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An adaptive output feedback controller for robot arms: stability and experiments☆

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An adaptive output feedback controller for robot arms is developed in this paper. A nonlinear observer based on desired joint velocities and bounded joint position error is used to estimate joint velocities. Experimental results validate the effectiveness of the proposed adaptive output feedback controller.

Abstract

An adaptive output feedback controller for robot arms is developed in this paper. To estimate the joint velocities, a simple nonlinear observer based on the desired velocity and bounded position tracking error is proposed. The closed-loop system formed by the adaptive controller, observer and the robot system is shown to be semi-global asymptotically stable. Extensive experiments conducted on a two link robot manipulator confirm the effectiveness of the proposed controller–observer structure. To highlight the performance of the proposed scheme, it is compared via experiments with a well-known passivity based control algorithm. © 2001 Elsevier Science Ltd. All rights reserved.

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1. Introduction

Adaptive control of robot arms based on complete state measurements has been dealt in great detail in the literature. The feed-forward and passivity based algorithms for robot arms proposed in Sadegh and Horowitz (1990), Slotine and Li (1991) and Ortega and Spong (1989), and the references therein, have been extensively used. A comparative experimental study of the standard and new algorithms has been done in Whitcomb, Rizzi, and Koditschek (1993). Most of these algorithms need complete state measurements. A major drawback of such schemes is that both joint position and joint velocity measurements of the robot are required for feedback control. Sensors for measuring robot joint velocities are expensive. Further, measurements from these sensors are often contaminated by noise. Velocity estimated feedback control of robot arms can be used instead and the requirement of robots to be equipped with velocity sensors can be eliminated. Most of the robot adaptive schemes use velocity errors or modified velocity errors to drive the parameter adaptation algorithms. When the actual velocities are not available, estimated velocities and position errors have to be used to drive the parameter adaptation algorithms. This leads to an added difficulty in proving the stability of these algorithms.

Considerable research is being conducted in the area of output feedback control of nonlinear systems. Output feedback control of robot arms has been studied by many researchers. In Berghuis and Nijmeijer (1993), the authors consider passivity based controller–observer design for robots. A linear observer is designed to estimate the velocities. It is shown that the closed-loop system formed by the controller–observer and the robot is locally exponentially stable. A linear velocity observer is designed assuming complete knowledge of the structural parameters of the robot. A robust variable structure

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controller and a nonlinear observer is designed in Zhu, Chen, and Zhang (1992). Berghuis and Nijmeijer (1994) proposes a linear controller and a linear observer for robust control in the presence of parameter uncertainties. In Canudas de Wit and Fixot (1992), tracking control of robot manipulators is proposed by combining a passivity based controller and a nonlinear sliding observer. Local asymptotic convergence of the position tracking errors and the velocity estimation errors was shown.

A nonlinear observer based on the robot error dynamics was designed in Nicosia and Tomei (1990), and a control design that uses joint position measurements and estimated velocity is proposed. Repetitive and adaptive control of robot manipulators with velocity estimation is presented in Kaneko and Horowitz (1997). In the case of repetitive control, the robot achieves tracking of the desired periodic trajectory through repeated learning trials. An adaptive controller is also designed. A linear observer is designed to estimate the velocities. Local asymptotic stability is shown for both the repetitive and the adaptive cases.

In this work, an adaptive feedback controller for robot arms is designed using partial state feedback, i.e., only joint position measurements are used to design the adaptive controller. A simple nonlinear observer is designed to estimate the robot joint velocities. The closed-loop system formed by the adaptive controller, observer and the robot system is shown to be semi-global asymptotically stable, i.e., the region of attraction can be increased arbitrarily by increasing the controller and the observer gains.

Convergence of the estimated parameters to the true parameters depends on whether the regressor matrix satisfies the persistence of excitation condition. In the proposed adaptive controller the regressor matrix entirely depends on the desired trajectory. Hence, the persistence of excitation condition is satisfied by choosing a persistently exciting desired trajectory. Experiments were conducted on a two link planar arm for the proposed controller–observer. Successful experimental results show the validity of the proposed controller and observer. The proposed scheme is compared, via experiments, with a well-known passivity based controller. The passivity based controller used for this comparison assumes that the parameters are exactly known and a first-order numerical differentiation of joint position measurements is used to estimate velocities.

The remainder of this paper is organized as follows. In Section 2, robot dynamics and problem formulation is given. Section 3 gives the proposed adaptive controller and observer. Closed-loop error dynamics is also derived in Section 3. Stability of the closed-loop system is shown in Section 4. Section 5 discusses the experimental platform and the experimental results. Some concluding remarks with a summary of this paper are given in Section 6.

2. Robot dynamics and problem formulation

Consider the dynamics of an n degree of freedom robot arm

$$\dot{\mathbf{x}}_1 = \mathbf{x}_2,$$

$$\mathbf{M}(\mathbf{x}_1)\dot{\mathbf{x}}_2 + \mathbf{C}(\mathbf{x}_1, \mathbf{x}_2)\mathbf{x}_2 + \mathbf{g}(\mathbf{x}_1) = \boldsymbol{\tau}, \tag{1}$$

where $\mathbf{x}_1 \in \mathbb{R}^n$, $\mathbf{x}_2 \in \mathbb{R}^n$ are the generalized position and velocity, respectively, $\mathbf{M}(\mathbf{x}_1) \in \mathbb{R}^{n\times n}$ is the inertia matrix, $\mathbf{C}(\mathbf{x}_1, \mathbf{x}_2) \in \mathbb{R}^{n\times n}$ is the matrix composed of Coriolis and centrifugal terms, $\mathbf{g}(\mathbf{x}_1) \in \mathbb{R}^n$ is the gravity vector, and $\boldsymbol{\tau} \in \mathbb{R}^n$ is the vector composed of joint torques. The structure of the robot dynamics satisfies the properties given in Appendix A.

Given a desired trajectory of the robot, the objective is to design a stable tracking controller that only requires joint position measurements for feedback. To achieve this objective, an adaptive controller together with a simple nonlinear observer to estimate joint velocities is proposed. Let $\mathbf{x}_1^{\mathrm{d}}(t)$ and $\mathbf{x}_2^{\mathrm{d}}(t)$ be the desired position and velocity, respectively. It is assumed that the desired state trajectory is twice continuously differentiable. Let $\hat{\mathbf{x}}_1(t)$ and $\hat{\mathbf{x}}_2(t)$ denote the estimated position and estimated velocity, respectively. Let $\boldsymbol{\theta} \in \mathbb{R}^p$ denote the actual parameter vector as given by property (iv) in Appendix A. Let $\hat{\boldsymbol{\theta}}(t)$ denote the estimate of $\boldsymbol{\theta}$. Define the tracking and the estimation errors by

$$\mathbf{e}_1(t) := \mathbf{x}_1(t) - \mathbf{x}_1^{\mathrm{d}}(t), \qquad \hat{\mathbf{e}}_1(t) := \hat{\mathbf{x}}_1(t) - \mathbf{x}_1(t),$$

$$\mathbf{e}_2(t) := \mathbf{x}_2(t) - \mathbf{x}_2^{\mathrm{d}}(t), \qquad \hat{\mathbf{e}}_2(t) := \hat{\mathbf{x}}_2(t) - \mathbf{x}_2(t),$$

$$\mathbf{e}_v(t) := \mathbf{e}_2(t) + \mathbf{e}_c(t), \qquad \tilde{\boldsymbol{\theta}}(t) := \hat{\boldsymbol{\theta}}(t) - \boldsymbol{\theta},$$

$$\mathbf{e}_c(t) := \boldsymbol{\Lambda}_c(\mathbf{e}_1)\mathbf{e}_1(t),$$

where $\mathbf{e}_1(t)$ and $\mathbf{e}_2(t)$ are the position and velocity tracking errors, respectively, $\hat{\mathbf{e}}_1(t)$ and $\hat{\mathbf{e}}_2(t)$ are the estimated position and estimated velocity errors, respectively, $\mathbf{e}_v(t)$ is the reference velocity error, $\tilde{\boldsymbol{\theta}}(t)$ is the parameter estimation error, $\mathbf{e}_c(t)$ is an auxiliary bounded position tracking error, and $\boldsymbol{\Lambda}_c(\mathbf{e}_1(t))$ is a positive definite diagonal matrix given by

$$\boldsymbol{\Lambda}_c(\mathbf{e}_1(t)) = \mathrm{diag}\left(\frac{\lambda_c}{1 + |e_{11}(t)|}, \ldots, \frac{\lambda_c}{1 + |e_{1n}(t)|}\right), \tag{2}$$

where $e_{11}(t), \ldots, e_{1n}(t)$ are the components of the position error vector $\mathbf{e}_1(t)$ and λ_c is a positive gain. Notice that this choice of $\boldsymbol{\Lambda}_c(\mathbf{e}_1)$ renders $\mathbf{e}_c(t)$ to be bounded by λ_c. In the remainder of the paper, whenever it is clear from the context, explicit dependence of variables on time is not shown. Also, throughout the paper $\|A\|$ denotes the 2-norm of A. The following section gives the adaptive controller, observer, and the closed-loop error dynamics.

3. Adaptive controller and observer

The following control scheme is proposed:

$$\boldsymbol{\tau} = \mathbf{Y}_{\mathrm{d}}(\mathbf{x}_1^{\mathrm{d}}, \mathbf{x}_2^{\mathrm{d}}, \dot{\mathbf{x}}_2^{\mathrm{d}})\hat{\boldsymbol{\theta}} - \mathbf{K}_{\mathrm{d}}(\mathbf{e}_v + \hat{\mathbf{e}}_2) - \mathbf{K}_{\mathrm{p}}\mathbf{e}_1, \quad (3)$$

where $\mathbf{K}_{\mathrm{d}}$, $\mathbf{K}_{\mathrm{p}}$ are positive definite gain matrices and $\hat{\boldsymbol{\theta}}$ is the estimated parameter vector of the robot. Note that the second term in the control law is a function of estimated velocity, desired velocity, and actual position error, i.e., $\mathbf{e}_v + \hat{\mathbf{e}}_2 = \hat{\mathbf{x}}_2 - \mathbf{x}_2^{\mathrm{d}} + \mathbf{e}_c$. The desired regressor matrix, $\mathbf{Y}_{\mathrm{d}}(\mathbf{x}_1^{\mathrm{d}}, \mathbf{x}_2^{\mathrm{d}}, \dot{\mathbf{x}}_2^{\mathrm{d}})$, is given by

$$\mathbf{Y}_{\mathrm{d}}(\mathbf{x}_1^{\mathrm{d}}, \mathbf{x}_2^{\mathrm{d}}, \dot{\mathbf{x}}_2^{\mathrm{d}})\hat{\boldsymbol{\theta}} = \hat{\mathbf{M}}(\mathbf{x}_1^{\mathrm{d}})\dot{\mathbf{x}}_2^{\mathrm{d}} + \hat{\mathbf{C}}(\mathbf{x}_1^{\mathrm{d}}, \mathbf{x}_2^{\mathrm{d}})\mathbf{x}_2^{\mathrm{d}} + \hat{\mathbf{g}}(\mathbf{x}_1^{\mathrm{d}}),$$

where $\hat{\mathbf{M}}(\mathbf{x}_1^{\mathrm{d}})$, $\hat{\mathbf{C}}(\mathbf{x}_1^{\mathrm{d}}, \mathbf{x}_2^{\mathrm{d}})$, and $\hat{\mathbf{g}}(\mathbf{x}_1^{\mathrm{d}})$ are the estimates of $\mathbf{M}(\mathbf{x}_1)$, $\mathbf{C}(\mathbf{x}_1,\mathbf{x}_2)$, and $\mathbf{g}(\mathbf{x}_1)$, respectively. The desired regressor matrix depends only on the desired trajectory and can be pre-computed. The parameter adaptation law is chosen as follows:

$$\hat{\boldsymbol{\theta}}(t) = \hat{\boldsymbol{\theta}}(0) - \boldsymbol{\Gamma}\left\{\mathbf{Y}_{\mathrm{d}}^{\mathrm{T}}\mathbf{e}_{\mathrm{r}}(t) - \int_0^t \dot{\mathbf{Y}}_{\mathrm{d}}^{\mathrm{T}}\mathbf{e}_r(\omega)\,\mathrm{d}\omega\right\}, \quad (4)$$

where $\hat{\boldsymbol{\theta}}(0)$ is the initial estimate of the unknown parameter vector, $\boldsymbol{\Gamma}$ is a positive definite gain matrix, and $\mathbf{e}_{\mathrm{r}}$ is given by

$$\mathbf{e}_r(t) = \mathbf{e}_1(t) - \hat{\mathbf{e}}_1(t) + \int_0^t \mathbf{e}_c(\omega)\,\mathrm{d}\omega - \eta_1\int_0^t \hat{\mathbf{e}}_1(\omega)\,\mathrm{d}\omega.$$

The following observer is proposed to estimate the states:

$$\dot{\hat{\mathbf{x}}}_1 = -\eta_1\hat{\mathbf{e}}_1 + \hat{\mathbf{x}}_2, \quad (5)$$

$$\hat{\mathbf{x}}_2 = \mathbf{x}_2^{\mathrm{d}} - \eta_2\hat{\mathbf{e}}_1 + \mathbf{e}_c, \quad (6)$$

where η_1 and η_2 are positive gains. Thus, by rearranging terms, the observer error dynamics is given by

$$\begin{aligned} \dot{\hat{\mathbf{e}}}_1 &= -\eta_1\hat{\mathbf{e}}_1 + \hat{\mathbf{e}}_2, \\ \dot{\hat{\mathbf{e}}}_2 &= -\eta_2\hat{\mathbf{e}}_2 + \eta_1\eta_2\hat{\mathbf{e}}_1 + 2\dot{\mathbf{e}}_c - \dot{\mathbf{e}}_v. \end{aligned} \quad (7)$$

The closed-loop error dynamics is derived in the following section.

3.1. Error dynamics

Noting that $\dot{\mathbf{e}}_v = \dot{\mathbf{e}}_2 + \dot{\mathbf{e}}_c$ and $-\mathbf{e}_v + \mathbf{e}_2 + \mathbf{e}_c = 0$, $\mathbf{M}(\mathbf{x}_1)\dot{\mathbf{e}}_v$ can be expressed as

$$\begin{aligned} \mathbf{M}(\mathbf{x}_1)\dot{\mathbf{e}}_v = &-\mathbf{C}(\mathbf{x}_1,\mathbf{x}_2)\mathbf{e}_v + \mathbf{M}(\mathbf{x}_1)\dot{\mathbf{e}}_2 + \mathbf{C}(\mathbf{x}_1,\mathbf{x}_2)\mathbf{e}_2 \\ &- \mathbf{g}(\mathbf{x}_1) + \mathbf{M}(\mathbf{x}_1)\dot{\mathbf{e}}_c + \mathbf{C}(\mathbf{x}_1,\mathbf{x}_2)\mathbf{e}_c. \end{aligned} \quad (8)$$

From (1) and (8), we obtain

$$\begin{aligned} \mathbf{M}(\mathbf{x}_1)\dot{\mathbf{e}}_v = &-\mathbf{C}(\mathbf{x}_1,\mathbf{x}_2)\mathbf{e}_v + \boldsymbol{\tau} - \mathbf{M}(\mathbf{x}_1)\dot{\mathbf{x}}_2^{\mathrm{d}} \\ &+ \mathbf{C}(\mathbf{x}_1,\mathbf{x}_2)\mathbf{x}_2^{\mathrm{d}} + \mathbf{M}(\mathbf{x}_1)\dot{\mathbf{e}}_c + \mathbf{C}(\mathbf{x}_1,\mathbf{x}_2)\mathbf{e}_c. \end{aligned} \quad (9)$$

Using the control law and noting that $\dot{\mathbf{e}}_c = \mathbf{E}'_c\mathbf{e}_2$, where $\mathbf{E}'_c = \lambda_c^{-1}\boldsymbol{\Lambda}_c^2$, the error equation is

$$\begin{aligned} \mathbf{M}(\mathbf{x}_1)\dot{\mathbf{e}}_v = &-\mathbf{C}(\mathbf{x}_1,\mathbf{x}_2)\mathbf{e}_v + \mathbf{Y}_{\mathrm{d}}\tilde{\boldsymbol{\theta}} - \Delta\mathbf{W} - \mathbf{K}_{\mathrm{d}}(\mathbf{e}_v + \hat{\mathbf{e}}_2) \\ &- \mathbf{K}_{\mathrm{p}}\mathbf{e}_1 + \mathbf{M}(\mathbf{x}_1)\mathbf{E}'_c\mathbf{e}_v - \mathbf{M}(\mathbf{x}_1)\mathbf{E}'_c\mathbf{e}_c, \end{aligned} \quad (10)$$

where $\Delta\mathbf{W}$ is given by

$$\begin{aligned} \Delta\mathbf{W} = &[\mathbf{M}(\mathbf{x}_1) - \mathbf{M}(\mathbf{x}_1^{\mathrm{d}})]\dot{\mathbf{x}}_2^{\mathrm{d}} + \mathbf{g}(\mathbf{x}_1) - \mathbf{g}(\mathbf{x}_1^{\mathrm{d}}) \\ &+ [\mathbf{C}(\mathbf{x}_1,\mathbf{x}_2)(\mathbf{x}_2^{\mathrm{d}} - \mathbf{e}_c) - \mathbf{C}(\mathbf{x}_1^{\mathrm{d}},\mathbf{x}_2^{\mathrm{d}})\mathbf{x}_2^{\mathrm{d}}]. \end{aligned}$$

Using Eq. (7), the observer error equation can be derived as follows:

$$\begin{aligned} \mathbf{M}(\mathbf{x}_1)\dot{\hat{\mathbf{e}}}_2 = &-\eta_2\mathbf{M}(\mathbf{x}_1)\hat{\mathbf{e}}_2 + \eta_1\eta_2\mathbf{M}(\mathbf{x}_1)\hat{\mathbf{e}}_1 \\ &+ 2\mathbf{M}(\mathbf{x}_1)\dot{\mathbf{e}}_c - \mathbf{M}(\mathbf{x}_1)\dot{\mathbf{e}}_v \\ = &-\mathbf{C}(\mathbf{x}_1,\mathbf{x}_2)\hat{\mathbf{e}}_2 - \{\mathbf{M}(\mathbf{x}_1)\dot{\mathbf{e}}_v + \mathbf{C}(\mathbf{x}_1,\mathbf{x}_2)\mathbf{e}_v\} \\ &- \eta_2\mathbf{M}(\mathbf{x}_1)\hat{\mathbf{e}}_2 + \mathbf{C}(\mathbf{x}_1,\mathbf{x}_2)(\hat{\mathbf{e}}_2 + \mathbf{e}_v) \\ &+ 2\mathbf{M}(\mathbf{x}_1)\dot{\mathbf{e}}_c + \eta_1\eta_2\mathbf{M}(\mathbf{x}_1)\hat{\mathbf{e}}_1. \end{aligned} \quad (11)$$

On substitution of the robot error dynamics (10) and using $\mathbf{x}_2 = \mathbf{e}_v - \mathbf{e}_c + \mathbf{x}_2^{\mathrm{d}}$, we obtain

$$\begin{aligned} &\mathbf{M}(\mathbf{x}_1)\dot{\hat{\mathbf{e}}}_2 \\ &= -\mathbf{C}(\mathbf{x}_1,\mathbf{x}_2)\hat{\mathbf{e}}_2 - \mathbf{Y}_{\mathrm{d}}\tilde{\boldsymbol{\theta}} + \Delta\mathbf{W} + \mathbf{K}_{\mathrm{d}}(\mathbf{e}_v + \hat{\mathbf{e}}_2) + \mathbf{K}_{\mathrm{p}}\mathbf{e}_1 \\ &\quad + \mathbf{M}(\mathbf{x}_1)\mathbf{E}'_c\mathbf{e}_v - \mathbf{M}(\mathbf{x}_1)\mathbf{E}'_c\mathbf{e}_c - \eta_2\mathbf{M}(\mathbf{x}_1)\hat{\mathbf{e}}_2 \\ &\quad + \eta_1\eta_2\mathbf{M}(\mathbf{x}_1)\hat{\mathbf{e}}_1 + \{\mathbf{C}(\mathbf{x}_1,\mathbf{e}_v) - \mathbf{C}(\mathbf{x}_1,\mathbf{e}_c)\}(\hat{\mathbf{e}}_2 + \mathbf{e}_v) \\ &\quad + \mathbf{C}(\mathbf{x}_1,\mathbf{x}_2^{\mathrm{d}})(\hat{\mathbf{e}}_2 + \mathbf{e}_v). \end{aligned} \quad (12)$$

4. Stability

First, define an extended vector $\mathbf{z}$ given by $\mathbf{z}^{\mathrm{T}} := [\mathbf{e}_1^{\mathrm{T}}, \mathbf{e}_v^{\mathrm{T}}, \hat{\mathbf{e}}_1^{\mathrm{T}}, \hat{\mathbf{e}}_2^{\mathrm{T}}, \tilde{\boldsymbol{\theta}}^{\mathrm{T}}]$. The following theorem gives the stability of the closed-loop system with the proposed controller–observer structure.

Theorem 1. *For the robot dynamics given in* (1), *using the adaptive controller* (3) *together with the update law* (4) *and the observer* (6), *it is always possible to choose feedback gains* $\mathbf{K}_{\mathrm{p}}$, $\mathbf{K}_{\mathrm{d}}$ *and* λ_c *and the observer gains* η_1 *and* η_2 *such that* $\mathbf{z} = 0$ *is locally uniformly stable*, $\mathbf{e}_v$, $\mathbf{e}_1$, $\hat{\mathbf{e}}_1$ *and* $\hat{\mathbf{e}}_2$ *locally asymptotically converge to zero. Further, the closed-loop system is semi-globally asymptotically stable, i.e., the region of attraction can be arbitrarily increased by increasing the controller and the observer gains.*

Proof. Consider the following Lyapunov function candidates, $V_c(\mathbf{e}_v, \mathbf{e}_1, \tilde{\boldsymbol{\theta}})$ for the robot error equation (10) and

$V_o(\hat{\mathbf{e}}_1, \hat{\mathbf{e}}_2)$ for observer error equation (12):

$$V_c(\mathbf{e}_v, \mathbf{e}_1, \tilde{\boldsymbol{\theta}}) = \tfrac{1}{2}\{\mathbf{e}_v^{\mathrm{T}}\mathbf{M}(\mathbf{x}_1)\mathbf{e}_v + \mathbf{e}_1^{\mathrm{T}}\mathbf{K}_{\mathrm{p}}\mathbf{e}_1 + \tilde{\boldsymbol{\theta}}^{\mathrm{T}}\boldsymbol{\Gamma}^{-1}\tilde{\boldsymbol{\theta}}\}, \quad (13)$$

$$V_o(\hat{\mathbf{e}}_1, \hat{\mathbf{e}}_2) = \tfrac{1}{2}\{\hat{\mathbf{e}}_2^{\mathrm{T}}\mathbf{M}(\mathbf{x}_1)\hat{\mathbf{e}}_2 + \eta_1\eta_2\sigma_{\mathrm{M}}\hat{\mathbf{e}}_1^{\mathrm{T}}\hat{\mathbf{e}}_1\}. \quad (14)$$

Taking the time derivative of these Lyapunov function candidates along the trajectories of (10) and (12) and simplifying gives

$$\begin{aligned}\dot{V}_c &= \mathbf{e}_v^{\mathrm{T}}\mathbf{Y}_{\mathrm{d}}\tilde{\boldsymbol{\theta}} - \mathbf{e}_v^{\mathrm{T}}\Delta\mathbf{W} - \mathbf{e}_v^{\mathrm{T}}\mathbf{K}_{\mathrm{d}}(\mathbf{e}_v + \hat{\mathbf{e}}_2) + \tilde{\boldsymbol{\theta}}^{\mathrm{T}}\boldsymbol{\Gamma}^{-1}\dot{\tilde{\boldsymbol{\theta}}} \\ &\quad + \mathbf{e}_v^{\mathrm{T}}\mathbf{M}(\mathbf{x}_1)\mathbf{E}'_c\mathbf{e}_v - \mathbf{e}_v^{\mathrm{T}}\mathbf{M}(\mathbf{x}_1)\mathbf{E}'_c\mathbf{e}_c \\ &\quad + (\mathbf{e}_1^{\mathrm{T}}\mathbf{K}_{\mathrm{p}}\mathbf{e}_2 - \mathbf{e}_v^{\mathrm{T}}\mathbf{K}_{\mathrm{p}}\mathbf{e}_1), \quad (15)\\ \dot{V}_o &= -\hat{\mathbf{e}}_2^{\mathrm{T}}\mathbf{Y}_{\mathrm{d}}\tilde{\boldsymbol{\theta}} + \hat{\mathbf{e}}_2^{\mathrm{T}}\Delta\mathbf{W} + \hat{\mathbf{e}}_2^{\mathrm{T}}\mathbf{K}_{\mathrm{d}}(\mathbf{e}_v + \hat{\mathbf{e}}_2) \\ &\quad + \hat{\mathbf{e}}_2^{\mathrm{T}}\mathbf{K}_{\mathrm{p}}\mathbf{e}_1 - \eta_1^2\eta_2\sigma_{\mathrm{M}}\hat{\mathbf{e}}_1^{\mathrm{T}}\hat{\mathbf{e}}_1 + \hat{\mathbf{e}}_2^{\mathrm{T}}\mathbf{M}(\mathbf{x}_1)\mathbf{E}'_c\mathbf{e}_v \\ &\quad - \hat{\mathbf{e}}_2^{\mathrm{T}}\mathbf{M}(\mathbf{x}_1)\mathbf{E}'_c\mathbf{e}_c - \eta_2\hat{\mathbf{e}}_2^{\mathrm{T}}\mathbf{M}(\mathbf{x}_1)\hat{\mathbf{e}}_2 + \eta_1\eta_2\sigma_{\mathrm{M}}\hat{\mathbf{e}}_1^{\mathrm{T}}\hat{\mathbf{e}}_2 \\ &\quad + \hat{\mathbf{e}}_2^{\mathrm{T}}\{\mathbf{C}(\mathbf{x}_1,\mathbf{e}_v) - \mathbf{C}(\mathbf{x}_1,\mathbf{e}_c) + \mathbf{C}(\mathbf{x}_1,\mathbf{x}_2^{\mathrm{d}})\}(\hat{\mathbf{e}}_2 + \mathbf{e}_v). \quad (16)\end{aligned}$$

Now, consider the composite Lyapunov function candidate $V = V_c + V_o$ for the closed-loop system. Using the parameter adaptation law given in (4), $\dot{V}$ is given by

$$\begin{aligned}\dot{V} &= -\mathbf{e}_v^{\mathrm{T}}\mathbf{K}_{\mathrm{d}}\mathbf{e}_v - \eta_2\hat{\mathbf{e}}_2^{\mathrm{T}}\mathbf{M}(\mathbf{x}_1)\hat{\mathbf{e}}_2 + \hat{\mathbf{e}}_2^{\mathrm{T}}\mathbf{K}_{\mathrm{d}}\hat{\mathbf{e}}_2 - \mathbf{e}_1^{\mathrm{T}}\mathbf{K}_{\mathrm{p}}\mathbf{e}_c \\ &\quad - \eta_1^2\eta_2\sigma_{\mathrm{M}}\hat{\mathbf{e}}_1^{\mathrm{T}}\hat{\mathbf{e}}_1 + \hat{\mathbf{e}}_2^{\mathrm{T}}\mathbf{M}(\mathbf{x}_1)\mathbf{E}'_c\mathbf{e}_v - \mathbf{e}_v^{\mathrm{T}}\mathbf{M}(\mathbf{x}_1)\mathbf{E}'_c\mathbf{e}_c \\ &\quad - \hat{\mathbf{e}}_2^{\mathrm{T}}\mathbf{M}(\mathbf{x}_1)\mathbf{E}'_c\mathbf{e}_c + \hat{\mathbf{e}}_2^{\mathrm{T}}\mathbf{K}_{\mathrm{p}}\mathbf{e}_1 - (\mathbf{e}_v^{\mathrm{T}} - \hat{\mathbf{e}}_2)\Delta\mathbf{W} \\ &\quad + \mathbf{e}_v^{\mathrm{T}}\mathbf{M}(\mathbf{x}_1)\mathbf{E}'_c\mathbf{e}_v + \eta_1\eta_2\sigma_{\mathrm{M}}\hat{\mathbf{e}}_1^{\mathrm{T}}\hat{\mathbf{e}}_2 + \hat{\mathbf{e}}_2^{\mathrm{T}}\{\mathbf{C}(\mathbf{x}_1,\mathbf{e}_v) \\ &\quad - \mathbf{C}(\mathbf{x}_1,\mathbf{e}_c) + \mathbf{C}(\mathbf{x}_1,\mathbf{x}_2^{\mathrm{d}})\}(\hat{\mathbf{e}}_2 + \mathbf{e}_v). \quad (17)\end{aligned}$$

Noting that $\|\mathbf{E}'_c\| \leqslant \lambda_c$, $\|\mathbf{e}_c\| \leqslant \lambda_c\|\mathbf{e}_1\|$, and using the properties given in Appendix A results in

$$\begin{aligned}\dot{V} &\leqslant -k_{\mathrm{dm}}\|\mathbf{e}_v\|^2 - (\eta\sigma_{\mathrm{M}} - k_{\mathrm{dM}})\|\hat{\mathbf{e}}_2\|^2 \\ &\quad + k_{\mathrm{pM}}\|\mathbf{e}_1\|\,\|\hat{\mathbf{e}}_2\| + \eta_1\eta_2\sigma_{\mathrm{M}}\|\hat{\mathbf{e}}_1\|\,\|\hat{\mathbf{e}}_2\| \\ &\quad + \sigma_{\mathrm{M}}\lambda_c\|\mathbf{e}_v\|^2 + c_{\mathrm{m}}v_{\mathrm{d}}\|\hat{\mathbf{e}}_2\|^2 + \sigma_{\mathrm{M}}\lambda_c^2\|\mathbf{e}_1\|\,\|\mathbf{e}_v\| \\ &\quad + \sigma_{\mathrm{M}}\lambda_c\|\mathbf{e}_v\|\,\|\hat{\mathbf{e}}_2\| + \sigma_{\mathrm{M}}\lambda_c^2\|\mathbf{e}_1\|\,\|\hat{\mathbf{e}}_2\| \\ &\quad + c_{\mathrm{m}}\|\mathbf{e}_v\|\,\|\hat{\mathbf{e}}_2\|^2 + c_{\mathrm{m}}\lambda_c\|\hat{\mathbf{e}}_2\|^2 + c_{\mathrm{m}}\|\mathbf{e}_v\|^2\|\hat{\mathbf{e}}_2\| \\ &\quad + c_{\mathrm{m}}\lambda_c\|\mathbf{e}_v\|\,\|\hat{\mathbf{e}}_2\| + c_{\mathrm{m}}v_{\mathrm{d}}\|\mathbf{e}_v\|\,\|\hat{\mathbf{e}}_2\| \\ &\quad + c_{\mathrm{m}}v_{\mathrm{d}}\|\mathbf{e}_v\|\,\|\hat{\mathbf{e}}_2\|\,\|\mathbf{e}_1\| \\ &\quad + (\|\mathbf{e}_v\| + \|\hat{\mathbf{e}}_2\|)\|\Delta\mathbf{W}\| \\ &\quad - k_{\mathrm{pm}}\lambda_{\mathrm{cm}}(\mathbf{e}_1)\|\mathbf{e}_1\|^2 - \eta_1^2\eta_2\sigma_{\mathrm{M}}\|\hat{\mathbf{e}}_1\|^2, \quad (18)\end{aligned}$$

where k_{pm} and k_{pM} are the smallest and the largest eigenvalues, respectively, of $\mathbf{K}_{\mathrm{p}}$, k_{dm} and k_{dM} are the smallest and the largest eigenvalues, respectively, of $\mathbf{K}_{\mathrm{d}}$, and $\lambda_{\mathrm{cm}}(\mathbf{e}_1)$ is the smallest eigenvalue of the time-varying matrix $\boldsymbol{\Lambda}_c(\mathbf{e}_1)$, which can be lower bounded by

$$\lambda_{\mathrm{cm}}(\mathbf{e}_1) := \min\left\{\frac{\lambda_c}{1 + |e_{11}|}, \ldots, \frac{\lambda_c}{1 + |e_{1n}|}\right\} \geqslant \frac{\lambda_c}{1 + \|\mathbf{e}_1\|}.$$

Using the upper bound on $\|\Delta\mathbf{W}\|$, derived in Appendix B, and grouping similar terms together, the time derivative of V can be bounded by

$$\begin{aligned}\dot{V} &\leqslant -(k_{\mathrm{dm}} - \sigma_{\mathrm{M}}\lambda_c - c_{\mathrm{m}}v_{\mathrm{d}} - c_{\mathrm{m}}\lambda_c - c_{\mathrm{m}}\|\hat{\mathbf{e}}_2\| \\ &\quad - c_{\mathrm{m}}v_{\mathrm{d}}\|\mathbf{e}_1\|)\|\mathbf{e}_v\|^2 - (\eta_2\sigma_{\mathrm{m}} - k_{\mathrm{dM}} - c_{\mathrm{m}}v_{\mathrm{d}} - c_{\mathrm{m}}\lambda_c \\ &\quad - c_{\mathrm{m}}\|\mathbf{e}_v\| - c_{\mathrm{m}}v_{\mathrm{d}}\|\mathbf{e}_1\|)\|\hat{\mathbf{e}}_2\|^2 \\ &\quad - \frac{k_{\mathrm{pm}}\lambda_c}{1 + \|\mathbf{e}_1\|}\|\mathbf{e}_1\|^2 - \eta_1^2\eta_2\sigma_{\mathrm{M}}\|\hat{\mathbf{e}}_1\|^2 \\ &\quad + (\sigma_{\mathrm{M}}\lambda_c + 2c_{\mathrm{m}}\lambda_c + 2c_{\mathrm{m}}v_{\mathrm{d}})\|\mathbf{e}_v\|\,\|\hat{\mathbf{e}}_2\| \\ &\quad + (k_{\mathrm{pM}} + \sigma_{\mathrm{M}}\lambda_c^2 + 2c_{\mathrm{m}}v_{\mathrm{d}}\lambda_c + c_{\mathrm{m}}\lambda_c^2)\|\mathbf{e}_1\|\,\|\hat{\mathbf{e}}_2\| \\ &\quad + (\sigma_{\mathrm{M}}\lambda_c^2 + 2c_{\mathrm{m}}v_{\mathrm{d}}\lambda_c + c_{\mathrm{m}}\lambda_c^2)\|\mathbf{e}_1\|\,\|\mathbf{e}_v\| \\ &\quad + \eta_1\eta_2\sigma_{\mathrm{M}}\|\hat{\mathbf{e}}_1\|\,\|\hat{\mathbf{e}}_2\|, \quad (19)\end{aligned}$$

where the inequality $abc \leqslant ab^2 + ac^2$ has been used with $a = \|\mathbf{e}_1\|$, $b = \|\hat{\mathbf{e}}_2\|$, $c = \|\mathbf{e}_v\|$. By choosing $\mathbf{K}_{\mathrm{d}}$, $\mathbf{K}_{\mathrm{p}}$, η_1, and η_2 such that

$$k_{\mathrm{dm}} \geqslant (\alpha_1^2 + \alpha_2^2 + \alpha_3^2 + \sigma_{\mathrm{M}}\lambda_c + c_{\mathrm{m}}v_{\mathrm{d}} + c_{\mathrm{m}}\lambda_c), \quad (20)$$

$$\eta_2 \geqslant \frac{1}{\sigma_{\mathrm{M}}}(\beta_1^2 + \beta_2^2 + \beta_3^2 + \beta_4^2 + k_{\mathrm{dM}} + c_{\mathrm{m}}v_{\mathrm{d}} + c_{\mathrm{m}}\lambda_c), \quad (21)$$

$$k_{\mathrm{pm}} \geqslant \frac{1}{\lambda_c}(\gamma_1^2 + \gamma_2^2 + \gamma_3^2), \quad (22)$$

$$\eta_1^2 \geqslant \frac{1}{\eta_2\sigma_{\mathrm{M}}}(\zeta_1^2 + \zeta_2^2) \quad (23)$$

the time derivative of V can be bounded by

$$\begin{aligned}\dot{V} &\leqslant -(\alpha_1^2 + \alpha_2^2 + \alpha_3^2 - c_{\mathrm{m}}\|\hat{\mathbf{e}}_2\| - c_{\mathrm{m}}v_{\mathrm{d}}\mathbf{e}_1)\|\mathbf{e}_v\|^2 \\ &\quad - (\beta_1^2 + \beta_2^2 + \beta_3^2 + \beta_4^2 - c_{\mathrm{m}}\|\mathbf{e}_v\| - c_{\mathrm{m}}v_{\mathrm{d}}\mathbf{e}_1)\|\hat{\mathbf{e}}_2\|^2 \\ &\quad - (\gamma_1^2 + \gamma_2^2 + \gamma_3^2)\frac{\|\mathbf{e}_1\|^2}{1 + \|\mathbf{e}_1\|} - (\zeta_1^2 + \zeta_2^2)\|\hat{\mathbf{e}}_1\|^2 \\ &\quad + \xi_1\|\mathbf{e}_v\|\,\|\hat{\mathbf{e}}_2\| + \xi_2\|\mathbf{e}_1\|\,\|\mathbf{e}_v\| \\ &\quad + \xi_3\|\mathbf{e}_1\|\,\|\hat{\mathbf{e}}_2\| + \xi_4\|\hat{\mathbf{e}}_1\|\,\|\hat{\mathbf{e}}_2\|, \quad (24)\end{aligned}$$

where $\xi_1 := (\sigma_{\mathrm{M}}\lambda_c + 2c_{\mathrm{m}}\lambda_c + 2c_{\mathrm{m}}v_{\mathrm{d}})$, $\xi_2 := (\sigma_{\mathrm{M}}\lambda_c^2 + b_{\mathrm{m}} + b_c + b_g + 2c_{\mathrm{m}}v_{\mathrm{d}}\lambda_c + c_{\mathrm{m}}\lambda_c^2)$, $\xi_3 := (\sigma_{\mathrm{M}}\lambda_c^2 + k_{\mathrm{pM}} + b_{\mathrm{m}} + b_c + b_g + 2c_{\mathrm{m}}v_{\mathrm{d}}\lambda_c + c_{\mathrm{m}}\lambda_c^2)$, and $\xi_4 := \eta_1\eta_2\sigma_{\mathrm{M}}$. Notice that the positive constants α_i's, β_i's, γ_i's, and ζ_i's can be chosen arbitrarily by choosing k_{dm}, η_2, k_{pm}, and η_1. Next, $\dot{V}$ is shown to be locally negative semi-definite by choosing the constants α_i's, β_i's, γ_i's, and ζ_i's. Choose α_2, β_2 such that $2\alpha_2\beta_2 \geqslant \xi_1$, hence

$$-\alpha_2^2\|\mathbf{e}_v\|^2 + \xi_1\|\mathbf{e}_v\|\,\|\hat{\mathbf{e}}_2\| - \beta_2^2\|\hat{\mathbf{e}}_2\|^2$$
$$= -(\alpha_2\|\mathbf{e}_v\| - \beta_2\|\hat{\mathbf{e}}_2\|)^2 - (2\alpha_2\beta_2 - \xi_1)\|\mathbf{e}_v\|\,\|\hat{\mathbf{e}}_2\| \leqslant 0.$$

Similarly, choose β_4 and ζ_2 such that $2\beta_4\zeta_2 \geqslant \xi_4$, hence

$$-\beta_4^2\|\hat{\mathbf{e}}_2\|^2 + \xi_4\|\hat{\mathbf{e}}_1\|\,\|\hat{\mathbf{e}}_2\| - \zeta_2^2\|\hat{\mathbf{e}}_1\|^2$$
$$= -(\zeta_4\|\hat{\mathbf{e}}_1\| - \beta_4\|\hat{\mathbf{e}}_2\|)^2 - (2\zeta_2\beta_4 - \xi_4)\|\mathbf{e}_v\|\,\|\hat{\mathbf{e}}_2\| \leqslant 0.$$

Consider the following:

$$-\alpha_3^2\|\mathbf{e}_v\|^2 + \xi_2\|\mathbf{e}_1\|\,\|\mathbf{e}_v\| - \gamma_3^2\frac{\|\mathbf{e}_1\|^2}{1+\|\mathbf{e}_1\|}$$
$$= -(\alpha_3\|\mathbf{e}_v\| - \mu_1\|\mathbf{e}_1\|)^2 - (\gamma_3^2 - \mu_1^2 - \mu_1^2\|\mathbf{e}_1\|)\,\frac{\|\mathbf{e}_1\|^2}{1+\|\mathbf{e}_1\|},$$

where $\mu_1 = \xi_2/2\alpha_3$. If $\|\mathbf{e}_1\| \leqslant (\gamma_3^2 - \mu_1^2)/\mu_1^2$ then

$$-\alpha_3^2\|\mathbf{e}_v\|^2 + \xi_2\|\mathbf{e}_1\|\,\|\mathbf{e}_v\| - \gamma_3^2\frac{\|\mathbf{e}_1\|^2}{1+\|\mathbf{e}_1\|} \leqslant 0.$$

Similarly, if $\mu_2 = \xi_3/2\beta_3$ and $\|\mathbf{e}_1\| \leqslant (\gamma_2^2 - \mu_2^2)/\mu_2^2$ then

$$-\beta_3^2\|\hat{\mathbf{e}}_2\|^2 + \xi_3\|\mathbf{e}_1\|\,\|\hat{\mathbf{e}}_2\| - \gamma_2^2\frac{\|\mathbf{e}_1\|^2}{1+\|\mathbf{e}_1\|} \leqslant 0.$$

Thus, if $\|\mathbf{e}_1\| \leqslant \min\{(\gamma_3^2 - \mu_1^2)/\mu_1^2, (\gamma_2^2 - \mu_2^2)/\mu_2^2\} \triangleq \delta_1$, then

$$\dot{V} \leqslant -(\alpha_1^2 - c_m\|\hat{\mathbf{e}}_2\| - c_m v_d\|\mathbf{e}_1\|)\|\mathbf{e}_v\|^2 - \frac{\gamma_1^2}{1+\delta_1}\|\mathbf{e}_1\|^2$$
$$-\zeta_1^2\|\hat{\mathbf{e}}_1\|^2 - (\beta_1^2 - c_m\|\mathbf{e}_v\| - c_m v_d\|\mathbf{e}_1\|)\|\hat{\mathbf{e}}_2\|^2. \quad (25)$$

Suppose $\|\mathbf{e}_1\| < \delta_1$, $\|\mathbf{e}_v\| < (\beta_1^2 - \delta_1)/c_m$, and $\|\hat{\mathbf{e}}_2\| < (\alpha_1^2 - \delta_1)/c_m$, then there exist constants $\kappa_1 > 0$, $\kappa_2 > 0$, $\kappa_3 > 0$ such that

$$\dot{V} \leqslant -\kappa_1^2\|\mathbf{e}_v\|^2 - \kappa_2^2\|\hat{\mathbf{e}}_2\|^2 - \kappa_3^2\|\mathbf{e}_1\|^2 - \zeta_1^2\|\hat{\mathbf{e}}_1\|^2. \quad (26)$$

This implies that $\mathbf{e}_1, \mathbf{e}_v, \hat{\mathbf{e}}_1, \hat{\mathbf{e}}_2 \in L_2 \cap L_\infty$, $\tilde{\boldsymbol{\theta}} \in L_\infty$, and $V \in L_\infty$. Further, from (10) and (12) we can conclude that $\dot{\mathbf{e}}_v, \dot{\hat{\mathbf{e}}}_2 \in L_\infty$. Thus, asymptotic convergence of $\mathbf{e}_1, \mathbf{e}_v, \hat{\mathbf{e}}_1, \hat{\mathbf{e}}_2$ follows from Barbalat's lemma. To find the region of attraction, define $V_{\min} = \min\{k_{pm}, \boldsymbol{\Gamma}_m, \eta_1\eta_2\sigma_M\}$ and $V_{\max} = \max\{k_{pM}, \Gamma_M, \eta_1\eta_2\sigma_M\}$, where Γ_m and Γ_M are the minimum and maximum eigenvalues, respectively, of the matrix Γ. Then the Lyapunov function $V(\mathbf{z})$ can be bounded as

$$\tfrac{1}{2}V_{\min}\|\mathbf{z}\|^2 \leqslant V(\mathbf{z}) \leqslant \tfrac{1}{2}V_{\max}\|\mathbf{z}\|^2. \quad (27)$$

Suppose

$$\mu = \min\left\{\frac{\alpha_1^2 - \delta_1}{c_m}, \frac{\beta_1^2 - \delta_1}{c_m}, \delta_1\right\},$$

then the region of attraction is given by

$$\|\mathbf{z}\| < \mu\sqrt{\frac{V_{\min}}{V_{\max}}} \quad (28)$$

Remark. The region of attraction given by (28) is very conservative. The region of attraction can be increased arbitrarily by increasing the controller gains $\mathbf{K}_p$, $\mathbf{K}_d$ and the observer gains η_1, η_2. Thus, the closed-loop system is semi-globally stable. Notice that μ in (28) depends on the gains k_{pm}, k_{dm}, η_1, and η_2.

5. Experiments

5.1. Experimental platform

The experimental platform consists of a two-link direct drive planar manipulator shown in Fig. 1. Each axis is driven by an NSK-Megatorque direct drive servo-motor which is capable of up to 3 revolutions per second maximum velocity and position feedback resolution of up to

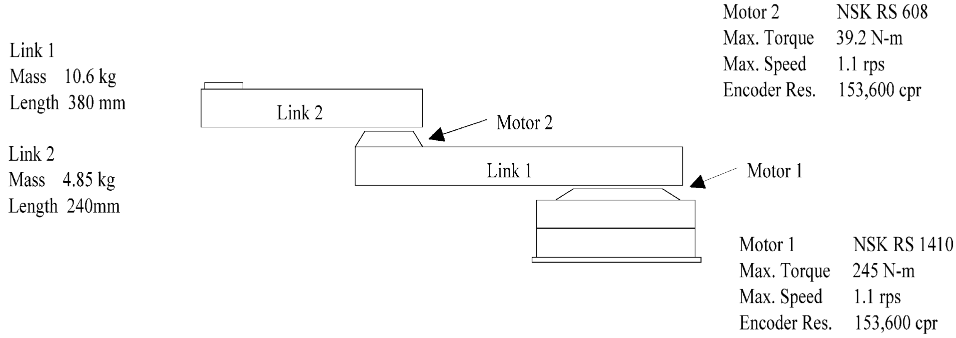

Fig. 1. Two-link robot manipulator.

156,400 counts per revolution. The base motor is rated to deliver up to 245 N m of torque output, and the elbow motor produces torque output up to 40 N m. The NSK-Megatorque motor system consists of motor and its driver unit. This is a stand-alone system that contains all the elements needed for complete closed-loop servo motor control.

The NSK motor consists of a high torque direct drive brush-less actuator, a high resolution brush-less resolver, and a high precision bearing. The Megatorque motor is capable of producing extremely high torque at low speeds suitable for direct drive applications. The heavy-duty bearing eliminates the need for separate mechanical support since the motor case can often support the load directly. The direct drive actuator eliminates the need for gear reduction, so repeatability is limited only by the resolution of the position feedback. Also, direct coupling of the motor and load permits tighter and more direct control of the load. Real-time control is performed with a host computer (Pentium PC) and a servo DSP (TMS320C30). The integer and floating-point arithmetic units equipped on the TMS320C30 DSP can obtain a peak arithmetic performance of 33.3 million floating point computations per second. This allows complex algorithms to be executed using very small sampling periods.

The inertia matrix, $\mathbf{M}(\mathbf{x}_1)$, and the matrix composed of Coriolis and centrifugal terms, $\mathbf{C}(\mathbf{x}_1,\mathbf{x}_2)$, for the two-link manipulator are given by

$$\mathbf{M}(\mathbf{x}_1) = \begin{bmatrix} p_1 + 2p_3c_2 & p_2 + p_3c_2 \\ p_2 + p_3c_2 & p_2 \end{bmatrix},$$

$$\mathbf{C}(\mathbf{x}_1,\mathbf{x}_2) = \begin{bmatrix} -p_3x_{21}s_2 & -p_3(x_{21}+x_{22})s_2 \\ p_3x_{21}s_2 & 0 \end{bmatrix},$$

where $c_i = \cos(x_{1i})$, $s_i = \sin(x_{1i})$ and x_{1i} and x_{2i} denote the components of the vectors $\mathbf{x}_1$ and $\mathbf{x}_2$, respectively, and p_1, p_2 and p_3 are coupled inertial parameters, which are treated as unknowns and estimated by the adaptive controller. The gravity term for this robot configuration is zero, i.e., $\mathbf{g}(\mathbf{x}_1) = 0$. The mathematical model for the robot is derived in Kao (1990). Exact expressions for the inertial parameters p_i can be found in Kao (1990). The desired regressor for the two-link manipulator is

$$\mathbf{Y}_\mathrm{d}(\mathbf{x}_1^\mathrm{d},\mathbf{x}_2^\mathrm{d},\dot{\mathbf{x}}_2^\mathrm{d}) = \begin{bmatrix} \dot{x}_{21}^\mathrm{d} & \dot{x}_{22}^\mathrm{d} & (2\dot{x}_{21}^\mathrm{d} + \dot{x}_{22}^\mathrm{d})\cos(x_{12}^\mathrm{d}) - ((x_{22}^\mathrm{d})^2 + 2x_{21}^\mathrm{d}x_{22}^\mathrm{d})\sin(x_{12}^\mathrm{d}) \\ 0 & \dot{x}_{21}^\mathrm{d} + \dot{x}_{22}^\mathrm{d} & \dot{x}_{21}^\mathrm{d}\cos(x_{12}^\mathrm{d}) + (x_{21}^\mathrm{d})^2\sin(x_{12}^\mathrm{d}) \end{bmatrix}.$$

The desired position, velocity, acceleration, and jerk (derivative of acceleration) trajectories for the two joint angles used in the experiments are given in Fig. 2. These represent desired joint angles for 14 cycles of circle trajectory in the Cartesian space. The first and the last cycle are of duration 2 s each, and the middle 12 cycles are of 1 s duration. In the first cycle the manipulator is accelerated such that a constant Cartesian velocity magnitude is reached, in the middle 12 cycles the constant Cartesian velocity is maintained, and the manipulator is decelerated to a stop in the last cycle.

The proposed controller–observer is compared with a well-known passivity based control algorithm. Exact knowledge of the parameters is assumed for the passivity based control scheme and a first-order (one-step) numerical differentiation of the joint position measurements has been used to obtain joint velocities. The following passivity based control algorithm is chosen:

$$\boldsymbol{\tau} = \mathbf{M}(\mathbf{x}_1)\dot{\mathbf{x}}_{2\mathrm{r}} + \mathbf{C}(\mathbf{x}_1,\mathbf{x}_2)\mathbf{x}_{2\mathrm{r}} + \mathbf{F}_v(\mathbf{x}_{2\mathrm{r}} - \mathbf{x}_2), \tag{29}$$

where $\dot{\mathbf{x}}_{2\mathrm{r}} = \dot{\mathbf{x}}_2^\mathrm{d} - \mathbf{K}_\mathrm{d}(\mathbf{x}_2 - \mathbf{x}_2^\mathrm{d}) - \mathbf{K}_\mathrm{p}(\mathbf{x}_1 - \mathbf{x}_1^\mathrm{d})$, and $\mathbf{F}_v$, $\mathbf{K}_\mathrm{d}$, $\mathbf{K}_\mathrm{p}$ are positive definite gain matrices.

5.2. Experimental results

Experimental results with the proposed adaptive controller–observer with a sampling period of 4 ms are shown in Figs. 3–6. Experimental results with the passivity based control algorithm with a sampling period of 4 ms are shown in Figs. 7–9. Since exact knowledge of the true parameters is assumed and joint velocity is obtained by finite difference of joint position for the passivity based algorithm, it does not involve parameter estimation and velocity estimation via an observer. Thus, potentially lower sampling rates could be used for the passivity based control algorithm as it involves less computation. Experiments were conducted with the passivity based scheme with a sampling period of 2 ms for the same trajectory. These experimental results are shown in Figs. 10–12.

The position tracking errors and the velocity estimation errors are given in Figs. 3 and 4, respectively. Fig. 3 shows that the absolute maximum position tracking error for joint 1 is about 0.01 rad (0.5°) and it is about 0.015 rad (0.75°) for joint 2. The velocity estimation errors are given in Fig. 4. There is an initial spike in the velocity estimation error at $t = 0$ because of the initial condition error. Fig. 4 shows that the velocity estimation errors are bounded within 0.2 rad/s for both links 1 and 2 after the initial errors settle down. These values correspond to about 10% of the desired velocity for link 1 and 4% for link 2. Notice that whenever there is change in direction of velocity the tracking errors take a sudden abrupt variation. This is typically due to the presence of low velocity friction in the motors.

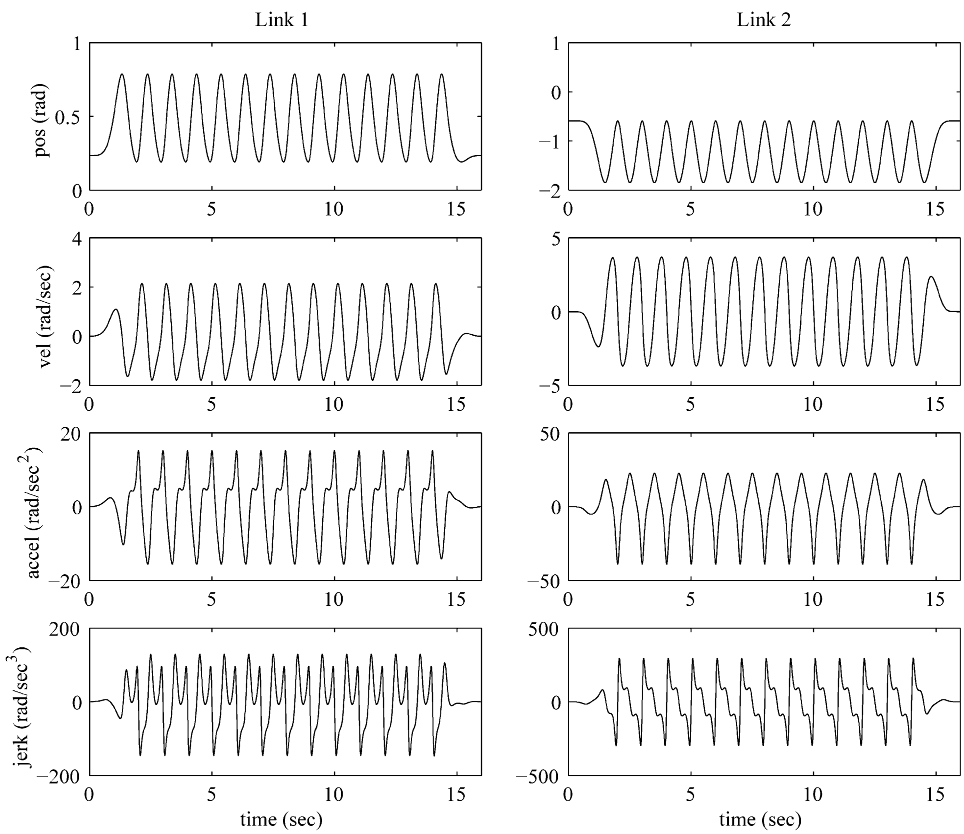

Fig. 2. Desired joint space trajectory.

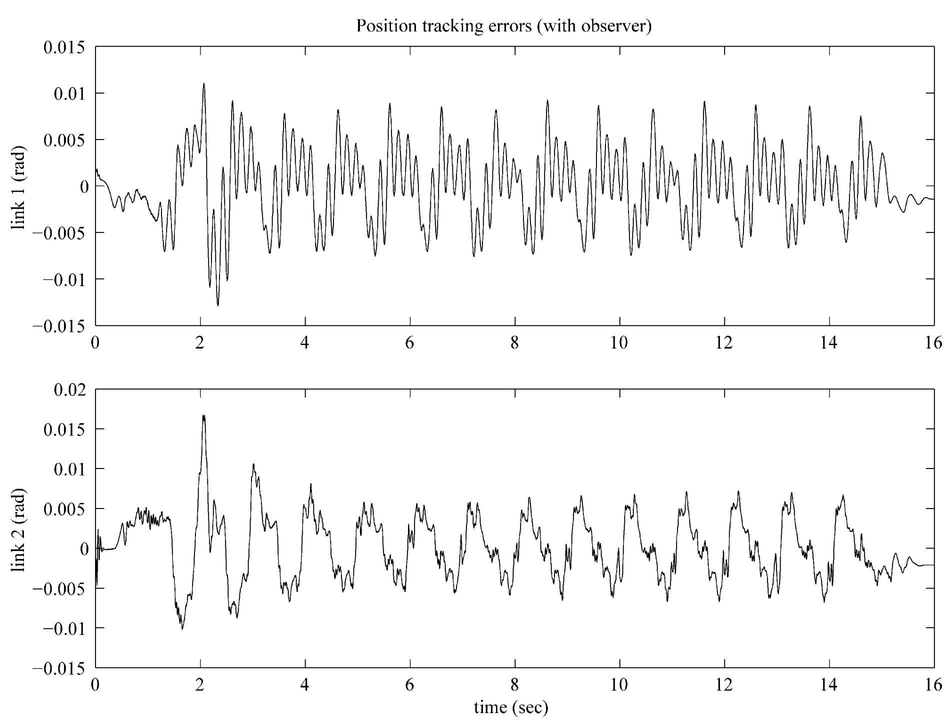

Fig. 3. Position tracking errors (proposed adaptive controller with observer).

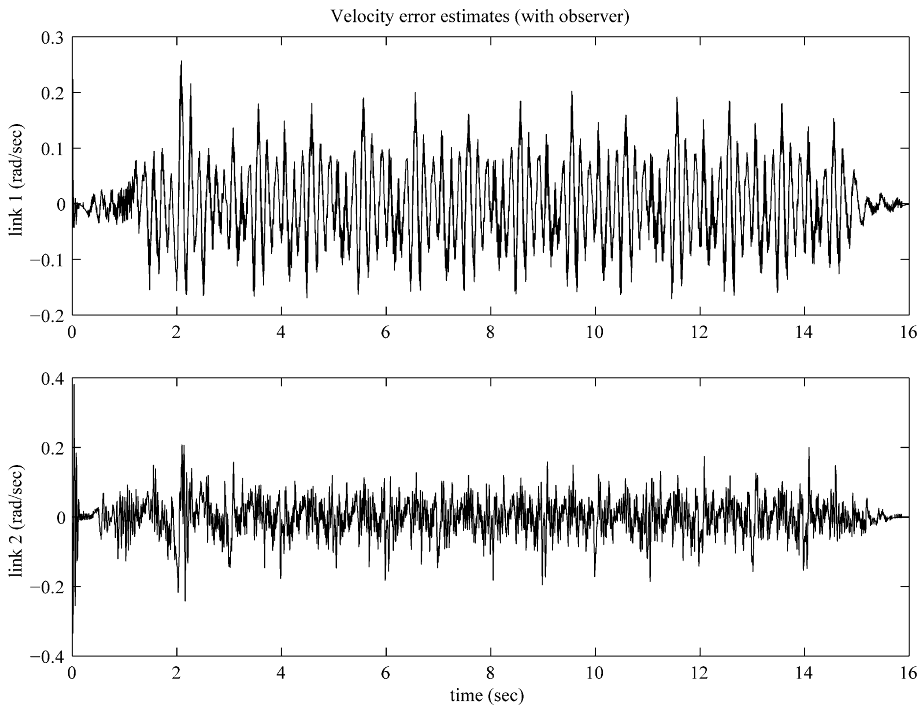

Fig. 4. Estimated velocity errors (proposed adaptive controller with observer).

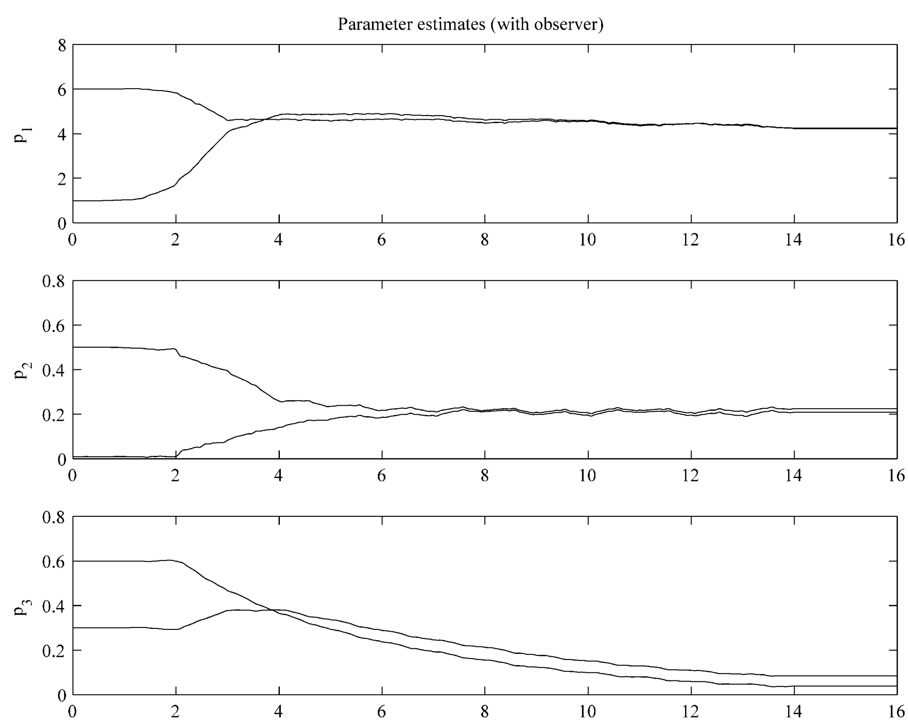

Fig. 5. Parameter estimates (proposed adaptive controller with observer).

The true parameters of the manipulator are $p_1 = 3.6$, $p_2 = 0.2$, and $p_3 = 0.15$. Fig. 5 shows parameter estimates for two sets of initial conditions, i.e., $(1.5, 0.01, 0.3)$ and $(6.0, 0.5, 0.6)$ were used as initial estimates of (p_1, p_2, p_3). The parameter estimates converge to values that are close to those of the true

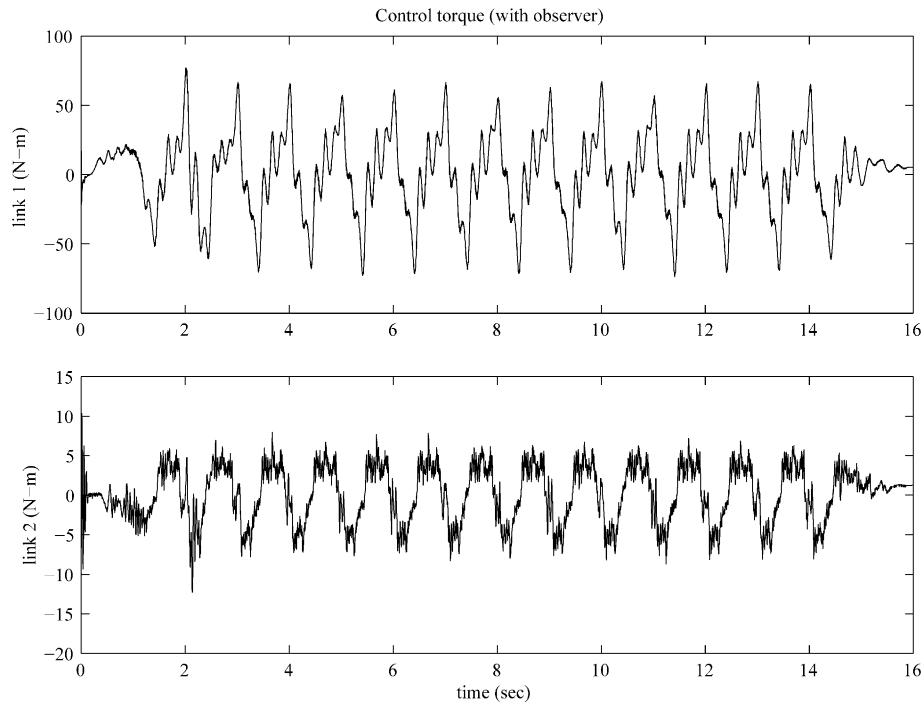

Fig. 6. Control input torque (proposed adaptive controller with observer).

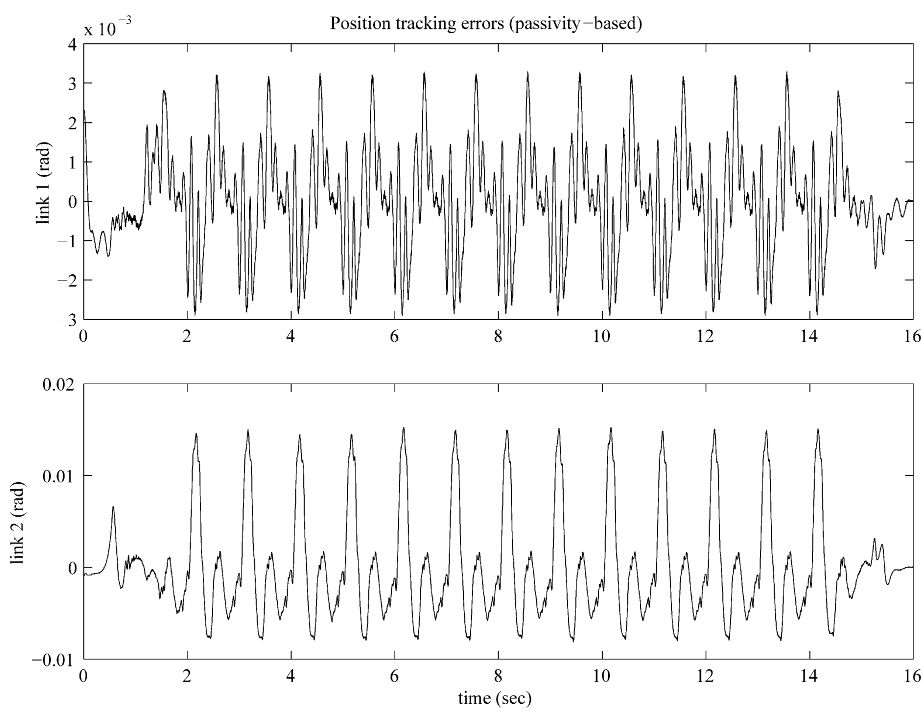

Fig. 7. Position tracking errors (passivity based scheme, $T_s = 4$ ms).

parameters. There is a slight bias for the converged value of the estimate of p_1. This can be noticed in parameter estimates obtained using different trajectories and is mainly attributed to friction. It is reasonable to attribute the smooth convergence of the parameter estimates to the use of the desired regressor matrix which completely depends on the desired joint trajectory.

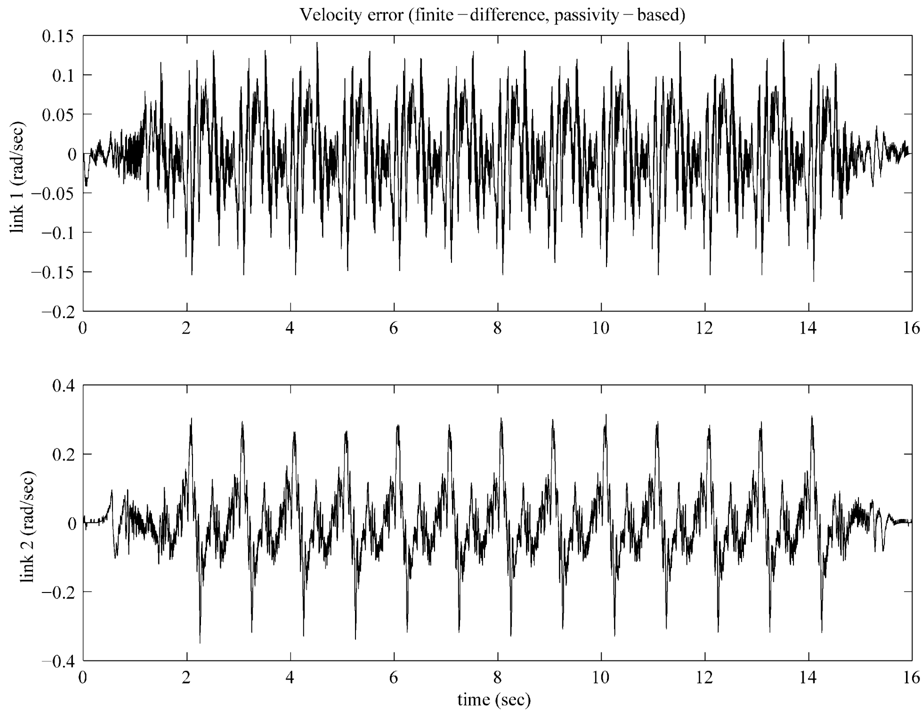

Fig. 8. Velocity errors (finite difference, passivity based scheme, $T_s = 4$ ms).

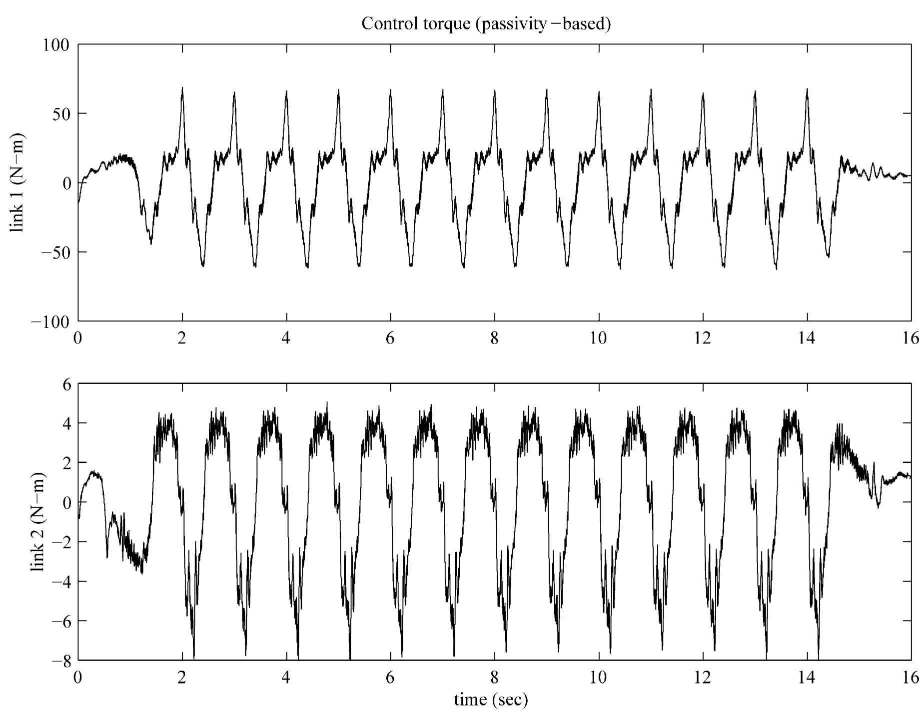

Fig. 9. Control torque (passivity based scheme, $T_s = 4$ ms).

Figs. 7–9 show the position error, the velocity error (finite-difference) and the control torque, respectively, for the passivity based algorithm with 4 ms sampling time. The feedback gains are well tuned to obtain the lowest possible errors for this scheme. Exact knowledge of the true robot parameters is assumed for the passivity based scheme. From the experimental results, we can conclude that the tracking errors for the proposed controller

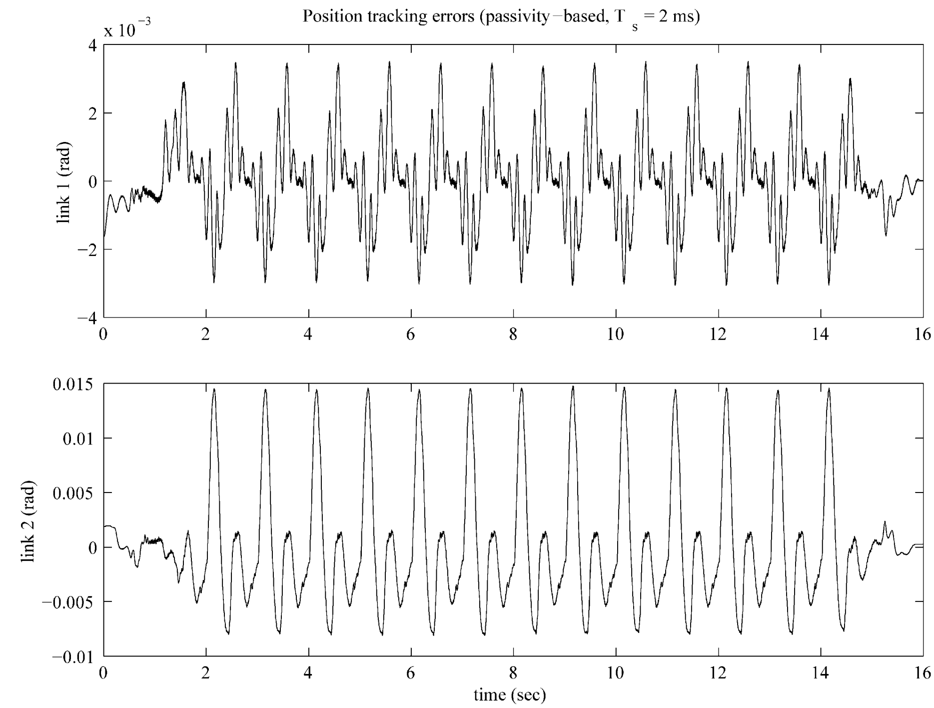

Fig. 10. Position tracking errors (passivity based scheme, $T_s = 2$ ms).

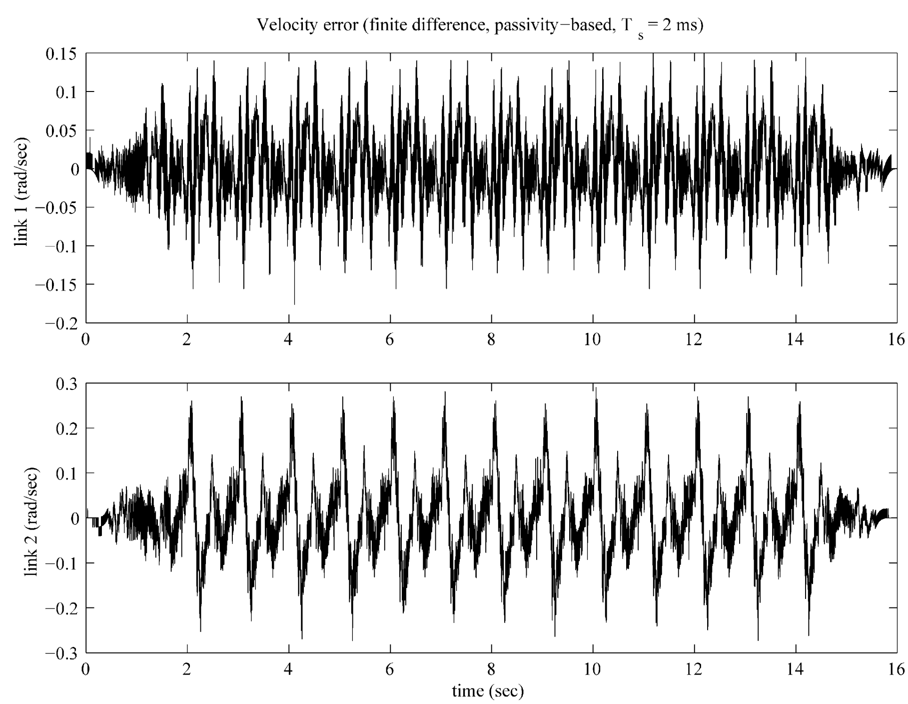

Fig. 11. Velocity errors (finite difference, passivity based scheme, $T_s = 2$ ms).

without velocity measurements and with uncertainty in the robot inertial parameters are similar to the passivity based algorithm with exact knowledge of the parameters. Figs. 10–12 show the position error, the velocity error (finite-difference) and the control torque, respectively, for the passivity based algorithm with 2 ms sampling time. These results are very similar to the ones with 4 ms sampling time.

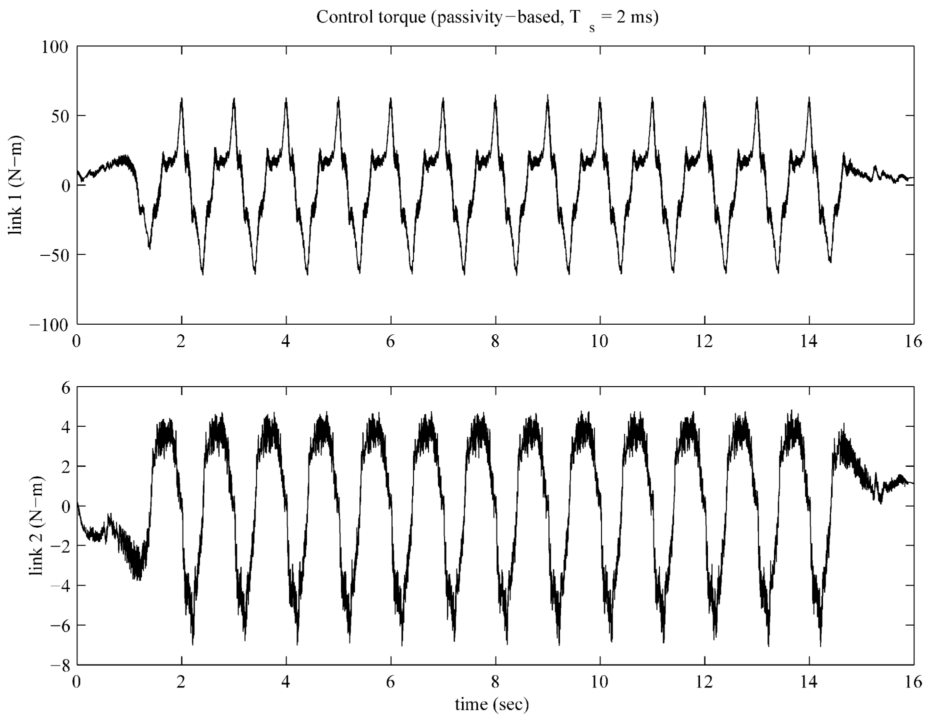

Fig. 12. Control torque (passivity based scheme, $T_s = 2$ ms).

6. Conclusions

In this paper, an adaptive control algorithm for robot manipulators without joint velocity measurements is given. A simple nonlinear observer was proposed to estimate the joint velocities utilizing the desired velocities and position tracking errors. The uniqueness of the proposed observer stems from the fact that it is robust to initial condition errors, since a modified bounded position tracking error is used. Lyapunov stability analysis is conducted to show that the proposed controller–observer structure renders the closed-loop system semi-global asymptotically stable. Experimental results validate the effectiveness of the proposed algorithm in the presence of unknown robot parameters. Comparison with a passivity based control algorithm, implemented with exact knowledge of the true parameters, shows that the proposed algorithm gives similar results even under large uncertainties in the robot parameters.

Appendix A. Properties of robot dynamics

(1) The inertia matrix $\mathbf{M}(\mathbf{x}_1)$ is positive definite and is bounded from above and below by positive constants σ_M and σ_m, that is

$$\sigma_m \leqslant \|\mathbf{M}(\mathbf{x}_1)\| \leqslant \sigma_M \quad \forall \mathbf{x}_1.$$

(2) The matrix $\mathbf{C}(\mathbf{x}_1,\mathbf{x}_2)$ is bounded and satisfies the following:

$$\|\mathbf{C}(\mathbf{x}_1,\mathbf{x}_2)\| \leqslant c_m \|\mathbf{x}_2\| \quad \forall \mathbf{x}_1,\mathbf{x}_2,$$

$$\mathbf{C}(\mathbf{x}_1,\mathbf{x}_2)\mathbf{z} = \mathbf{C}(\mathbf{x}_1,\mathbf{z})\mathbf{x}_2 \quad \forall \mathbf{x}_2,\mathbf{z}.$$

(3) The matrix $\dot{\mathbf{M}}(\mathbf{x}_1) - 2\mathbf{C}(\mathbf{x}_1,\mathbf{x}_2)$ is skew-symmetric.

(4) The dynamics is linear in the unknown parameters and can be expressed as

$$\mathbf{M}(\mathbf{x}_1)\dot{\mathbf{x}}_2 + \mathbf{C}(\mathbf{x}_1,\mathbf{x}_2)\mathbf{x}_2 + \mathbf{g}(\mathbf{x}_1) = \mathbf{Y}(\mathbf{x}_1,\mathbf{x}_2,\dot{\mathbf{x}}_2)\boldsymbol{\theta},$$

where $\boldsymbol{\theta} \in \mathbb{R}^p$ is the unknown parameter vector and $\mathbf{Y}(\mathbf{x}_1,\mathbf{x}_2,\dot{\mathbf{x}}_2) \in \mathbb{R}^{n\times p}$ is the known regressor matrix.

Appendix B. Upper bound of $\|\Delta\mathbf{W}\|$

A bound for $\|\Delta\mathbf{W}\|$ to be used in (18) is obtained in this section. First, $\Delta\mathbf{W}$ is written as

$$\Delta\mathbf{W} = [\mathbf{M}(\mathbf{x}_1) - \mathbf{M}(\mathbf{x}_1^d)]\dot{\mathbf{x}}_2^d + \mathbf{g}(\mathbf{x}_1) - \mathbf{g}(\mathbf{x}_1^d) \tag{B.1}$$

$$+ [\mathbf{C}(\mathbf{x}_1,\mathbf{x}_2)(\mathbf{x}_2^d - \mathbf{e}_c) - \mathbf{C}(\mathbf{x}_1^d,\mathbf{x}_2^d)\mathbf{x}_2^d]. \tag{B.2}$$

$\Delta\mathbf{W}$ can be also be written as

$$\Delta\mathbf{W} = [\mathbf{M}(\mathbf{x}_1) - \mathbf{M}(\mathbf{x}_1^d)]\dot{\mathbf{x}}_2^d + [\mathbf{C}(\mathbf{x}_1,\mathbf{x}_2^d) - \mathbf{C}(\mathbf{x}_1^d,\mathbf{x}_2^d)]\mathbf{x}_2^d + (\mathbf{g}(\mathbf{x}_1) - \mathbf{g}(\mathbf{x}_1^d)) + [\mathbf{C}(\mathbf{x}_1,\mathbf{x}_2)(\mathbf{x}_2^d - \mathbf{e}_c) - \mathbf{C}(\mathbf{x}_1,\mathbf{x}_2^d)\mathbf{x}_2^d]. \tag{B.3}$$

A bound on the first part of the above equation can be derived as in Sadegh and Horowitz (1990):

$$[\mathbf{M}(\mathbf{x}_1) - \mathbf{M}(\mathbf{x}_1^{\mathrm{d}})]\dot{\mathbf{x}}_2^{\mathrm{d}} + [\mathbf{C}(\mathbf{x}_1, \mathbf{x}_2^{\mathrm{d}}) - \mathbf{C}(\mathbf{x}_1^{\mathrm{d}}, \mathbf{x}_2^{\mathrm{d}})]\mathbf{x}_2^{\mathrm{d}} + (\mathbf{g}(\mathbf{x}_1) - \mathbf{g}(\mathbf{x}_1^{\mathrm{d}})) \leqslant (b_{\mathrm{m}} + b_c + b_g)\|\mathbf{e}_1\|.$$

The last part of equation (B.3) can be written as

$$[\mathbf{C}(\mathbf{x}_1, \mathbf{x}_2)(\mathbf{x}_2^{\mathrm{d}} - \mathbf{e}_c) - \mathbf{C}(\mathbf{x}_1, \mathbf{x}_2^{\mathrm{d}})\mathbf{x}_2^{\mathrm{d}}] = \mathbf{C}(\mathbf{x}_1, \mathbf{x}_2^{\mathrm{d}})\mathbf{e}_v - \mathbf{C}(\mathbf{x}_1, \mathbf{e}_c)\mathbf{e}_v - 2\mathbf{C}(\mathbf{x}_1, \mathbf{x}_2^{\mathrm{d}})\mathbf{e}_c + \mathbf{C}(\mathbf{x}_1, \mathbf{e}_c)\mathbf{e}_c.$$

This term can be bounded as

$$\|\mathbf{C}(\mathbf{x}_1, \mathbf{x}_2)(\mathbf{x}_2^{\mathrm{d}} - \mathbf{e}_c) - \mathbf{C}(\mathbf{x}_1, \mathbf{x}_2^{\mathrm{d}})\mathbf{x}_2^{\mathrm{d}}\| \leqslant (c_{\mathrm{m}} v_{\mathrm{d}} + c_{\mathrm{m}} \lambda_c)\|\mathbf{e}_v\| + (2c_{\mathrm{m}} v_{\mathrm{d}} \lambda_c + c_{\mathrm{m}} \lambda_c^2)\|\mathbf{e}_1\|.$$

Hence the bound on $\Delta\mathbf{W}$ is given by

$$\|\Delta\mathbf{W}\| \leqslant (b_{\mathrm{m}} + b_c + b_g + 2c_{\mathrm{m}} v_{\mathrm{d}} \lambda_c + c_{\mathrm{m}} \lambda_c^2)\|\mathbf{e}_1\| + (c_{\mathrm{m}} v_{\mathrm{d}} + c_{\mathrm{m}} \lambda_c)\|\mathbf{e}_v\|. \quad \text{(B.4)}$$

References

Berghuis, H., & Nijmeijer, H. (1993). A passivity approach to controller–observer design for robots. *IEEE Transactions on Robotics and Automation*, *9*(6), 740–754.

Berghuis, H., & Nijmeijer, H. (1994). Robust control of robots via linear estimated state feedback. *IEEE Transactions on Robotics and Automation*, *39*(10), 2159–2162.

Canudas de Wit, C., & Fixot, N. (1992). Trajectory tracking in robot manipulators via nonlinear estimated state feedback. *IEEE Transactions on Robotics and Automation*, *8*(1), 138–142.

Kaneko, K., & Horowitz, R. (1997). Repetitive and adaptive control of robot manipulators with velocity estimation. *IEEE Transactions on Robotics and Automation*, *13*(2), 204–217.

Kao, W.W. (1990). Learning control of robot manipulators, Ph.D. thesis, University of California, Berkeley, CA.

Nicosia, S., & Tomei, P. (1990). Robot control using only joint position measurements. *IEEE Transactions on Automatic Control*, *35*, 1058–1061.

Ortega, R., & Spong, M. W. (1989). Adaptive motion control of rigid robots: A tutorial. *Automatica*, *25*(6), 877–888.

Sadegh, N., & Horowitz, R. (1990). Stability and robustness analysis of a class of adaptive controllers for robotic manipulators. *The International Journal of Robotics Research*, *9*(3), 74–92.

Slotine, J. J. E., & Li, W. (1991). *Applied nonlinear control*. Englewood Cliffs, NJ: Prentice-Hall.

Whitcomb, L. L., Rizzi, A. A., & Koditschek, D. E. (1993). Comparative experiments with a new adaptive controller for robot arms. *IEEE Transactions on Robotics and Automation*, *9*(1), 59–70.

Zhu, J. H., Chen, H. T., & Zhang, Z. J. (1992). A variable structure robot control algorithm with an observer. *IEEE Transactions on Robotics and Automation*, *8*(4), 486–492.

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