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Hartmann Newtonian radiating MHD flow for a rotating vertical porous channel immersed in a Darcian Porous Regime: An exact solution

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An exact solution

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Abstract

Purpose – The purpose of this paper is to develop and correct the problem studied by Makinde and Mhone (2005) to a rotating vertical porous channel immersed in a Darcian porous regime in presence of a strong transverse magnetic filled and with the application of thermal radiation. In this investigation, the fluid is considered to be of viscous, electrically conducting, Newtonian and radiating and is optically thin with a relatively low density. Excellent agreement is obtained for exact solutions with those of previously published works.

Design/methodology/approach – In this investigation, a closed form analytical method based on the complex notations for the velocity, temperature and the pressure is developed to solve the governing coupled, non-linear partial differential equations. The accuracy and effectiveness of the method are demonstrated.

Findings – Interestingly observed that, the Lorentizian body force is not act as a drag force as in conventional MHD flows, but as an aiding body force and this will serve to accelerate the flow and boost the primary velocities. Due to the large rotation of the channel, the primary velocities are become flattered and shift towards the walls of the channel. With a rise in Darcian drag force, flow velocity and shear stress are found to reduce. Moreover, increasing thermal radiation and rotation of the channel strongly depress the shear stress, and maximum flow reversal, i.e. back flow is observed due to large Darcian resistance, thermal radiation and rotation.

Research limitations/implications – The analysis is valid for unsteady, two-dimensional laminar flow of an optically thick no-gray gas, electrically conducting, and Newtonian fluid past an isothermal vertical surface adjacent to the Darcian regime with variable surface temperature. An extension to three-dimensional flow case is left for future work.

Practical implications – Practical interest of such study includes applications in magnetic control of molten iron flow in the steel industry, liquid metal cooling in nuclear reactors, magnetic suppression of molten semi-conducting materials and meteorology and in many branches of engineering and science. It is well known that the effect of thermal radiation is important in space technology and high-temperature processes. Thermal radiation also plays an important role in controlling heat transfer process in polymer processing industry.

Originality/value – The paper presents useful conclusions with the help of graphical results obtained from studying exact solutions based on complex notations for Darcian drag force, rotation of

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Dedicated respectfully to the late Professor Eric Laithwaite (1921-1997), born Atherton, Lancashire, formerly of the Electrical Engineering Department, Manchester University for his pioneering developments in Magnetic Levitation and Magnetic Propulsion Systems.

the channel and conduction-radiation heat transfer interaction by unsteady rotational flow in a vertical porous channel embedded in a Darcian porous regime under the application hydromagnetic force. The results of this study may be of interest to engineers for heat transfer augmentation and drag reduction in heat exchangers as well as MHD boundary layer control of re-entry vehicles, etc.

Keywords Darcian resistance, Oscillatory flow, Heat radiation, Vertical porous channel, Hydromagnetic flow, Rotating fluid, Astronautical flows

Paper type Research paper

Nomenclature

u, v, w	Velocity components along X, Y, Z-directions
x, y, z	Variables along X, Y, Z-directions
w_0	Injection/suction velocity
K_r	Darcian resistance parameter
M	Hartmann number
Gr	Grashof number for heat transfer
Pr	Prandtl number
qr	Radiative heat flux
N	Radiation parameter
P	A constant
$\bar{p}$	The modified pressure
T_0	Mean temperature
B_0	Applied magnetic field along the $\bar{z}$-axis
g	Acceleration due to gravity
C_P	Specific heat at constant pressure
t	Time variable
$\overline{T}$	Dimensional fluid temperature
$\bar{\omega}$	Dimensional frequency of oscillations
$\bar{t}$	Dimensional time

Greek symbols

α	Mean radiation absorption coefficient
β	Coefficient of volume expansion
λ	Injection/suction parameter
η	Space coordinate
ω	Frequency of oscillations
v	Kinematic viscosity
σ	Electric conductivity
ρ	Fluid density
κ	Thermal conductivity
Ω	Rotation parameter
τ_L	Skin-friction at the left wall
θ	Fluid temperature
θ_0	Mean non-dimensional temperature

1. Introduction

The theory of rotating fluids (Greenspan, 1969) is highly important due to its occurrence in various natural phenomena and for its applications in various technological situations which are directly governed by the action of Coriolis force. The broad subjects of oceanography, meteorology, atmospheric science and limnology all contain some important and essential features of rotating fluids. In astrophysics it is applied to study the stellar and solar structure, inter planetary and inter stellar matter, solar storms and flares, etc. In engineering, it finds its application in MHD generators, ion propulsion, MHD bearings, MHD pumps, MHD boundary layer control of reentry vehicles, etc. Hartmann (1937) first studied the flow of a viscous incompressible fluid under transverse magnetic field in a MHD channel flow. Literature related to hydromagnetic channel flows is reported by several scholars, namely, Crammer and Pai (1973), Ferraro and Plumpton (1966), Shercliff (1965) on account of their varied importance.

Several investigations are carried out on the problem of hydrodynamic flow of a viscous incompressible fluid in rotating medium considering various variations in the problem. Mention may be made of the studies of Greenspan and Howard (1963), Holton (1965), Walin (1969), Siegmann, 1971), Hayat and Hutter (2004), Singh *et al.* (2005). The problem of magnetohydrodynamic flow of a viscous incompressible electrically conducting fluid in a rotating medium is studied by many researchers, namely, Ghosh (2001), Singh (2000), Hossain *et al.* (2001), Ghosh and Pop (2002), Hayat *et al.* (2008), Hayat and Abelman (2007), Abelman *et al.* (2009), Wang and Hayat (2004) under different conditions and configurations to analyse various aspects of the problem and to find its application in Science and Engineering. Seth *et al.* (2011) studied the unsteady hydromagnetic Couette flow of a viscous incompressible electrically conducting fluid in a rotating system in the presence of a uniform transverse magnetic field.

Convective flows in channels driven by temperature differences of bounding walls have been studied and reported, extensively in literature. On account of their varied importance, such flows have been studied by Rapits and Perdikis (1982). Hamadah and Wirtz (1991) have investigated the free convective flow in vertical channel for ordinary medium. Attia and Kotb (1996) studied the MHD flow between two parallel porous plates. Singh and Sharma (2001) have analysed the three-dimensional Couette flow through a porous medium with heat transfer. The study of vertical channel flow bounded by a wavy wall and a vertical flat plate filled with porous medium was presented by Ahmed (2008). Later on Ahmed (2009) also investigated the effects of free convection heat transfer on the three-dimensional channel flow through a porous medium with periodic injection velocity. Recently, Fasogbon (2010) studied the simultaneous buoyancy force effects of thermal and species diffusion through a vertical irregular channel by using parameter perturbation technique. Free convection flows in vertical slots were discussed by Weidman (2006), Magyari (2007), Weidman and Medina (2008). Very recently, Ahmed and Zueco (2011) investigate the effects of Hall current, magnetic field, rotation of the channel and suction-injection on the oscillatory free convective MHD flow in a rotating vertical porous channel when the entire system rotates about an axis normal to the channel plates and a strong magnetic field of uniform strength is applied along the axis of rotation. Makinde and Mhone (2005) investigated the combined effects of transverse magnetic field and radiative heat transfer in unsteady flow of a conducting optically thin fluid through a channel filled with porous medium. Magnetohydrodynamic oscillatory convection flow of a viscous, incompressible and electrically conducting fluid in a rotating vertical porous channel is analytically presented by Singh (2012). Chamkha and Camille (2000) presented the magnetohydrodynamic transient convective radiative heat transfer one-dimensional flow in an isotropic, homogenous porous regime adjacent to a hot vertical plate. Seddeek (2004) studied the transient-free convection magnetohydrodynamic boundary layer flow in a fluid-saturated porous medium channel, and consider the influence of temperature-dependent properties and inertial effects on the convection regime. Cogley *et al.* (1968) studied the effects of melting and thermal radiation on mixed convection from a vertical surface embedded in a non-Newtonian fluid saturated non-Darcy porous medium.

In the present analysis, an oscillatory convection flow of an electrically conducting viscous incompressible Newtonian fluid in a vertical porous channel filled with Darcian porous regime is studied. Constant injection and suction is applied at the left and the right infinite porous plates, respectively. The entire system rotates about an axis perpendicular to the planes of the plates of the channel and a uniform magnetic field is also applied along this axis of rotation. Such a study is found useful in magnetic control of molten iron flow in the steel industry, liquid metal cooling in nuclear reactors, magnetic suppression of molten semi-conducting materials and meteorology. During mathematical analysis it is found that the study presented by Makinde and Mhone (2005) is incorrect and the modified model is presented here. Excellent agreement is obtained for exact solutions with those of Singh (2012).

1.1 Makinde and Mhone (2005) model and accuracy
During mathematical analysis it is found that the mathematical formulation of the problem by Makinde and Mhone (2005) is not in consistency with the geometry of the physical problem shown in their Figure 1. Geometrically the channel is horizontal whereas the mathematical formulation is for the vertical channel where the Boussinesq incompressible fluid model is assumed to include buoyancy force in momentum Equation (1). Boundary conditions (3) and (4) are not according to the choice of the Cartesian coordinate system with Ox-axis lying along the centerline of the channel. The temperatures of the walls are also not non-uniform as mentioned in the abstract. For this purely oscillatory flow the boundary conditions (13) and (14) cannot be obtained after the substitution of (11) into the boundary conditions (9) and (10). The energy Equation (8) is incorrect and its solution (15) obtained under wrong boundary conditions (13) and (14) is obviously incorrect. This solution is further used in the Equation of motion (12) which consequently yields a wrong solution again. For $\Omega = 0$ in Equation (31) of the present analysis gives the correct form of the velocity distribution and Equation (32) gives the correct form of temperature distribution of the problem by Makinde and Mhone (2005) for the case of ordinary medium. Therefore, in this paper we try to develop and correct the problem studied by Makinde and Mhone (2005) to a rotating vertical porous channel immersed in a Darcian porous regime in presence of transverse magnetic filled. Moreover, the accuracy of the present study is excellent with those of Singh (2012).

2. Basic equations

Consider the flow of a viscous, incompressible and electrically conducting fluid in a rotating vertical channel in a Darcian porous regime. In order to derive the basic equations for the problem under consideration following assumptions are made:

- the two infinite vertical parallel plates of the channel are permeable and electrically non-conducting;
- the flow considered is fully developed, laminar and oscillatory;
- the fluid is viscous, incompressible and finitely conducting;

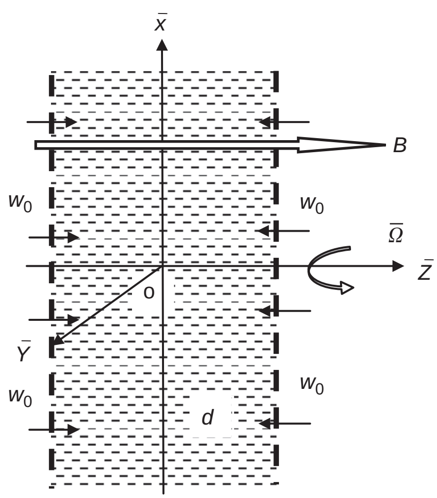

Figure 1.
Coordinate system and the flow configuration

- all fluid properties are assumed to be constant except that of the influence of density variation with temperature is considered only in the body force term;
- the pressure gradient in the channel oscillates periodically with time;
- a magnetic field of uniform strength $\boldsymbol{B}$ is applied perpendicular to the plates of the channel;
- the magnetic Reynolds number is taken to be small enough so that the induced magnetic field is neglected;
- hall effect, electrical and polarization effects are also neglected;
- the temperature of a plate is non-uniform and oscillates periodically with time;
- the temperature difference of the two plates is also assumed to be high enough to induce heat transfer due to radiation;
- the fluid is assumed to be optically thin with relatively low density; and
- the entire system (consisting of channel plates and the fluid) rotates about an axis perpendicular to the plates.

Under these assumptions we write hydromagnetic governing equations of motion and continuity in a rotating frame of reference as:

$$\nabla \cdot \boldsymbol{V} = 0 \tag{1}$$

$$\frac{\partial \boldsymbol{V}}{\partial \bar{t}} + (\boldsymbol{V} \cdot \nabla)\boldsymbol{V} + 2\bar{\Omega} \times \boldsymbol{V} = -\frac{1}{\rho}\nabla \bar{p} + v\nabla^2 V - \frac{v}{\overline{\overline{K}}}\boldsymbol{V} + \frac{1}{\rho}(\boldsymbol{J} \times \boldsymbol{B}) + \boldsymbol{F} \tag{2}$$

$$\rho C_P\left[\frac{\partial \overline{T}}{\partial \bar{t}} + (V \cdot \nabla)\bar{T}\right] = \kappa \nabla^2 \bar{T} - \nabla q_r \tag{3}$$

In Equation (2) the last term on the left hand side is the Coriolis force. On the right hand side of (2) the last term $F\left(= g\beta\overline{T}\right)$ accounts for the force due to buoyancy and the second last term is the Lorentz force due to magnetic field $\boldsymbol{B}$ and is given by:

$$J \times B = \sigma(V \times B) \times B \tag{4}$$

and the modified pressure $\bar{p} = p' - \frac{\rho}{2}|\Omega \times R|^2$, where $\boldsymbol{R}$ denotes the position vector from the axis of rotation, p' denotes the fluid pressure, $\boldsymbol{J}$ is the current density, $\overline{K}$ is the permeability of the porous medium and all other quantities have their usual meaning.

2.1 Formulation of the problem

In the present analysis, we consider an unsteady flow of a viscous incompressible and electrically conducting Newtonian fluid bounded by two infinite vertical porous plates distance "d" apart as shown in Figure 1. A coordinate system is chosen such that the $\overline{X}$-axis is oriented upward along the centerline of the channel filled with saturated Darcian porous regime and $\overline{Z}$-axis taken perpendicular to the planes of the plates lying in $\overline{Z} = \pm d/2$ planes. The fluid is injected through the porous plate at $\overline{Z} = -d/2$ with constant velocity w_0 and simultaneous sucked through the other porous plate at $\overline{Z} = +d/2$ with the same velocity w_0. The non-uniform temperature of the plate at $\overline{Z} = +d/2$ is assumed to be varying periodically with time. The temperature difference between the plates is high enough to induce the heat due to radiation. The $\overline{Z}$-axis is considered to be the axis of rotation about which the fluid and the plates are assumed to be rotating as a solid body with a constant angular velocity $\overline{\Omega}$. A transverse magnetic field of uniform strength $\boldsymbol{B}\,(0, 0, B_0)$ is also applied along the axis of rotation. All physical quantities depend on $\bar{z}$ and $\bar{t}$ only for this problem of fully developed laminar flow. The equation of continuity $\nabla.\ \boldsymbol{V} = 0$ gives on integration $\overline{w} = w_0$. Then the velocity may reasonably be assumed with its components along $\overline{X}, \overline{Y}, \overline{Z}$ directions as $\boldsymbol{V}(\overline{u}, \overline{v}, w_0)$.

Using the velocity and the magnetic field distribution as stated above the magnetohydrodynamic flow in the rotating channel is governed by the following Cartesian equations:

$$\frac{\partial \overline{u}}{\partial \bar{t}} + w_0 \frac{\partial \overline{u}}{\partial \bar{z}} = -\frac{1}{\rho}\frac{\partial \overline{p}}{\partial \overline{x}} + v\frac{\partial^2 \overline{u}}{\partial \bar{z}^2} + 2\overline{\Omega}\overline{u} - \frac{v}{\overline{K}}\overline{u} - \frac{\sigma B_0^2 \overline{u}}{\rho} + g\beta\overline{T}, \tag{5}$$

$$\frac{\partial \overline{v}}{\partial \bar{t}} + w_0 \frac{\partial \overline{v}}{\partial \bar{z}} = -\frac{1}{\rho}\frac{\partial \overline{p}}{\partial \overline{y}} + v\frac{\partial^2 \overline{v}}{\partial \bar{z}^2} + 2\overline{\Omega}\overline{u} - \frac{v}{\overline{K}}\overline{v} - \frac{\sigma B_0^2 \overline{v}}{\rho}, \tag{6}$$

$$0 = -\frac{1}{\rho}\frac{\partial \overline{p}}{\partial \bar{z}}, \tag{7}$$

$$\frac{\partial \overline{T}}{\partial \bar{t}} + w_0 \frac{\partial \overline{T}}{\partial \bar{z}} = \frac{\kappa}{\rho C_P}\frac{\partial^2 \overline{T}}{\partial \bar{z}^2} - \frac{1}{\rho C_P}\frac{\partial^2 q_r}{\partial \bar{z}^2} \tag{8}$$

where "–" represents the dimensional physical quantities.

The last term in Equation (8) is the radiative heat flux.

Following Cogley *et al.* (1968) it is assumed that the fluid is optically thin with a relatively low density and the heat flux due to radiation in Equation (8) is given by:

$$\frac{\partial q_r}{\partial \bar{z}} = 4\alpha^2 \overline{T} \tag{9}$$

where α is the mean radiation absorption coefficient. After the substitution of Equation (9), Equation (8) becomes:

$$\frac{\partial \overline{T}}{\partial \bar{t}} + w_0 \frac{\partial \overline{T}}{\partial \bar{z}} = \frac{\kappa}{\rho C_P}\frac{\partial^2 \overline{T}}{\partial \bar{z}^2} - \frac{4\alpha^2}{\rho C_P}\overline{T} \tag{10}$$

Equation (7) shows the constancy of the hydrodynamic pressure along the axis of rotation. We shall assume now that the fluid flows under the influence of pressure gradient varying periodically with time in the $\overline{X}$-axis is of the form:

$$-\frac{1}{\rho}\frac{\partial \overline{p}}{\partial \overline{x}} = P cos \overline{\omega}\bar{t} \tag{11}$$

The boundary conditions for the problem are:

$$\overline{Z} = \frac{\overline{d}}{2}: \quad \overline{u} = \overline{v} = 0, \quad \overline{T} = T_0 cos\overline{\omega}\bar{t}, \tag{12}$$

$$\overline{Z} = -\frac{\overline{d}}{2}: \quad \overline{u} = \overline{v} = 0, \quad \overline{T} = 0. \tag{13}$$

Introducing the following non-dimensional quantities:

$$\begin{aligned} &\eta = \frac{\bar{z}}{d},\ x = \frac{\overline{x}}{d},\ y = \frac{\overline{y}}{d},\ u = \frac{\overline{u}}{w_0},\ v = \frac{\overline{v}}{w_0},\ \theta = \frac{\overline{T}}{T_0},\ t = \overline{\omega}\bar{t}, \\ &p = \frac{\overline{p}}{\rho w_0^2},\ K_r = \frac{w\overline{K}}{d^2},\ \lambda = \frac{w_0\overline{d}}{v},\ \boldsymbol{\Omega} = \frac{\overline{\Omega}d^2}{v},\ M = B_0 d\sqrt{\frac{\sigma}{\mu}}, \\ &Gr = \frac{g\beta d^2 T_0}{v w_0},\ Pr = \frac{\mu c_p}{k},\ N = \frac{2\alpha d}{\sqrt{\kappa}},\ \omega = \frac{\overline{\omega}d^2}{v} \end{aligned} \tag{14}$$

In view of (14), Equations (5), (6) and (10) with denoting $\Lambda = M^2 + K_r^{-1}$, become:

$$\frac{\partial u}{\partial t} + \lambda\frac{\partial u}{\partial \eta} = -\lambda\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial \eta^2} + 2\Omega v - \Lambda u + Gr\,T \tag{15}$$

$$\frac{\partial v}{\partial t} + \lambda\frac{\partial v}{\partial \eta} = -\lambda\frac{\partial p}{\partial y} + \frac{\partial^2 v}{\partial \eta^2} - 2\Omega u - \Lambda v \tag{16}$$

$$\omega Pr\frac{\partial \theta}{\partial t} + \lambda Pr\frac{\partial \theta}{\partial \eta} = \frac{\partial^2 \theta}{\partial \eta^2} - N^2\theta \tag{17}$$

The boundary conditions in the dimensionless form become:

$$\eta = \frac{1}{2}: \quad u = v = 0, \quad \theta = cos\,t, \tag{18}$$

$$\eta = -\frac{1}{2}: \quad u = v = 0, \quad \theta = 0, \tag{19}$$

For the oscillatory internal flow we shall assume that the fluid flows under the influence of a non-dimension pressure gradient varying periodically with time in the direction of x-axis only which implies that:

$$-\frac{\partial p}{\partial x} = Pcos\,t, \text{ and } \quad -\frac{\partial p}{\partial x} = 0. \tag{20}$$

2.2 Solution of the problem

Now combining Equations (15) and (16) into single equation by introducing a complex function of the form $F = u + iv$ and with the help of Equation (20), we get:

$$\omega\frac{\partial F}{\partial t} + \lambda\frac{\partial F}{\partial \eta} = \lambda P \cos t + \frac{\partial^2 F}{\partial \eta^2} - (\Lambda + 2i\Omega)F + Gr\,\theta \tag{21}$$

with corresponding boundary conditions as:

$$\eta = \frac{1}{2}: \quad F = 0, \quad \theta = \cos t, \tag{22}$$

$$\eta = -\frac{1}{2}: \quad F = 0, \quad \theta = 0, \tag{23}$$

In order to solve Equations (21) and (17) under boundary conditions (22) and (23), it is convenient to adopt complex notations for the velocity, temperature and the pressure as under:

$$F(\eta, t) = F_0(\eta)e^{it}, \quad \theta = \theta_0(\eta)e^{it} - \frac{\partial y}{\partial x} = Pe^{it}. \tag{24}$$

The solutions will be obtained in terms of complex notations, the real part of which will have physical significance.

The boundary conditions (22) and (23) in complex notations can also be written as:

$$\eta = \frac{1}{2}: \quad F = 0, \quad \theta = e^{i\,t}, \tag{25}$$

$$\eta = -\frac{1}{2}: \quad F = 0, \quad \theta = 0, \tag{26}$$

Substituting expressions (24) in Equations (21) and (17), we get:

$$\frac{d^2 F_0}{d\eta^2} - \lambda\frac{dF_0}{d\eta} - (\Lambda + 2i\Omega + i\omega)F_0 = -\lambda P - Gr\,\theta_0, \tag{27}$$

$$\frac{d^2\theta_0}{d\eta^2} - \lambda Pr\frac{d\theta_0}{d\eta} - \left(N^2 + i\omega Pr\right)\theta_0 = 0. \tag{28}$$

The transformed boundary conditions reduce to:

$$\eta = \frac{1}{2}: \quad F_0 = 0, \quad \theta_0 = 1, \tag{29}$$

$$\eta = -\frac{1}{2}: \quad F_0 = 0, \quad \theta_0 = 0, \tag{30}$$

The solution of the ordinary differential Equation (27) under the boundary conditions (29) and (30) gives the velocity field as:

$$F(\eta,t)=\left[\begin{array}{l}\frac{1}{2sinh\left(\frac{m-n}{2}\right)}\left[\begin{array}{l}\frac{Gr}{2sinh\left(\frac{r-s}{2}\right)}\left\{\begin{array}{l}\left(\frac{e^{\frac{r-s}{2}}}{C_1}-\frac{e^{-\frac{r-s}{2}}}{C_2}\right)\left(e^{m\eta-\frac{n}{2}}-e^{n\eta-\frac{m}{2}}\right)\\+\left(\frac{C_1-C_2}{C_1C_2}\right)\left(e^{m\eta+\frac{n}{2}}-e^{n\eta+\frac{m}{2}}\right)e^{-\frac{\lambda Pr}{2}}\end{array}\right\}\\+\frac{2\lambda P}{(\Lambda+2i\Omega+i\omega)}\left(e^{m\eta}sinh\frac{n}{2}-e^{n\eta}sinh\frac{m}{2}\right)\end{array}\right]\\+\frac{\lambda P}{(\Lambda+2i\Omega+i\omega)}-\frac{Gr}{2sinh\left(\frac{r-s}{2}\right)}\left(\frac{e^{r\eta-\frac{s}{2}}}{C_1}-\frac{e^{S\eta-\frac{r}{2}}}{C_2}\right)\end{array}\right]e^{it} \qquad (31)$$

where $C_1=r^2-\lambda r-(\Lambda+2i\Omega+i\omega),\quad C_2=s^2-\lambda s-(\Lambda+2i\Omega+i\omega),$

$$m=\frac{\lambda+\sqrt{\lambda^2+4(\Lambda+2i\Omega+i\omega)}}{2},\quad n=\frac{\lambda-\sqrt{\lambda^2+4(\Lambda+2i\Omega+i\omega)}}{2}$$

$$r=\frac{\lambda Pr+\sqrt{\lambda^2Pr^2+4(N^2+i\omega Pr)}}{2},\quad s=\frac{\lambda Pr-\sqrt{\lambda^2Pr^2+4(N^2+i\omega Pr)}}{2}$$

Similarly, the solution of Equation (28) for the temperature field can be obtained under the boundary conditions (29) and (30) as:

$$\theta(\eta,t)=\frac{1}{2sinh\left(\frac{r-s}{2}\right)}\left(e^{r\eta-\frac{s}{2}}-e^{s\eta-\frac{r}{2}}\right)e^{it}. \qquad (32)$$

Now, it is convenient to write the primary (u) and secondary (v) velocity fields, in terms of the fluctuating parts, separating the real and imaginary part from Equation (31) and taking only the real parts as they have physical significance, the velocity distributions of the flow field can be expressed in fluctuating parts as given below:

$$u(\eta,t)=u_0\,cos\,t-v_0\,sin\,t\quad and\quad v(\eta,t)=u_0\,sin\,t+v_0\,cos\,t,\quad F_0=u_0+i\,v_0 \qquad (33)$$

From the velocity field obtained in Equation (31) we can get the skin-friction τ_L at the left plate ($\eta=-0.5$) as:

$$\tau=\left(\frac{\partial F}{\partial\eta}\right)_{\eta=-0.5}=\frac{1}{2sinh\left(\frac{m-n}{2}\right)}\left[\begin{array}{l}\frac{Gr}{2sinh\left(\frac{r-s}{2}\right)}\left\{\begin{array}{l}\left(\frac{e^{\frac{r-s}{2}}}{C_1}-\frac{e^{-\frac{r-s}{2}}}{C_2}\right)(m-n)e^{-\frac{\lambda}{2}}+\\\left(\frac{C_1-C_2}{C_1C_2}\right)\left(me^{-\frac{m-n}{2}}-ne^{\frac{m-n}{2}}\right)e^{-\frac{\lambda Pr}{2}}\end{array}\right\}\\+\frac{2\lambda P}{(\Lambda+2i\Omega+i\omega)}\left(me^{-\frac{m}{2}}sinh\frac{n}{2}-ne^{-\frac{n}{2}}sinh\frac{m}{2}\right)\end{array}\right]$$
$$-\frac{Gr}{2sinh\left(\frac{r-s}{2}\right)}\left(\frac{r}{C_1}-\frac{s}{C_2}\right)e^{-\frac{\lambda Pr}{2}} \qquad (34)$$

2.3 Validity and accuracy

We have obtained a comprehensive range of solutions to the transformed conservation equations. To test the validity of the present computations, we have compared the flow velocity in Table I with the Singh (2012). It is clearly seen from Table I that the results are in excellent agreement. As the accuracy of the numerical solutions is very good, the values of u corresponding to analytical solutions are very close to each other. Table I show that the flow velocity is found to decelerate with heat radiation N from 1.0 through 5.0-10.0 for small rotation in a Darcian regime.

3. Results and discussions

To have better insight of the physical problem the variations of the velocity, temperature, skin-friction are evaluated numerically for different sets of the values of rotation parameter Ω, Hartmann number M, injection/suction parameter λ, Darcian resistance K_r, pressure gradient P, Grashof number Gr, radiation parameter N and the frequency of oscillations ω and these numerical values are then shown graphically to assess the effect of each parameter.

From Equations (31) and (32), it is observed that the steady part of the velocity field has two layer characters. These layers may be identified as the thermal layer arising due to interaction of the thermal field and the velocity field and is controlled by the Prandtl number; and the suction layer as modified by the rotation and the porosity of the medium. On the other hand, the oscillatory part of the velocity field exhibits a two-layer character. These layers may be identified as the modified suction layers, arising as a result of the triangular interaction of the Coriolis force and the unsteady convective forces with the porosity of medium. In all the Figures 2-5, we have investigated the

Table I. Comparison of values of the flow velocity (u) for the present results with Singh (2012) when $Gr=2$, $P=5$, $K_r=0.2$, $\omega=5$, $\lambda=1$, $\Omega=5$, $M=5.0$, $t=1$ and $Pr=0.71$

	Present results			Singh (2012)		
y	$n=1$	$n=5$	$n=10$	$n=1$	$n=5$	$n=10$
−0.5	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
−0.25	0.3483	0.3307	0.3201	0.3497	0.3351	0.3283
0.0	0.5258	0.5032	0.4836	0.5302	0.5131	0.4925
0.25	0.3571	0.3419	0.3280	0.3482	0.3454	0.3307
0.5	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

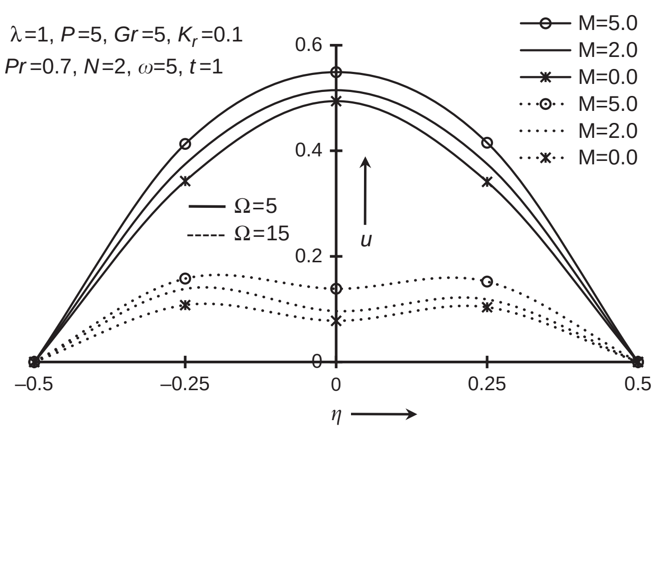

Figure 2. Primary velocity distribution for **M** and **Ω**

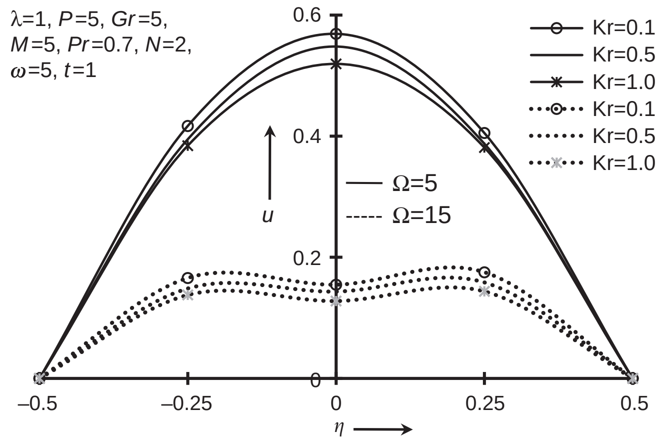

Figure 3.
Primary velocity distribution for K and Ω

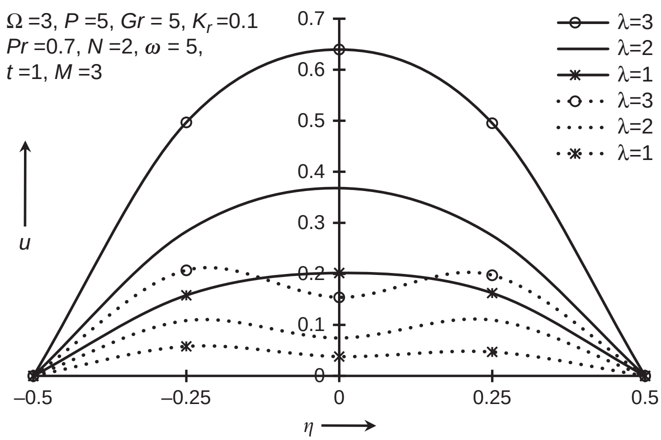

Figure 4.
Primary velocity distribution for λ and Ω

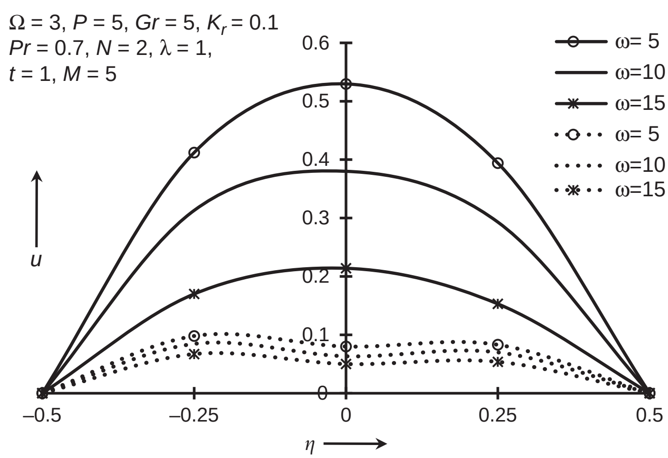

Figure 5.
Primary velocity distribution for ω and Ω

physical behaviour of the primary velocity (u) for two cases of rotation $\Omega = 5$ (small) and $\Omega = 15$ (large). The effects of the magnetic field on the primary velocity field (u) are depicted in Figure 2. With an increase in $\boldsymbol{M}$ from the non-conducting, i.e. purely hydrodynamic case ($\boldsymbol{M} = \mathbf{0}$) through $\mathbf{2.0}$ to $\mathbf{5.0}$ there is a clear increase in velocity, i.e. flow is accelerated. In the momentum Equation (21), the hydromagnetic term, $-\boldsymbol{M}^2\boldsymbol{F}$ deviates from classical magnetohydrodynamic flat-plate boundary layer flow owing to the presence of a negative sign in the magnetic term. The applied magnetic field, $\boldsymbol{B}_0$, is therefore effectively moving with the motion of the flow caused by the rotation of the system. The resulting Lorentizian body force will therefore not act as a drag force as in conventional MHD flows, but as an aiding body force. This will serve to accelerate the flow and boost the primary velocities. This result has also been found by Chamkha and Camille (2000), Seddeek (2004) and Zueco *et al.* (2009). The maximum effect is achieved at intermediate distances from the vertical porous plates into the boundary layer transverse to the wall. Figure 3 indicates the behaviour of primary velocity with the variations in Darcy resistance (Kr). The primary velocity shows a decrement with the increases in Darcy resistance for $Kr = 0.1, 0.5, 1.0$. It is because that the presence of a porous medium increases the resistance to flow and thus reduces the fluid velocity. The variation of the primary velocity profiles with the injection/suction parameter $\boldsymbol{\lambda}$ is presented in Figure 4. For small $\boldsymbol{\Omega}(=\mathbf{5})$ the velocity goes on increasing with increasing $\boldsymbol{\lambda}$ and remains parabolic with maximum at the centerline. However, for large $\boldsymbol{\Omega}(=\mathbf{15})$ although velocity increases with increasing $\boldsymbol{\lambda}$ but the maximum of the velocity shift towards the walls of the channel. The effects of the frequency of oscillations $\boldsymbol{\omega}$ on the velocity are exhibited in Figure 5. It is noticed that velocity decreases with increasing frequency $\boldsymbol{\omega}$ for either case of channel rotation large or small. Figure 6 illustrates the variation of the primary velocity (u). It is quite obvious from this figure that velocity goes on decreasing with increasing rotation $\boldsymbol{\Omega}$ of the entire system. The velocity profiles initially remain parabolic with maximum at the centre of the channel for small values of rotation parameter $\boldsymbol{\Omega}$ and then as rotation increases the velocity profiles flatten. For further increase in $\boldsymbol{\Omega}(=\mathbf{15})$ the maximum of velocity profiles no longer occurs at the centre but shift towards the walls of the channel. It means that for large rotation there arise boundary layers on the walls of the channel.

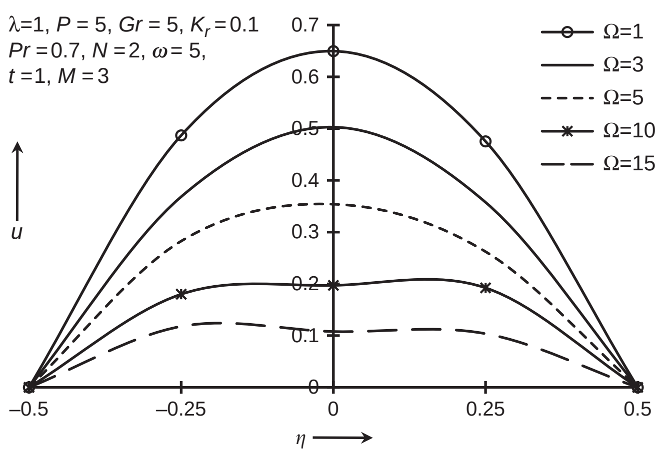

Figure 6.
Primary velocity distribution for Ω

Figure 7 illustrates the transient shear stress variations (τ_L) at the left plate ($\eta = -0.5$) with Hartmann number (M) and Darcian resistance (K_r). As explained earlier, since the applied magnetic field is translating with the flow velocity, it induces an acceleration effect in the flow and thereby primary velocities are increased and the shear stress at the wall ($\eta = -0.5$) will therefore be enhanced with a rise in Hartmann number (M) which is proportional to the magnetic field (B_0). For all the flow cases of M and K_r, a significant flow reversal is sustained with the increasing effect of frequency of oscillation, i.e. shear stresses become negative, i.e. back flow arises. Maximum back flow effect is observed for $K_r = 1$ and $M = 1$ in the Darcian regime. No flow reversal, however, arises at the plate for small frequency of oscillations except for $K_r = 0.1$ and $M = 10$. However, for $M = 5{,}10$, at $K_r = 0.1$, positive shear stresses arise at the plate and back flow effect is still present. Moreover, with the frequency of oscillations, shear stresses are found to reduce, i.e. the flow is retarded. An increase in K_r also strongly reduces the shear stress, in consistency with earlier discussion for the velocity response.

The distribution of shear stresses with rotation parameter (Ω) and thermal radiation (N) is shown in Figure 8 at $t = 1$. Inspection shows that increasing radiation

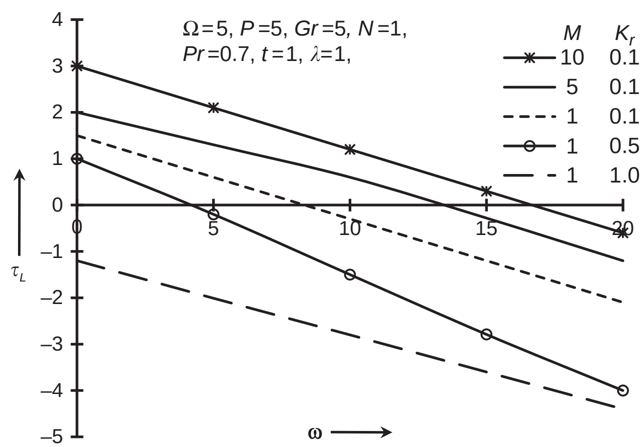

Figure 7.
Skin-friction for M and K at $t = 1$

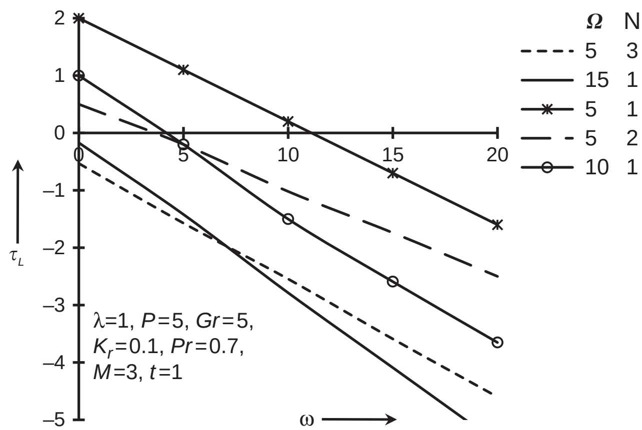

Figure 8.
Skin-friction for $\boldsymbol{\Omega}$ and N at $t = 1$

decelerates the flow, i.e. reduces shear stress, flow reversal is observed, i.e. shear stress becomes negative. Clearly all profiles decay as $\boldsymbol{\omega}$ increases. Increasing $\boldsymbol{N}$ is found also to decrease shear stress very sharply in Darcian regime. The dotted and solid curves show that the shear stresses are attained at maximum negative values due to large rotation ($\boldsymbol{\Omega} = \mathbf{15}$) and radiation ($\boldsymbol{N} = \mathbf{3}$), i.e. back flow is observed to be large.

We note that in all cases $\lambda > 0$ in our computations indicating uniform suction at the wall.

Figure 9 illustrate the effect of suction parameter λ, and radiation parameter $\boldsymbol{N}$ on the temperature distribution (θ) across the vertical channel in Darcian regime. The temperature, θ, is reduced in this Darcian regime with increasing thermal radiation. A strong decrease in temperature accompanies an increase in thermal radiation. For $N = 1$ thermal radiation and thermal conduction contributions are equivalent. For $N > 1$ thermal radiation is dominant over conduction and vice versa. Moreover, the temperature is escalated for the increasing effect of suction/injection parameter and having positive values in the regime for all λ. Due to rotation of the system, negative temperatures have been observed in the region $-0.5 \leqslant \eta \leqslant 0.25$ of the channel corresponding to $N = 5$ and 3. Temperatures are observed to grow strongly only from some distance in the left channel half space ($\eta = -1/2$) and follow a non-linear path to the right channel plate ($\eta = 1/2$). The temperature values are clearly much less for $N = 5$ than for $N = 2$. Therefore, although an initial growth in thermal radiation serves to reduce temperatures; further escalation in the thermal radiation/suction parameter, in fact has a positive effect on the Darcian regime, stabilizes the temperature field and leads to a steep growth in the temperature field to the right channel plate ($\eta = 1/2$). Therefore, the negative bulk temperature occurring in Figure 9 is associated with flow reversal due to the rotation of the vertical channel.

4. Conclusions

The problem of oscillatory magnetohydrodynamic convective and radiative MHD flow for a vertical porous channel in a Darcian porous regime is analysed. The fluid is injected through one of the porous plates and simultaneously removed through the other porous plate with the same velocity. The entire system (consisting of porous channel plates and the fluid) rotates about an axis perpendicular to the plates. The closed form solutions for the velocity and temperature fields are obtained

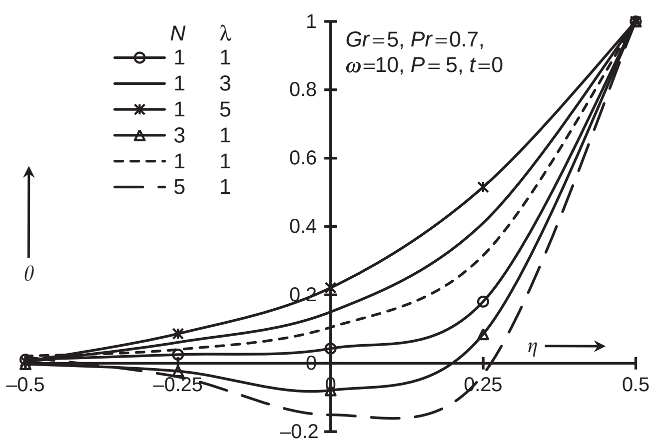

Figure 9.
Temperature distribution for **N** and λ at $t = 0$

analytically and then evaluated numerically for different values of parameters appeared in the equations. The above analysis brings out the following results of physical interest on the velocity, temperature and shear stress profiles of the flow field:

(1) It is immediately apparent that all the profiles for primary velocities are symmetrical about the channel centre line ($\eta = 0$) of the vertical porous channel. Due to small rotation of the channel, the velocity profiles are looking Parabolic, but for large rotation the profiles are seen to be flattered.

(2) Due to rotation of the system in a Darcian regime, temperature is observed to attain at maximum value for $\lambda = 5$, and it has a least value corresponding to $N = 5$ at ($\eta = 1/2$).

(3) It is found that with the increasing rotation of the channel the velocity decreases and the maximum of the parabolic velocity profiles at the centre of the channel shifts towards the walls of the channel.

(4) The Hartmann number due to magnetohydrodynamic flow has the effect of accelerating the primary velocity profiles, and the shear stress.

(5) The velocity increases with the increase of the injection/suction parameter.

(6) The rotation parameter and the frequency parameter have the effect of decreasing the primary velocity profiles as well as the magnitude of the skin-friction.

(7) The Darcian resistance has the influence of decreasing the primary velocity and the primary skin-friction.

(8) Due to large rotation of the channel, the maximum flow reversal is observed in the shear stress.

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