

# Path Tracking of dynamics of a Chaotic Memristor Circuit

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## Abstract

Since its recent invention, the Memristor has proven to be a revolutionary device due to the versatility of its applications. Among these, the study of chaotic circuits using Memristors is a fertile area to be explored. However, despite several ways to track the chaotic paths for memristive circuits have been previously proposed, all of these provide merely numeric or qualitatively solutions. As it could be necessary for some applications to have all the possible information about their dynamics, a new methodology to path tracking the chaotic trajectories for memristive circuits using the Multistage Homotopy Perturbation Method is proposed. This methodology provides a straightforward semi-analytic expression to obtain the chaotic path in an iterative way, providing both, the numeric and qualitative behaviour of the circuit.

**Keywords:** Homotopy perturbation method, Chaotic circuits, Memristor

## 1 Introduction

Industrial efforts to push electronics beyond the technological limits has resulted in a vertiginous evolution of electronic devices. As a result of this tendency, new devices like single electron transistors [3, 1, 2] have been developed. Within these, the memristor highlights for being a promising alternative to substitute the MOS transistors in modern circuits [24, 8, 9, 10, 11, 18, 26, 19, 32, 23, 27, 28, 12, 14, 7, 16, 17, 13, 25, 4, 20, 31, 6, 21, 29, 15, 30, 22, 5]; high scale of integration, memory functions and some other attractive characteristics make the memristor to be considered the most revolutionary device since the invention of the transistor [33].

As expected, many research has been carried to find applications to this innovative two-terminal device. At present, the memristor has been proposed to be used in circuits such as:

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1. Artificial neural networks [37, 34, 39, 40, 38, 36, 35].
2. Memories [42, 43, 41].
3. Image encryption [44].
4. Oscillators [45, 47, 46].
5. Chaotic circuits [49, 48, 50, 52, 51].
6. Filters [53].

Due to their nature, one of the most straightforward applications for memristors is in the construction of chaotic circuits, fundamental part of several high-end applications such as random number generators and cryptography for secure communications [54].

Along with the research on the application of memristive circuits, new mathematical and numerical tools to simulate them in the static and dynamic domains have to be proposed. Researchers in the chaos area commonly use numerical methods to path tracking the chaotic trajectories. However, numerical methods strictly provide the quantitative behaviour of the circuit, while in some cases, it is also necessary to know their qualitative behaviour. Nevertheless some other methods provides a purely qualitative solution, a method capable to provide all the information, qualitative and quantitative, of the chaotic trajectories of memristive circuits, based on the Multistage Homotopy Perturbation Method (MuHPM) [55], is proposed. This methodology is capable to deliver a valid semi-analytic expression in the vicinity of specific time instants to iteratively track the chaotic trajectories. A comparison of this method against Runge-Kutta is realized to determine their accuracy.

This article has been organized as follows.

Section 2 provides an introduction to the Multistage HPM Method;

while Section 3 shows how to track the path of a chaotic memristor circuit using the proposed method.

In Section 4, numerical simulations are provided and results will be thoroughly discussed.

Finally, a brief conclusion is given in Section 5.

## 2 Introduction to Multistage HPM Method

Homotopy Perturbation Methods (HPM) [58, 59, 66, 65, 62, 67, 63, 64, 57, 56, 60, 61, 68, 69, 70, 71] are mathematical tools to find an approximate or exact solution of a related problem. Such methods are based on both, perturbation [72, 73] and homotopy methods [85, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 86, 87, 88, 89, 90] to provide a set of non-linear differential equations in a systematic and straightforward way. Thereby, the characteristics of Multistage HPM makes this numerical method ideal to track the trajectories of a chaotic circuit. Thus, in this section the basics of Multistage HPM will be explored.

### 2.1 The Homotopy Perturbation Method

The basic idea of HPM is to introduce an homotopy parameter  $p$  which takes values ranging from 0 up to 1. When parameter  $p = 0$ , the equation usually reduces to a simple, or trivial, equation to solve. Then,  $p$  is gradually increased to 1, producing a sequence of deformations where every solution is closer to the last one. Eventually, at  $p = 1$ , the system takes the original form of the equation and the final stage of deformation provides the desired solution. Therefore, only few iterations are needed to achieve good accuracy.

The HPM method considers that a nonlinear differential equation can be expressed as

$$A(u) - f(r) = 0, \quad r \in \Omega, \quad (2.1)$$

having the boundary condition

$$B\left(u, \frac{\partial u}{\partial \eta}\right) = 0, \quad r \in \Gamma, \quad (2.2)$$

where  $A$  is a general differential operator,  $f(r)$  is a known analytic function,  $B$  is a boundary operator, and  $\Gamma$  is the boundary of the  $\Omega$  domain. The  $A$  operator, generally, can be divided into two operators,  $L$  and  $N$ , where  $L$  is the linear operator and  $N$  is the nonlinear operator. Hence, (2.1) can be rewritten as

$$L(u) + N(u) - f(r) = 0. \tag{2.3}$$

Now, the homotopy function is

$$H(v, p) = (1 - p)[L(v) - L(u_0)] + p(L(v) + N(v) - f(r)) = 0, \quad p \in [0, 1] \tag{2.4}$$

where  $u_0$  is the initial approximation solution for (2.3) which satisfies the boundary conditions and  $p$  is known as the perturbation homotopy parameter. Analysing (2.4) can be concluded that

$$H(v, 0) = L(v) - L(u_0) = 0, \tag{2.5}$$

$$H(v, 1) = L(v) + N(v) - f(r) = 0. \tag{2.6}$$

We assume that the solution for (2.4) can be written as a power series of  $p$

$$v = p^0 v_0 + p^1 v_1 + p^2 v_2 + \dots \tag{2.7}$$

Adjusting  $p = 1$  results that the approximate solution for (2.1) is

$$u = \lim_{p \rightarrow 1} v = v_0 + v_1 + v_2 + \dots \tag{2.8}$$

The series (2.8) is convergent on most cases [58, 59, 91, 92].

## 2.2 Basic Procedure of Multistage HPM method

The basic MuHPM [55] process consists of

1. Setup.  $M = (t_f - t_0)/\Delta t$ .  $M$  is defined as the number of path tracking steps or the number of HPM segments,  $t_0$  is considered as the initial time,  $t_f$  the final time, and  $\Delta t$  is the step size of time for path tracking.
2. Apply HPM method to obtain an approximate solution  $y(t)$  to the nonlinear differential equation.
3. Set  $k = 0$ .
4.  $t^* = t_0 + k\Delta t$  and initial condition  $y(t^*) = y_k$  are updated in HPM solution.
5. A prediction is performed for  $t_{k+1} = t_0 + (k + 1)\Delta t$ . That is,  $y_{k+1} = y(t_{k+1})$ .
6. Update  $k = k + 1$ .
7. Repeat steps 4, 5, and 6 until  $t^* > t_f$  or  $k > M$ .

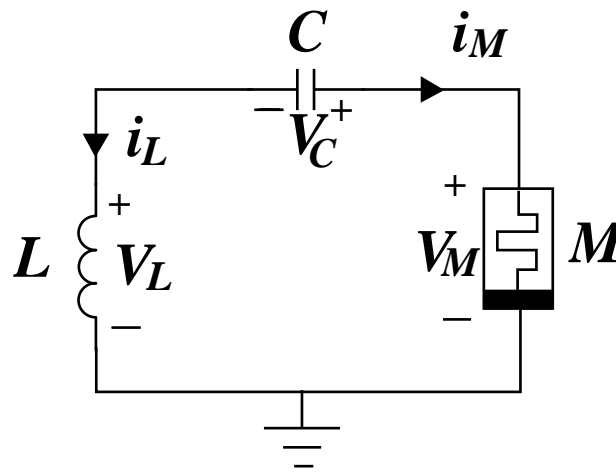


Figure 1: Chaotic circuit containing one Memristor.

### 3 Tracking the Path of Chaotic Memristor Circuits

In order to show the effectiveness of the proposed HPM method, the chaotic circuit in [51] is used as an example of the application of the proposed method to trace the chaotic paths of a memristive circuit. The circuit is composed only by three elements: two energy storage elements (an inductor and a capacitor) and a non-linear active Memristor, as shown in Figure 1. In accordance with [51], the Memristor behaviour can be modelled by

$$\begin{aligned} V_M &= \beta(z^2 - 1)i_M, \\ \frac{dz}{dt} &= i_M - \alpha - i_M z, \end{aligned} \quad (3.9)$$

where  $V_M$  is the potential drop at the Memristor,  $i_M$  represents the current flowing through the Memristor,  $\alpha$  and  $\beta$  are parameters of the device, and  $z(t)$  is the internal state of the memristive system.

The differential equation system representing the dynamic behaviour of the chaotic circuit is derived follows: the first equation is obtained from the capacitor current formulation described by

$$i_C = C \frac{dV_C}{dt}, \quad (3.10)$$

The second equation is obtained applying Kirchoff's voltage law around the closed LCM loop as

$$\begin{aligned} -V_L - V_C + V_M &= 0, \\ -L \frac{di_L}{dt} - V_C + \beta(z^2 - 1)i_M &= 0, \end{aligned} \quad (3.11)$$

Finally, a third equation is established for the variable  $z$  given by (3.9); in order to set the system

$$\begin{aligned} \frac{dx}{dt} &= \frac{y}{C}, & x(0) &= r_1, \\ \frac{dy}{dt} &= -\frac{1}{L} [x + \beta(z^2 - 1)y], & y(0) &= r_2, \\ \frac{dz}{dt} &= -y - \alpha z + yz, & z(0) &= r_3, \end{aligned} \quad (3.12)$$

where, for a simpler notation,  $x = V_C(t)$  is voltage across capacitor  $C$ ,  $y = i_M(t) = -i_L(t)$  is current through the loop,  $\alpha = 0.6$ ,  $\beta = 3/2$ ,  $L = 3$ , and  $C = 1$ .

The homotopy equation (HPM) is formulated as

$$\begin{aligned}
 (1-p) \left( \frac{dv_1}{dt} - \frac{dx_0}{dt} \right) + p \left( \frac{dv_1}{dt} - v_2 \right) &= 0, \\
 (1-p) \left( \frac{dv_2}{dt} - \frac{dy_0}{dt} \right) + p \left( \frac{dv_2}{dt} + \frac{1}{3} \left[ v_1 + \frac{3}{2} (v_3^2 - 1) v_2 \right] \right) &= 0, \\
 (1-p) \left( \frac{dv_3}{dt} - \frac{dz_0}{dt} \right) + p \left( \frac{dv_3}{dt} + v_2 + 0.6v_3 - v_2v_3 \right) &= 0,
 \end{aligned} \tag{3.13}$$

where

$$\begin{aligned}
 v_{1,0}(t) = x_0(t) = x(0) &= r_1, \\
 v_{2,0}(t) = y_0(t) = y(0) &= r_2, \\
 v_{3,0}(t) = z_0(t) = z(0) &= r_3,
 \end{aligned} \tag{3.14}$$

in order to fulfil initial conditions from (3.12).

According to the HPM method and (2.7), every variable in (3.13) can be approximated as

$$\begin{aligned}
 v_1 &= v_{1,0} + pv_{1,1} + p^2v_{1,2} + p^3v_{1,3} + \dots, \\
 v_2 &= v_{2,0} + pv_{2,1} + p^2v_{2,2} + p^3v_{2,3} + \dots, \\
 v_3 &= v_{3,0} + pv_{3,1} + p^2v_{3,2} + p^3v_{3,3} + \dots.
 \end{aligned} \tag{3.15}$$

Substituting (3.15) into (3.13) and arranging coefficients with  $p$  powers, we construct and solve the following system having 12 equations and 12 unknowns

$$\begin{aligned}
 \frac{dv_{11}}{dt} - r_2 = 0, v_{11}(t^*) = 0, \frac{dv_{12}}{dt} - v_{21} = 0, v_{12}(t^*) = 0, \\
 \frac{dv_{13}}{dt} - v_{22} = 0, v_{13}(t^*) = 0, \frac{dv_{14}}{dt} - v_{23} = 0, v_{14}(t^*) = 0, \\
 \frac{dv_{21}}{dt} + \frac{1}{2}r_3^2r_2 + \frac{1}{3}r_1 - \frac{1}{2}r_2 = 0, v_{21}(t^*) = 0, \\
 \frac{dv_{22}}{dt} + r_3v_{31}r_2 + \frac{1}{2}r_3^2v_{21} - \frac{1}{2}v_{21} + \frac{1}{3}v_{11} = 0, v_{22}(t^*) = 0, \\
 \frac{dv_{23}}{dt} + r_3v_{32}r_2 + \frac{1}{3}v_{12} + \frac{1}{2}r_3^2v_{22} + r_3v_{31}v_{21} - \frac{1}{2}v_{22} \\
 + \frac{1}{2}v_{31}^2r_2 = 0, v_{23}(t^*) = 0, \\
 \frac{dv_{24}}{dt} + \frac{1}{3}v_{13} + v_{31}v_{32}r_2 + r_3v_{32}v_{21} + \frac{1}{2}r_3^2v_{23} + \frac{1}{2}v_{31}^2v_{21} - \frac{1}{2}v_{23} + r_3v_{33}r_2 \\
 + r_3v_{31}v_{22} = 0, v_{24}(t^*) = 0, \\
 \frac{dv_{31}}{dt} + 0.6r_3 + r_2 - r_2r_3 = 0, v_{31}(t^*) = 0, \\
 \frac{dv_{32}}{dt} - r_2v_{31} - v_{21}r_3 + .6v_{31} + v_{21} = 0, v_{32}(t^*) = 0, \\
 \frac{dv_{33}}{dt} - r_2v_{32} + 0.6v_{32} - v_{21}v_{31} + v_{22} - v_{22}r_3 = 0, v_{33}(t^*) = 0, \\
 \frac{dv_{34}}{dt} - r_2v_{33} - v_{21}v_{32} + v_{23} + 0.6v_{33} - v_{23}r_3 - v_{22}v_{31} = 0, v_{34}(t^*) = 0.
 \end{aligned}
 \tag{3.16}$$

Following the MuHPM method, the step size is set at  $\Delta t = 0.1$ ,  $t_i = 0$  as initial point, and final tracing point as  $t_f = 100$  and  $M = 1000$ . Next, the first three iterations for the MuHPM method are shown

1. First, we solve (3.16) to obtain the following four order approximation (see equation 3.15)

$$\begin{aligned}
 x(t) &= \lim_{p \rightarrow 1} v_1(t) = \lim_{p \rightarrow 1} \left( \sum_{k=0}^4 p^k v_{1,k}(t) \right), \\
 y(t) &= \lim_{p \rightarrow 1} v_2(t) = \lim_{p \rightarrow 1} \left( \sum_{k=0}^4 p^k v_{2,k}(t) \right), \\
 z(t) &= \lim_{p \rightarrow 1} v_3(t) = \lim_{p \rightarrow 1} \left( \sum_{k=0}^4 p^k v_{3,k}(t) \right).
 \end{aligned}
 \tag{3.17}$$

Expression (3.17) is too cumbersome to be written here, nonetheless, we will show the particular solutions for each segment of MuHPM.

2. For  $k = 0$ . We set  $t^* = 0$ ,  $r_1 = 0.1$ ,  $r_2 = 0.1$ , and  $r_3 = 0$  in (3.17), to obtain segment 0

$$\begin{aligned}
 x_{s0}(t) &= 0.1 + 0.1t + 0.008333333333t^2 - 0.004166666666t^3 \\
 &\quad - 0.000793981481t^4, \\
 y_{s0}(t) &= 0.1 + 0.0166666666667t - 0.0125t^2 - 0.00317592592t^3 \\
 &\quad - 0.000028935185t^4, \\
 z_{s0}(t) &= -0.1t + 0.0166666666667t^2 + 0.0008333333333t^3 \\
 &\quad + 0.00107175925926t^4,
 \end{aligned}
 \tag{3.18}$$

Evaluating (3.18) at  $t = 0.1$ , we obtain the following prediction

$x_{s_0}(0.1) = 0.110079087269$ ,  $y_{s_0}(0.1) = 0.101538487847$ , and  $z_{s_0}(0.1) = -0.009832392824$ . Besides, (3.18) would be the result of applying the standard HPM method [58] to solve (3.12).

3. For  $k = 1$ . We set  $t^* = 0.1$ ,  $r_1 = x_{s_0}(0.1)$ ,  $r_2 = y_{s_0}(0.1)$ , and  $r_3 = z_{s_0}(0.1)$  in (3.17), in order to obtain segment 1

$$\begin{aligned} x_{s_1}(t) &= .100000000111 + 0.09999999951t + 0.008333326655t^2 \\ &\quad - 0.004166543972t^3 - 0.000795168818t^4, \\ y_{s_1}(t) &= 0.016666631327t + .100000000063 - 0.012499302163t^2 \\ &\quad - 0.003182873641t^3 + 0.549591616344E-5t^4, \\ z_{s_1}(t) &= -8.0420E-11 - 0.0999999582585t + 0.01666584134t^2 \\ &\quad + 0.000841521281t^3 + 0.001031474990t^4, \end{aligned} \tag{3.19}$$

Evaluating (3.19) at  $t = 0.2$ , the next prediction is

$x_{s_1}(0.2) = 0.120298728458$ ,  $y_{s_1}(0.2) = 0.102807900043$ , and  $z_{s_1}(0.2) = -0.019324975548$ .

4. For  $k = 2$ . We set  $t^* = 0.2$ ,  $r_1 = x_{s_1}(0.2)$ ,  $r_2 = y_{s_1}(0.2)$ , and  $r_3 = z_{s_1}(0.2)$ . Updating (3.17), we obtain the approximate solution for segment 2. Furthermore, the process is repeated until  $t^* > t_f$  or  $k > M$ .

The initial condition and predictions generated by the MuHPM process traces the trajectory for the differential equation in (3.12); which successfully describes the memristive chaotic circuit.

#### 4 Numerical Simulation and Discussion

As explained in previous sections, MuHPM is a numerical-analytical method that divides the chaotic behaviour into a series of segments to obtain a set of analytical approximations (one by segment) that, as a whole, serve to track the path of the non-linear differential equation of the memristive circuit ((3.18) and (3.19)).

Figures 2(a), 2(b), and 2(c) show how MuHPM accurately trace the curves  $x(t)$ ,  $y(t)$ , and  $z(t)$ . In order to provide a reference point, the obtained results were compared to the numerical solution obtained using the Fehlberg fourth-fifth order Runge-Kutta method with degree four interpolant (RKF45) [93, 94] built-in routine from Maple 13 Software. The routine was configured using an absolute error of  $10^{-7}$  and a relative error of  $10^{-6}$ . According to the results depicted in Figures 3(a) and 3(b), can be seen that the chaotic behaviour of the circuit in Fig. 1 is accurately calculated by the MuHPM method.

Tables 1, 2 and 3 show the dynamic behaviour of variables  $x(t)$ ,  $y(t)$ , and  $z(t)$  respectively to show, quantitatively, a comparison of the MuHPM, the basic HPM (3.18) and the RKF45 methods. From the tables can be noticed that the HPM method rapidly loses accuracy after  $t = 3s$  while MuHPM remains accurate up to  $t = 100s$  (see the relative error column), using as reference the results from applying the RKF45 method. The relative error (RE) increases gradually for all three variables, starting with an error of zero at  $t = 0$  until reaching their maximum value at  $t = 100s$ , that is,  $RE[x(100)] = -0.0185651$ ,  $RE[y(100)] = -0.000446$ , and  $RE[z(100)] = -0.000169$ . This increasing tendency of the error can be diminished by reducing the step size  $t$  or by improving the resultant approximations from the MuHPM method. In this work, the trial function ( $x_0(t)$ ,  $y_0(t)$ , and  $z_0(t)$ ) of the proposed HPM method was based in constants calculated from initial conditions and afterwards by predictive steps. Nevertheless, it is possible to propose a trial function more efficient, giving as result an increase of the path tracking range; this option will be addressed in a future work.

Finally, for a later work, we will improve the methodology in order to significantly decrease the number of segments required to complete the path tracking; this will lead to increase the qualitative understanding about how the circuital parameters affect the chaotic path for longer time segments.

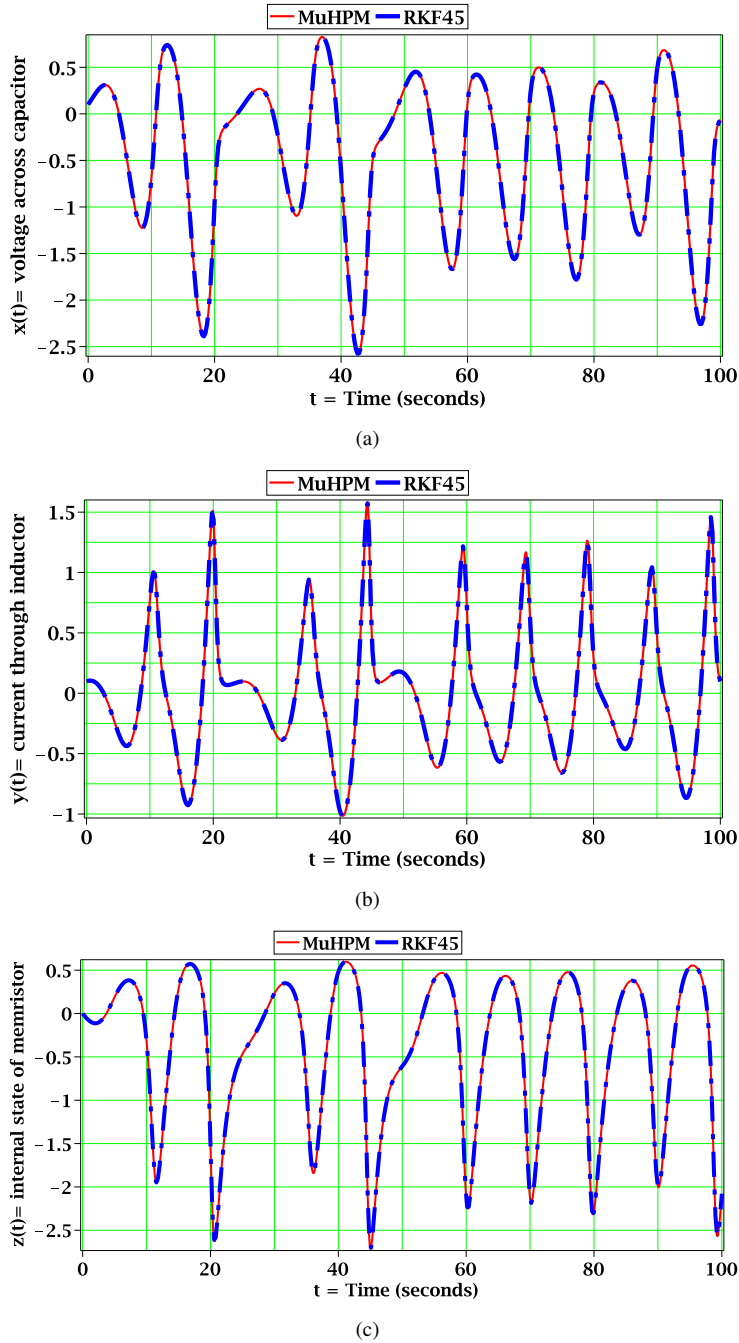
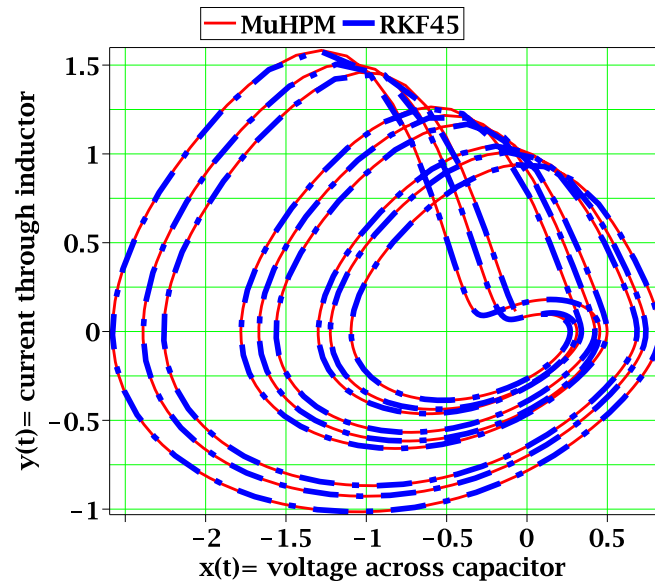
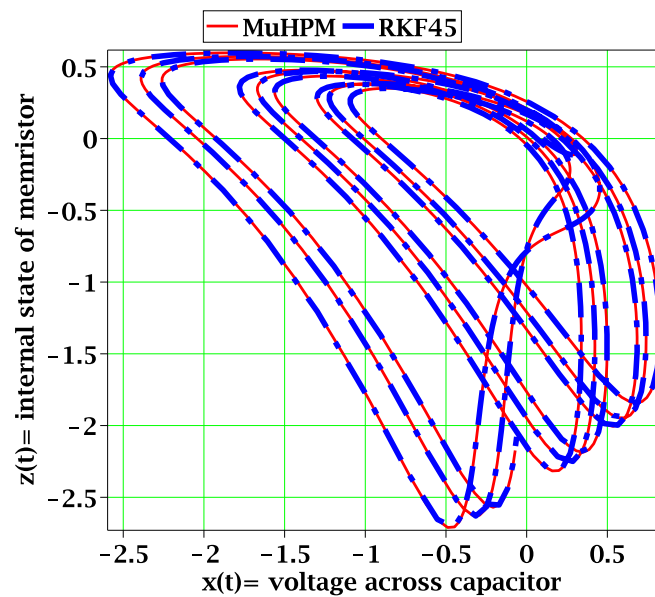


Figure 2: Path tracking for variables a)  $x(t)$ , b)  $y(t)$ , and c)  $z(t)$  from the differential equation of the chaotic memristive circuit. Time, in seconds, is given in the range of  $[0, 100]$ .





(a) Path tracking for  $x(t)-y(t)$



(b) Path tracking for  $x(t)-z(t)$

Figure 3: Paths for the chaotic circuit containing Memristor calculated using MuHPM (continuous line) and RKF45 (dashed line). Time, in seconds, is given in the range of  $[0, 100]$ .

$t$	MuHPM	HPM	RKF45	Relative Error
0.00*	0.100000000000	0.100000000000	0.100000000000	0.000000000000
0.10	0.110079087269	0.110079087269	0.110079086421	0.000000007695
0.20	0.120298728458	0.120298729630	0.120298724155	0.000000035772
0.40	0.141046327958	0.141046340741	0.141046323725	0.000000030010
0.80	0.182875982722	0.182874785185	0.182875974611	0.000000044355
1.00	0.203378837869	0.203372685185	0.203378823527	0.000000070520
3.00	0.305718760880	0.298187500000	0.305718783609	-0.000000074346
4.00	0.206504194694	0.163407407407	0.206504289111	-0.000000457219
5.00	-0.043266596321	-0.2087384259	-0.043266350794	0.000005674747
8.00	-1.170592436579	-3.9521481481	-1.170592466989	-0.000000025978
10.00	-0.695419650446	-10.173148148	-0.695422443741	-0.000004016704
20.00	-0.945002521428	-154.93703703	-0.945017919602	-0.000016294320
30.00	-0.233265692384	-745.02500000	-0.233272243142	-0.000028082816
50.00	0.269375020008	-5457.2842592	0.269392158892	-0.000063624621
70.00	0.254343941220	-20444.7287037	0.254406289432	-0.000245133467
100.00	-0.060559066640	-83471.381481	-0.061683355359	-0.018565159291

Table 1: Numerical path tracking (3.12) for voltage across capacitor  $x(t)$  by using MuHPM, HPM (3.18), and RKF45. The relative error is calculated from MuHPM and RKF45. Time is given in seconds. \*At  $t=0$ , the error is expressed as absolute error.

$t$	MuHPM	HPM	RKF45	Relative Error
0.00*	0.100000000000	0.100000000000	0.100000000000	0.000000000000
0.10	0.101538487847	0.101538487847	0.101538491360	-0.000000034598
0.20	0.102807900043	0.10280787963	0.102807906516	-0.000000062954
0.40	0.104463374390	0.104462666667	0.104463377498	-0.000000029758
0.80	0.103718813856	0.103695407407	0.103718825364	-0.000000110953
1.00	0.101034080285	0.100961805556	0.101034084188	-0.000000038636
3.00	-0.032886721969	-0.05059375000	-0.032886661610	0.000001835387
4.00	-0.171681208941	-0.24400000000	-0.171681107541	0.000000590629
5.00	-0.325990251363	-0.544241898148	-0.325990172567	0.000000241712
8.00	-0.191676695554	-2.31125925926	-0.191677058287	-0.000001892422
10.00	0.771986844312	-4.44861111111	0.771983020391	0.000004953350
20.00	1.476340347896	-34.6037037037	1.476468413169	-0.000086745088
30.00	-0.350692776287	-119.83750000	-0.350681308519	0.000032700325
50.00	0.168182847308	-608.152314815	0.168196427098	-0.000080744202
70.00	0.595184038800	-1844.05972222	0.595278584883	-0.000158851846
100.00	0.097351950300	-6192.67777778	0.097391339729	-0.000404608528

Table 2: Numerical path tracking (3.12) for current through inductor  $y(t)$  by using MuHPM, HPM (3.18), and RKF45. The relative error is calculated from MuHPM and RKF45. Time is given in seconds. \*At  $t=0$ , the error is expressed as absolute error.

$t$	MuHPM	HPM	RKF45	Relative Error
0.00*	0.000000000000	0.000000000000	0.000000000000	0.000000000000
0.10	-0.009832392824	-0.009832392824	-0.009832394654	-0.000000186141
0.20	-0.019324975548	-0.019324951852	-0.019324974914	0.000000032822
0.40	-0.037253399522	-0.037252562963	-0.037253403134	-0.000000096965
0.80	-0.068496653970	-0.068467674074	-0.068496667893	-0.000000203266
1.00	-0.081520266961	-0.081428240741	-0.081520269905	-0.000000036117
3.00	-0.073204466845	-0.040687500000	-0.073204521431	-0.000000745665
4.00	0.039945585312	0.194370370370	0.039945531733	0.000001341289
5.00	0.190476126786	0.690682870370	0.190476108878	0.000000094017
8.00	0.351138007655	5.083259259259	0.351138071694	-0.000000182375
10.00	-0.321206117603	12.217592592593	-0.321203789045	0.000007249421
20.00	-1.521808805572	182.8148148148	-1.521699428035	0.000071873377
30.00	0.231529303107	902.6250000000	0.231526168244	0.000013539810
50.00	-0.608498144889	6839.328703703	-0.608546966734	-0.000080233350
70.00	-2.135554762700	26093.43981481	-2.135410008407	0.000067782993
100.00	-2.078796951000	108165.9259259	-2.079149484448	-0.000169585321

Table 3: Numerical path tracking (3.12) for internal state of Memristor  $z(t)$  by using MuHPM, HPM (3.18), and RKF45. The relative error is calculated from MuHPM and RKF45. Time is given in seconds. \*At  $t=0$ , the error is expressed as absolute error.

## 5 Conclusions

In this work, a methodology to accurately predict the dynamic behaviour of chaotic memristive circuits using the multistage perturbation method was proposed. As shown in the previous analyses, the proposed methodology provides a semi-analytical solution which is easily manipulable to obtain both, the quantitative and qualitative information of the dynamics of the memristive circuit. According to a comparison against similar methodologies, the proposed solution has proven to be highly accurate along the simulation time ( $t \in [0, 100]$ ). While after this range the accuracy begins to decrease, future work aims to increase the precision within a wider range of time by changing the homotopy map and by augmenting the order of the HPM approximation.

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