Candidates, Credibility, and Re-election Incentives

Richard Van Weelden*

March 9, 2011

Abstract

I analyze a model of repeated elections in which a representative voter selects among candidates with known policy preferences in each period. In addition to having preferences over policy, the elected candidate would like to use their position to secure rents at the voter's expense. I restrict attention to equilibria which are consistent with retrospective voting in which the candidates' strategies are stationary. I consider the policies different candidates would implement and the amount of rent-seeking they would engage in if elected, and, consequently, which candidates will be elected by the voter. I show that, when utilities are concave over policy choices, the voter will not want to delegate to candidates who share their policy-preferences. The voter is better off delegating to candidates with preferences on opposite sides of their own, and using these candidates to discipline each other. The resulting policy divergence decreases the amount of rent-seeking by elected candidates, resulting in a better outcome for the voter. I show that the best equilibrium, from the voter's perspective, involves two candidates, symmetric about the median, who are elected at some history of the equilibrium. The policies the candidates pursue in office do not converge but are more moderate than the candidates' own preferences.

Keywords: political competition; repeated elections; endogenous candidates; divergent platforms; rent-seeking.

---

*Max Weber Fellow, European University Institute and Assistant Professor of Economics, University of Chicago. Phone: +39-346-8354-093. Email: richard.vanweelden@gmail.com. This article is based on Chapter 1 of my Dissertation from Yale University. I am grateful to Dirk Bergemann, Justin Fox, Dino Gerardi, Johannes Hörner and Larry Samuelson for the extremely helpful advice they gave me. I would also like to thank Evrim Aydin, Marco Battiglini, Alessandro Bonatti, Rahul Deb, Eduardo Faingold, Mikhail Golosov, Anne Guan, Fuhito Kojima, Alessando Lizzeri, Lucas Maestri, Eric Maskin, Massimo Morelli, Stephen Morris, Ben Polak, Herakles Polemarchakis, Arturas Rozenas, Kareen Rozen, Konstantin Sonin, Francesco Squintani, Filippo Taddei, Emanuele Tarantino, Juuso Välimäki, Lin Zhang and various seminar audiences for helpful comments. Financial support from the Cowles Foundation, the Carl Arvid Anderson Fellowship, the Max Weber Programme, and Professor Larry Samuelson was greatly appreciated when pursuing this work.

1 Introduction

In elections voters choose between two (or more) competing platforms and candidates. A defining feature of American elections is that there are usually two candidates – one Republican and one Democrat – with a realistic chance of winning election. In addition, in contrast to the median voter theorem, the platforms that the two candidates run on do not converge. In fact, the candidates not only run on different platforms but represent different interests and ideologies. In this paper I provide a novel explanation for these features of elections: namely that polarization of candidate preferences creates greater incentives for candidates to act in the voters' interests when in office in order to secure re-election. Consequently, voters will seek out candidates with preferences away from the median as such candidates will engage in less rent-seeking behavior when in office; when utilities are concave with respect to policies, the resulting decrease in rent seeking will more than compensate voters for the policy being pulled away from the center.

Since the seminal results of Downs (1957)[1] a large literature has emerged to explain the lack of convergence in candidate platforms.[2] These models typically make two key assumptions: the parties are exogenously specified and the parties can make binding commitments to implement any platform. As such, the more fundamental question of why parties represent the interests they do,[3] is not addressed. On the other end of the spectrum, citizen candidate models (Osborne and Slivinski 1996, Besley and Coate 1997), which endogenize the candidates' decision of whether or not to run for office, assume that candidates cannot make binding commitments, and so will always implement their ideal policy if elected. The only decision a potential candidate makes in these models, then, is whether or not to run for office. There is then a large gap, in the existing theories of political competition, between the polar opposite assumptions of perfect commitment with exogenous parties and endogenous parties with no commitment. In this paper, I develop a model in which both the candidates and the platforms are endogenously determined.

Neither the assumption that candidates can make binding commitments to any platform or that candidates will always implement their most-preferred platform if elected is particularly natural: elected officials have considerable discretion over which policies to pursue when in office, but their behavior is surely influenced by the desire to be re-elected. As Persson and Tabellini (2000) describe it, "It is thus somewhat schitzophrenic to study either extreme: where promises have no meaning or where they are all that matter. To bridge the two models is an important challenge." If the set of platforms that candidates would be willing to implement if elected – and, hence, could credibly promise to the voters – is to emerge endogenously, the candidate must have an incentive

[1] See also Hotelling (1929) and Black (1958).

[2] See Palfrey (1984), Calvert (1985), Roemer (1994) among others.

[3] As opposed, for example, to having a median voter party. In the original Downsian framework such considerations did not apply since parties were motivated purely by the desire to hold office, an assumption later authors relaxed. See Wittman (1983) for an overview.

to implement platforms other than their ideal point when in office in order to influence future elections; that is, a platform is credible if the candidate would be willing to implement it to secure re-election. This requires a model of repeated elections. In the existing literature on repeated elections, however, all candidates are identical (e.g. Ferejohn 1986) or the voters know nothing about the candidate's preferences before electing them (e.g. Duggan 2000), and, if the incumbent is not re-elected, the replacement is drawn at random from the population of candidates. As such, while this literature provides interesting insights about how re-election pressures affect the policy choices of incumbent candidates, it cannot address the question of which candidates will be elected in the first place.

In this paper I study the impact of re-election incentives on political competition. I consider an environment in which the preferences of the candidates are known to the voter prior to the election, and the elected candidate will come up for re-election after implementing the platform for that period. I can then look at the set of platforms each type of candidate would be willing to implement to secure re-election. This not only provides a theory of the credibility of campaign promises, but, once we know which platforms a candidate would be willing to implement, we can consider which candidates would be willing to implement more desirable platforms than other candidates. This, in turn, determines who the voter should elect. The key insight is that candidates who have preferences different from the voter are willing to implement more desirable platforms, since the voter can punish such a candidate for a deviation without punishing themselves. As the voter is indifferent between policies equally spaced from her ideal point – but a candidate with non-median preferences is not indifferent between those policies – the voter can discipline a non-median candidate with the credible threat of electing a candidate who will implement a policy on the opposite side of the median, thereby motivating the candidate to engage in less rent-seeking behavior when in office.

I show that the best equilibrium involves two, and only two, candidates, and that these candidates implement different platforms if elected. This provides, to my knowledge, the first result in the literature with endogenous candidates, diverging platforms, and an ability for the candidates to implement a platform other than their ideal point.[4]

I consider a setting in which a (representative) voter chooses from a continuum of potential candidates with known ideal points in every period. Further, I assume that there is some opportunity for shirking, such as corruption or lack of effort, on the part of the elected candidate. I model the rents that the elected candidate could secure at the voter's expense as a monetary transfer from the electorate to the candidate. Since the candidate will come up for re-election, it is possible for the voter to induce the candidate to engage in less than full rent-seeking by making their re-election

[4]Fedderson et al. (1990) allows for endogenous candidates, motivated only by holding office, who can make binding commitments, and shows that the candidates locate at the median in the pure strategy equilibrium. Roemer (2001) specifies the objective of the party to be that of the average (or median) member, assuming that all citizens are part of one of the two parties. In addition there are several papers (e.g. Palfrey (1984), Callander (2005)) with two exogenously specified candidates but the possibility of candidate entry.

conditional on their behavior in office. If the voter always elects a candidate with the median policy preferences, then all that a deviating candidate can be punished with is the lack of future rents, so, in equilibrium, the amount of rent-seeking must be strictly positive. If utilities are concave with respect to the implemented policy, however, candidates with a non-median ideal point would be willing to implement a more desirable platform. Such a candidate could promise to implement a policy that is slightly in the direction of their ideal point while engaging in significantly less rent-seeking than a median candidate would engage in. If they deviate, the voter would elect a candidate on the other side of the median and allow them to implement a policy slightly in the direction of their ideal point. Since utility functions are concave over policy, the difference between the two policies, even if they are very close to the voter's ideal point, have a significant disciplining effect on candidates with preferences different than the voter; the disutility to the voter from a non-median policy is then more than off-set by the decrease in rent-seeking behavior. Consequently, there are non-median candidates who have an incentive to implement more desirable platforms than a median candidate would. Further, I show that there are exactly two types of candidates, symmetric about the median, who are willing to implement more desirable platforms than any other candidates.

This framework then provides many interesting insights. First, it provides a theory of the credibility of campaign promises between the polar opposite assumptions of full commitment in the Hotelling/Downs framework and the no commitment assumption in citizen candidate models.[5] Second, I provide a model of endogenous candidates with a richer set of strategies than whether or not to run for office. I derive endogenous candidates with non-median policies and, more fundamentally, non-median preferences. While the policies the candidates implement in office do not converge, they are more moderate than the candidates would implement in the absence of electoral concerns. The baseline model also produces an extremely strong incumbency advantage, as the incumbent is always re-elected. I show, however, that when the platform implemented is imperfectly observed by the voter, the incumbent will be defeated with positive probability.

In the extensions I show that the results continue to hold when some of the assumptions of the baseline model are relaxed. First, I show that the results go through with short-lived, or alternatively term-limited, candidates, provided the candidates care about the policy implemented after leaving office. Similarly, I replace the representative voter with a continuum of voters with heterogeneous preferences, and derive analogous results to the representative voter case. I then extend the baseline model to allow for imperfect monitoring, so the voter cannot observe perfectly the platform the candidate implemented while in office, and show that the incumbent is then re-elected with probability greater than $\frac{1}{2}$ but less than 1. Finally, I endogenize candidate entry, extending the stage game to allow candidates to decide whether to run. In equilibrium, candidates who have no possibility of winning the election will not run for office.

While this model and many of its implications are novel, it, of course, incorporates many existing

[5] Banks (1990) and Callander and Wilke (2007) also provide an intermediate degree of credibility by assuming that lying is costly for the candidates.

ideas in the literature. That repeated elections can induce politicians to implement policies different from their ideal point in order to be re-elected is well known. Barro (1973) and Ferejohn (1986) consider how voters can use re-election incentives to discipline homogenous candidates in a model of moral hazard. Alesina (1988) considers the moderating effects of re-election incentives in a model of repeated elections between ideologically motivated, and exogenously specified, candidates who have preferences on the left and right respectively. Duggan (2000) shows that there exists a unique equilibrium that is consistent with symmetry, candidate-stationarity, retrospective voting and prospective voting in an infinitely repeated elections model where the candidates' preferences are private information. Aragones et al. (2007) considers a model of repeated elections with exogenously given candidates, and shows that there exists an equilibrium, using trigger strategies, in which the candidates announce polices different from their ideal point and keep their promises. Bernhardt et al. (2009) shows that politicians will be willing to implement more moderate policies to secure re-election if they know that their replacement will have preferences from the other side of political spectrum, and Bernhardt et al. (2011) extends this model of repeated elections to allow for heterogeneity in candidates' competence or valence. Unlike the other papers in this literature, I allow voters to choose among candidates with known, and heterogenous, policy preferences, and so endogenize the candidates. Further, while in the above papers, all else equal, more moderate candidates are electorally advantaged, I find that voters may actually prefer candidates who hold non-median policy positions to those who are more centrist.

My results also relate to the literature on the possible benefits of biased agents. Felli and Merlo (2006, 2007) consider a model of endogenous lobbying, where, due to the concavity of utility functions, candidates bargain with lobby groups furthest from their ideal point as those lobby groups have the greatest incentive to contribute in order to influence the candidates' policies. Che and Kartik (2009) show that a decision maker can benefit from consulting an expert with different beliefs than herself as such an expert will have greater incentive to acquire information to convince the decision maker to change her beliefs. Dewatripont and Tirole (1999) consider the advantages of having information acquired by "advocates" rather than an unbiased expert. Finally, Caillaud and Tirole (1999) model the internal organization of political parties, and show that an "ideological" party may be preferred to a centrist one, since, though all candidates ultimately locate at the center, a candidate for an ideological party is induced to exert more effort in designing a high-quality platform, due the credible threat of the rank and file breaking with the candidate if they do not.

The paper proceeds as follows: In Section 2 I describe the model; Section 3 provides examples of subgame perfect equilibria which highlight the benefits of ideological competition, and motivate the results to come; Section 4 presents the results for the baseline model; Section 5 extends the baseline model, and Section 6 concludes. The proofs of the results for the baseline model are in the Appendix.

2 Model

I assume there is a single "representative" voter, and normalize this voter's type to be 0. I allow this voter to choose from a continuum of candidates of known types, $\kappa \in [-1, 1]$, in every period. I further assume, to highlight the fact that the results depend on the heterogeneity of the candidates' preferences as opposed to the additional competition from additional candidates, that there are an infinite number of candidates of each type.[6] Formally, each candidate can be indexed as

$$k = (\kappa, i) \in [-1, 1] \times \mathbb{N},$$

where κ represents the type and i indexes which type-κ candidate k is.

In every period the voter chooses one candidate to elect.[7] The elected candidate chooses which policy to implement from a one-dimensional policy space, $x \in [-1, 1]$. In addition, there is an opportunity for the elected candidate to secure rents, $m \in [0, M]$, for themselves at the voter's expense. These rents could reflect outright corruption, such as taking bribes, or simply a lack of effort – either taking time off work or not addressing difficult but important issues – by the candidate when in office. For example, the candidate may receive some benefit from holding office but this benefit is eroded if the candidate works excessively long hours, denies favors to family and friends, or takes actions which antagonize powerful lobby groups or vocal minorities. I assume that voters and candidates have utility functions which are concave over the implemented policy. To simplify the analysis I assume utilities are linear in the amount of rents secured, though I discuss the case where utilities are concave over rents in Section 4.3. I will refer to a policy-rents pair, $p = (x, m)$, as a platform.

The utility to the voter, if platform $p = (x, m)$ is implemented in the stage game, is

$$u(p) = -|x|^{\lambda} - m,$$

where $\lambda > 1$ reflects the degree of concavity. That utility functions are strictly concave with respect to policy will be the key assumption driving the results.[8] The elected candidate also has preferences over the policy implemented but benefits from securing rents for themselves. So the utility to the

[6] In particular this guarantees that the voter always has the option of selecting a different candidate with the same policy preferences as themselves in every period.

[7] I do not allow the candidates to communicate with the voter before the election. Since candidates' types are common knowledge, adding (non-binding) communication would add additional complexity to the model without providing any new insights. I discuss the possibility of communication in Section 4.3

[8] The assumption that utility functions are concave is one of the most widely made assumptions in economics. While utility functions being concave over income is uncontroversial, and strict concavity of utility functions plays a key role in many models of elections (e.g. Ledyard 1984, Snyder and Ting 2002, Callander and Wilke 2007), it is less clear that utility functions are, in fact, concave over policy (see Osborne 1995 for a discussion). One possible interpretation of concavity is that, if there are two issues, if the solution to to the first issue isn't to an individual's liking, the individual will get more upset if the other issue goes against them than if they were happy with the resolution to the first issue. Such an assumption seems likely to hold for most voters in most situations.

candidate in office if platform $p = (x, m)$ is implemented is

$$\bar{g}_\kappa(p) = -|\kappa - x|^\lambda + \gamma m,$$

if their type is κ. I assume that $\gamma \geq 1$ so the elected candidate cares about the amount of rents secured at least as much as the voter and candidates who are not in office. Although not necessary for the main results, this reduces the number of cases to consider. Moreover, because the rents accrue to the elected candidate while the costs of rent-seeking are distributed across the electorate, this is a natural range of parameters to focus on.

If the candidate is not in office their utility is identical to the voter's utility, except they have ideal point κ. The candidate's utility is then

$$g_\kappa(p) = -|\kappa - x|^\lambda - m.$$

In particular, $g_0(p) = u(p)$. In specifying the preferences of the candidates out of office to be the same as the voters, I am following in the citizen-candidate tradition (Osborne and Slivinski 1996, Besley and Coate 1997). As candidates in my model take no action except when elected by the voter, what is important for the results is that an elected candidate has preferences over the policy implemented when in office, and also cares about the policy implemented after leaving office. While it seems natural that candidates should care about the policy implemented, the candidate would also have reason to care about the policy implemented after leaving office even if we take a more cynical view of candidate motives: if the candidate cares about policy only to the extent that it creates a "legacy" for themselves, this legacy is surely eroded if the successor immediately reverses the implemented policy.[9]

Notice also that, in this specification of preferences, candidates do not value being in office for its own sake but as a means to an end: being elected allows the candidate to implement a policy more to their liking and to secure rents for themselves. The results go through if candidates derive some intrinsic benefit, B, from holding office, provided this benefit is not so large that candidates would be willing to forgo all rent-seeking in order to secure re-election, in which case the the problem of motivating candidates to act in the voter's interest when in office would be trivial.[10]

The timing of the game is as follows:

1. In each period the voter elects a candidate, k_t.
2. The elected candidate implements platform $p_t = (x_t, m_t)$ for that period.
3. The voter observes the p_t implemented.

[9]Some evidence that public officeholders care who succeeds them comes from the timing of judicial retirements: the literature (e.g. Hagle 1993, Spriggs and Walbeck 1995, Hansford et al. 2010, though there are dissenting findings (e.g. Yoon 2006)) shows that judges often time their retirement so that the same party which appointed them can appoint their successor, making it more likely their replacement will share their judicial philosophy.

[10]Specifically, the results go through provided $B < \frac{1-\delta}{\delta}\gamma M$. This guarantees that there does not exist a subgame perfect equilibrium in which, at every history, all type-0 candidates implement the voter's most preferred platform $p = (0, 0)$.

4. The game is repeated with the voter deciding whether to re-elect the candidate. If the incumbent candidate is not re-elected the voter decides which candidate to elect instead.

I assume that the game is infinitely repeated with discount factor $\delta < 1$. Since each period in this model corresponds to an election cycle, we would expect δ to be significantly below 1, and we will be concerned with more than just the limiting properties as $\delta \to 1$.

In each period the voter chooses a candidate, k_t, and the elected candidate chooses platform $p_t = (x_t, m_t)$. A history then consists of the previously elected candidates and the platforms they implemented when in office. That is, a history is

$$h^t = ((k_0, p_0), \ldots, (k_{t-1}, p_{t-1})).$$

Define $h_k^t = (h^t, k)$ to be the history in which candidate k is elected after history h^t, H^t and H_k^t to be the set of all t period histories h^t and h_k^t respectively, and $H = \cup_{t \geq 0} H^t$ and $H_k = \cup_{t \geq 0} H_k^t$. A strategy for the voter is to elect one candidate at every history,

$$\begin{aligned} s_v &: H \to [-1, 1] \times \mathbb{N}, \\ s_v(h^t) &= k_t. \end{aligned}$$

In addition, I define $s_v|_{h^t}$ to be the voter's strategy in the subgame beginning at history h^t. A strategy for each candidate consists of the platform they would implement, if elected by the voter, at every history,

$$\begin{aligned} s_k &: H_k \to [-1, 1] \times [0, M], \\ s_k(h_k^t) &= p_t. \end{aligned}$$

For any profile of strategies, s, we can then calculate the payoffs in the repeated game for each player. The payoff for the voter is

$$U(s) = (1 - \delta) \sum_{t=0}^{\infty} \delta^t u(p_t),$$

and for each candidate k the payoff is

$$U_k(s) = (1 - \delta) \sum_{t=0}^{\infty} \delta^t u_k(k_t, p_t),$$

where

$$u_k(k_t, p_t) = \begin{cases} \bar{g}_\kappa(p_t) & \text{if } k = k_t, \\ g_\kappa(p_t) & \text{if } k \neq k_t. \end{cases}$$

For simplicity I restrict attention to subgame perfect equilibria in pure strategies. In addition, since we are concerned not only with the equilibrium platforms, but also with the candidates elected, I introduce the notion of *credible* candidates. These are the candidates who the voter would elect at some history – either on or off the equilibrium path – in a given equilibrium, s^*.

Definition 1 ***Credible Candidates***
A type, κ, of candidate is credible in subgame perfect equilibrium s^* if a candidate of type κ is elected at some history under strategies s^*.

Before proceeding with the analysis I pause to discuss some of the main elements of the model. The most striking feature is that I have specified a single representative voter, but a continuum of candidates. The assumption of a single voter not only simplifies the analysis, but also biases against finding divergent platforms, since there is no notion of different constituencies that the candidates could be appealing to.[11] In this model policy divergence can appear even when there is no disagreement among the voters (or moderate voters have mass 1).[12] In the extensions, I consider a model with a continuum of voters who have heterogeneous preferences. Because the specified preferences are single-peaked, I show that the existence of a representative voter is equivalent to requiring that the voters choose a Condorcet winner in every period. The assumption of a continuum of candidates, while highly stylized, reflects the idea that there are many potential candidates out there who desire to hold office, so, if voters wanted to elect a candidate with a certain set of preferences, such a candidate could be found. In the extensions I make this explicit by treating this as the set of potential candidates who make the decision of whether or not to run for office.

Finally, as the game is infinitely repeated, candidates in this model are not term-limited and so can be re-elected forever. While there are many offices, such as the Presidency, which are term limited, there are many others which are not. There is no limit on the number of terms which can be served in the U.S. House or Senate, or, in some states, as Governor.[13] In the extensions I consider the case where the candidates are short-lived or, alternatively, term limited, and show that the results go through if the candidates seek to see candidates of the same type elected in future periods.

3 Examples

In this section I present examples to highlight the benefit of delegating to candidates with heterogenous preferences. As this is a repeated game it will, of course, admit many subgame perfect equilibria. Many of these equilibria, however, do not incorporate the benefits of competition between the candidates. In this section I show that the candidate stationary equilibrium with converging candidates and platforms provides lower utility to the voter than equilibria with divergence. I then

[11] See, for example, Glaeser et al. (2005) and Virag (2008).

[12] It is often argued that parties represent relatively extreme policies while the voters themselves are more moderate. See Fiorina (2006): "[P]olarized political ideologues are a distinct minority of the American population. For the most part Americans continue to be ambivalent, moderate, and pragmatic, in contrast to the cocksure extremists and ideologues who dominate our political life." (Preface to 2nd Edition)

[13] There are no term-limits for the governors of Connecticut, Idaho, Illinois, Iowa, Massachusetts, Minnesota, New Hampshire, New York, North Dakota, Texas, Vermont, Washington, or Wisconsin.

calculate the amount of divergence resulting in the equilibrium which generates the highest payoff to the voter – I refer to this as the *Divergent Platforms Equilibrium.*

Notice that, if the game were not repeated, all candidates would implement their ideal policy with maximal rent-seeking, M, if elected; the voter would then elect a candidate of type 0. This would, of course, form a subgame perfect equilibrium if played at every history. This equilibrium does not, however, take advantage of the competition between the candidates. If the voter held the candidates to a more rigorous standard for re-election the candidates would be willing to implement more desirable platforms. In what follows, I restrict attention to equilibria which satisfy retrospective voting – so the incumbent is re-elected if and only if the platform they implemented gives the voter sufficient utility – and platform stationarity – so the platform the candidate implements is independent of the history at which they are elected. I will make these notions precise in the next section.

I begin by considering equilibria in which the voter always elects candidates who share their policy-preferences, $\kappa = 0$, who then implement the median policy, $x = 0$. For the voter to be willing to condition their re-election decision on the candidate's behavior in office they must be indifferent between re-electing the incumbent and kicking them out of office. Hence, the level of rent-seeking will not be affected by whether or not the incumbent is re-elected. We can then calculate the minimum amount of rent-seeking that can be supported from the equation

$$\bar{g}_0(0, m_0) = (1 - \delta)\gamma M + \delta g_0(0, m_0).$$

That is, we solve for the level of rent-seeking, m_0, which makes a type-0 candidate indifferent between being elected and securing rents m_0 in every period (which gives the candidate utility $\bar{g}_0(0, m_0)$ in every period) and securing maximal rents today with some other candidate engaging in that level of rent-seeking in all future periods (which gives the candidate utility γM in the first period, and $g_0(0, m_0)$ in all future periods). It is straightforward to show that the minimal level of rent-seeking is then $(1 - \delta)\gamma M/(\gamma + \delta)$. So, if the voter always delegates to candidates who share their policy preferences, the following is the best such equilibrium for the voter.

Median Candidates Equilibrium

The following strategies constitute a subgame perfect equilibrium:

The voter elects a candidate of type-0 at every history. If the voter elected a candidate of type-0 in the previous period the voter will re-elect that candidate if and only if the platform they implemented gave the voter utility at least $u(0, \frac{(1-\delta)\gamma M}{\gamma+\delta})$ in the previous period, and elects a different type-0 candidate otherwise.

Candidates of type $\kappa = 0$ implement their ideal policy while securing rents $(1 - \delta)\gamma M/(\gamma + \delta)$, and all other candidates implement their ideal policy with full rent-seeking.

While this takes advantage of the competition between candidates it does not exploit the advantages from the heterogeneity in the candidates' preferences. Consider a candidate of type $\kappa > 0$, where κ is small, and consider the most desirable platform such a candidate would be willing to implement to secure re-election. Note that the most desirable platform, from the voter's perspective, would trade off policy and rents secured by the candidate in the same ratio for the candidate and the voter. As utilities are linear with respect to rents the ratio of marginal utilities over rent-seeking is a constant, γ. So we must have the ratio of the marginal utilities for the candidate and the voter with respect to policy equal to γ. This implies that the candidate must implement a platform (x, m) which satisfies,

$$\frac{-\frac{\partial \bar{g}_\kappa(x,m)}{\partial x}}{\frac{\partial u(x,m)}{\partial x}} = (\frac{\kappa - x}{x})^{\lambda - 1} = \gamma.$$

Consequently, candidate κ will implement policy

$$x_\kappa = \frac{\kappa}{1 + \gamma^{\frac{1}{\lambda-1}}} = \frac{\gamma^{\frac{1}{1-\lambda}}\kappa}{1 + \gamma^{\frac{1}{1-\lambda}}}.$$

Note that, as the candidate cares about policy, the punishment for not securing re-election is harshest if the candidate who would replace them comes from the opposite side of the political spectrum. I then consider the equilibrium in which the continuation equilibrium from not securing re-election is a candidate of type $-\kappa$ being elected and implementing policy $-\kappa/(1 + \gamma^{\frac{1}{\lambda-1}})$. We can calculate the lowest level of rent-seeking that can be supported from the equation

$$\bar{g}_\kappa(x_\kappa, m_\kappa) = (1 - \delta)\gamma M + \delta g_\kappa(-x_\kappa, m_\kappa).$$

Solving for the level of rent-seeking, m_κ, the voter's utility from candidates of type κ is then,

$$w(\kappa) = -\frac{(1-\delta)\gamma M}{\gamma + \delta} + \frac{1}{\gamma + \delta}(1 + \gamma^{\frac{1}{1-\lambda}})^{-\lambda}[\delta((1 + 2\gamma^{\frac{1}{1-\lambda}})^\lambda - \gamma^{\frac{\lambda}{1-\lambda}}) - (1 + \gamma^{\frac{1}{1-\lambda}})]\kappa^\lambda.$$

The utility to the voter is then increasing in κ whenever $\delta \geq \underline{\delta} \equiv (1 + 2\gamma^{\frac{1}{1-\lambda}})^{1-\lambda}$.[14] Since γ is positive and λ is greater than 1, we have $\underline{\delta} \in (0, 1)$. In the Appendix I calculate the values of $\underline{\delta}$ for different parameters γ and λ. For reasonable parameters there is a benefit to moving away from the center even when δ is much lower than 1.[15]

Delegating to candidates of type $\kappa > 0$ has both advantages and disadvantages to the voter. First, the candidate will implement policy $x_\kappa > 0$ in every period, resulting in a utility loss to the

[14] The minimum δ required for this equation to be increasing in κ is lower than $\underline{\delta}$. By assuming that $\delta \geq \underline{\delta}$ we ensure that, if the continuation policy is $-\kappa/(1 + \gamma^{\frac{1}{1-\lambda}})$ if not re-elected, no candidate whose type is on the interval $(0, \kappa)$ would be willing to implement a more desirable platform than a candidate of type κ. This guarantees that there will only be two *credible* types in the best equilibrium when $\delta \geq \underline{\delta}$.

[15] Note that we could also use the *Median Candidates Equilibrium* to punish the deviating candidate. With sufficient patience the type κ candidate would be willing to implement a platform which gives the voter higher payoff than the *Median Candidates Equilibrium* while using the *Median Candidates Equilibrium* as punishment. This means that the *Median Candidates Equilibrium*, unlike the *Divergent Platforms Equilibrium*, is not robust to a mutually profitable renegotiation between the voter and a candidate out of office. This provides a second justification for considering the *Divergent Platforms Equilibrium*, as it is the only one to satisfy a strong form of retrospective voting and a renegotiation proofness requirement. See Van Weelden (2011a) for details.

voter. Additionally, the elected candidate has greater potential gains from deviating along the policy dimension as the equilibrium policy will not correspond to the candidate's ideal policy, increasing the amount of rents required to prevent the candidate from deviating. However, the candidate also has more to lose by deviating, since if they lose election the policy in the future moves from x_κ to $-x_\kappa$. This increased disciplining effect makes the incumbent willing to decrease the amount of rent-seeking to secure re-election. Because the implemented policy, x_κ, is proportional to κ the advantages and disadvantages are both proportional to κ^λ. So, when $\delta \geq \underline{\delta}$, the welfare of the voter is increasing in κ, and will continue to increase in κ, until either we reach maximum polarization, or the rents secured by the elected candidate are driven to 0.[16] We can then define

$$\bar{\kappa} = \min\{\kappa : \bar{g}_\kappa(x_\kappa, 0) \geq (1-\delta)\gamma M + \delta g_\kappa(-x_\kappa, 0)\} > 0,$$

to be the minimum degree of heterogeneity in candidate preferences necessary for the rents to be driven to 0. When $\bar{\kappa} < 1$, we have that, for all $\kappa \in [\bar{\kappa}, 1]$, the level of rent-seeking will be driven to 0. We must then solve for the most moderate policy that can be supported. I then define $x(\kappa)$ – the policy which makes a type κ candidate indifferent between implementing $x(\kappa)$ in every period without securing any rents, and implementing their ideal point today with full rent-seeking and having $-x(\kappa)$ implemented in all future periods – from the equation

$$\bar{g}_\kappa(x(\kappa), 0) = (1-\delta)\gamma M + \delta g_\kappa(-x(\kappa), 0).$$

I show in the Appendix that $x(\kappa)$ has a unique minimizer on $[\bar{\kappa}, 1]$, which I characterize. Define the degree of divergence in candidate preferences as

$$\kappa^* = \begin{cases} \arg\min_{\kappa \in [\bar{\kappa}, 1]} x(\kappa) & \text{if } \bar{\kappa} < 1, \\ 1 & \text{otherwise.} \end{cases} \tag{1}$$

Similarly, define the divergence in implemented policies,

$$x^* = \begin{cases} x(\kappa^*) & \text{if } \bar{\kappa} < 1, \\ \frac{1}{1+\gamma^{\frac{1}{\lambda-1}}} & \text{otherwise.} \end{cases} \tag{2}$$

Now define the amount of rent-seeking supported in equilibrium,

$$m^* = \begin{cases} 0 & \text{if } \bar{\kappa} < 1, \\ \frac{1}{\gamma+\delta}\{(1-\delta)\gamma M - \frac{\delta(1+2\gamma^{\frac{1}{1-\lambda}})^\lambda - 1}{(1+\gamma^{\frac{1}{1-\lambda}})^\lambda}\} & \text{otherwise.} \end{cases} \tag{3}$$

Finally, let

$$u^* = u(x^*, m^*) = -(x^*)^\lambda - m^*, \tag{4}$$

be the utility to the voter when the policy implemented is x^* and the elected candidate secures rents m^*. This determines the optimal divergence in platforms and candidate preferences.

[16] That it will be optimal to drive rents to 0 is due to the functional form assumption that rents enter linearly. I relax this assumption in Section 4.3.

Divergent Platforms Equilibrium
Suppose $\delta \geq \underline{\delta}$. The following strategies constitute a subgame perfect equilibrium:
At time-0 the voter elects a candidate of type κ^*. The incumbent candidate is re-elected if and only if the platform the candidate implements gives utility at least u^* and the candidate is of type κ^* or $-\kappa^*$. If the incumbent is of type $\kappa^*(-\kappa^*)$ and the utility is less than u^* the voter elects candidate of type $-\kappa^*(\kappa^*)$ in the next period. At any history where the incumbent is type $\kappa \notin \{\kappa^*, -\kappa^*\}$ the voter elects candidate of type κ^* in the next period.

Candidates of type κ^* choose platform (x^*, m^*), candidates of type $-\kappa^*$ choose platform $(-x^*, m^*)$, and all other candidates choose their ideal points while securing full rents in any period the voter elects them.

The *Divergent Platforms Equilibrium* then highlights the advantage of having candidates with heterogeneous preferences. When $\delta \geq \underline{\delta}$ the voter receives strictly higher payoff in this equilibrium than in the *Median Candidates Equilibrium.* Note that in this equilibrium there are two *credible* candidates who implement non-converging platforms if elected, with this polarization benefiting the voter. In the next section I show that, within the class of platform-stationary subgame perfect equilibria with retrospective voting, this equilibrium is the best one for the voter.

4 Results

4.1 Efficiency

As is standard in the literature (e.g. Duggan 2000, Bernhardt et al. 2009) I restrict attention to equilibria in which the candidates' strategies are stationary. My first restriction is that the platform the candidate implements is independent of the history at which they are elected.

Definition 2 ***Platform-Stationarity***
A subgame perfect equilibrium satisfies platform-stationarity if, for all candidates k, the platform they implement if elected does not depend on the history at which they are elected. That is, $s_k^*(h_k^t)$ is a constant function.

Aside from simplifying the analysis, platform-stationary equilibria have many attractive features. First, they reflect an environment in which the candidates (i.e. parties) have platforms that are stable over time – something observed in advanced democracies. It also rules out equilibria in which the voter induces good behavior by the candidate with the threat of extremely bad outcomes that harm both the voter and the candidate in future periods. I discuss non-stationary equilibria in Van Weelden (2011a).[17]

[17]Without restricting to platform stationary equilibria, with reasonable patience, it is possible to support any feasible payoff for the voter in a subgame perfect equilibrium, including equilibria in which the voter's most preferred platform is implemented in every period along the equilibrium path. Even without considering stationary equilibria,

In addition, I restrict attention to equilibria in which the voter votes retrospectively. Intuitively, retrospective voting means that the incumbent is re-elected if and only if they provide the voter with some threshold utility level. Retrospective voting is an important concept in political science (see Fiorina 1981), as it provides a simple strategy that closely approximates how voters are believed to behave. Achens and Bartels (2004b) summarize the theory of retrospective voting as follows: "The best current defense of democracy is the theory of retrospective voting. Citizens may not know much about the issues, the argument goes, but they can tell good from bad outcomes, and that allows them to remove incompetent or corrupt incumbents. Moreover, knowing that the voters use that rule, every government will have every incentive to do what they want, thus fulfilling the promise of democratic theory." Because of this, retrospective voting is heavily studied in the theoretical literature (e.g. Barro 1973, Ferejohn 1986) and it is standard in the repeated elections literature to restrict attention to this class of equilibria (e.g. Duggan 2000, Bernhardt et al. 2009).

One subtlety is that the simplest definition – the incumbent is re-elected at any history provided the platform they implemented in the previous period gives the voter utility at least $\bar{u}$ – could specify non-equilibrium behavior off the equilibrium path. At an off-path history in which an "extreme" type, who would always implement an undesirable platform if elected, were to be elected and implement a platform which gives the voter a high utility, it would not be equilibrium behavior for the voter to re-elect this candidate.[18] I then define retrospective voting for the candidate who is supposed to be elected at that history.

Definition 3 ***Retrospective Voting (R)***
A subgame perfect equilibrium, s^*, satisfies retrospective voting (R), if there exists a feasible utility level $\bar{u}$ such that, for all histories h^t, if $k_t = s_v^*(h^t)$, then the incumbent candidate is re-elected (that is $s_v^*(h^t, (k_t, p_t)) = k_t$) if and only if

$$u(p_t) \geq \bar{u}.$$

Further, for any platforms p_t, p_t' with $\mathbb{I}(u(p_t) \geq \bar{u}) = \mathbb{I}(u(p_t') \geq \bar{u})$),

$$s_v^*|_{(h^t,(k_t,p_t))} = s_v^*|_{(h^t,(k_t,p_t'))}.$$

That is, the candidate is re-elected if and only if they provide the voter with some utility standard, with the voter electing a different candidate otherwise. The voter evaluates the performance

---

however, there is still an advantage to delegating to non-median candidates. There is a range of δ under which it is possible to support the voter's most preferred platform in all periods by electing a non-median candidate along the equilibrium path, but there is no equilibrium in which the voter's most preferred policy can be supported by electing median candidates along the equilibrium path. See Van Weelden (2011a) for details.

[18] For example, consider the case where $\delta = 0$ and so any candidate must implement their ideal point while securing full rents at every history in any subgame perfect equilibrium, so the voter must elect a candidate of type $\kappa = 0$ at every history. Now, suppose we are at a history in which a candidate of type $\kappa \neq 0$ had implemented a policy which gives the voter sufficiently high utility in the previous period. The voter's strategy cannot call for them to re-elect the candidate at such a history. In papers such as Duggan (2000) and Bernhardt et al. (2009) this does not arise because the candidate type is private information.

of the candidate on whether or not this threshold utility level is achieved – the candidate is re-elected if, and only if, the utility of the voter in the last period was at least $\bar{u}$. The last line of the definition says that the voter conditions their behavior in future periods only on whether or not the candidate meets that standard, so who the voter will elect in future periods is not affected by how much the candidate exceeds or falls short of the utility threshold. Retrospective voting (R) rules out equilibria in which the voter re-elects the candidate after some platform but rejects them after a platform which gives the voter higher utility. The main result of this section is that the *Divergent Platforms Equilibrium* is the best equilibrium to satisfy platform stationarity and retrospective voting (R).[19]

Recall the definitions of κ^*, x^*, m^*, and u^* from equations (1) – (4) in the construction of the *Divergent Platforms Equilibrium.* I consider the strategy profile, s^*_D, with divergence, to be the platform-stationary subgame perfect equilibrium in which:

(a) The voter elects a candidate of type $\kappa \in \{-\kappa^*, \kappa^*\}$ at every history.

(b) For all histories h^t, if $k_t = s^*_v(h^t)$ they are re-elected ($s^*_v((h^t, (k_t, p_t))) = k_t$) if and only if $u(p_t) \geq u^*$; if $u(p_t) < u^*$ then if k_t is type κ^* ($-\kappa^*$) the voter elects a candidate of type $-\kappa^*$ (respectively κ^*) at history $(h^t, (k_t, p_t))$.

(c) The strategy for at least one candidate of type κ^* is to implement platform (x^*, m^*) at any history in which they are elected; the strategy for at least one candidate of type $-\kappa^*$ is to implement platform $(-x^*, m^*)$ at any history in which they are elected.

(d) Candidates of type $\kappa \notin \{-\kappa^*, \kappa^*\}$ implement a platform that gives the voter payoff strictly less than u^* if elected.

Notice that this strategy profile is unique up to the candidate elected in the first period, and the behavior – for both the voter and candidate – after an out of equilibrium election decision. All that is required is that behavior be consistent with equilibrium at these histories. I now state the main result of this section.

[19]Platform stationarity plays a much larger role in the results than retrospective voting. Even outside the class of equilibria consistent with retrospective voting, non-convergence in candidate platforms is still beneficial. Retrospective voting rules out equilibria in which the voter rejects an incumbent for implementing a more desirable platform than one that would have secured re-election (see Van Weelden 2011a). If the amount of rent-seeking is driven to 0, which will happen when $\bar{\kappa} < 1$, there is only one relevant dimension, and there is no other equilibrium which gives the voter higher payoff than the *Divergent Platforms Equilibrium* which is consistent with platform stationarity. I focus on equilibria which satisfy retrospective voting for robustness to cases with positive rent-seeking, and because it describes natural voting behavior. In addition, retrospective voting, when combined with a renegotiation proofness requirement, provides an additional justification of the *Divergent Platform Equilibrium.* See Van Weelden (2011a) for the details.

Theorem 1 ***Efficiency***
If $\delta \geq \underline{\delta}$, then:

(1) there does not exist a subgame perfect equilibrium satisfying retrospective voting (R) and platform-stationarity which gives the voter payoff higher than u^*.

(2) any subgame perfect equilibrium satisfying retrospective voting (R) and platform-stationarity in which the voter achieves payoff u^* must be of the form s_D^*.

So we have that the best equilibrium in this class has only candidates of type κ^* and $-\kappa^*$ elected at any history. In any platform-stationay equilibrium with retrospective voting any candidate of type $\kappa \notin \{-\kappa^*, \kappa^*\}$ would strictly prefer to implement their ideal point while securing full rents in this period, followed by any continuation equilibrium that gives the voter utility u^*, to implementing any platform which gives the voter payoff u^* and securing re-election for themselves. Consequently, if elected, such a candidate must implement a platform which gives the voter utility strictly less than u^*. Recognizing this, at all histories, the voter will have a strict incentive not to elect any candidate of type $\kappa \notin \{-\kappa^*, \kappa^*\}$. So we have the following corollary.

Corollary 1 *The best subgame perfect equilibrium satisfying retrospective voting (R) and platform-stationarity involves exactly two credible types of candidates.*

So the "best" equilibrium involves two types of candidates implementing non-converging platforms. Notice that the punishment for each candidate deviating is that the other candidate is elected: the candidates both punish each other. Voters break the tie between candidates they are indifferent between in order to discipline politicians. This provides a reason for voters to change which party they elect from period to period even in the absence of shifts in preferences of either the party or the voters.[20]

Notice one more attractive feature of the *Divergent Platforms Equilibrium.* Because the candidates' strategies are stationary, the voter must be choosing a static best response at every history. As such, the discount factor of the voter is irrelevant: even if the voter were completely myopic the results would go through unchanged. All that matters for the analysis is that the voter prefers relatively extreme candidates, recognizing that they engage in less rent-seeking behavior when in office. That more centrist candidates also engage in more rent-seeking is an implication of this model which could be tested empirically. Candidates with moderate policy preferences are less sensitive to differences in the policy implemented and so are unwilling to forgo the opportunity to secure rents for themselves in order to be able to influence the policy implemented in the future. This lack of "trust" in candidates with moderate policies has received much attention in the recent

[20]In the baseline model, of course, the candidate always secures re-election for themselves and so the punishment of switching parties is never implemented. I show in the extensions that when the candidate's platform is not perfectly observed this switching will occur with positive probability along the equilibrium path.

literature; it has been proposed that non-median policies can signal character/honesty (Kartik and McAfee 2007, Callander and Wilke 2007) or competence (Carrillo and Castanheira 2008).

This model is not a signaling story, however, and the effect identified here is quite different. While the previous literature has argued that voters may be suspicious of candidates who claim to be moderate, as announcing a moderate policy could also be a sign of dishonesty, I have shown that a centrist voter may rationally prefer a non-centrist candidate to one they know to be moderate, even if the non-centrist candidate is neither more competent nor more honest than the more moderate one. Rather, simply because of the incentives provided by the desire to secure re-election, candidates with preferences less similar to the voter may actually choose policies more to the voter's liking.

4.2 Comparative Statics

I now turn to analyzing the properties of the *Divergent Platforms Equilibrium.* First note that, in order to get non-median candidates, we must have sufficient patience. The greater the concavity, that is the larger λ, the less patience required, as small changes in policy have a greater relative effect. The more candidates benefit from securing rents the more patient candidates must be, as greater differences in policy are needed to induce the elected candidate to forgo rents.

Theorem 2 ***Discounting***

(1) Divergence is more easily achieved the more concave utilities are with respect to policy ($\frac{\partial \underline{\delta}}{\partial \lambda} < 0$), but more difficult to achieve with rent-motivated candidates ($\frac{\partial \underline{\delta}}{\partial \gamma} > 0$).

(2) Concavity drives divergence:

$$\lim_{\lambda \to 1} \underline{\delta} = 1.$$

(3) Divergent platforms can always be supported if utilities are sufficiently concave:

$$\lim_{\lambda \to \infty} \underline{\delta} = 0.$$

This result then means that it is the concavity of utility functions that drives the divergent platforms.[21] Interestingly, in static models, for example in the literature on probabilistic voting (e.g. Ledyard 1984, Kamada and Kojima 2010), concavity often creates additional pressures for convergence. My results show, however, that in a dynamic environment, concavity is the key assumption which generates divergent platforms. Further, regardless of the level of discounting, we get a benefit to divergence provided utilities are sufficiently concave. I now consider how the parameters of the model affect the candidates elected and the platform implemented.

[21]When $\gamma < 1$, divergent platforms can be supported even without $\lambda > 1$. Notice also that while the assumed patience $\delta \geq \underline{\delta}$ is more than is necessary for an equilibrium with divergence to be better than the *Median Candidate Equilibrium* it is straightforward to show that, for any δ, the voter's welfare is decreasing in κ, when λ is sufficiently close to 1.

Theorem 3 ***Properties of Divergent Platforms Equilibrium***
If $\delta \geq \underline{\delta}$, then:

(1) if $\bar{\kappa} > 1$ (i.e. $m^* > 0$), the amount of rent-seeking is increasing in the opportunity to secure rents and decreasing in patience,

$$\frac{\partial m^*}{\partial M} > 0; \qquad \frac{\partial m^*}{\partial \delta} < 0.$$

(2) if $\kappa^* < 1$, then the amount of divergence is increasing in the opportunity for rent-seeking,

$$\frac{\partial \kappa^*}{\partial M} > 0; \qquad \frac{\partial x^*}{\partial M} > 0,$$

and candidates' taste for rents,

$$\frac{\partial \kappa^*}{\partial \gamma} > 0; \qquad \frac{\partial x^*}{\partial \gamma} > 0,$$

but decreasing in patience,

$$\frac{\partial \kappa^*}{\partial \delta} < 0; \qquad \frac{\partial x^*}{\partial \delta} < 0.$$

So we see that when there is more opportunity for the elected candidate to secure rents for themselves (M larger) there will be more rents secured by the candidates – provided the equilibrium rent-seeking is positive. Perhaps more interesting is that greater opportunity for rent-seeking leads to greater polarization of both candidates preferences and policies. Similarly, if the candidates have greater taste for rents, relative to policy, the greater the divergence – though by Theorem 2 more patience is required to support any divergence at all. Unsurprisingly, more patient candidates will engage in less rent-seeking and implement more moderate policies. These policies are, in fact, implemented by more moderate candidates as well.

While κ^* is decreasing in δ, it is bounded away from 0. As $\delta \to 1$ the payoff to the voter will converge to their ideal point, but the preferences of the elected candidates will not converge to the preferences of the voter. That the voter's utility goes to 0 as δ goes to 1 is not surprising as this was true even without heterogeneity in candidate preferences. With divergent platforms, however, the utility of the voter approaches the utility from the voter's ideal point at a much faster rate: the ratio of the utility loss to the voter from the *Divergent Platform Equilibrium* to the utility loss in the *Median Candidates Equilibrium* goes to 0.

Theorem 4 ***Patient Players***

(1) Platforms converge to the voter's ideal point as candidates become patient:

$$\lim_{\delta \to 1} (x^*, m^*) = (0, 0).$$

(2) The candidates elected in equilibrium do not converge:

$$\lim_{\delta \to 1} \kappa^* = \min\{((\lambda - 1)\gamma M)^{\frac{1}{\lambda}}, 1\} > 0.$$

(3) There is faster convergence to the voter's ideal point with divergence than median candidates:

$$\lim_{\delta \to 1} \frac{u(0,0) - u^*}{u(0,0) - u(0, m_0)} = 0.$$

4.3 Discussion

We have identified an equilibrium which incorporates many central features of American elections. There are exactly two types of candidates, who implement non-converging platforms, who are ever elected. While the platforms do not converge the candidates implement more moderate platforms than they would in the absence of re-election incentives.[22] I showed that, among the stationary equilibria with retrospective voting, this is the equilibrium which gives the voter the highest utility. That is, a two party outcome leads to more efficient outcomes. In this equilibrium the voter is choosing the standard for re-election which results in the highest payoff – at least among pure strategy equilibria. This equilibrium then is similar to the one considered by Ferejohn (1986), who considers the optimal standard for re-election when candidates are homogenous. In my paper, the voter's decision is more complicated, as the voter not only needs to determine the standard for re-election but also which candidates are willing to meet it. In Ferejohn's model the threatened punishment of not re-electing the incumbent is always credible, because the candidates are identical so the voter is indifferent between the candidates. Here, there is heterogeneity in the candidates, but the voter must still be indifferent between the candidates who are elected at different histories. This is an equilibrium result, rather than an assumption, in my model however – if the voter strictly preferred to re-elect the incumbent the threat of switching candidates would not be credible and the candidate could engage in full rent-seeking with impunity.[23]

In this model I have abstracted away from campaign communications: the voter simply infers what the candidate will do if elected. As the candidates have no private information, any equilibrium

[22] While the non-convergence of candidate platforms is well-established, comparing the policy a candidate implements to their most preferred policy is more difficult. Still, the literature on legislator behavior (e.g. Levitt 1996) finds that both legislator ideology and their constituent's preferences have a significant effect on the legislator's voting record.

[23] In an equilibrium where the two candidates where not symmetric around the voter, the candidate who implemented the more moderate policy would also have to engage in more rent-seeking. As previously noted, that more moderate candidates would also engage in more rent-seeking, is a testable prediction of this model.

with communication is also an equilibrium without communication. It would be possible, of course, to extend the game to allow candidates to announce the platform they intend to implement before the election takes place; in equilibrium the announcement would only be "believed" if it corresponds to the candidate's equilibrium action. One possible role of communication in this environment is in facilitating a renegotiation by the candidate and the voter. In real-world elections, candidates spend an enormous amount of time (and money) communicating their platforms to voters. In the midst of this communication, it should be possible for candidates to communicate to voters if there is a mutually profitable renegotiation. In this sense, the communication we observe in elections strengthens the rationale for considering renegotiation-proof equilibria. In Van Weelden (2011a) I consider the possibility of renegotiation. I show that the *Divergent Platforms Equilibrium* not only satisfies renegotiation proofness requirements, but is the unique equilibrium to be robust to this possibility.[24] This provides a second justification of the *Divergent Platforms Equilibrium*.

While the baseline model provides many interesting implications there are other implications which seem less reasonable. Under a range of parameters the level of rent-seeking in the best equilibrium will be driven to 0. This is due to the assumption that preferences over the amount of rent-seeking enter the utility functions of the players linearly – if utilities are concave over rents, the amount of rent-seeking will not be driven to 0. However, there is still a benefit to policy divergence. Consider utility functions,

$$\bar{g}_k(p) = -|\kappa - x|^\lambda + \gamma m^{\nu_1},$$
$$g_k(p) = -|\kappa - x|^\lambda - m^{\nu_2},$$
$$u(p) = g_0(p) = -|x|^\lambda - m^{\nu_2}.$$

When $\nu_1 = \nu_2 = 1$ the payoffs are linear in m. Utility functions are concave over rents if $\nu_1 < 1 < \nu_2$. Now, define m_0 to be the minimum level of rents that can be supported without heterogeneity,

$$\bar{g}_0(0, m_0) = (1-\delta)\gamma M + g_0(0, m_0).$$

We can then define γ_0 to be the ratio of marginal utilities in rent-seeking between the candidate and the voter when the level of rent-seeking is m_0,

$$\gamma_0 = \gamma \frac{\nu_1}{\nu_2} m_0^{\nu_1 - \nu_2}.$$

By the previous argument, the utility of the voter would then be increasing in κ (at 0) when

$$\delta \geq (1 + 2\gamma_0^{\frac{1}{1-\lambda}})^{1-\lambda}.$$

Note first that, from the above equation, for any δ, γ, M, ν_1, and ν_2, divergence will always be beneficial when λ is sufficiently large. Next, notice that, as m_0 is increasing in M, divergence is

[24] Specifically it is the only equilibrium to satisfy a stronger version of retrospective voting and to be internally renegotiation proof (Ray 1994).

more easily supported when there are greater opportunities for rent-seeking behavior – in particular, for any parameters, divergence will always be beneficial when M is sufficiently high. However, as m_0 goes to 0 as δ goes to 1, it is no longer always the case that increasing patience will increase the benefits of platform divergence. Still, it is straightforward to show that, when $\lambda - 1 > \nu_2 - \nu_1$, so that utility functions are sufficiently concave over policy relative to rents, divergence is beneficial whenever the players are sufficiently patient.

Finally, note that the level of rent-seeking would never be driven to 0. In any equilibrium satisfying retrospective voting the voter conditions their re-election decision only on the utility they received in the previous period. Therefore, the candidate must implement a platform that trades off policy and rents in the same ratio as the voter. So we must have

$$(\frac{\kappa - x_\kappa}{x_\kappa})^{\lambda-1} = \gamma \frac{\nu_1}{\nu_2} m_\kappa^{\nu_1-\nu_2}.$$

As $\nu_1 < \nu_2$ this guarantees that $m_\kappa > 0$, and so the amount of rent-seeking will be positive.

In addition, although it is true that there is an incumbency advantage in elections (e.g. Ansolabehere and Snyder 2002), the incumbency advantage described in the baseline model is unreasonably strong with the incumbent candidate re-elected forever. This is due to the perfect information assumption. Since we are restricting attention to pure strategies and perfect information it is possible for all players to anticipate what will happen in all subsequent periods. In particular, the candidate knows exactly what they must do to secure re-election. Since the voter would never want to elect a candidate who would fall short of the standard for re-election, the candidate elected in the first period will be re-elected forever along the equilibrium path. If we relax the perfect monitoring assumption, this is no longer true. Even if the candidate does what the voter expects there is still some possibility of a poor outcome resulting. Still, in order to maintain incentives for the candidate to implement desirable platforms, the voter must condition their re-election decision on the outcome in the previous period. Consequently, there will be turnover in equilibrium. I consider this in the extensions.

5 Extensions

In this section I consider four extensions of the baseline model. In the first extension I consider short-lived candidates who can only hold office for one term. This allows me to address the behavior of candidates who are term-limited and so cannot be rewarded with re-election. I then relax the assumption of a representative voter, and consider a model with a continuum of voters who have heterogenous preferences. I show that, under natural conditions about who the voters' would elect, their choices can be represented by those of a single voter. I then return to the baseline model, but allow for imperfect monitoring. As discussed above, this introduces a source of randomness into the model, generating turnover in equilibrium. After introducing imperfect monitoring, I extend the stage game to allow candidates to make the decision of whether or not to run for office. I show that there will then be two candidates who run for office in every period, with both candidates elected with positive probability.

Since the conditions – platform stationarity, retrospective voting – are defined analogously in the new environments I omit the formal definitions. In each section I discuss how the model adapts and then state the main result of the section. The proofs of these results, which closely mimic the proofs for the baseline results, are omitted. As in the baseline model, it is possible to provide an additional justification of this equilibrium based on the possibility of renegiation. All the details can be found in Van Weelden (2011a).

5.1 Short-Lived Players

I now consider what would happen if the candidates only run in one election and cannot be re-elected. This could be either because they live for only one period or they are term limited.[25] While parties are long-lived entities I assume for this section that parties cannot induce the candidate to implement a specified platform once elected.[26] This doesn't necessarily preclude equilibria in which the candidates select platforms other than their ideal point, however, if the platform the candidate implements affects which candidate is elected in the next period, and the candidate cares about the platform implemented in the future. This requires that voters reward and punish candidates with their choice of candidate after the incumbent leaves office. There is at least some evidence that this happens. Brownstein (2007), describes the difficulty the Republicans faced in the 2008 election: "Bush won't be on the ballot in 2008, of course, but throughout American history outgoing presidents have cast a long shadow over the campaign to succeed them. And when the departing president has been as unpopular as Bush is now, his party has usually lost the White House in the next election."

The candidates can then be indexed by (κ, t) where κ denotes the candidate's type and t refers to the period in which they are alive.[27] The strategies and payoffs can be defined precisely as in the baseline model except that, at each history, the voter is restricted to elect a candidate who is alive in that period ($s_v(h^t) \in [-1, 1] \times \{t\}$). I again restrict attention to platform-stationary equilibria, which I define to be equilibria in which all candidates of the same type implement the same platform regardless of the period they are alive. I then adapt the notion of retrospective voting to refer to the voter electing a candidate of the same type in the next period if the incumbent meets the specified utility standard, $\bar{u} = u_s^*$.

When candidates are short-lived, since they cannot receive rents in future periods, the amount

[25] In the most common examples of term limits, most notably the U.S. Presidency, the number of terms is two. There are some examples, such as the Virginia Governorship and Mexican Presidency, in which the candidate can only be elected to one (consecutive) term.

[26] Alesina and Spear (1988) consider a model in which parties can make transfers to candidates after they leave office in order to induce them to take action in the party's interest when in office. Such transfers are not possible in this model, and, I show, not necessary for divergent platforms to be beneficial.

[27] With short-lived candidates there is no reason to have multiple candidates of the same type.

of rent-seeking in equilibrium will be higher with short-lived than long-lived candidates. Notice also that, though the rents from being in office can only be received in one period, policy changes can persist indefinitely. In addition, note that, in order to induce the same persistent change in policy, a short-lived candidate would be willing to decrease the amount of rent-seeking in one period by more than a long-lived candidate would be willing to reduce rent-seeking in this and all future periods. As such, by inducing polarization, the amount of rent-seeking in equilibrium will decrease more quickly with short-lived candidates than long lived ones, and so less patience will be necessary for divergence to be beneficial. So we can define $\underline{\delta}_s$ with $\underline{\delta}_s < \underline{\delta}$. As in the baseline model, define s_D^* to be the platform-stationary subgame perfect equilibrium strategies in which:

(a) At time-0 the voter elects candidate $\{(-\kappa_s^*, 0), (\kappa_s^*, 0)\}$, where $\kappa_s^* > 0$.

(b) If, in period t, the incumbent is (κ_s^*, t) then the voter elects candidate $(\kappa_s^*, t+1)$ in the next period if the platform the candidate implements gives the voter utility at least u_s^*, and candidate $(-\kappa_s^*, t+1)$ otherwise. If the incumbent is $(-\kappa_s^*, t)$ then the voter elects candidate $(-\kappa_s^*, t+1)$ in the next period if the platform the candidate implements gives the voter utility at least u_s^*, and candidate $(\kappa_s^*, t+1)$ otherwise. If the incumbent is type $\kappa \notin \{-\kappa_s^*, \kappa_s^*\}$ then the voter elects a candidate from the set $\{(-\kappa_s^*, t+1), (\kappa_s^*, t+1)\}$ in the next period.

(c) Candidates of type κ_s^* implement platform (x_s^*, m_s^*), candidates of type $-\kappa_s^*$ implement platform $(-x_s^*, m_s^*)$ in any period they are elected.

(d) Candidates of type $\kappa \notin \{-\kappa_s^*, \kappa_s^*\}$ implement a platform which gives the voter payoff strictly lower than u_s^* in any period they are elected.

The above strategy profile is almost identical to the strategies with long-lived candidates. If $\delta \geq \underline{\delta}$ there will be sufficient patience to support divergence with either short or long-lived candidates and the degree of divergence will be the same in either case. However, if rents are not driven to 0, short-lived candidates will engage in more rent-seeking.

Theorem 5 ***Divergent Platforms with Short-Lived Candidates***
For $\delta \geq \underline{\delta}_s$, there is no other subgame perfect equilibrium which satisfies retrospective voting (R_s) and platform-stationarity and gives the voter higher utility than s_D^*.

When candidates are only in office for one term they cannot be rewarded with future rents from being in office, so, in the absence of policy divergence, there is no reason for the candidate not to secure full rents. However, with divergent platforms, they can be rewarded for good behavior with a more desirable policy in future periods. If we interpret candidates with the same preferences as a party, then candidates will implement moderate policies and engage in less rent-seeking to help elect their party's candidate in the future.[28] This would hold even without the possibility of coercion from the party. The value of policy divergence is much greater with short lived candidates than with infinitely lived ones. With no policy divergence the utility to the voter is $-M$ regardless of δ. With divergence, however,

$$\lim_{\delta\to 1} u(x_s^*, m_s^*) = 0,$$

so we have convergence to the voter's ideal point as the candidates become patient.

Another nice feature of the short-lived candidate interpretation is that the equilibrium platforms are not affected if the candidates derive an intrinsic benefit, B, from holding office – even if B gets arbitrarily large. So even if candidates would be willing to promise anything to be elected, the inability to make binding commitments means that the candidates will still implement non-median policies.

5.2 Continuum of Voters

So far I have assumed a single "representative" voter who selects a candidate in every period. In this section I adapt the model to allow for a continuum of voters with ideal points drawn from the interval $[-1, 1]$. A voter of type $v \in [-1, 1]$ derives utility in the stage game,

$$u_v(p) = -|x - v|^\lambda - m.$$

I assume that the preferences of voters are drawn from distribution G on the interval $[-1, 1]$ with $G(0) = \frac{1}{2}$ and $G' > 0$ everywhere. Hence the preferences of the median voter in this model correspond to the preferences of the representative voter in the baseline model. While I analyze voters as distinct from candidates we could allow for all players to be voters and candidates simultaneously as in citizen candidate models (Osborne and Slivinski 1996, Besley and Coate 1997). This paper could then be viewed as a repeated version of these models.

A model with a continuum of voters raises additional issues about how the voters coordinate on specific candidates. With a continuum of voters, the voters cannot cast their vote in order to influence the outcome of elections as no voter is ever pivotal. If the voters are not attempting to influence the outcome of elections it is not clear how they coordinate on one candidate or another. Rather than specify the strategies of each individual voter I instead assume that the voters can elect a candidate at every history. That is, I assume there exists a choice function, ϕ, which maps histories into the candidate elected by the voters at that history. For any choice function, ϕ, and profile of candidate strategies, s, we can then define the payoff to voter v as

$$U_v(\phi, s) = (1 - \delta) \sum_{t=0}^{\infty} \delta^t u_v(p_t),$$

[28] That polarization of parties can provide incentives to short-lived candidates is also considered by Testa (2005). She considers a model in which long-lived parties must delegate to short-lived candidates in every period. Candidates always implement their ideal policy but decide whether to implement it at low cost or at a high cost, securing rents for themselves in the process. She shows that the candidates will only implement the policy at low-cost if there is sufficient polarization between the candidates selected by each party.

with the candidate payoffs defined exactly as in the baseline model. The key to this section is that, because preferences are single peaked, there is no possibility of Condorcet cycles. In the stage game, voters choosing a candidate from the Condorcet win set is then equivalent to a representative voter playing a best response. I define a collective best response as a choice function for which no deviation would make more than half the voters strictly better off.

Definition 4 ***Collective Best Response***
A choice function, ϕ, is a collective best response to the candidates' strategies if, given the candidates' strategies, s, there does not exist a choice function $\phi^{'}$ such that

$$G(\{v : U_v(\phi^{'}, s) > U_v(\phi, s)\}) > \frac{1}{2}.$$

I then define a subgame perfect equilibrium in the game with a continuum of voters exactly as in the baseline model, replacing the requirement that a "representative voter" is playing a best response with the requirement that the voters as a whole are playing a *collective best response* to the candidates' strategies. As in the baseline model, I maintain the assumption that the candidates' strategies are platform-stationary,[29] and the voters can then make their choice at each history without concern for how this will affect play in future periods. Given that preferences are single-peaked, a majority of voters will prefer one stage-game outcome to another if and only if the median voter prefers that outcome. Requiring that the voters are playing a *collective best response* then means that the voters always choose a (weak) Condorcet winner at every history. As in the baseline model, which candidate is elected from this set will depend on the platform implemented in the previous period.

With a continuum of voters, the notion of retrospective voting must now be defined in terms of sets of platforms rather than utility levels, as different voters will receive different utilities from a given platform. I then assume that there exists a set of platforms P, with $\emptyset \neq P \subsetneqq [-1, 1] \times [0, M]$, such that the incumbent is re-elected if and only if they implement a platform in P. I require that this set of platforms, P, be *monotone*. That is, for all platforms $p, p' \in [-1, 1] \times [0, M]$, if $p \in P$ and $G(\{v : u_v(p') \geq u_v(p)\}) \geq \frac{1}{2}$ then $p' \in P$.

That the set of platforms which secure re-election is monotone rules out equilibria in which the candidate would be re-elected if they implement a certain platform but rejected for implementing a platform which gives higher utility to more than half the voters. So, in essence, retrospective voting (R_c) means that the incumbent will be re-elected if and only if a majority of the voters were sufficiently happy with the candidate's performance in the previous period. Again, by single-peakedness, a majority of voters will "approve" of the candidate's behavior if and only if the median voter does.

[29]As preferences are single-peaked when the candidates' strategies are stationary a collective best response must exist.

I consider strategies in which the behavior of the candidates and the choices of the voters are identical to the baseline model. As before I refer to this platform-stationay subgame perfect equilibrium strategy profile as s^*_D:

(a) The voters elect a candidate of type $\kappa \in \{-\kappa^*, \kappa^*\}$ at every history.

(b) For all histories h^t, if the elected candidate k_t is type κ^* $(-\kappa^*)$ they are re-elected ($s^*_v(h^t, (k_t, p_t)) = k_t$) if and only if $p_t \in P^* \equiv \{p : u_0(p) \geq u^*\}$; if $p_t \notin P^*$ the voter selects a candidate of type $-\kappa^*$ (respectively κ^*) at history $(h^t, (k_t, p_t))$.

(c) Candidates of type κ^* implement platform (x^*, m^*) at any history at which they are elected. Candidates of type $-\kappa^*$ implement platform $(-x^*, m^*)$ at any history at which they are elected.

(d) All candidates of type $\kappa \notin \{-\kappa^*, \kappa^*\}$ implement a platform $p_t \notin P^*$ at any history at which they are elected.

I now present the result for the model with a continuum of voters.

Theorem 6 ***Divergent Platforms with a Continuum of Voters***
For $\delta \geq \underline{\delta}$, there is no other subgame perfect equilibrium which satisfies retrospective voting (R_c) and platform-stationarity and gives higher utility to at least half the voters than s^*_D.

So we see that the representative voter can be replaced with a continuum of voters who always choose from the set of Condorcet winners. The voters decide which candidate from this set to elect based on the performance of the candidate in office in the previous period. I do not address how the voters coordinate on this outcome. Notice also that the discussion of welfare is more nuanced with more voters. While a majority of the voters benefit from the polarization of platforms, with a utilitarian welfare function, the welfare effects can be ambiguous. Depending on the distribution of voter preferences, the disutility to voters on the opposite side of the spectrum from the elected candidate can be larger than the utility gains to all voters from the decrease in rent-seeking. Van Weelden (2011b) provides a fuller analysis of the welfare implications of policy divergence with heterogenous voters. In particular, with sufficient patience, divergence is welfare enhancing for any symmetric distribution of voter ideal points.

5.3 Imperfect Monitoring

In the previous analysis I assumed that voters knew the candidates' types before electing them and observed precisely the platform that the elected candidate implemented. The limitation of this is that, with pure strategies and perfect information, there is no source of randomness in the model. The incumbent candidate is then re-elected forever. I now consider what would happen if, instead, the utility the voter received from the platform implemented were not deterministic.

So, while more moderate policies and less rent-seeking by elected candidates lead to higher utility, on average, there are still shocks (such as shocks to the economy) that affect the voter's utility. I assume that the voter does not observe the platform the candidate implemented but only the utility they derived from it. That is, the voter observes

$$u_t = u(p_t) + \varepsilon_t,$$

where ε_t is an i.i.d. draw from some distribution F(y).

Histories and strategies can then be defined as in the baseline model, except that the voter only observes the past utilities and not the policies, and the players seek to maximize expected payoffs. As in the baseline model I restrict attention to equilibria in which the candidates' strategies are independent of history. Since the voter only observes the public history I consider pure strategy public perfect equilibria in which the candidates' strategies satisfy platform-stationarity.

If the candidates are to have an incentive to implement any platform other than their ideal point the voter must still condition their decision on the outcome in the previous period. The notion of retrospective voting is easily adapted – the voter already conditioned their behavior only on the utility level in the baseline model. Retrospective voting remains natural behavior even if voters are imperfectly informed.[30] As Cost (2008) describes it, "The average voter doesn't understand the intricacies of economic policy. Heck, when you think about it, nobody really understands the economy. So, voters often rely on simple yet sensible metrics to make political decisions about the economy. One of them has been more or less operative since the election of 1840: if the economy tanks during a Republican administration, vote Democrat. If it tanks during a Democratic administration, vote Republican. Applying this rule to 2008, we get the following. McCain, because he is of the incumbent party, gets the political harm. Obama, because he is of the out party, gets the political benefit. That's all there is to it."

I now consider the best equilibrium satisfying retrospective voting (R_u) and platform-stationarity. In order to make the analysis tractable, and to parameterize the noise to allow for comparative statics, I assume the noise has the following functional form,

$$Pr(\varepsilon_t \leq y) = F(y) = \begin{cases} 0, & \text{if } y < -\beta, \\ \frac{1}{2}(\frac{\beta+y}{\beta})^2, & \text{if } y \in [-\beta, 0], \\ 1 - \frac{1}{2}(\frac{\beta-y}{\beta})^2, & \text{if } y \in [0, \beta], \\ 1, & \text{if } y > \beta. \end{cases}$$

Therefore, the associated density function is

$$f(y) = F'(y) = \begin{cases} \frac{\beta-|y|}{\beta^2}, & \text{if } y \in [-\beta, \beta], \\ 0, & \text{otherwise.} \end{cases}$$

[30] As the utility to the voter is non-deterministic but the voter conditions their re-election decision on their own utility this means that candidates are rewarded for luck. Political scientists argue that this, in fact, happens. See, for example, Achen and Bartels (2004a, 2004b) and Wolfers (2006).

Notice that ε_t is then a symmetric, mean-0 random variable and that the probability of observing a specific utility level, u_t, given implemented platform, p_t, is decreasing in $|u(p_t) - u_t|$. The parameter β determines the amount of noise in policy-making, with higher β corresponding to a setting in which the voter's utility is more random. That ε_t does not have full support will greatly simplify the analysis since, provided β is not too large, it will be possible to support equilibria in which a candidate who engages in full rent-seeking will be defeated with certainty. In addition, notice that by using a noise without full support I am biasing against generating turnover in equilibrium – if the noise had full support then, regardless of platform implemented and the standard for re-election, the incumbent would be defeated with positive probability.[31]

I begin by considering the best equilibrium involving only type-0 candidates. If there is not too much noise (β not too large) it will be possible to support equilibria with less than full rent-seeking. Notice that in any platform-stationary public perfect equilibrium each candidate who is elected at any history must implement a platform which gives the voter the same utility. As the utility standard for re-election is history independent this implies that the probability of re-election, q, is the same in every period.

Theorem 7 ***Median Candidates Equilibrium with Imperfect Monitoring***
For any $\delta \in (0,1)$, there exists $\beta^*(\delta) > 0$ such that for all $\beta \in (0, \beta^*(\delta))$ there exist public perfect equilibria satisfying platform-stationarity and retrospective voting (R_u) in which only candidates of type $\kappa = 0$ are elected and these candidates engage in rent-seeking $m \in (0, M)$. The incumbent will be re-elected with probability $q \in (\frac{1}{2}, 1)$. Further, the best equilibrium involves level of rents-seeking m^0_β and probability of re-election q^0_β with,

$$\frac{\partial m^0_\beta}{\partial \beta} > 0; \qquad \frac{\partial q^0_\beta}{\partial \beta} < 0,$$

and

$$\lim_{\beta \to 0} m^0_\beta = \frac{(1-\delta)\gamma M}{\gamma + \delta}; \qquad \lim_{\beta \to 0} q^0_\beta = 1.$$

First notice that, as the density goes continuously to 0, for any standard for re-election, the resulting rent-seeking will lead to a positive probability of the incumbent being defeated. In addition, as the rents enter the utility function linearly, there cannot be an equilibrium with less than full rent-seeking in which the incumbent is re-elected with probability less than or equal to $\frac{1}{2}$. When the elected candidate decides how much rent-seeking to engage in they face a trade-off between the benefits of additional rents today against the associated decrease in re-election probability this

[31] When the noise has finite support, if there are signals which would never be observed without a deviation, the punishments supporting the equilibrium could be off the equilibrium path. Here the assumption of a finite support noise is made only to simplify the algebra and the results do not rely on such a construction. In particular, the only punishment the voter has of the candidate – of not re-electing them – will be implemented with positive probability in equilibrium.

induces. If the standard for re-election, $\bar{u}_\beta$, is such that $u(0, m^0_\beta) \leq \bar{u}_\beta$, the gains to the candidate from deviating are linear in the amount of the deviation, but the marginal cost is decreasing in the size of the deviation, since the probability the voter receives a specific utility level u_t in each period is decreasing in $|u(0, m) - u_t|$. Hence, if the candidate cannot profit from a marginal deviation – a necessary condition for the candidate to be optimizing – they would strictly prefer to deviate by a large amount. Hence, in any equilibrium, we must have $u(0, m^0_\beta) > \bar{u}_\beta$, which implies that the incumbent will be re-elected with probability greater than $\frac{1}{2}$. So, because the noise is symmetric and single peaked, there will be an incumbency advantage. As the noise increases, the outcome becomes more random, and the re-election decision is then less responsive to the amount of rent-seeking. Consequently the incumbent will engage in more rent-seeking. As the noise disappears (β goes to 0), the equilibrium converges to the *Median Candidates Equilibrium*, with the incumbent never defeated along the equilibrium path.

I now consider equilibria involving candidates of type $\kappa \neq 0$. That is, I consider the best equilibrium that can be supported using only candidates of type $\kappa \geq 0$ and $-\kappa$. Note that when $\kappa = 0$ we have converging candidates and platforms as in the equilibrium in the previous theorem. I then look for the best κ for the voter. Unlike the baseline model I am assuming that there are at most two types of candidates who are ever elected.

Since the voter only observes the utility level, the implemented platform must trade rents and policy in the same ratio for the candidate and the voter. When κ and β are not too large, because rents enter the payoff linearly, the implemented policies will be the same as in the perfect monitoring case, x_κ and $-x_\kappa$. We can then determine the lowest level of rent-seeking that can be supported with candidates κ and $-\kappa$. When $\delta > \underline{\delta}$ the voter's welfare will be increasing in κ provided the noise is not too large. As in the perfect monitoring case, the welfare of the voter increases until the rents are driven to 0 or we reach maximum polarization. We can then define κ^*_β to be unique degree of polarization that maximizes the voter's utility, with (x^*_β, m^*_β) the associated platform candidate κ^*_β would implement.

I now define the following strategy profile, which I refer to as s^*_D as in the previous sections, to be a platform-stationary subgame perfect equilibrium in which:

(a) The voter only ever elects candidates of type $\kappa \in \{-\kappa^*_\beta, \kappa^*_\beta\}$.

(b) Candidates of type κ^*_β implement platform (x^*_β, m^*_β) and candidates of type $-\kappa^*_\beta$ implement platform $(-x^*_\beta, m^*_\beta)$. All other candidates implement platforms which give the voter utility no higher than u^*_β.

(c) If the incumbent is type κ^*_β (respectively $-\kappa^*_\beta$) the voter re-elects the candidates as long as the voter's payoff in that period is at least $\bar{u}_\beta$ and elects a candidate of type $-\kappa^*_\beta$ (κ^*_β) otherwise.

Letting $q^*_\beta = F(u(x^*_\beta, m^*_\beta) - \bar{u}_\beta)$, the associated probability of re-election, we have the main result of this section.

Theorem 8 ***Divergent Platforms with Imperfect Monitoring***
When $\delta > \underline{\delta}$ there exists $\bar{\beta}(\delta) > 0$ such that, for all $\beta \in (0, \bar{\beta}(\delta))$, the best two-type equilibrium satisfying platform-stationarity and retrospective voting (R_u) is of the form s_D^*. The incumbent will be re-elected with probability $q_\beta^* \in (\frac{1}{2}, 1)$ in all periods. Further:

(1) greater noise results in lower welfare and more turnover,

$$\frac{\partial u_\beta^*}{\partial \beta} < 0; \qquad \frac{\partial q_\beta^*}{\partial \beta} < 0.$$

(2) as the noise disappears the equilibrium converges to the perfect monitoring case,

$$\lim_{\beta \to 0} (x_\beta^*, m_\beta^*) = (x^*, m^*); \quad \lim_{\beta \to 0} \kappa_\beta^* = \kappa^*; \quad \lim_{\beta \to 0} q_\beta^* = 1.$$

So we see that, when there isn't too much noise, the voter still benefits from divergence in platforms and candidate preferences. Among the class of "two-type" equilibria the optimal divergence can be uniquely determined. With positive noise there will be turnover in equilibrium with the incumbent re-elected with probability between $\frac{1}{2}$ and 1. So there is an incumbency advantage but one less extreme than with perfect monitoring. Higher levels of noise lead to worse platforms being implemented and a higher probability of the incumbent being defeated. As the noise disappears the best equilibrium converges to the *Divergent Platforms Equilibrium* from the perfect monitoring case.

5.4 Candidates' Decision to Run for Office

So far, the candidates have not made the decision of whether or not to run for office: the voter could choose any candidate. In this section I allow candidates to decide whether or not to run. In period-0, the voter elects any candidate in $[-1, 1] \times \mathbb{N}$, and in all subsequent periods the voter chooses among the candidates who decided in the previous period to run, with K_t the set of candidates who decided to run for office in period t.

I give the candidates lexiographic preferences: they first seek to maximize payoffs as in the previous analysis, but prefer to be elected to not running, and prefer not running to being defeated. In particular, I assume that two strategies which give the same expected utility are evaluated based on the discounted number of periods being elected less the discounted number of unsuccessful runs for office,

$$\mathbb{E}[(1-\delta) \sum_{t=0}^{\infty} \delta^t (\mathbb{I}(k = k_t) - \mathbb{I}(k \in K_t \setminus \{k_t\}))].$$

I assume that there is positive noise, $\beta > 0$, so that the outcome of future elections will be uncertain. Further, I assume that candidates must make the decision of whether or not to run for office before the realization of the noise is revealed – given the length of the election process in the United States this is a reasonable assumption. This is a repeated extensive form game with the following order

of play in each stage game.
1. The voter elects candidate $k_t \in K_t$, where $K_0 \equiv [-1,1] \times \mathbb{N}$.
2. The elected candidate, k_t, implements platform p_t; all candidates simultaneously decide whether to run for office in the next period. K_{t+1} is the set of candidates who choose to run.
3. $u_t = u(p_t) + \epsilon_t$ is publicly observed.
4. This game is then repeated with the voter deciding which candidate in K_{t+1} to elect.
To rule out the case where no candidate runs for office, assume that if $K_t = \emptyset$ then the voter cannot elect anyone and the payoff to all players is $-\infty$. Histories and strategies are defined as before, except that the voter can only elect candidates who have run for office. I then define a public perfect equilibrium as before. I maintain the assumption that candidates' strategies are platform stationary. That is, although I allow the candidates to condition their decision of whether to run for office on the history, the platform they implement if elected must still be history independent.

As before I restrict attention to equilibria in which the voter votes retrospectively. Of course, the voter can only elect a candidate if they have decided to run for office. I then specify retrospective voting to be that the voter re-elects the incumbent if they meet the utility threshold and run for re-election in the next period. In addition I incorporate into the notion of retrospective voting (R_e) the requirement that voters' choices are not influenced by unsuccessful runs for office by candidates. This rules out situations in which candidates run for office knowing they wont win in order to affect who will be elected, either in this period or in the future.

I now look for two-candidate equilibria as in the previous section – equilibria in which the voter would always elect a candidate of type $\kappa \geq 0$ or $-\kappa$ if such a candidate is available, and in which at least one candidate of type κ and $-\kappa$ runs for office in every period. I further assume that all candidates of the same type would implement the same platform if elected. That is, for any two candidates $k_1 = (\kappa, i)$, and $k_2 = (\kappa, j)$ and any history h^t,

$$s_{k_1}(h^t_{k_1}) = s_{k_2}(h^t_{k_2}).$$

I consider the best equilibrium to satisfy these requirements. I can define the following platform-stationary equilibrium strategy profile, s^*_D:

(a) The voter only ever elects candidates of type $\kappa \in \{-\kappa^*_\beta, \kappa^*_\beta\}$, provided such a candidate is available.

(b) Candidates of type κ^*_β implement platform (x^*_β, m^*_β) and candidates of type $-\kappa^*_\beta$ implement platform $(-x^*_\beta, m^*_\beta)$ if elected.

(c) If the incumbent is type $\kappa \in \{-\kappa^*_\beta, \kappa^*_\beta\}$ they will run for re-election. If the incumbent is type κ^*_β (respectively $-\kappa^*_\beta$) the voter re-elects the candidates as long as their payoff in that period is at least $\bar{u}_\beta$ and elects a candidate of type $-\kappa^*_\beta$ (κ^*_β), if available, otherwise.

(d) At every history, one candidate of type κ^*_β and one candidate of type $-\kappa^*_\beta$ run for office and are both elected with positive probability; if $m^*_\beta > 0$, if the incumbent is defeated in any period they will not run for office again along the equilibrium path. Candidates of type $\kappa \notin \{-\kappa^*_\beta, \kappa^*_\beta\}$ do not run for office at any history.

I now state the main result from this section.

Theorem 9 ***Divergent Platforms with Candidate Entry***
When $\delta > \underline{\delta}$, for $\beta \in (0, \bar{\beta}(\delta))$, the best two-type equilibrium satisfying platform-stationarity and retrospective voting (R_e) is of the form s^*_D.

So we see that when there is positive noise and the candidates make the decision of whether or not to run for office, there will be one candidate of both types who runs for office in each period. The incumbent will be re-elected with probability greater than $\frac{1}{2}$, but the challenger will win election with positive probability as well.

6 Conclusion

In this paper I presented a model of repeated elections in order to consider which platforms candidates would be willing to implement in order to secure re-election. I then showed that electing non-median candidates results in a better outcome for the voter. I derived an equilibrium with two, endogenously determined, candidates, symmetric about the median, who implement non-converging platforms and have non-converging policy-preferences. These predictions are consistent with what is observed in US elections: two candidates with different ideal points running on non-converging platforms that are, in fact, more moderate than the candidates' ideal points. It is also, to my knowledge, the first model to derive all of these implications. Even though candidates never deviate, if there is imperfect monitoring, candidates will not always be re-elected, giving us a reasonable model of incumbency advantage and turnover in elections.

Models of political competition generally make very stark assumptions: candidates can make binding commitments to any platform or will always implement their ideal point if elected; candidates are exogenously specified and compete by choosing platforms or candidates are endogenous but can only implement their ideal point. This paper bridges the gap between these literatures. By studying a model of repeated elections we can consider which platforms candidates would be willing to implement in order to secure re-election, and, in turn, which candidates would be willing to implement the most desirable ones. Therefore, by bridging the gap between full commitment and no commitment we can also bridge the gap between the models of endogenous candidates and strategic platform selection.

Finally, while I have only considered a model of voters and candidates, the key insights could be applied in other settings. For example, when delegating to members of a committee, the principal could delegate to the more extreme members of the committee – in contrast to the "ally principle" – in order to induce more effort. This avenue is left for future research.

7 References

Achen, Christopher and Larry Bartels. 2004a. "Blind Retrospection: Electoral Responses to Drought, Flu, and Shark Attacks." Working Paper.

Achen, Christopher and Larry Bartels. 2004b. "Musical Chairs." Working Paper.

Alesina, Alberto. 1988. "Credibility and Policy Convergence in a Two-Party System with Rational Voters." *American Economic Review*, 78 (4): 796-805.

Alesina, Alberto and Stephen Spear. 1988. "An Overlapping Generations Model of Electoral Competition." *Journal of Public Economics*, 37 (3): 359-379.

Ansolabahere, Stephen and James Snyder. 2002. "The Incumbency Advantage in U.S. Elections: An Analysis of State and Federal Offices 1942-2000." *Election Law Journal: Rules, Politics, and Policy*, 1 (3): 315-338.

Aragones, Enriqueta, Thomas Palfrey, and Andrew Postlewaite. 2007. "Reputation and Rhetoric in Elections." *Journal of the European Economic Association*, 5: 846-884.

Banks, Jeffrey. 1990. "A Model of Electoral Competition with Incomplete Information." *Journal of Economic Theory*, 50 (2): 309-325.

Barro, Robert. 1973. "The Control of Politicians: An Economic Model." *Public Choice*, 14: 19-42.

Bernhardt, Dan, Larissa Campuzano, Odilon Camara, and Francesco Squintani. 2009. "On the Benefits of Party Competition." *Games and Economic Behavior*, 66 (2): 685-707.

Bernhardt, Dan, Odilon Camara, and Francesco Squintani. 2011. "Competence and Ideology." *Review of Economic Studies*, forthcoming.

Besley, Timothy and Steven Coate. 1997. "An Economic Model of Representative Democracy." *Quarterly Journal of Economics*, 112 (1): 85-114.

Black, Duncan. 1958. The Theory of Committees and Elections. Cambridge, Cambridge University Press.

Brownstein, Ronald. 2007. "Bush the Albatros; He's not running in '08, but History Shows His Bad Ratings Can Swamp the GOP." Los Angeles Times, July 18, 2007, A21.

Callander, Steven. 2005. "Electoral Competition in Heterogenous Districts." *Journal of Political Economy*, 113 (5): 1116-1145.

Callander, Steven and Simon Wilkie. 2007. "Lies, Damn Lies, and Political Campaigns." *Games and Economic Behavior*, 60 (2): 262-286.

Calvert, Randall. 1985. "Robustness of the Multidimensional Voting Model: Candidate Motivations, Uncertainty, and Convergence." *American Journal of Political Science*, 29 (1): 69-95.

Carrillo, Juan and Micael Castanheira. 2008. "Information and Strategic Political Polarization." *The Economic Journal*, 118 (6): 845-874.

Caillaud, Bernard and Jean Tirole. 1999. "Party Governance and Ideological Bias." *European Economic Review*, 43, 779-789.

Che, Yeon-Koo and Navin Kartik. 2009. "Opinions as Incentives." *Journal of Political Economy*, 117 (5): 815-860.

Cost, Jay. 2008. "Why no Traction for McCain?" realclearpolitics.com

Dewatripont, Mathias and Jean Tirole. 1999. "Advocates." *Journal of Political Economy*, 107 (1): 1-39.

Downs, Anthony. 1957. An Economic Theory of Democracy. New York, Harper and Row.

Duggan, John. 2000. "Repeated Elections with Asymmetric Information." *Economics and Politics*, 12 (2): 109-135.

Fedderson, Timothy, Itai Sened, and Steven Wright. 1990. "Rational Voting and Candidate Entry under Plurality Rule." *American Journal of Political Science*, 34 (4): 1005-1016.

Felli, Leonardo and Antonio Merlo. 2006. "Endogenous Lobbying." *Journal of the European Economic Association*, 4: 180-215.

Felli, Leonardo and Antonio Merlo. 2007. "If You Cannot Get Your Friends Elected Lobby Your Enemies." *Journal of the European Economic Association*, 5: 624-635.

Ferejohn, John. 1986. "Incumbent Performance and Electoral Control." *Public Choice*, 50: 5-25.

Fiorina, Morris. 1981. Retrospective Voting in American Elections. Yale University Press.

Fiorina, Morris. 2006. Culture War? The Myth of a Polarized America. Pearson Longman.

Glaeser, Edward, Giacomo Ponzetto, and Jesse Shapiro. 2005. "Strategic Extremism: Why Republicans and Democrats Divide on Religious Values." *Quarterly Journal of Economics*, 120 (4): 1283-1330.

Hagle, Timothy. 1993. "Strategic Retirements: A Political Model of Turnover in the United States Supreme Court." *Political Behavior*, 15, 1: 25-48.

Hansford, Thomas, Elisha Savchak, and Donald Songer. 2010. "Politics, Careerism, and the Voluntary Departure of U.S. District Court Judges." *American Politics Research*, 38, 6: 986-1014.

Hotelling, Harold. 1929. "Stability in Competition." *The Economic Journal*, 39 (153): 41-57.

Kamada, Yuichiro and Fuhito Kojima. 2010. "Voter Preferences, Polarization, and Electoral Policies." Working Paper.

Kartik, Navin and Preston McAfee. 2007. "Signaling Character in Electoral Competition." *American Economic Review*, 97 (3): 852-870.

Ledyard, John. 1984. "The Pure Theory of Large Two-Candidate Elections." *Public Choice*, 44 (1): 7-41.

Levitt, Steven. 1996. "How Do Senators Vote? Disentangling the Role of Voter Preferences, Party Affiliation, and Senate Ideology." *American Economic Review*, 86 (3): 425-441.

Osborne, Martin. 1995. "Spatial Models of Political Competition Under Plurality Rule: A Survey of Some Explanations of The Number of Candidates and The Positions They Take." *Canadian Journal of Economics*, 27: 261-301.

Osborne, Martin and Al Slivinski. 1996. "A Model of Political Competition with Citizen Candidates." *Quarterly Journal of Economics*, 111 (1): 65-96.

Palfrey, Thomas. 1984. "Spatial Equilibrium with Entry." *Review of Economic Studies*, 51 (1): 139-156.

Persson, Torsten and Guido Tabellini. 2000. Political Economics: Explaining Economic Policy. MIT Press, Cambridge, MA.

Ray, Debraj. 1994. "Internally Renegotiation-Proof Equilibrium Sets: Limit Behavior with Low Discounting." *Games and Economic Behavior*, 6 (2): 162-177.

Roemer, John. 1994. "A Theory of Policy Differentiation in Single-Issue Electoral Politics." *Social Choice and Welfare*, 11 (4): 355-380.

Roemer, John. 2001. Political Competition. Harvard University Press, Cambridge, MA.

Spriggs, James and Paul Walbeck. 1995. "Calling it Quits: Strategic Retirement on the Federal Court of Appeals, 1893-1991." *Political Research Quarterly*, 48, 3: 573-597.

Snyder, James and Michael Ting. 2002. "An Informational Rationale for Political Parties." *American Journal of Political Science*, 46 (1): 90-110.

Testa, Cecilia. 2005. "Political Polarization and Electoral Accountability." Working Paper.

Van Weelden, Richard. 2011a. "Candidates, Credibility, and Re-election Incentives." *SSRN Working Paper No. 1465504*.

Van Weelden, Richard. 2011b. "Moderate Voters, Polarized Parties". Working Paper.

Virag, Gabor. 2008. "Playing to your Own Audience: Extremism in Two-Party Elections." *Journal of Public Economic Theory*, 10 (5): 891-922.

Wittman, Donald. 1983. "Candidate Motivation: A Synthesis of Alternative Theories." *American Political Science Review*, 77 (1): 142-157.

Wolfers, Justin. 2006. "Are Voters Rational?: Evidence from Gubernatorial Elections." Working Paper.

Yoon, Albert. 2006. "Pensions, Politics, and Judicial Tenure: An Empirical Study of Federal Judges, 18692002." *American Law and Economics Review*, 8, 1: 143-180.

8 Appendix

8.1 κ^* and x^*

For any $\kappa \geq \bar{\kappa}$ we can define $x(\kappa)$ to be the unique solution to

$$(1-\delta)\gamma M - \delta(\kappa + x(\kappa))^\lambda = -(\kappa - x(\kappa))^\lambda.$$

Differentiating with respect to κ and re-arranging we have

$$[\delta(\kappa + x(\kappa))^{\lambda-1} + (\kappa - x(\kappa))^{\lambda-1}]x'(\kappa) = (\kappa - x(\kappa))^{\lambda-1} - \delta(\kappa + x(\kappa))^{\lambda-1}.$$

So we have that $x'(\kappa) = 0$ if and only if

$$\delta(\kappa + x(\kappa))^{\lambda-1} = (\kappa - x(\kappa))^{\lambda-1},$$

which implies that

$$x(\kappa) = \frac{1 - \delta^{\frac{1}{\lambda-1}}}{1 + \delta^{\frac{1}{\lambda-1}}}\kappa.$$

Plugging this into the original expression for $x(\kappa)$ we get that

$$(1-\delta)\gamma M = \frac{2^\lambda(\delta - \delta^{\frac{\lambda}{\lambda-1}})}{(1 + \delta^{\frac{1}{\lambda-1}})^\lambda}\kappa^\lambda.$$

As this equation has a unique solution,

$$\kappa^{**} = \frac{1+\delta^{\frac{1}{\lambda-1}}}{2}\left(\frac{(1-\delta)\gamma M}{\delta(1-\delta^{\frac{1}{\lambda-1}})}\right)^{\frac{1}{\lambda}},$$

we see that $x'(\kappa) = 0$ at one and only one point on $[\bar{\kappa}, \infty)$.

Next, note that taking derivative with respect to κ again we get

$$[\delta(\kappa + x(\kappa))^{\lambda-1} + (\kappa - x(\kappa))^{\lambda-1}]x^{''}(\kappa) = (\lambda-1)[(\kappa - x(\kappa))^{\lambda-2} - \delta(\kappa + x(\kappa))^{\lambda-2}] > 0,$$

when $\kappa = \kappa^{**}$. So $x^{''}(\kappa^{**}) > 0$ and we have a local min. Combining these two observations, we can conclude that $x(\kappa)$ is decreasing for $\kappa < \kappa^{**}$ and increasing for $\kappa > \kappa^{**}$. Hence we know that there exist a unique minimizer,

$$\kappa^{*} = \arg\min_{\kappa \in [\bar{\kappa}, 1]} x(\kappa).$$

8.2 Proofs of Results for Baseline Model

Before proceeding to the main results I show a simple Lemma: any candidate not of type κ^* would strictly prefer to implement their ideal policy while securing full rents in this period with $(-x^*, m^*)$ being implemented in every future period to implementing any platform which gives the voter utility u^* in every period.

Lemma 1 *If $\delta \geq \underline{\delta}$ then for all $\kappa \neq \kappa^*$ and any (x, m) such that $u(x, m) \geq u^*$,*

$$\bar{g}_{\kappa}(x, m) < (1-\delta)\gamma M + \delta g_{\kappa}(-x^*, m^*).$$

Proof. There are three cases to consider: candidates of type $\kappa < 0$, types $\kappa \in [0, \min\{\bar{\kappa}, 1\})$, and types $\kappa \in [\bar{\kappa}, 1] \backslash \{\kappa^*\}$.

First consider $\kappa \in [\bar{\kappa}, 1] \backslash \{\kappa^*\}$. If this set is non-empty this implies that $\kappa^* \geq \bar{\kappa}$ and so $m^* = 0$. Further, candidates of type κ are indifferent between implementing platform $(x(\kappa), 0)$ in every period and implementing their ideal point with full rent-seeking today and having $(-x(\kappa), 0)$ implemented in all future periods. In addition, as $x(\kappa)$ has a unique minimizer,

$$x^* < x(\kappa) \leq \frac{\kappa}{1+\gamma^{\frac{1}{1-\lambda}}},$$

from the definition of $\bar{\kappa}$. So for candidates of type κ, for any platform (x, m) with $u(x, m) \geq u(x(\kappa), 0)$ we have

$$\bar{g}_{\kappa}(x, m) \leq \bar{g}_{\kappa}(x(k), 0) = (1-\delta)\gamma M + g_{\kappa}(-x(\kappa), 0) < (1-\delta)\gamma M + \delta g_{\kappa}(-x^*, m^*).$$

As $u(x(\kappa), 0) < u^*$ we are done for $\kappa \in [\bar{\kappa}, 1] \backslash \{\kappa^*\}$.

Now consider $\kappa \in [0, \min\{\bar{\kappa}, 1\})$. We now consider the most desirable platform, from the voter's perspective, that candidate κ would be willing to implement. Note that the payoff from the most desirable platform can be no higher than the payoff from policy $x_\kappa = \kappa/(1+\gamma^{\frac{1}{\lambda-1}})$ and associated level of rents given by

$$\bar{g}_\kappa(x_\kappa, m_\kappa^{'}) = \gamma m_\kappa^{'} - (\kappa - x_\kappa)^\lambda = (1-\delta)\gamma M - \delta(m^* + (\kappa + x^*)^\lambda) = (1-\delta)\gamma M - g_\kappa(-x^*, m^*).$$

We can then define $w(\kappa) = -m_\kappa^{'} - x_\kappa^\lambda$ as the welfare to the voter from candidate κ. We must show that $w(\kappa) < u^*$. Note that

$$w(\kappa) = -(1-\delta)M + \frac{1}{\gamma}[\delta((\kappa + x^*)^\lambda + m^*) - (1+\gamma^{\frac{1}{1-\lambda}})^{1-\lambda}\kappa^\lambda].$$

First consider the case where $\bar{\kappa} \geq 1$. Then $\kappa^* = 1$ and $x^* = 1/(1+\gamma^{\frac{1}{\lambda-1}})$. Therefore

$$w^{'}(\kappa) = \frac{\lambda}{\gamma}[\delta(\kappa + x^*)^{\lambda-1} - (1+\gamma^{\frac{1}{1-\lambda}})^{1-\lambda}\kappa^{\lambda-1}],$$

and we can conclude that $w(\kappa)$ is increasing on the interval $[0,1)$ as

$$\delta \geq \underline{\delta} = (1+\gamma^{\frac{1}{1-\lambda}})^{1-\lambda}(\frac{\kappa}{\kappa + \frac{\kappa}{1+\gamma^{\frac{1}{\lambda-1}}}})^{\lambda-1} > (1+\gamma^{\frac{1}{1-\lambda}})^{1-\lambda}(\frac{\kappa}{\kappa + x^*})^{\lambda-1}.$$

As $w(1) = u^*$ when $\bar{\kappa} \geq 1$ we can conclude that $w(\kappa) < u^*$ on the interval $\kappa \in [0, \min\{\bar{\kappa}, 1\})$.

Finally consider the case where $\bar{\kappa} < 1$. Then we have $x^* \leq x(\bar{\kappa}) = \frac{\bar{\kappa}}{1+\gamma^{\frac{1}{\lambda-1}}}$ and $m^* = 0$. Then

$$w(\kappa) - u^* = -(1-\delta)M + (x^*)^\lambda + \frac{1}{\gamma}[\delta(\kappa + x^*)^\lambda - (1+\gamma^{\frac{1}{1-\lambda}})^{1-\lambda}\kappa^\lambda].$$

Note that we have seen that when $x^* = \frac{\bar{\kappa}}{1+\gamma^{\frac{1}{\lambda-1}}}$ we have $w(\kappa) - u^* < 0$ from the previous analysis. As $w(\kappa) - u^*$ is clearly increasing in x^* we can conclude that $w(\kappa) - u^* < 0$ for all $\kappa \in [0, \bar{\kappa})$ when $\bar{\kappa} < 1$. So we can conclude that for all $\kappa \in [0, \min\{\bar{\kappa}, 1\})$, $w(\kappa) < u^*$ and so,

$$\bar{g}_\kappa(x, m) < (1-\delta)\gamma M + g_\kappa(-x^*, m^*).$$

Finally note that for all $\kappa < 0$ the payoff from platform $(-x^*, m^*)$ in future periods is strictly higher than for candidate $-\kappa > 0$. Hence, as candidate $-\kappa$ does not strictly prefer any platform which gives the voter u^* to their ideal point today followed by $(-x^*, m^*)$ in future periods candidates of type κ must strictly prefer to implement their ideal point. ■

Clearly, by symmetry, this also implies that no type other than $-\kappa^*$ is willing to implement a platform which gives the voter utility u^* if the punishment is that (x^*, m^*) would be implemented in all subsequent periods.

Proof of Theorem 1. Part (1). Suppose there exists a SPE satisfying (R) and platform-stationarity which gives the voter utility $u^{'} > u^* > -M$. Let s^* be a SPE such that $U(s^*) = u^{'}$. By platform-stationarity, the voter has the same options available to her at all histories, and so must receive the same payoff, $u^{'}$ in every period regardless of the history.

First note that the incumbent can never be defeated in such an equilibrium along the equilibrium path. If the candidate would be re-elected if they implement their ideal point with full rents then they would clearly choose that platform and be re-elected forever. If the incumbent is to be defeated in equilibrium the candidate cannot meet the standard for re-election when they implement their ideal platform with full rent-seeking. By condition (R), then, in any period where the candidate does not secure re-election they must implement their ideal point while securing full rents. As the payoff to the voter in any period where the candidate engages in full rent-seeking is no higher than $-M$ at no history can the voter elect a candidate who will not secure re-election. Further, because the payoff to the voter is higher than $-M$, the candidate cannot be re-elected after implementing their ideal policy with full rent-seeking.

I now turn to showing that, given that every continuation equilibrium must give the same payoff to the voter, no candidate will have an incentive to implement a platform which gives the voter payoff $u' > u^*$. WLOG assume the candidate elected in the first period, k_0, is of type $\kappa_0 \geq 0$. Now consider the continuation equilibrium where that candidate is not re-elected, $s^*|_{(k_0,(\kappa_0,M))}$. Denote by k_1 the candidate elected in the first period of that equilibrium and let κ_1 be their type. By the above arguments this continuation equilibrium must give the voter utility $u^{'}$ and result in the same platform (x^1, m^1) being implemented in all future periods along the equilibrium path. Recall, by Lemma 1, that no candidate would be willing to implement a platform that gives the voter utility $u^{'}$ if the outcome path from the continuation equilibrium if not elected has $(-x^*, m^*)$ implemented in all future periods so we must have

$$g_{\kappa_0}(x^1, m^1) < g_{\kappa_0}(-x^*, m^*).$$

Combined with the fact that

$$u(x^1, m^1) = u^{'} > u^* = u(x^*, m^*),$$

we can conclude that $x^1 < -x^*$ and $m^1 < m^*$. This clearly implies that $m^* > 0$ and so $\kappa^* = 1$ and $x^* = \frac{1}{1+\gamma^{\frac{1}{\lambda-1}}}$. However, since candidate k_1 is evaluated strictly on the utility they provide to the voter, they must select a platform which trades off policy and rent-seeking in the same ratio as the voter, $m^1 < M$, and $\kappa_1 \in [-1, 1]$,

$$|x^1| \leq \frac{|\kappa_1|}{1+\gamma^{\frac{1}{\lambda-1}}} \leq \frac{1}{1+\gamma^{\frac{1}{\lambda-1}}}.$$

So clearly we cannot have $x^1 < -x^*$, and we have a contradiction. Therefore, there cannot exist a SPE satisfying (R) and platform-stationarity which gives the voter utility greater than u^*.

Part (2). Now consider the SPEs which achieve u^*. We can repeat the above argument except we now have $g_{\kappa_0}(x^1, m^1) \leq g_{\kappa_0}(-x^*, m^*)$ and $u(x^1, m^1) = u(-x^*, m^*)$ which implies that $x^1 \leq -x^*$ and $m^1 \leq m^*$. Similarly, as above, we can rule out $x^1 < -x^*$, so $(x^1, m^1) = (-x^*, m^*)$. Hence, by Lemma 1, we have that $\kappa_0 = \kappa^*$ is the only candidate with $\kappa_0 \geq 0$ who could be elected. By symmetry, $-\kappa^*$ is the only candidate with $\kappa < 0$ who can be elected in the first period of any SPE which gives the voter utility u^*. So we can conclude that no type $\kappa \notin \{-\kappa^*, \kappa^*\}$ can be elected at any history.

As all players' utility functions are strictly concave with respect to policy, if a candidate of type $\kappa \in \{-\kappa^*, \kappa^*\}$ is implementing platforms other than (x^*, m^*) (or $(-x^*, m^*)$) this platform must give the voter utility less than u^*. Since the continuation payoff to the voter must be u^* at all histories such candidates can never be elected at any history. Finally, at least one type κ^* candidate is implementing (x^*, m^*) and at least one type $-\kappa^*$ candidate is implementing $(-x^*, m^*)$ in order for the threatened punishment if the elected candidate deviates to be credible. ■

Proof of Theorem 2. We first calculate the comparative statics for part (1). We begin by noting that $\log \underline{\delta} = (1-\lambda)\log(1 + 2\gamma^{\frac{1}{1-\lambda}})$ so

$$\begin{aligned} \frac{\partial \log \underline{\delta}}{\partial \gamma} &= \frac{2\gamma^{\frac{-\lambda}{1-\lambda}}}{1 + 2\gamma^{\frac{1}{1-\lambda}}} > 0, \\ \frac{\partial \log \underline{\delta}}{\partial \lambda} &= -\log(1 + 2\gamma^{\frac{1}{1-\lambda}}) + \frac{1}{1 + 2\gamma^{\frac{1}{1-\lambda}}} 2\gamma^{\frac{1}{1-\lambda}} \frac{\log \gamma}{(1-\lambda)} < 0, \end{aligned}$$

as $\lambda > 1$ and $\gamma \geq 1$.

We now consider the limits. Consider first the limit as $\lambda \to 1$. Note that when $\lambda < 2$, $\gamma^{\frac{1}{1-\lambda}} \leq \gamma^{-1}$ and so

$$(1 + 2\gamma^{-1})^{1-\lambda} \leq \underline{\delta} < 1.$$

As $(1 + 2\gamma^{-1})^{1-\lambda}$ clearly goes to 1 as $\lambda \to 1$ we can conclude that

$$\lim_{\lambda \to 1} \underline{\delta} = 1.$$

Finally we show that $\underline{\delta}$ goes to 0 as $\lambda \to \infty$. Note that when $\lambda \geq 2$, $\gamma^{\frac{1}{1-\lambda}} \geq \gamma^{-1}$ and so

$$0 < \underline{\delta} < (1 + \gamma^{-1})^{1-\lambda}.$$

As $(1 + \gamma^{-1})^{1-\lambda}$ clearly goes to 0 as $\lambda \to \infty$ we can conclude that

$$\lim_{\lambda \to \infty} \underline{\delta} = 0.$$

■

Proof of Theorem 3. Part (1) is immediate so I focus on part (2). We begin by calculating some comparative statics on κ^{**} and x^{**}. The equations

$$x^{**} = \frac{1 - \delta^{\frac{1}{\lambda-1}}}{1 + \delta^{\frac{1}{\lambda-1}}} \kappa^{**},$$

$$\kappa^{**} = \frac{1+\delta^{\frac{1}{\lambda-1}}}{2}(\frac{(1-\delta)\gamma M}{\delta(1-\delta^{\frac{1}{\lambda-1}})})^{\frac{1}{\lambda}},$$

immediately imply that

$$\frac{\partial \kappa^{**}}{\partial M} > 0; \qquad \frac{\partial x(\kappa^{**})}{\partial M} > 0,$$

and

$$\frac{\partial \kappa^{**}}{\partial \gamma} > 0; \qquad \frac{\partial x(\kappa^{**})}{\partial \gamma} > 0.$$

I now show that κ^{**} is strictly decreasing in δ, which will immediately imply that x^{**} is also strictly decreasing. It is sufficient to show that

$$\frac{(1-\delta)(1+\delta^{\frac{1}{\lambda-1}})^{\lambda}}{\delta(1-\delta^{\frac{1}{\lambda-1}})},$$

is decreasing in δ. Define $\rho = \delta^{\frac{1}{\lambda-1}}$. Taking log and differentiating w.r.t ρ gives

$$\begin{aligned} & \frac{-(\lambda-1)}{1-\rho^{\lambda-1}}\rho^{\lambda-2} + \frac{\lambda}{1+\rho} - \frac{\lambda-1}{\rho} + \frac{1}{1-\rho} \\ = \; & \frac{-(\lambda-1)}{1-\rho^{\lambda-1}}\rho^{\lambda-2} + \frac{1-\lambda+\rho+\lambda\rho}{\rho(1-\rho^2)}. \end{aligned}$$

So we have that κ^{**} is decreasing if

$$f(\rho) \equiv (\lambda-1)(1-\rho^{\lambda+1}) - \rho(\lambda+1)(1-\rho^{\lambda-1}) > 0.$$

Note that $f(1) = 0$, and for all $\rho \in (0,1)$,

$$f'(\rho) = (\lambda+1)[-(\lambda-1)\rho^{\lambda} - 1 + \lambda\rho^{\lambda-1})] < 0,$$

as $-(\lambda-1)\rho^{\lambda} - 1 + \lambda\rho^{\lambda-1}$ in increasing in ρ and equal to 0 when $\rho = 1$. So $f(\rho) > 0$ for all $\rho < 1$ and we can conclude that κ^{**} is decreasing in δ. Hence we can conclude that

$$\frac{\partial \kappa^{**}}{\partial \delta} < 0, \qquad \frac{\partial x^{**}}{\partial \delta} < 0.$$

So we have that k^{**} and x^{**} are increasing in M and γ and decreasing in δ. Finally, since $\kappa^* < 1$ we have that $\kappa^* = \kappa^{**}$ and $x^* = x^{**}$ so we are done. ■

Proof of Theorem 4. Now we can consider the limiting case where $\delta \to 1$. Since $\lim_{\delta \to 1} \bar{\kappa} = 0$, we know that when δ is close to 1 we have $\kappa^* = \min\{\kappa^{**}, 1\}$, $m^* = 0$, with $x^* = x^{**}$ if $\kappa^{**} < 1$ and $x^* = x(1)$ otherwise. Recall that

$$(\kappa^{**})^{\lambda} = \gamma M \frac{(1-\delta)(1+\delta^{\frac{1}{\lambda-1}})^{\lambda}}{2^{\lambda}\delta(1-\delta^{\frac{1}{\lambda-1}})},$$

and since the top and bottom of the fraction both converge to 0 as δ goes to 1 we can use L'Hopital's rule to get

$$\lim_{\delta\to 1}(\kappa^{**})^\lambda = \lim_{\delta\to 1}\frac{\gamma M}{2^\lambda}\frac{-(1+\delta^{\frac{1}{\lambda-1}})^\lambda + \lambda(1-\delta)(1+\delta^{\frac{1}{\lambda-1}})^{\lambda-1}\frac{1}{\lambda-1}\delta^{\frac{-\lambda}{\lambda-1}}}{1-\delta^{\frac{1}{\lambda-1}}\frac{\lambda}{\lambda-1}} = \frac{\gamma M}{2^\lambda}(2^\lambda)(\lambda-1) = \gamma M(\lambda-1).$$

Consequently, we have

$$\lim_{\delta\to 1}\kappa^* = \lim_{\delta\to 1}\min\{\kappa^{**},1\} = \min\{(\gamma M(\lambda-1))^{\frac{1}{\lambda}},1\}.$$

Next, we consider the limiting policy, x^*. First note that $x^* \le x(1)$ and $x(1)$ solves

$$-(1-x(1))^\lambda = (1-\delta)\gamma M - \delta(1+x(1))^\lambda,$$

so x^* obviously goes to 0 as δ goes to 1. More interestingly, dividing by $m_0 = (1-\delta)\gamma M/(\gamma+\delta)$ we have

$$-(m_0^{-1/\lambda} - x(1)m_0^{-1/\lambda})^\lambda = \gamma+\delta-\delta(m_0^{-1/\lambda}+x(1)m_0^{-1/\lambda})^\lambda.$$

Note that, as $m_0^{-1/\lambda}$ goes to ∞ as δ goes to 1, this implies that $x(1)m_0^{-1/\lambda}$ goes to 0. Hence, we can conclude that

$$\lim_{\delta\to 1}\frac{u(0,0)-u^*}{u(0,0)-u(0,m_0)} = \lim_{\delta\to 1}\frac{(x^*)^\lambda}{m_0} = 0.$$

■

8.3 Examples of $\underline{\delta}$

The following table shows $\underline{\delta}$ for different values of γ and λ. As we can see, provided λ is reasonably large the lower bound on δ is not very restrictive.

$\gamma\backslash\lambda$	2	2.5	3	3.5	4	5
1	0.33	0.19	0.11	0.06	0.04	0.01
5	0.71	0.46	0.28	0.17	0.10	0.03
10	0.83	0.58	0.38	0.23	0.14	0.05
50	0.96	0.81	0.61	0.42	0.27	0.11
100	0.98	0.88	0.69	0.50	0.34	0.14
500	0.99+	0.95	0.84	0.68	0.51	0.24