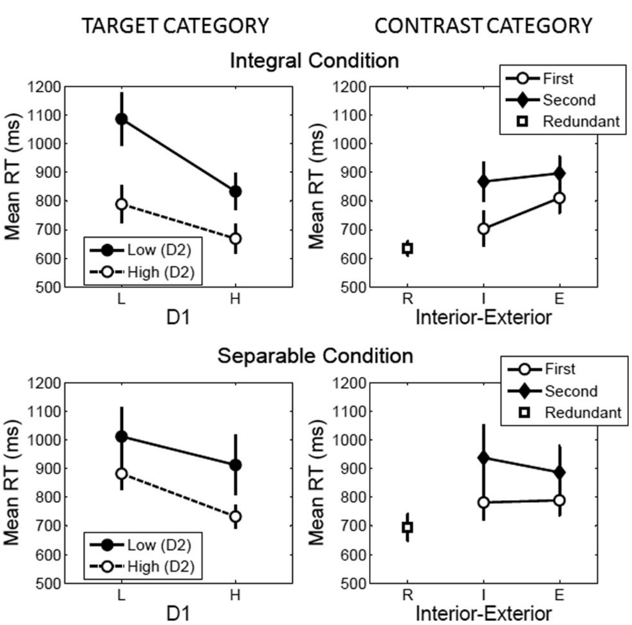

Figure 8. Observed mean response times (RTs) for the averaged subjects and stimuli from Experiment 2. Error bars represent ±1 *SE*. The top panels show the results for the integral stimulus condition; the bottom panels show the results for the separable stimulus condition. The left panels show the results for the target-category stimuli, and the right panels show the results for the contrast-category stimuli. L = low-salience dimension value; H = high-salience dimension value; D1 = Dimension 1; D2 = Dimension 2; R = redundant stimulus; I = interior stimulus; E = exterior stimulus.

More specifically, the integral condition displayed a positive MIC ($MIC_{Integral}$ = 132.91 ms), but the separable condition displayed a negative MIC ($MIC_{Separable}$ = –51.70 ms). Statistical analyses commensurate with those from Experiment 1 were conducted separately within each condition (see Table 6). The critical result is that the Brightness × Saturation interaction was significant for the integral condition, but the Bar Position × Saturation interaction was not significant for the separable condition. In addition, a direct comparison across the integral and separable conditions revealed a significant difference in the observed MICs ($MIC_{Integral} > MIC_{Separable}$), $t(22) = 3.79$, $p < .001$.

Next, consider the contrast-category RTs (see Figure 8, right panels). Recall that the coactive model is the only member of the set of logical-rule models that predicts that RTs should get slower as the stimulus moves from the interior to the exterior of the stimulus space (see Figures 3 and 4). As can be seen in Figure 8 (top-right panel), precisely this pattern of results was observed in the integral condition. By contrast, if processing is serial or parallel, then mean RTs should either be roughly constant for the interior and exterior stimuli or else slower for the interior stimuli. This pattern of results is the one observed in the separable condition (see Figure 8, bottom-right panel).

This contrast in the pattern of interior versus exterior RTs across the integral and separable conditions is corroborated by statistical test. In particular, the average E-I difference was significantly greater in the integral-dimension condition than in the separable-dimension condition, $t(22) = 1.79$, $p < .05$ (one-tailed). A one-

Table 6
Statistical Test Results for the Average Data From the Integral and Separable Stimulus Conditions in Experiment 2

	Target category	
Variable	*df*	*F*
Integral stimulus condition		
Brightness	1	96.19***
Saturation	1	23.24***
Brightness × Saturation	**1**	**10.47****
Error	11	
Separable stimulus condition		
Bar Position	1	5.3*
Saturation	1	29.28***
Bar Position × Saturation	**1**	**3.93**
Error	11	

Note. The mean-interaction-contrast test is highlighted in boldface.
* $p < .05$. ** $p < .01$. *** $p < .001$.

tailed test is justified because our prior hypothesis is clearly directional in nature: The coactive model hypothesized for the integral condition predicts a positive value of E-I, whereas the serial/parallel models hypothesized for the separable condition predict a value of E-I that is either zero or negative.

Although the main patterns of results for Experiment 2 corroborate our hypotheses, it is important to acknowledge that we observed considerable variability in the patterns across the individual rotations. Whether this variability is due to the small numbers of subjects at each rotation or to stimulus-related factors involving the category configuration itself remains an open question. In follow-up analyses, we found that the factor of rotation did not interact significantly with stimulus type (E vs. I) in either the integral-dimension condition or the separable-dimension condition. However, the lack of a significant interaction may simply reflect a lack of statistical power due to the small number of subjects in each cell of the design. We pursue the potential role of the different category rotations in theoretical analyses reported in our General Discussion.

In summary, the results from the present experiment lend considerable generality to our earlier findings from Experiment 1. Taken together, the results support the hypothesis that classification of integral-dimension stimuli into rule-based categories is accomplished via a coactive processing mechanism. We expand upon these results in our General Discussion.

General Discussion

Summary and Conclusions

The results of this study highlight a fundamental distinction in the architectures underlying the processing of separable- versus integral-dimension stimuli in tasks of rule-based categorization. Past tests of the logical-rule models indicated that in cases involving separable-dimension stimuli, rule-based classification decision making operates via serial or mixed serial/parallel processing of the individual dimensions that compose the stimuli (Little et al., 2011). From a psychological perspective, the idea is that the observer makes separate decisions along each individual dimension regarding the category region in which a stimulus falls. These separate decisions are then combined to determine which logical rule has been satisfied. By contrast, the present results suggest that a dramatically different *coactive* process operates when people classify integral-dimension stimuli into rule-based categories. In coactive processing, rather than making separate decisions along each individual dimension, information from the individual dimensions is instead *pooled* into a *single* channel that governs categorization decision making. These results converge with past classic ones in the field that suggest that integral-dimension stimuli are perceived and processed in "holistic" fashion (e.g., Garner, 1974). The work goes well beyond classic previous results, however, by formalizing the manner in which the pooled, holistic perceptual information is accumulated to yield classification decisions in rule-based categorization tasks.

Contribution of the Current Work

We obtained several sources of converging evidence in favor of the coactivation hypothesis for the integral-dimension stimuli. First, we replicated the earlier results of Fifić et al. (2008) that demonstrated a positive MIC for members of the conjunctive-rule target category. Second, we obtained support for the novel theoretical prediction that the interior members of the disjunctive-rule contrast category would be classified faster than the exterior members. These results were observed for both a small number of highly experienced observers (Experiment 1) and a larger group of relatively unpracticed observers (Experiment 2). Third, the coactive model provided outstanding quantitative accounts, at the individual-subject level, of the complete RT distributions observed for each of the individual stimuli in the categorization task. In our view, this ability of the coactive model to account quantitatively for these rich sets of data with relatively few free parameters is extremely important. It suggests that, beyond accounting for the focused qualitative effects of interest (i.e., the positive MIC for the target category and the interior-stimuli advantage for the contrast category), the model is likely capturing a wide assortment of other effects yet to be brought to light. In a nutshell, our verification of the model's a priori qualitative predictions combined with its excellent quantitative fits provides impressive converging evidence in support of the model. Moreover, none of the other logical-rule models of categorization could predict the focused qualitative effects, and none came close to the coactive model in their ability to quantitatively fit the individual-stimulus RT distributions across the individual subjects.

Alternative Perceptual-Representation Assumptions

As acknowledged in our introduction, for simplicity in the present investigation, we adopted the assumption that the integral-dimension stimuli in the category structure occupied locations in a rectangular grid (see Figure 3). Because the saturation/brightness values of colors from the Munsell system are derived from psychological judgments, this assumption about the structure of the psychological space in which the colors are organized seems like a reasonable starting point. Nevertheless, an interesting question is whether some of the alternative logical-rule architectures could account for the present data if alternative perceptual-representation assumptions were introduced. Because there are an infinite variety of different possible perceptual-representation assumptions, we are limited in our ability to draw any very general conclusions regarding this issue. Nevertheless, we did conduct investigations involving one type of alternative perceptual representation with particular theoretical importance. Various researchers have suggested that the perceptual representation for integral-dimension stimuli may display "mean-shift integrality" (e.g., Ashby & Maddox, 1994, Figure 4). The idea is that as values increase, say, along Dimension x, there is a correlated increase in the perceived values along Dimension y (and vice versa), leading to a distorted rhombic configuration of the type illustrated in Figure 9. In the present investigation, we were interested to test the predictions of the serial and parallel-processing architectures when applied to a configuration with mean-shift integrality, rather than the rectangular configuration assumed in the present work. We report the details of our preliminary investigation in the Appendix. In short, we found that the serial and parallel models could sometimes predict coactive-looking results for subsets of the four 90-degree rotations of the Figure 9 category boundaries. (For example, the parallel model might predict coactive-looking RT results if the target

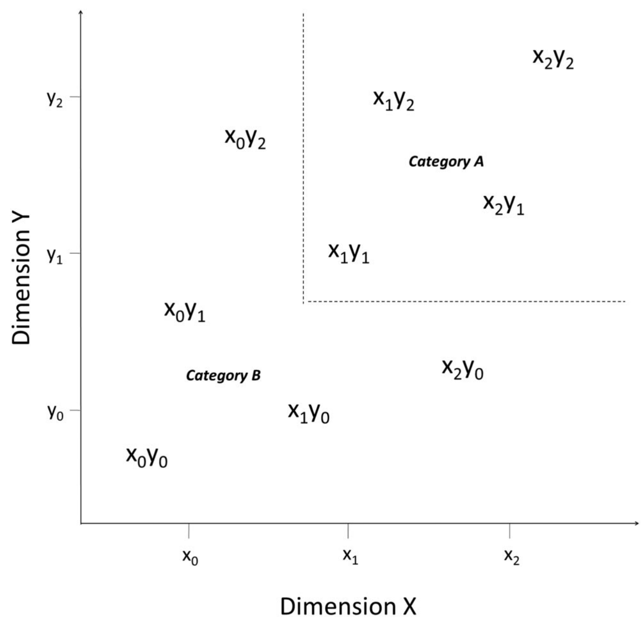

Figure 9. Schematic illustration of the category structure with stimuli displaying "mean-shift" integrality.

category occupied the upper right quadrant of the 3 × 3 stimulus space.) Crucially, however, the serial and parallel models never predicted coactive-looking results after averaging predictions across the four rotations. Thus, the serial and parallel models of logical-rule-based categorization would be unable to account for the data reported in the present experiments simply by adopting the assumption of mean-shift integrality.

Although the assumption of mean-shift integrality is not *sufficient* to allow the serial or parallel models to account for the data, we do not, of course, conclude that more complex perceptual-representation assumptions are not necessary. Almost certainly, achieving a complete quantitative account of the present results would require the derivation of more detailed perceptual maps for these integral-dimension stimuli, regardless of the information-processing architecture that operates.

Toward a More Complete Theory of the Representation and Processing of Integral- and Separable-Dimension Stimuli

More generally, a crucial goal for future research is to achieve a deeper theoretical understanding of inter-connections among the wide assortment of converging operations associated with the representation and processing of integral-dimension stimuli, including the present evidence for coactivation. In this final section, we take a step toward that goal by considering results from Garner's (1974; Garner & Feldfoldy, 1970) hugely influential speeded classification tasks. Because these tasks provide examples of speeded classification tasks involving rule-based category structures, the present theory should have some direct things to say about them.

Two of the most fundamental of Garner's (1974; Garner & Feldfoldy, 1970) speeded classification tasks are the *control* and

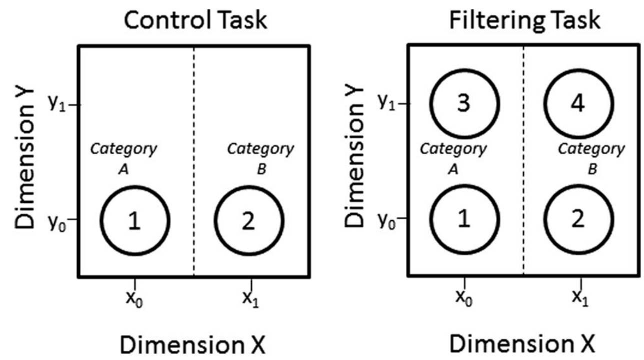

Figure 10. Garner's (1974; Garner & Felfoldy, 1970) control and filtering conditions. The circles represent isoprobability contours for the perceptual distribution of each stimulus.

filtering tasks illustrated in Figure 10. The overall set of four stimuli is constructed by orthogonally combining two values along each of two dimensions. In the control task, the value on one dimension is held constant, and participants classify stimuli according to values on a single relevant dimension. For example, in a case in which Dimension x is relevant, participants might be required to classify Stimulus 1 into Category A and Stimulus 2 into Category B. In the comparison filtering task, values are allowed to vary along the irrelevant dimension. Thus, participants would be required to classify Stimuli 1 and 3 into Category A, and Stimuli 2 and 4 into Category B. The classic result is that, when stimuli are composed of separable dimensions, there is no interference in the filtering task, i.e., participants classify objects in the filtering task as rapidly as they classify stimuli in the control task. By contrast, when stimuli are composed of integral dimensions, there is interference. One interpretation is that, when stimuli are composed of separable dimensions, participants are able to attend selectively to the single relevant dimension and to "filter" the irrelevant one. Thus, from a psychological standpoint, there are only two stimuli in the filtering task, defined by the values on the single relevant dimension. By contrast, such filtering cannot be achieved in cases involving integral-dimension stimuli.

The question arises, however, in the case of integral-dimension stimuli, why should classification of the four stimuli in the filtering

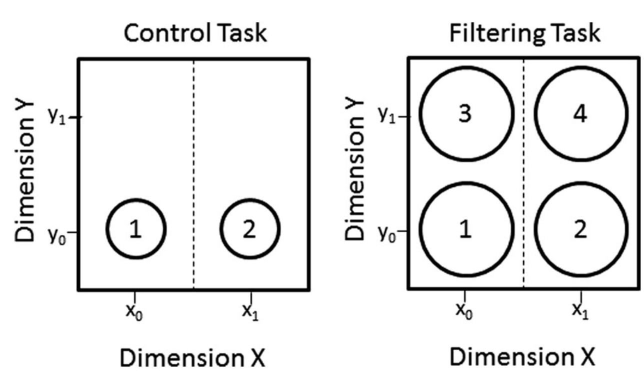

Figure 11. Schematic diagram showing increased perceptual variance (representing increased stimulus uncertainty) in the filtering task relative to the control task.

task be any slower than classification of the two stimuli in the control task? It is worth noting that, in and of itself, the coactive rule-based model does *not* provide an explanation of the result. Presumably, in both tasks, the observer would establish a single rule-based boundary, such as illustrated in Figure 10. Because the perceptual distributions are the same distance from the boundary in the control and filtering tasks, the random-walk process that governs decisions along that relevant dimension would finish equally quickly in both tasks.

In a previous attempt to use decision-bound theory to explain such results, Ashby and Maddox (1994) offered the following idea. They suggested that because of increased stimulus uncertainty in the filtering task, the perceptual distributions may have increased variances in the filtering task relative to the control task, as we illustrate schematically in Figure 11. They combined this idea with their RT-distance hypothesis, which we described earlier in this article. According to that hypothesis, RT is a nonlinearly decreasing function of the distance of a percept from the decision boundary. As can be seen in the right panel of Figure 11, with the increase in variance, many more percepts will lie very close to the decision bound in the filtering task compared to the control task. And because of the nonlinear relation between RT and distance, the overall mean RT will be slowed in the filtering task compared with the control task.[4]

Nosofsky and Palmeri (1997a) criticized that interpretation. They noted that although the model could predict a slowing of the *mean* RTs, it failed to predict the properties of the complete RT distributions. In particular, as can be seen in Figure 11, although more percepts are very close to the rule-based boundary in the filtering task compared to the control task, it is also the case that more percepts are farther away. Therefore, according to the RT-distance hypothesis, the *fastest* individual-trial RTs in the filtering task should be as fast as or faster than the fastest individual-trial RTs in the control task. Nosofsky and Palmeri (1997a) collected detailed RT-distribution data that falsified that prediction.

In light of our newly developed logical-rule models framework, it is worth noting that the alternative random-walk processing approach does not have this same problem as does the RT-distance hypothesis. According to the present approach, multiple percepts from a stimulus distribution are sampled on each trial, and each sampled percept drives the random walk to either one category criterion or the other (see also Ashby, 2000). Each step of the random walk is the same size, and its direction is decided only by the side of the decision boundary to which the sampled percept falls. Therefore, the random-walk process would predict slowed mean RTs in the filtering task, because an increased proportion of the stimulus distributions would fall to the wrong side of the rule-based decision boundary. However, as just explained, it would not make the same incorrect distributional predictions as did the RT-distance hypothesis. In a nutshell, by combining Ashby and Maddox's (1994) increased-variance idea with the present logical-rule processing approach, a rigorous and comprehensive explanation of the filtering interference may be forthcoming.

We offer this possibility only as a single example, and numerous other converging operations that distinguish between integral and separable dimensions remain to be explained. In view of the highly successful accounts of speeded rule-based classification performance obtained by the logical-rule models in the present study (involving integral-dimension stimuli) and the previous studies of Fifić et al. (2010) and Little et al. (2011; involving separable-dimension stimuli), we are now primed to pursue this ambitious goal.

[4] Ashby and Maddox (1994) also considered mean-shift integrality, illustrated in our Figure 9, as another possible explanation of filtering interference.

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(*Appendix follows*)

Appendix

Logical-Rule Models Applied to Configurations With Mean-Shift Integrality

We conducted simulations of the serial self-terminating, parallel self-terminating and coactive logical-rule models in cases in which the stimuli had the mean-shift-integrality structure depicted in Figure 9. Across different sets of simulations, we modulated the degree of mean-shift integrality as follows. We started with the baseline (square) configuration depicted in Figures 2 and 3 and assumed that adjacent dimension values were separated by unit distance. Let O_{ij} denote the object that has logical-value i on dimension x (x_i) and logical-value j on dimension y (y_j), where the logical values are 0, 1, and 2. In the mean-shift integrality representation, the mean y'value of object O_{ij} would be set equal to $y_j' = y_j + \delta \cdot i$. Likewise, the mean x' value of object O_{ij} would be set equal to $x_i' = x_i + \delta \cdot j$. Across different sets of simulations, we varied the magnitude of the shift parameter δ from .00 to .20 in increments of .05 to vary the degree of mean-shift integrality. We set the variances along each dimension to be equal to one another and presumed zero correlation between the x' and y' values *within* each individual stimulus distribution.

In all cases, the decision boundaries D_x and D_y were assumed to be orthogonal to the coordinate axes of the space, as would be presumed in applying the present types of logical rules. For each configuration, the values of D_x and D_y were set midway between the adjacent stimuli from contrasting categories, as depicted, for example, in Figure 9. (For example, in Figure 9, D_x would be set equal to the mean x value among x_0y_1, x_0y_2, x_1y_1, and x_1y_2.) All other parameters were set at representative values based on results of fitting the models to the individual-subject data from Experiment 1. The specific parameter settings for each model are reported in Table A1.

For each mean-shift-integrality configuration, the models were used to simulate data from each of the 90-degree rotations of the category boundaries, as depicted in Figure 3. That is, across rotations, the target category occupied either the upper right quadrant, upper left quadrant, lower right quadrant, or lower left quadrant. The results reported below were based on 50,000 simulated trials per individual stimulus per condition. For each combination of mean-shift-integrality (δ) and category-boundary rotation, we then computed the MIC for the target-category members, and the value of $E_1 + E_2 - I_1 - I_2$ for the contrast category. (In all cases, the predicted error rates for each individual stimulus were less than .10.) Finally, we averaged across the predictions from the four rotations to yield the predicted average MIC and $E_1 + E_2 - I_1 - I_2$ values. These predictions are reported in Table A2. As can be seen, the average predictions from the models in cases involving mean-shift-integrality are essentially the same as for the square configuration (i.e., the case in which $\delta = 0$). The serial model predicts an average MIC essentially equal to zero and predicts a negative value of $E_1 + E_2 - I_1 - I_2$. The parallel model predicts a negative average MIC and an average value of $E_1 + E_2 - I_1 - I_2$ essentially equal to zero. Only the coactive model predicts both a positive MIC and a positive value of $E_1 + E_2 - I_1 - I_2$.

Table A1
Parameter Values Used in Mean-Shift-Integrality Simulations of the Logical-Rule Models

Model	p_x	σ_x	σ_y	A	B	μ_r	σ_r	k	R
Serial	.50	.75	.75	4	4	5.85	0.16	40	1.5
Parallel		.75	.75	4	4	5.85	0.16	40	1.0
Coactive		.50	.50	5	5	5.85	0.16	30	1.5

Table A2
Mean Predictions of MIC and E-I Values from Simulations of the Serial, Parallel, and Coactive Logical-Rule Models Under Conditions of Mean-Shift Integrality

	Shift Parameter δ				
Model	0.00	0.05	0.10	0.15	0.20
Serial					
MIC	0.11	1.11	0.04	0.56	0.66
E-I	−114.71	−117.31	−121.80	−128.71	−137.02
Parallel					
MIC	−42.15	−41.90	−41.73	−43.10	−44.21
E-I	2.99	2.80	3.98	5.29	6.39
Coactive					
MIC	51.35	52.72	57.21	62.68	68.90
E-I	26.72	25.86	25.43	25.39	23.69

Note. MIC = mean interaction contrast; E = exterior; I = interior; E-I = $E_1 + E_2 - I_1 - I_2$. Holding the stimulus configuration fixed, predictions are averaged across the four 90-degree rotations of the category boundaries.

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