Ramsey Numbers Involving Cycles

Stanisław P. Radziszowski
Department of Computer Science
Rochester Institute of Technology
Rochester, NY 14623, spr@cs.rit.edu

ABSTRACT: We gather and review general results and data on Ramsey numbers involving cycles. This survey is based on the author's 2009 revision #12 of the *Dynamic Survey* DS1, "Small Ramsey Numbers", at the *Electronic Journal of Combinatorics.*

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1. Scope and Notation

There is a vast amount of literature on Ramsey type problems starting in 1930 with the original paper of Ramsey [Ram]. Graham, Rothschild and Spencer in their "Ramsey Theory" book [GRS], and Soifer in the 2009 "The Mathematical Coloring Book - Mathematics of coloring and the colorful life of its creators" [Soi] present exciting developments in the history, results and people of Ramsey theory. The subject has grown amazingly, in particular with regard to asymptotic bounds for various types of Ramsey numbers (for example see the survey papers [GrRö, Neš, ChGra2]), but the progress on evaluating the basic numbers themselves has been very unsatisfactory for a long time.

Ramsey Theory studies the conditions of when a combinatorial object necessarily contains some smaller given objects. The role of Ramsey numbers is to quantify some of the general existential theorems in Ramsey Theory. In the case of the so called generalized graph Ramsey numbers one studies partitions of the edges of the complete graph, under the condition that each of the parts avoids some prespecified arbitrary graph, in contrast to classical Ramsey numbers when the avoided graphs are complete.

This survey is a compilation of results on Ramsey numbers for the cases when one (or most, or all) of the avoided graphs is a cycle. The results commented on here are taken from a much broader general Ramsey numbers survey [Rad] by the author, which since 1994 has been updated periodically as a living article at the *Electronic Journal of Combinatorics.* Thus, while the results and data gathered here are subsumed by the August 2009 revision #12 of [Rad], the latter has only minimal comments associated with the results. This is remedied here. Furthermore, it seems that recent years have brought the new vigorous attention of many researchers, especially to the cases involving cycles, to the extent that now it merits its own overview. For the ease of use, and to avoid potential confusions, we employ the same labels of references when from [Rad], even in cases when it forces us to use nonconsecutive labels, when doing so would seem in order. Similarly, the definitions and notation of this survey are entirely those from [Rad]. For deeper exposition of basic concepts and intuition behind them consult the main surveys of this volume, and [GRS] or [Soi].

We do not attempt complete coverage of asymptotic results on Ramsey numbers with cycles, but rather concentrate on cases where exact formulas or concrete values have been obtained or significant work towards them was done. Hence, only the main facts on asymptotic behavior are presented, but with many pointers to further literature. The complete graph on 3 vertices and the cycle of length 3 is the same graph, $K_3 = C_3$. The study of K_3 in the context of Ramsey numbers is very rich in itself and is often considered separately from longer cycles. Here, we point to the results involving C_3 mainly in the context of longer cycles. Also, the bipartite graph $K_{2,2}$ is the same as quadrilateral C_4, and many papers discuss C_4 implicitly under the header of bipartite graphs. Similarly as for triangles, but now to a lesser extent, we decided to skip a number of results originating in bipartite graph theory. The surveys mentioned above give many more pointers to results on K_3 and $K_{2,2}$, and thus indirectly also for cycles.

Let $G_1, G_2, \ldots, G_m$ be graphs or s-uniform hypergraphs (s is the number of vertices in each edge). $R(G_1, G_2, \ldots, G_m; s)$ denotes the m-color **Ramsey number** for s-uniform graphs/hypergraphs, avoiding G_i in color i for $1 \le i \le m$. It is defined as the least integer n such that, in any coloring with m colors of the s-subsets of a set of n elements, for some i the s-subsets of color i contain a sub-(hyper)graph isomorphic to G_i (not necessarily induced). The value of $R(G_1, G_2, \ldots, G_m; s)$ is fixed under permutations of the first m arguments.

If $s = 2$ (standard graphs) then s can be omitted. The complete graph on n vertices will be denoted by K_n. If $G_i = K_k$, then we can write k instead of G_i, and if $G_i = G$ for all i we can use the abbreviation $R_m(G; s)$ or $R_m(G)$. For $s = 2$, $K_k - e$ denotes a K_k without one edge. P_i is a **path** on i vertices, C_i is a **cycle** of length i, and W_i is a **wheel** with $i-1$ spokes, i.e. a graph formed by some vertex x, connected to all vertices of some cycle C_{i-1}, or $W_i = K_1 + C_{i-1}$. $K_{n,m}$ is a complete n by m bipartite graph, in particular $K_{1,n}$ is a **star** graph. The **book** graph $B_i = K_2 + \overline{K}_i = K_1 + K_{1,i}$ has $i+2$ vertices, and can be seen as i triangular pages attached to a single edge. Finally, for a graph G, let nG stand for the graph formed by n vertex disjoint copies of G.

2. Two Color Numbers Involving Cycles

The history of knowledge of graph Ramsey numbers $R(G,H)$ seems to indicate that the difficulty of computing or estimating $R(G,H)$ increases with the density of edges in G. Thus, evaluation of the classical Ramsey numbers $R(k,l)$, when the avoided graphs are complete, is considered one of the hardest tasks, while we know significantly more when graphs G and/or H become sparse. An interesting famous case is formulated in the first theorem, which gives the exact value of $R(G,H)$ when G and H are such extremes. Of course, for all graphs G and H, $R(G,H) = R(H,G)$.

Theorem 1 (Chvátal [Chv], 1977)

$R(T_n, K_m) = (n-1)(m-1)+1$ *for any tree* T_n *with* n *vertices.*

Theorem 1 has a relatively easy proof. It holds in particular for $T_n = P_n$. Note that adding just one closing edge to P_n forms a cycle C_n. However, perhaps surprisingly, the corresponding problem of Ramsey numbers $R(C_n, K_m)$ is far from being well understood. There has been remarkable progress in this area in the last twenty years, but still many of the basic questions remain open. We will address them in more detail in section 2.2.

2.1. Cycles

Arguably the most widely known classical Ramsey number $R(3,3)$ was mentioned implicitly by Bush [Bush] who was reporting that in the 1953 William Lowell Putnam Mathematical Competition question #2 in Part I asks for the proof of what can be denoted by $R(3,3) \le 6$. The 1955 paper by Greenwood and Gleason [GG] includes the result $R(3,3) = 6$ with proofs. This is also the first reported case of cycle Ramsey numbers $R(C_3, C_3)$, since clearly $C_3 = K_3$. Chvátal and Harary [CH1] were the first to give the value $R(C_4, C_4) = 6$. The initial general result for cycles, $R(C_3, C_n) = 2n-1$ for $n \ge 4$, was obtained by Chartrand and Schuster [ChaS] in 1971. The complete solution of the case $R(C_n, C_m)$ was obtained soon afterwards, independently by Faudree and Schelp [FS1] and Rosta [Ros1]. Both these proofs are somewhat complicated, however a new simpler proof by Károlyi and Rosta was published recently [KáRos], in 2001.

Theorem 2 (Faudree, Schelp [FS1], 1974; Rosta [Ros1], 1973)

$$R(C_n, C_m) = \begin{cases} 2n-1 & \text{for } 3 \le m \le n,\ m \text{ odd},\ (n,m) \ne (3,3) \\ n-1+m/2 & \text{for } 4 \le m \le n,\ m \text{ and } n \text{ even},\ (n,m) \ne (4,4) \\ \max\{n-1+m/2,\ 2m-1\} & \text{for } 4 \le m < n,\ m \text{ even and } n \text{ odd} \end{cases}$$

Burr, Erdős and Spencer [BES] in 1975 studied a variety of two color cases for multiple disjoint copies of several small graphs, and among those for short cycles. Their work includes a particularly elegant proof of $R(nC_3, mC_3) = 3n + 2m$ for $n \geq m \geq 1$, $n \geq 2$. This was extended by Li and Wang to $R(nC_4, mC_4) = 2n + 4m - 1$ for $m \geq n \geq 1$, $(n, m) \neq (1,1)$ [LiWa1]. The same authors derived further formulas for $R(nC_4, mC_5)$ [LiWa2]. The general problem of nC_m, more formulas and bounds for various cases were studied also by Mizuno and Sato [MiSa], Denley [Den], Burr and Rosta [BuRo3], and Bielak [Biel1].

2.2. Cycles versus complete graphs

The Ramsey numbers $R(C_n, K_m)$ pose different problems when different restricted relationships between n and m are assumed. For fixed $n = 3$ it becomes the study of the classical numbers $R(3, k)$, which attracted efforts of many researchers until in a 1995 breakthrough Kim proved that $R(3, k) = \Theta(k^2/\log k)$ [Kim] (for history of this result see the chapter by Spencer [Spe3] in this volume). The exact values of $R(3, k)$ are known for $k \leq 9$ (see column C_3 of Table 1). Computation of the exact values for $k \geq 10$ is still elusive and well beyond known theoretical and computational methods. For more comments on the smallest open case $R(3,10)$ see the problem section of this volume. The other end of the problem seems to be much easier for fixed m. In particular, Theorem 2 gives $R(C_3, C_n) = 2n - 1$. Actually, this row of Table 1 seems to generalize to the following simple but apparently hard to prove conjecture.

Conjecture 1 (Faudree, Schelp [FS4], 1976)

$R(C_n, K_m) = (n-1)(m-1) + 1$ *for all* $n \geq m \geq 3$, *except* $n = m = 3$.

The authors of [EFRS2], while studying many similar problems, also restate the same conjecture. Over last three decades there was a steady sequence of papers proving it for increasing sets of pairs of n and m. The parts of Conjecture 1 were proved as follows.

First observe that the lower bound is easy, since the graph $(m-1)K_{n-1}$, formed by $m-1$ vertex-disjoint copies of K_n, clearly provides a witness for $R(C_n, K_m) > (n-1)(m-1)$, even without the exception $n = m = 3$. We note that this same construction similarly gives easily lower bounds for other cases in further sections of this survey.

The hard part is to derive the upper bound. Bondy and Erdős proved it for $n \geq m^2 - 2$ [BoEr] in 1973, Chartrand and Schuster for $n > 3 = m$ [ChaS] in 1971, Yang, Huang and Zhang for $n \geq 4 = m$ [YHZ1] in 1999, Bollobás, Jayawardene, Yang, Huang, Rousseau, and Zhang for $n \geq 5 = m$ [BJYHRZ] in 2000, Schiermeyer for $n \geq 6 = m$, and for $n \geq m \geq 7$ with $n \geq m(m-2)$ [Schi1] in 2003, Nikiforov for $n \geq 4m + 2$, $m \geq 3$ [Nik] in 2005, and finally Chen, Cheng and Zhang for $n \geq 7 = m$ [ChenCZ1] in 2008, All these developments, and a numbers of special small cases which had to be proved on the way, are summarized in Table 1. Still open conjectured cases are marked by "conj." The result $R(C_8, K_8) = 50$, which is a necessary starting point for confirming this conjecture for $m = 8$, was recently proved independently by Jaradat-Alzaleq [JarAl] and by Zhang-Zhang [ZZ3]. The proofs of the latter consist of quite intricate considerations of many subcases. Some new unifying approach to all rows of Table 1 would be very welcome. Let us also note that a stronger version of Conjecture 1 is likely true, since as one can now see in further rows of Table 1, the general formula holds even for some m slightly larger than n.

	C_3	C_4	C_5	C_6	C_7	C_8	...	C_n for $n \geq m$
K_3	6 GG	7 ChaS	9 ChaS	11 ChaS	13 ChaS	15 ChaS	...	$2n-1$ ChaS
K_4	9 GG	10 CH2	13 He2/JR4	16 JR2	19 YHZ1	22 YHZ1	...	$3n-2$ YHZ1
K_5	14 GG	14 Clan	17 He2/JR4	21 JR2	25 YHZ2	29 BJYHRZ	...	$4n-3$ BJYHRZ
K_6	18 Kéry	18 Ex2-RoJa1	21 JR5	26 Schi1	31 Schi1	36 Schi1	...	$5n-4$ Schi1
K_7	23 Ka2-GY	22 RT-JR1	25 Schi2	31 CheCZN	37 CheCZN	43 JarBa/Ch+	...	$6n-5$ ChenCZ1
K_8	28 GR-MZ	26 RT		36 ChenCX	43 ChenCZ1	50 JarAl/ZZ3	...	$7n-6$ conj.
K_9	36 Ka2-GR	30-32 RT-XSR1					...	$8n-7$ conj.
K_{10}	40-43 Ex5-RK2	34-39 RT-XSR1					...	$9n-8$ conj.

Table 1. Known Ramsey numbers $R(C_n, K_m)$.
(Ch+ abbreviates ChenCZ1, see also comments on joint credits below)

Joint credit [He2/JR4] in Table 1 refers to two cases in which Hendry [He2] announced the values without presenting the proofs, which later were given in [JR4]. The special cases of $R(C_6, K_5) = 21$ [JR2] and $R(C_7, K_5) = 25$ were solved independently in [YHZ2] and [BJYHRZ]. The double pointer [JarBa/ChenCZ1] refers to two independent papers, similarly as [JarAl/ZZ3]. For joint credits marked in Table 1 with "-", the first reference is for the lower bound and the second for the upper bound.

Erdős et al. [EFRS2] asked what is the minimum value of $R(C_n, K_m)$ for fixed m. Interestingly, even without knowledge of most of the data gathered in Table 1, the authors suggested that it might be possible that $R(C_n, K_m)$ first decreases monotonically, then attains a unique minimum, then increases monotonically with n. What we now know, more than 30 years later, provides some strong evidence confirming their intuition.

For the column-wise (with fixed n) asymptotic behavior, beyond the impressive Kim's result $R(3,k) = \Theta(k^2/\log k)$ [Kim] mentioned earlier and discussed in other chapters of this volume, we present in the next theorem the known bounds for $n = 4$.

Theorem 3 ([Spe2] 1977; 1980, [CLRZ] 2000)

There exist positive constants c_1 *and* c_2 *such that*

$$c_1(m/\log m)^{3/2} \le R(C_4, K_m) \le c_2(m/\log m)^2.$$

The lower bound was obtained by Spencer [Spe2] using the probabilistic method. The upper bound is presented in a paper by Caro, Li, Rousseau and Zhang [CRLZ], who in turn give the credit to an unpublished work by Szemerédi from 1980. Erdős, in 1981, in the Ramsey problems section of the paper [Erd2] formulated a challenge by asking for a proof of $R(C_4, K_m) < m^{2-\varepsilon}$, for some $\varepsilon > 0$. Erdős placed this problem among the problems on which he "spent lots of time". No proof of this bound is known to date. The asymptotics of the general and odd n cases of $R(C_n, K_m)$ were studied by several authors including those of [BoEr, FS4, EFRS2, Sud1, LiZa2, AlRö].

2.3. Cycles versus wheels

We remind the reader that in this survey the wheel graph $W_n = K_1 + C_{n-1}$ has n vertices. This is different from some authors who use the definition $W_n = K_1 + C_n$ with $n+1$ vertices. While in the Table 2 of known small values of $R(W_n, C_m)$ this difference just shifts the values between adjacent rows, the general formulas are affected a bit more.

Both wheel and cycle graphs are sparse, so we expect that the corresponding Ramsey numbers $R(W_n, C_m)$ will be smaller and easier to compute. Indeed, the linear functions for all fixed n and for fixed odd m, while the other parameter is large enough, are known. Yet proving the ranges for which these linear functions hold, and finding the concrete values for small cases, seem to be quite independent and challenging tasks. We gather what is known in Table 2 below, and then comment on some of the results therein as well as point to some open problems.

Since $W_4 = K_4$, the first data row in Table 2 is the same as the second data row of Table 1 for K_4. Similarly, the first row of Table 1 for $K_3 = W_3$ could be prepended to Table 2 as is, but we didn't do it for the sake of brevity. As in Table 1, the rows of Table 2 are easier to deal with for large m. Similarly, the full solutions for $n = 3, 4$ are the same as for $R(K_n, C_m)$, which were given in the previous section. The almost complete general row-wise solution is presented in Theorem 4.

Theorem 4 ([SuBT1, ZhaCC, ChenCN])

(a) $R(W_n, C_m) = 3m - 2$ *for even* $n \ge 4$ *with* $m \ge n - 1$, $m \ne 3$,
(b) $R(W_n, C_m) = 2m - 1$ *for odd* $n \ge 3$ *with* $2m \ge 5n - 7$.

Theorem 4(a) was conjectured in a few papers by Surahmat et al. [SuBT1, SuBT2, Sur]. Parts of this conjecture were proved in [SuBT1, ZhaCC], and the proof was completed by

Chen, Cheng and Ng [ChenCN] in 2009. Theorem 4(b) was proved in 2006 by Surahmat, Baskoro and Tomescu [SuBT1], but Surahmat conjectured that it also holds for odd $n \geq 3$ with $m \geq 5$ and $m > n$ [Sur]. The latter stronger version of (b) remains open.

	C_3	C_4	C_5	C_6	C_7	C_8	C_m	for
W_4	9 GG	10 CH2	13 He2	16 JR2	19 YHZ1	22 ...	$3m-2$ ...	$m \geq 4$ YHZ1
W_5	11 Clan	9 Clan	9 He4	11 JR2	13 SuBB2	15 ...	$2m-1$ ...	$m \geq 5$ SuBB2
W_6	11 BE3	10 JR3	13 ChvS	16 SuBB2	19 ...	22 ...	$3m-2$ ...	$m \geq 4$ SuBB2
W_7	13 BE3	9 Tse1					$2m-1$	$m \geq 14$ SuBT1
W_8	15 BE3	11 Tse1			19^* ChenCN	22^* ...	$3m-2^*$ ...	$m \geq 7$ ChenCN
W_9	17 BE3	12 Tse1					$2m-1$	$m \geq 19$ SuBT1
W_n for	... $2n-1$ $n \geq 6$ BE3		$2n-1$ $n \geq 19$ Zhou2		$2n-1$ $n \geq 29$ Zhou2		large wheels	cycles

Table 2. Ramsey numbers $R(W_n, C_m)$, for $n \leq 9$, $m \leq 8$.
(results from unpublished manuscript are marked with a *)

With fixed m the analysis seems harder. We give only one result for odd m.

Theorem 5 (Zhou [Zhou2], 1995)

$R(W_n, C_m) = 2n - 1$ *for odd* m *with* $n \geq 5m - 6$.

The special case of Theorem 5 with $m = 3$ was obtained by Burr and Erdős [BE3] in 1983. For these Ramsey numbers even a general formula for the number of critical graphs has been derived, in particular the critical graphs for $R(W_n, C_3)$ are unique for $n = 3, 5$, and for no other n [RaJi]. The next column for $m = 4$ already poses open questions, both regarding concrete small values and the behavior for large n. Only an upper bound $R(C_4, W_n) \leq n + \lceil (n-1)/3 \rceil$ for $n \geq 7$ was obtained in [SuBUB]. Furthermore, besides the values recorded in Table 2, it is known that $R(C_4, W_n) = 13, 14, 16, 17$ for $n = 10, 11, 12, 13$, respectively [Tse1].

Finally, we note that the formula for Ramsey numbers involving C_m again depends on the parity of m. Since C_m is a subgraph forming much of the wheel, it should be no surprise that in the case of $R(W_n, C_m)$ we need to consider four distinct situations with respect to parity of n and m. We will see more such dependencies on parity in further sections as well.

2.4. Cycles versus books

We recall that the book graph of n triangular pages is defined as $B_n = K_2 + \bar{K}_n$. The book-complete and book-book Ramsey numbers have been studied extensively, and we direct the reader to the survey [Rad] for related results. The somewhat less overall studied case of book-cycle numbers, however, has attracted much recent attention. In this section we overview known results about the Ramsey numbers $R(B_n, C_m)$.

	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}	C_m	for
B_2	7 RS1	7 F*al*6	9 C*al*	11 F*al*8	13 ...	15	17	19	21	$2m-1$ ...	$m \geq 2$ F*al*8
B_3	9 RS1	9 F*al*6	10 F*al*8	11 JR2	*13*	15 F*al*8	17 ...	19	21	$2m-1$ ...	$m \geq 8$ F*al*8
B_4	11 RS1	11 F*al*6	11 F*al*8	*12*	*13*	*15*	*17*	19 F*al*8	21 ...	$2m-1$ ...	$m \geq 10$ F*al*8
B_5	13 RS1	12 F*al*6	13 F*al*8	*14*	*15*	*15*	*17*	*19*	*21*	$2m-1$	$m \geq 12$ F*al*8
B_6	15 RS1	13 F*al*6	15 F*al*8	*16*	*17*	*18*	*18*		*21*	$2m-1$	$m \geq 14$ F*al*8
B_7	17 RS1	16 F*al*6	17 F*al*8	*16*	*19*	*20*	*21*			$2m-1$	$m \geq 16$ F*al*8
B_8	19 RS1	17 Tse1	19 F*al*8	*17*	*19*	*22*	≥ *23*			$2m-1$	$m \geq 18$ F*al*8
B_9	21 RS1	18 Tse1	21 F*al*8	*18*			≥ *25*	≥ *26*		$2m-1$	$m \geq 20$ F*al*8
B_{10}	23 RS1	19 Tse1	23 F*al*8	*19*				≥ *28*		$2m-1$	$m \geq 22$ F*al*8
B_{11}	25 RS1	20 Tse1	25 F*al*8							$2m-1$	$m \geq 24$ F*al*8
B_n for	... $2n+3$ $n \geq 2$ RS1	$\approx n$ some	... $2n+3$ $n \geq 4$ F*al*8		$2n+3$ $n \geq 15$ F*al*8		$2n+3$ $n \geq 23$ F*al*8		$2n+3$ $n \geq 31$ F*al*8	large books	cycles

Table 3. Ramsey numbers $R(B_n, C_m)$ for $n, m \leq 11$.
(using *et al.* abbreviations, F*al* for FRS and C*al* for CRSPS)

Since $B_1 = K_3$, the cases of Ramsey numbers for B_1 versus C_m are the same as those for $R(K_3, C_m)$ presented in section 2.2. The case of $B_2 = K_4 - e$ versus C_m is completely solved: small cases were given by different authors as marked in the first row of Table 3, and the general case was solved in the 1978 and 1991 papers by Faudree, Rousseau and Sheehan [FRS6] and [FRS8] (abbreviated in Table 3 as F*al*6 and F*al*8). Actually, in [FRS8], extending the results of [FRS6], the authors proved some theorems shedding much light on other more general cases. The main results are presented as Theorems 6 and 7. Note that now we have distinct cases only with respect to the parity of m.

Theorem 6 (Faudree, Rousseau, Sheehan [FRS8], 1991)

(a) $R(B_n, C_m) = 2m - 1$ *for* $n \geq 1$, $m \geq 2n + 2$,
(b) $R(B_n, C_m) = 2n + 3$ *for odd* $m \geq 5$ *with* $n \geq 4m - 13$.

The centered entries in italics in the middle of Table 3 are from personal communication and manuscripts by Shao. The latter also include proofs of inequalities $R(B_n, C_n) \geq 3n - 2$, $R(B_{n-1}, C_n) \geq 3n - 4$ for $n \geq 3$, and an improvement to the bound on m in Theorem 6(a) to $m \geq 2n - 1 \geq 7$ [Shao].

The column-wise situation is more difficult. Theorem 6(b) gives the values for odd m and n large enough, but likely the range of n for which it holds can be extended. The special case for $m = 3$ was solved completely by Rousseau and Sheehan [RS1], and that for $m = 5$ is included in [FRS8]. For even m, already the smallest case of C_4 is very difficult, since it is related to the existence of certain combinatorial designs. In particular, Faudree, Rousseau and Sheehan in 1978 proved the following Theorem 7. More theorems about asymptotics and bounds on $R(B_n, C_m)$ can be found in the papers [NiRo4, Zhou1].

Theorem 7 (Faudree, Rousseau, Sheehan [FRS6], 1978)

For any prime power q, $q^2 + q + 2 \leq R(C_4, B_{q^2 - q + 1}) \leq q^2 + q + 4$.

The authors of [FRS6] characterize the special conditions under which the so called locally friendly graphs, whose existence is in question for larger q, are witnesses that the upper bound of Theorem 7 holds exactly. Since B_n is a subgraph of B_{n+1}, hence likely $R(C_4, B_n) = n + O(\sqrt{n})$. This would be similar to the behavior of $R(C_4, K_{2,n})$ (see section 3.2 of [Rad]). Finally we note that, besides the values recorded in Table 3 for $m = 4$, Tse obtained the exact values $R(C_4, B_{12}) = 21$ [Tse1], $R(C_4, B_{13}) = 22$ and $R(C_4, B_{14}) = 24$ [Tse2], using computer algorithms.

2.5. Cycles versus other graphs

Technically C_3 and C_4 are cycle graphs, yet in graph theory, and in particular in Ramsey theory, they are very often seen as K_3 and a special bipartite graph $K_{2,2}$, respectively. For example, numerous papers whose references are gathered in sections 3.2 and 4.8 of the survey [Rad] consider the Ramsey numbers involving them in the context of K_3 and complete bipartite graphs. First we present a sample result from this area concerning quadrilateral-star numbers $R(K_{2,2} = C_4, K_{1,n})$. The value of the latter, in other words, is 1 plus the order of the largest C_4-free graph whose complement has the maximum degree less than n.

Theorem 8 (Parsons [Par3], 1975; Burr et al. [BEFRS5], 1989)

(a) $n + \sqrt{n} - 6n^{11/40} \le R(C_4, K_{1,n}) = f(n) \le n + \sqrt{n} + 1$, *and*

(b) *for every prime power* q, $f(q^2) = q^2 + q + 1$ *and* $f(q^2+1) = q^2 + q + 2$.

While Theorem 8 gives pretty good bounds on $R(K_{2,2}, K_{1,n})$, many concrete cases are still evasive. For more bounds and values of $f(n)$ see [Par4, Par5, Chen, ChenJ, GoMC, MoCa, HaMe4]. For the results on C_4 versus trees consult [EFRS4, Bu7, BEFRS5, Chen], and for many other results involving bipartite graphs refer to section 3.2 of [Rad], in particular to several papers by Lortz et al. referenced there. For general cases of cycles versus stars consult [Clark, Par6], cycles versus trees [BEFRS2, FSS1], and cycles versus $K_{n,m}$ and multipartite complete graphs [BoEr].

Next we present a solution to the basic problem of cycles versus paths. Note similarity of the formula of Theorem 9 to that in the cycle-cycle problem of Theorem 2. Some small specific subcases derived earlier by other authors are listed in [FLPS].

Theorem 9 (Faudree, Lawrence, Parsons, Schelp [FLPS], 1974)

$$R(P_n, C_m) = \begin{cases} 2n-1 & \text{for } 3 \le m \le n,\ m \text{ odd}, \\ n-1+m/2 & \text{for } 4 \le m \le n,\ m \text{ even}, \\ \max\{m-1+\lfloor n/2 \rfloor, 2n-1\} & \text{for } 2 \le n \le m,\ m \text{ odd}, \\ m-1+\lfloor n/2 \rfloor & \text{for } 2 \le n \le m,\ m \text{ even.} \end{cases}$$

The classical result by Gerencsér and Gyárfás [GeGy] gives a formula for path numbers $R(P_n, P_m) = m + \lfloor n/2 \rfloor - 1$, for all $m \ge n \ge 2$. It is tempting to compare it in detail to Theorems 2 and 9. Merging together the conditions in the three formulas is routine but somewhat tedious. Obviously, for all n and m it holds that $R(P_n, P_m) \le R(P_n, C_m) \le R(C_n, C_m)$. Each of the two inequalities can become an equality, and, as derived in [FLPS], all four possible combinations of $<$ and $=$ hold for an infinite number of pairs (n, m). For example, if both n and m are even, and at least one of them is greater than 4, then $R(P_n, P_m) = R(P_n, C_m) = R(C_n, C_m)$. The full specification of four cases would require several more lines of details.

Between 1997 and 2004, Rousseau and Jayawardene wrote several papers concerning Ramsey numbers for short cycles versus other graphs, where besides general theoretical results they computed many new exact values of $R(C_m, G)$ for specific G's (some of them were pointed to in Tables 1-3). They collected a large set of data which can give insights into new general claims (or refute them), namely, they found the values of Ramsey numbers for C_4 versus all graphs on six vertices [JR3], C_5 versus all graphs on six vertices [JR4], and C_6 versus all graphs on five vertices [JR2]. In addition, unfortunately only in an unpublished manuscript [RoJa2], the authors gave interesting upper bounds: $R(C_4, G) \le 2q + 1$ for any isolate-free graph G with q edges, and $R(C_4, G) \le p + q - 1$ for any connected graph G on p

vertices and q edges. In a similar direction, Burr et al. [BEFRS2] proved the equality $R(C_{2m+1}, G) = 2n - 1$ for sufficiently large sparse graphs G on n vertices, in particular $R(C_{2m+1}, T_n) = 2n - 1$ for all $n > 1512m + 756$, for n vertex trees T_n.

3. Multicolor Numbers for Cycles

3.1. Three colors

The first larger paper in this area by Erdős, Faudree, Rousseau and Schelp [EFRS1] appeared in 1976. It gives some formulas and bounds for multicolor Ramsey numbers of several simple graphs, including those for $R(C_m, C_n, C_k)$ and $R(C_m, C_n, C_k, C_l)$ for large m. The case of three colors is presented in Theorem 10.

Theorem 10 (Erdős, Faudree, Rousseau, Schelp [EFRS1], 1976)
For m large enough all of the following hold:

(a) $R(C_m, C_{2p+1}, C_{2q+1}) = 4m - 3$ *for* $p \geq 2,\ q \geq 1$,
(b) $R(C_m, C_{2p}, C_{2q+1}) = 2(m + p) - 3$ *and*
(c) $R(C_m, C_{2p}, C_{2q}) = m + p + q - 2$ *for* $p, q \geq 1$.

The three color case is thus clear when one of the cycles is sufficiently long. The situation is getting harder when we are closer to the diagonal. Several such cases which were solved for concrete small parameters, mostly with the help of computer algorithms, are listed in Table 4. The diagonal itself, i.e. the cases of $R_3(C_m)$, were much more studied and thus we know more there. The papers referenced in Theorems 11 and 12, and [GyRSS], used the powerful Szemerédi's Regularity Lemma [Szem] to prove the upper bounds. We present the main results in this direction in the sequel.

Theorem 11 (Triple even cycles)

(a) Figaj, Łuczak [FiŁu1], 2007.

$$R(C_{2\lfloor\alpha_1 n\rfloor}, C_{2\lfloor\alpha_2 n\rfloor}, C_{2\lfloor\alpha_3 n\rfloor}) = (\alpha_1 + \alpha_2 + \alpha_3 + \max\{\alpha_1, \alpha_2, \alpha_3\} + o(1))\,n, \quad \textit{for all} \quad \alpha_1, \alpha_2, \alpha_3 > 0.$$

In particular, for even n, we have $R(C_n, C_n, C_n) = (2 + o(1))\,n$.

(b) Benevides, Skokan [BenSk], 2009.

$$R(C_n, C_n, C_n) = 2n \quad \textit{for all sufficiently large } n.$$

Observe that (b) is an improvement of the second part of (a). We also note that Theorem 11(a) implies (cf. Corollary 2 in [FiŁu1]) a solution to the related longstanding open problem for paths, namely that $R(P_m, P_n, P_k) = m + (n + k)/2 + o(m)$ for $m \geq n, k$. By now, we know even more. In a recent large paper Gyárfás, Ruszinkó, Sárközy and Szemerédi [GyRSS] were able to prove the exact diagonal result for long triple paths. They proved that an amazingly simple formula $R(P_n, P_n, P_n) = 2n - 2 + n \bmod 2$ holds for all sufficiently large n. In a not yet published paper Figaj and Łuczak [FiŁu2] extend their result to triples of cycles of mixed parity, obtaining asymptotic values similar in form to the formula of Theorem 11(a).

$m\ n\ k$	$R(C_m, C_n, C_k)$	references	general results
3 3 3	**17**	GG	see [Rad] page 29
3 3 4	17	ExRe	
3 3 5	21	Sun1+/Tse3	$5k-4$ for $k \geq 5$, $m = n = 3$ [Sun1+]
3 3 6	26	Sun1+	
3 3 7	31	Sun1+	
3 4 4	12	Schu	
3 4 5	13	Sun1+/Rao/Tse3	
3 4 6	13	Sun1+/Tse3	
3 4 7	15	Sun1+/Tse3	
3 5 5	≥ 17	Tse3	
3 5 6	21	Sun1+	
3 5 7	25	Sun1+	
3 6 6			
3 6 7	21	Sun1+	
3 7 7			
4 4 4	**11**	BS	
4 4 5	12	Sun2+/Tse3	
4 4 6	12	Sun2+/Tse3	$k+2$ for $k \geq 11$, $m = n = 4$ [Sun2+]
4 4 7	12	Sun2+/Tse3	values for $k = 8, 9, 10$ are 12, 13, 13 [Sun2+]
4 5 5	13	Tse3	
4 5 6	13	Sun1+	
4 5 7	15	Sun1+	compare Theorem 10(b)
4 6 6	11	Tse3	
4 6 7	13	Sun1+/Tse3	
4 7 7			
5 5 5	**17**	YR1	
5 5 6	21	Sun1+	
5 5 7	25	Sun1+	
5 6 6			
5 6 7	21	Sun1+	
5 7 7			
6 6 6	**12**	YR2	$R_3(C_{2q}) \geq 4q$ for $q \geq 2$ [DzNS]
6 6 7	15	Sun1+	compare Theorem 10(c)
6 7 7			compare Theorem 10(a)
7 7 7	**25**	FSS2	$R_3(C_{2q+1}) = 8q+1$ for large q [KoSS]
8 8 8	**16**	Sun	$R_3(C_{2q}) = 4q$ for large q [BenSk]

Table 4. Ramsey numbers $R(C_m, C_n, C_k)$ for $m, n, k \leq 7$ and $m = n = k = 8$.
(Sun1+ abbreviates SunYWLX, Sun2+ abbreviates SunYLZ2,
the work in [SunYWLX] and [SunYLZ2] is independent from [Tse3])

A lower bound $R_3(C_{2m}) \geq 4m$ for all $m \geq 2$ follows from a more general construction by Dzido, Nowik and Szuca [DzNS], which is valid for any number of colors (see the next section 3.2). For small n, only the case $R_3(C_4) = 11$, solved by Bialostocki and Schönheim [BS] in 1984 by using elegant edge counting reasoning, seems to be special. The other two known exact values, $R_3(C_6) = 12$ obtained by Yang and Rowlinson [YR2] in 1993 and $R_3(C_8) = 16$ by Sun [Sun] in 2006, already required an intensive use of computations. These two cases follow the pattern proved for large n, so, it seems reasonable to pose the following Conjecture 2, which was actually done by Dzido [Dzi1]. The first currently open case is that of $R_3(C_{10})$. In order to settle it (as for all other open cases) one only needs to prove the upper bound $R_3(C_{10}) < 21$, since from the construction in [DzNS] we know that $R_3(C_{10}) \geq 20$.

Conjecture 2 (Triple even cycles - Dzido [Dzi1], 2005)
$R(C_n, C_n, C_n) = 2n$ *for all even* $n \geq 6$.

For the case of three odd cycles we begin with the well known conjecture by Bondy and Erdős.

Conjecture 3 (Triple odd cycles - Bondy and Erdős, cf. [Erd2], 1981)
$R(C_n, C_n, C_n) = 4n - 3$ *for all odd* $n \geq 5$.

Now the situation is somewhat different, though still Szemerédi's Regularity Lemma (RL) played a critical role in establishing the upper bound. The following Theorem 12 confirms Conjecture 3 for large n, however the derivation of really how large n needs to be is difficult because of the use of RL.

Theorem 12 (Triple odd cycles - Kohayakawa, Simonovits, Skokan [KoSS], 2005, 2009)
$R(C_n, C_n, C_n) = 4n - 3$ *for all sufficiently large odd* n.

We note that an equivalent formulation of the last theorem could be $R_3(C_{2m+1}) = 8m + 1$ for all sufficiently large m. Theorem 12 improves a well known Łuczak's result stating that $R(C_n, C_n, C_n) \leq (4 + o(1))n$, with equality for odd n [Łuc]. As observed by Erdős [Erd2] we really only need to prove the upper bound of Conjecture 3 (as in Conjecture 2), since the lower bound is easy. A classical case of $R_3(C_3) = 17$ [GG] is special, but the other two known exact initial values follow the pattern of Conjecture 3: $R_3(C_5) = 17$ obtained with computations by Yang and Rowlinson [YR1] in 1992, and an equality $R_3(C_7) = 25$ proved by Faudree, Schelten and Schiermeyer [FSS2] in 2003. The latter did not require any computer supported computations, however the proof is long and complicated. The first currently open case is that of $R_3(C_9)$. As in the even case, to solve it one only needs to show $R_3(C_9) \leq 33$.

Two interesting exact results for triple cycles were obtained by Sun et al., namely $R(C_3, C_3, C_k) = 5k - 4$ for $k \geq 5$ [SunYWLX], and $R(C_4, C_4, C_k) = k + 2$ for $k \geq 11$ [SunYLZ2]. All exceptions to these formulas for small k are listed in Table 4. Such results are definitely important steps towards a very difficult, but perhaps achievable, goal of exact knowledge of all three color Ramsey numbers for cycles. Almost all of the off-diagonal cases in Table 4 required the use of computer algorithms.

3.2. More colors

For more than three colors, we first present all known nontrivial concrete results, except those for $R_k(C_3) = R_k(K_3)$ which typically belong to the study of cases involving triangle K_3 (see sections 5.1 and 5.2 of [Rad]). Two of the cases listed below, namely those of 4-color C_4 and 5-color C_6, required large scale computations to prove the upper bound.

$R_4(C_4) = 18$	[Ex2] [SunYLZ1]
$18 \le R_4(C_6)$	[SunYJLS]
$27 \le R_5(C_4) \le 29$	[LaWo1]
$R_5(C_6) = 26$	[SunYJLS] [SunYW]
$21 \le R(C_3, C_4, C_4, C_4) \le 27$	[XuR2]
$28 \le R(C_3, C_3, C_4, C_4) \le 36$	[XuR2]
$49 \le R(C_3, C_3, C_3, C_4)$	see 5.6.n in [Rad]

General formulas for $R(C_m, C_n, C_k, C_l)$, for large m [EFRS1], were obtained in the same paper as the results listed in Theorem 10 for three colors. The results there for four colors are also quite similar in form, but significantly more complicated. For three colors there was steady follow up work for the off diagonal cases reported in the previous section, but it has yet to be done for more colors. However, the diagonal did attract attention - the study of $R_k(C_m)$, and in particular of the special case $R_k(C_4)$.

In the mid-seventies, Irving [Ir], Chung [Chu2] and Chung-Graham [ChGra1] established that $R_k(C_4) \le k^2 + k + 1$ for all $k \ge 1$, and $R_k(C_4) \ge k^2 - k + 2$ for all $k - 1$ which is a prime power. In 2000, Lazebnik and Woldar [LaWo1] improved the lower bound to $R_k(C_4) \ge k^2 + 2$ for odd prime power k, and finally the latter was extended to any prime power k by Ling [Ling] and Lazebnik-Mubayi [LaMu].

We summarize the bounds for k-color diagonal cases of even cycles in the following Theorem 13. Interestingly, all six claims below cover different situations and each is best in some respect (say, each of (b), (c), (d) is better than the other two for some k and m).

Theorem 13 (Multicolor even cycles)

All of the following hold:

(a) $R_k(C_{2m}) \ge (k+1)m$ *for odd* k *and* $m \ge 2$ [DzNS],

(b) $R_k(C_{2m}) \ge (k+1)m - 1$ *for even* k *and* $m \ge 2$ [DzNS],

(c) $R_k(C_{2m}) \ge 2(k-1)(m-1) + 2$ [SunYXL],

(d) $R_k(C_{2m}) \geq k^2 + 2m - k$ *for* $2m \geq k+1$ *and prime power* k [SunYJLS],
(e) $R_k(C_{2m}) = \Theta(k^{m/(m-1)})$ *for fixed* $m = 2, 3$ *and* 5 [LiLih],
(f) $R_k(C_{2m}) \leq 201km$ *for* $k \leq 10^m/201m$ [EG].

For multicolor odd cycles diagonal Ramsey numbers the most commonly cited bounds are those from the 1973 paper by Bondy and Erdős [BoEr]:

$$2^k m < R_k(C_{2m+1}) \leq (k+2)!\,(2m+1).$$

The lower bound follows again from natural canonical colorings. A somewhat better upper bound $R_k(C_{2m+1}) < 2(k+2)!\,m$ was obtained by Erdős and Graham [EG], but there is likely much more room for further improvements. This has been accomplished for the special case of C_5 by Li [Li3] in a recent exciting development.

Theorem 14 (Li [Li3], 2009)
$R_k(C_5) \leq \sqrt{18^k k!}/10$ *for all* $k \geq 3$.

More discussion of asymptotic bounds for $R_k(C_n)$ can be found in the papers [Bu1, GRS, ChGra2, LiLih]. There is still much to do. In particular, we know very little about upper bounds on $R_k(C_n)$. We also recommend the 2008 survey paper of multicolor cycle cases by Li [Li2], which nicely complements the discussion of this paper, in particular with respect to asymptotic bounds.

3.3. Cycles versus other graphs

Similarly as in previous sections, it seems easier to proceed when the length of one cycle parameter is large enough. Erdős, Faudree, Rousseau and Schelp [EFRS1] (this paper also contains the proof of Theorem 10) in 1976 studied the cases of $R(C_n, K_{t_1}, \ldots, K_{t_k})$ and $R(C_n, K_{t_1,s_1}, \ldots, K_{t_k,s_k})$ for large n. When the cycle lengths are kept fixed, the techniques needed are different. Alon and Rödl [AlRö] in 2005 obtained a surprising asymptotic result that for more colors involving C_4, and in general even cycles, the problem is more manageable. Some similar results in this direction were obtained in [ShiuLL]. For the numbers $R(C_4, K_n)$ the bounds of Theorem 3 are quite far apart, while the next theorem settles the exact asymptotics for more colors. The paper [AlRö] includes several other asymptotic results, including those for K_3 and other even cycles instead of C_4.

Theorem 15 (Alon, Rödl [AlRö], 2005)
For three colors $R(C_4, C_4, K_n) = \Theta(n^2 \operatorname{poly}\log n)$,
and for more colors $R(C_4, C_4, \ldots, C_4, K_n) = \Theta(n^2/\log^2 n)$.

Despite known exact asymptotics, we have rather poor understanding of small cases for this type of numbers. Below we list the bounds established in [XSR1] for the mixed cases involving C_3, C_4 and K_4 (see also the bounds from [XuR2] given in section 3.2). The lower bounds were obtained by a few different constructions, in contrast to several other numbers involving cycles for which the natural canonical colorings are normally used. The upper bounds follow from known bounds on the maximum number of edges in C_4-free graphs and known bounds for smaller Ramsey numbers. That's almost all of what we know for this type of concrete numbers. We challenge the reader to improve any of the following bounds:

$19 \le R(C_4, C_4, K_4) \le 22$,	$31 \le R(C_4, C_4, C_4, K_4) \le 50$,
$25 \le R(C_3, C_4, K_4) \le 32$,	$42 \le R(C_3, C_4, C_4, K_4) \le 76$,
$52 \le R(C_4, K_4, K_4) \le 72$,	$87 \le R(C_4, C_4, K_4, K_4) \le 179$.

We end the section on multicolor cycle numbers with a compilation of some promising initial exact results for three colors concerning mixture of cycles and paths. For two paths and a cycle it is known that $R(P_3, P_3, C_m) = m$ and $R(P_3, P_4, C_m) = m + 1$ for $m \ge 6$ [Dzi2], $R(P_4, P_4, C_m) = m + 2$ for $m \ge 6$ and $R(P_3, P_5, C_m) = m + 1$ for $m \ge 8$ [DzKP], $R(P_4, P_5, C_m) = m + 2$ for $m \ge 23$ and $R(P_4, P_6, C_m) = m + 3$ for $m \ge 18$ [ShaXSP], and $R(P_m, P_n, C_k) = 2n + 2\lfloor m/2 \rfloor - 3$ for large n and odd $m \ge 3$ [DzFi2]. For two cycles and a path we know that $R(P_3, C_m, C_m) = R(C_m, C_m) = 2m - 1$ for odd $m \ge 5$ [DzKP]. Most of the small cases not covered by the above formulas are listed in [Rad]. Also, Dzido and Fidytek [DzFi2] presented a table of exact values of $R(P_3, P_k, C_m)$ for all $3 \le k \le 8$ and $3 \le m \le 9$. All this may help in launching new conjectures for 3-color numbers involving these most basic mixed parameters.

4. Hypergraph Numbers for Cycles

We close this survey with some interesting recent results on hypergraph Ramsey numbers for so-called loose and tight cycles. A *loose* 3-uniform cycle C_n on the set $[n]$ = $\{1, 2, \ldots, n\}$ is the set of triples $\{123, 345, 567, \ldots, (n-1)n1\}$, forming a cycle with an overlap of consecutive edges of exactly one point. Note that for loose cycles n must be even. A 3-uniform cycle C_n formed by $\{123, 234, 345, \ldots, (n-1)n1, n12\}$, in which consecutive edges share two points, is called *tight*. Loose and tight paths are defined similarly.

For such loose cycles, Haxell, Łuczak, Peng, Rödl, Ruciński, Simonovits and Skokan [HaŁP1+] proved that $R(C_{4k}, C_{4k}; 3) > 5k - 2$ and $R(C_{4k+2}, C_{4k+2}; 3) > 5k + 1$. Furthermore, asymptotically these lower bounds are tight. Generalizations to r-uniform hypergraphs and graphs other than cycles were studied in [GySS].

For tight cycles, Haxell, Łuczak, Peng, Rödl, Ruciński and Skokan [HaŁP2+] proved that $R(C_{3k}, C_{3k}; 3) \approx 4k$ and $R(C_{3k+i}, C_{3k+i}; 3) \approx 6k$ for $i = 1$ or 2. For tight paths the same paper establishes $R(P_k, P_k; 3) \approx 4k/3$. We finally note that the tetrahedron, or four triples on the set of four points, is a tight 3-uniform hypergraph cycle C_4. The corresponding Ramsey number, $R(4,4;3) = 13$ [MR1], is the only nontrivial classical Ramsey number for hypergraphs whose exact value is known.

References

Papers containing results obtained with the help of computer algorithms have been marked with stars. The references are ordered alphabetically by the last name of the first author, and where multiple papers have the same first author they are ordered by the last name of the second author, etc. References' labels are the same as in [Rad] if they appear in this much more extensive survey, and thus numerical labels for some entries can be nonconsecutive.

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