A comparison between different optimization criteria for tuned mass dampers design

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ABSTRACT

Tuned mass sampers (TMDs) are widely used strategies for vibration control in many engineering applications, so that many TMD optimization criteria have been proposed till now. However, they normally consider only TMD stiffness and damping as design variables and assume that the tuned mass is a pre-selected value. In this work a more complete approach is proposed and then also TMD mass ratio is optimized. A standard single degree of freedom system is investigated to evaluate TMD protection efficiency in case of excitation at the support. More precisely, this model is used to develop two different optimizations criteria which minimize the main system displacement or the inertial acceleration. Different environmental conditions described by various characterizations of the input, here modelled by a stationary filtered stochastic process, are considered. Results show that all solutions obtained considering also the mass of the TMD as design variable are more efficient if compared with those obtained without it. However, in many cases these solutions are inappropriate because the optimal TMD mass is greater than real admissible values in practical technical applications for civil and mechanical engineering. Anyway, one can deduce that there are some interesting indications for applications in some actual contexts. In fact, the results show that there are some ranges of environmental parameters ranges where results attained by the *displacement criterion* are compatible with real applications requiring some percent of main system mass. Finally, the present research gives promising indications for complete TMD optimization application in emerging technical contexts, as micro-mechanical devices and nano resonant beams.

1. Introduction

New trends in materials and construction technologies have increased substantially systems performances in the last few decades in many engineering applications, such as mechanical, aeronautical and civil ones. As immediate consequences of these technologies, for example in civil engineering, structures tend to be lighter, more slender and have smaller natural damping capacity than those of their older counterparts.

This trend has increased the importance of damping technology to mitigate induced vibrations, and significant progresses have been made towards making structural control a practicable technology to enhance structural functionality and safety against natural and artificially generated vibrations (see for instance [1]). The concept of vibration control is now widely accepted and has been frequently applied in many different fields, such as civil, mechanical, automotive and aeronautical. Passive vibration control is nowadays a mature technology: among the numerous passive control methods available, the tuned mass damper (TMD) is one of the simplest and the most reliable control device. It offers a relatively simple and effective way of reducing excessive vibrations of high rise buildings, towers, chimneys, various mechanical systems and so on. By attaching a secondary mass to the main system to be protected, with approximately the same natural frequency, large relative displacements between the main system and the secondary mass will occur at resonance. In this way the vibrating energy of the system can be dissipated by placing a properly tuned damper between the two. Since this vibration control strategy was firstly proposed by Frahm [2], many different TMD configurations have been projected. With reference to TMD optimal solutions, the first design criterion was proposed by Ormondroyd and Den Hartog [3]. This criterion concerned the minimization of the system response with respect to stationary harmonic excitation with the most critical frequency. Alternatively, a stochastic stationary excitation can be considered, e.g. white noise.

Many optimum TMD methods have been developed aimed to opportunely design this vibration control technique under various types of excitation sources. On the basis of Den Hartog's method, Warburton and Ayorinde [4] have obtained the optimum parameters of the TMD for an undamped structure under harmonic support excitation, where the acceleration amplitude was set to be constant for all input frequencies, and also for other kinds of harmonic excitations. Analytical development of the TMD design has considered several types of optimization procedures, different mathematical models for the primary system and the associated external loading [5]. Sun et al. [6] provide an excellent review on the history of tuned vibration absorbers. Moreover, a number of solutions for various types of white noise excitation and various minimization criteria have been reported by Grigoriu and Soong [7]. Takewaki [8] developed a method for optimal viscous damper placement in building structures with a tuned mass damper, taking into account the response amplification due to the ground. Other authors considered unconstrained optimization of single nonlinear [9] and multiple linear [10] tuned mass dampers using as objective function the variance of the system displacement, modelling the input as a stationary white noise process.

Many recent optimization proposals have been produced in last few years, such as in two recent papers, where analytical and numerical approaches are used to obtain results that were afterwards represented in an approximate analytical format, so that a synthesis highlighting influence of system parameters are reported [11,12].

Increasing in passive TMDs performances are nowadays possible by using active or semi-active approach, which requires a prescribed active control algorithm and external power supply to generate the control force that drives the auxiliary mass. These aspects were first studied by Morison and Karnopp [13], and then investigated by other authors (see for instance [14]).

Anyway, just with reference to the passive strategies (the widely studied and applied approach) only few TMD optimizations have dealt with a complete design. This is because the commonly used design considers only the TMD frequency and damping ratio as design variables, whereas TMD mass or as usually the ratio γ_T between main system mass and TMD mass is usually assumed as a constant parameter defined in a pre-design stage. In addition, it can be noticed that optimum values required for the parameter γ_T by any optimization technique are very high and therefore they are incompatible with real applications due to economic and practical consideration. This outcome is in agreement with other literature results (see for example [15,16]).

For this reason TMD optimization was generally developed by a two elements design vector $\mathbf{b}=(\omega_T,\xi_T)^T$, which collects the TMD frequency and the TMD damping ratio, whereas γ_T is defined in a pre-design phase considering its practical range of variation which depends on the specific problem. There are a number of practical and economic factors which could influence its choice: γ_T usually does not exceed a few percentage of the global main system mass for both civil and mechanical applications.

It is known that for small values of TMD mass, the protection efficiency of this control strategy against vibrations monotonically grows with the mass ratio [16]. It must be noticed that there are some practical applications where the amount of TMD should be greater than usually. For example in civil engineering Matta and De Stefano [17,18] proposed to use an entire roof floor as TMD in a few floors building adopting a γ_T whose order of magnitude is included between 10% and 20%.

These authors thought some new and specific applications in micro and nanomechanics, where it can be possible to have greater values of mass ratio for practical applications. It is for instance the case of micro-electro-mechanical devices (MEMS); they are miniature mechanical and/or electromechanical systems designed to perform tasks that previously were done with much larger mechanical structures. MEMS benefits include smaller size, lower power consumption, accelerated time to market and significantly reduced costs. Their study, production and use are rapidly growing up in many applications, such as accelerometers and similar mechanical sensors. A major impetus behind MEMS technology stems from the fact that mechanical mechanisms benefit from the same scaling-based advantages that have driven the integrated circuit (IC) revolution in recent decades. Specifically, small size of mechanical beams leads to faster speed, lower power consumption, higher complexity, and lower cost and it does not so only in the electrical domain, but in virtually all other domains, including especially mechanical. Although many examples of this from all physical domains exist, vibrating RF MEMS resonators perhaps provide the most direct example of how small size leads to faster speed in the mechanical domain. For instance, a vibrating string tuned to a natural period by proper mechanical parameters will vibrate at those resonance frequencies cleaning all other frequencies. With reference to a guitar's string tuned to "A" note, that is made of nickel and steel and whose spanning is about 25″ in length, will vibrate at a resonance frequency of 110 Hz. In vibrating only at resonant frequency and no others, it is actually mechanically selecting exhibiting a so called quality factor or Q factor on the order of 350, which is $\sim 50\times$ more frequency selective than an on-chip electrical LC. The Q factor expresses the measure of response spectrum width for a general resonant system: from the physical point of view it is the ratio of the total energy stored divided by the energy lost in a single cycle.

In many electronic applications, this is an important result, that will be obtained for a different range of frequencies than those characterising standard vibration elements at the RF and IF. In those applications they typically work with much higher frequencies, from tens of MHz to well into the GHz range. Dimensional scaling is needed in order to achieve such frequencies with even better mechanical selectivity. In particular, by using a metallic string whose length is only 10 μm, constructing it in stiffer, IC-compatible materials (like polysilicon), supporting it at nodes rather than at its ends (to minimize anchor losses), and exciting it electro-statically or piezo-electrically rather than plucking it, it can be achieved a free–free beam (FF-beam) resonator which resonates at frequencies around 100 MHz with Q's in excess of 10,000.

In keeping with the scaling-based arguments presented so far, further scaling down to nano-dimensions does indeed yield frequencies in excess of 1 GHz. However, as with nanoelectronics in the electrical domain, there are issues in the mechanical domain that might hinder the use of nanomechanical vibrating resonators for today's communication purposes. It is clear how it is important to introduce TMS device in those micro-vibrating systems to limiting or drastically reduce undesired frequencies, as in micro-scale mechanical resonators, that have high sensitivity as well as fast response and are widely used as sensors and modulators.

On the base of these considerations, in this paper TMD optimum design is developed adopting a complete approach, which considers also TMD mass ratio as a design variable. Optimization is carried out by using standard genetic algorithm and Matlab solvers.

The main aim of this work is not to propose new approximate or exact analytical formulas to evaluate optimal TMD parameters under different design scenarios, but it is to evaluate if, when and how a *complete optimization* strategy is reasonable and applicable in practical engineering problems. For this reason, the comparison between solutions obtained considering two distinct TMD optimization criteria is performed. The criteria are based on the minimization of relative displacement and of inertial acceleration of the main system, respectively. The main system is represented by a single degree of freedom subjected to a coloured stationary white noise input acting at its support.

Optimal TMD performances and design parameters are evaluated for different environmental parameters, considering various input frequency contents and main system damping ratios. Three different spectral contents are used, correspondingly to a narrow, medium and broad band signal. Results immediately show that acceleration criterion presents optimal mass ratio greater than any practical applications, while the displacement criterion presents some interesting ranges of applications without a too great mass ratio. So that a more detailed analysis of optimal results are shown for these criteria, in terms of objective function (OF) and design vector (DV) parameters. Results are given for different main system/input frequencies, input frequency contents and main system damping ratios.

Finally, a comparison between the different possible solutions is showed and the possible ranges of the parameters where the optimal TMD solution can be adopted are described.

2. Statement of the TMD optimization problem

In this section, a simple but representative model for TMD applications is analysed. It is a linear main system, represented by its first mode [16], excited at its support by an acceleration process as illustrated in Fig. 1. The system is protected against excessive vibration levels by a simple linear TMD. The dynamic response of this combined system is governed by the dynamic equilibrium equation:

$$\mathbf{M}\ddot{\mathbf{Y}}(t)+\mathbf{C}\dot{\mathbf{Y}}(t)+\mathbf{K}\mathbf{Y}(t)=\mathbf{r}\ddot{Y}_b(t) \tag{1}$$

where $\mathbf{Y}=(Y_S,Y_T)^T$ is the relative base displacement vector, and $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are mass, damping and stiffness matrices, respectively, whereas $\mathbf{r}$ is the drag vector.

Introducing the reduced state space vector

$$\mathbf{Z}_s=(Y_T,Y_S,\dot{Y}_T,\dot{Y}_S)^T, \tag{2}$$

the system motion Eq. (1) can be replaced by

$$\dot{\mathbf{Z}}_s(t)=\mathbf{A}_s\mathbf{Z}_s(t)+\mathbf{r}_z\ddot{Y}_b(t) \tag{3}$$

where

$$\mathbf{A}_s=\begin{pmatrix}\mathbf{0} & \mathbf{I}\\ \mathbf{H}_s^1 & \mathbf{H}_s^2\end{pmatrix} \tag{4}$$

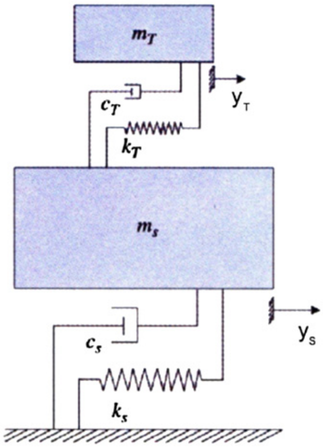

Fig. 1. Mechanical model of a linear TMD system.

is the structural system matrix, $\mathbf{r}_z$=(0,0,1,1)^T, **I** and **0** the unit and the zero 2×2 matrices, respectively, and

$$\mathbf{H}_s{}^1 = \mathbf{M}^{-1}\mathbf{K} = \begin{pmatrix} -\omega^2{}_T & +\omega^2{}_T \\ +\gamma_T\omega^2{}_T & -(\gamma_T\omega^2{}_T + \omega^2{}_0) \end{pmatrix} \tag{5}$$

$$\mathbf{H}_s^2 = \mathbf{M}^{-1}\mathbf{C} = \begin{pmatrix} -2\xi_T\omega_T & +2\xi_T\omega_T \\ +\gamma_T 2\xi_T\omega_T & -(\gamma_T 2\xi_T\omega_T + 2\xi_0\omega_0) \end{pmatrix} \tag{6}$$

The mechanical parameters of the system are

$$\omega_T = \sqrt{\frac{k_T}{m_T}} \tag{7}$$

$$\omega_0 = \sqrt{\frac{k_S}{m_S}} \tag{8}$$

$$\xi_T = \frac{c_T}{2\sqrt{m_T k_T}} \tag{9}$$

$$\xi_0 = \frac{c_S}{2\sqrt{m_S k_S}} \tag{10}$$

$$\gamma_T = \frac{m_T}{m_S} \tag{11}$$

$\ddot{Y}_b(t)$ is the acceleration that excites the system at its support and is represented by a filtered stochastic process which can be expressed as a second order linear filter by following equations:

$$\begin{cases} \ddot{Y}_f(t) + 2\xi_f\omega_f\dot{Y}_f + \omega_f^2 Y_f = -w(t) \\ \ddot{Y}_b(t) = \ddot{Y}_f(t) + w(t) = -(2\xi_f\omega_f\dot{Y}_f + \omega_f^2 Y_f) \end{cases} \tag{12}$$

where $w(t)$ is a stationary Gaussian zero-mean white noise process whose intensity is given by S_0,[1] ω_f is the base frequency and ξ_f the filter damping ratio.

The new state space vector is

$$\mathbf{Z} = (Y_T \quad Y_S \quad Y_f \quad \dot{Y}_T \quad \dot{Y}_S \quad \dot{Y}_f)^T \tag{13}$$

The space state covariance matrix $\mathbf{R}_{ZZ}$ is obtained as the solution of a *Lyapunov equation* [6]:

$$\mathbf{A}\mathbf{R}_{ZZ} + \mathbf{R}_{ZZ}\mathbf{A}^T + \mathbf{B} = \mathbf{0} \tag{14}$$

[1] $E[w(t)w(t-\tau)] = 2\pi S_0\delta(t-\tau)$.

where

$$\mathbf{A}=\begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -\omega_T^2 & +\omega_T^2 & +\omega_f^2 & -2\xi_T\omega_T & +2\xi_T\omega_T & +2\xi_f\omega_f \\ +\gamma_T\omega_T^2 & -(\gamma_T\omega_T^2+\omega_S^2) & +\omega_f^2 & +\gamma_T 2\xi_T\omega_T & -(\gamma_T 2\xi_T\omega_T+2\xi_0\omega_0) & +2\xi_f\omega_f \\ 0 & 0 & -\omega_f^2 & 0 & 0 & -2\xi_f\omega_f \end{pmatrix} \tag{15}$$

The 6×6 matrix $\mathbf{B}$ has all null elements except one:

$$[\mathbf{B}]_{6,6}=2\pi S_0. \tag{16}$$

In order to define an optimization criterion for the TMD mechanical parameters, it is useful to introduce a dimensionless index that considers the covariance response parameters.

A frequent way to optimize systems subject to random vibration is finding a DV able to maximize the performance efficiency. The problem is how to define the efficiency related to main system vibration control offered by TMD. In this context a common approach considers a dimensionless reduction factor ψ that is the ratio between the protected and unprotected main system variance, that is,

$$\psi=\frac{(\sigma_r)_{\text{protected}}}{(\sigma_r)_{\text{unprotected}}} \tag{17}$$

where the suffix r indicates an opportunely defined response quantity index which is in general an element of state space covariance matrix. In addition in order to obtain the response of the unprotected system we need to solve Eq. (14) also for this one, then obtaining the 4×4 covariance matrix $\mathbf{R}_{\mathbf{Z}_0\mathbf{Z}_0}$, which can be evaluated by solving the following equation:

$$\mathbf{A}_0\mathbf{R}_{\mathbf{Z}_0\mathbf{Z}_0}+\mathbf{R}_{\mathbf{Z}_0\mathbf{Z}_0}\mathbf{A}_0^T+\mathbf{B}_{\boldsymbol{0}}=\mathbf{0} \tag{18}$$

where the space state vector for the unprotected system and its system matrix are now given by

$$\mathbf{Z}_0=\{Y_S \quad Y_f \quad \dot{Y}_S \quad \dot{Y}_f\}^T \tag{19}$$

$$\mathbf{A}_0=\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\omega_0^2 & \omega_f^2 & -2\xi_0\omega_0 & 2\xi_f\omega_f \\ 0 & -\omega_0^2 & 0 & -2\xi_f\omega_f \end{pmatrix} \tag{20}$$

The 4×4 matrix $\mathbf{B}_0$ has all null elements except one, that is

$$[\mathbf{B}]_{4,4}=2\pi S_0. \tag{21}$$

Once the two quantities in Eq. (17) are known, it is possible to define the objective function (OF) that has to be minimized by opportunely designed design variables (DV). The optimization approach is then finally posed following the general formulation firstly proposed by Nigam [19]:

$$\text{find } \mathbf{b}\in\boldsymbol{\Omega}_b \tag{22}$$

$$\text{that minimizes } OF(\mathbf{b}) \tag{23}$$

$$\text{subject to } g_i(\mathbf{b})\le 0 \quad (i=1,2,\ldots,k) \tag{24}$$

The OF can be defined in a standard deterministic way (for example by the total structural weight or by the total volume) or in a stochastic one, as the ratio in Eq. (17). In this case the covariance or the spectral moments of the variables could be used (for example, displacement, acceleration or stress in relevant elements). Also constraints expressed by Eq. (24) could regard spectral or statistical moment or, in a more realistic way, they could express reliability limitations, as for example in the following form:

$$P_f(\mathbf{b})-P_f^{\text{adm}}\le 0, \tag{25}$$

where P_f^{adm} is the maximum admissible failure probability and $P_f(\mathbf{b})$ is the actual failure probability of system.

The optimal TMD mechanical parameters are collected in the design vector $\mathbf{b}$. If, as commonly assumed, the mass ratio γ_T is preliminary assigned, a reduced DV is defined by $\mathbf{b}_r=(\omega_T,\xi_T)^T$. In a wide amount of practical applications the maximum acceptable mass ratio is small (for technical or economical reasons), and in this case TMD efficiency monotonically increases with γ_T. On the contrary the complete TMD optimization problem, defined by a complete DV $\mathbf{b}_c=(\omega_T,\xi_T,\gamma_T)^T$ may provide an optimal solution corresponding to greater performances in vibration control. Unfortunately, the optimal mass ratio is normally extremely larger than practical engineering values; therefore the complete approach is often disregarded.

In order to compare different optimization criteria, the OF in Eq. (17) is here defined considering the main system displacement Y_S or the inertial acceleration $\ddot{X}_S = \ddot{Y}_S + \ddot{Y}_b$. This means that the main goal of optimization is to define the TMD mechanical characteristics that reduce the system-vibrating relative base displacement, or alternatively reduces the inertial forces acting on the main mass.

The choice of specific criteria is a function of required performances in the design process, and the dimensionless reduction factor ψ is used to obtain two different *OF*, one for *displacement* (*index r in* Eq. (17) *is the main system displacement*) and one for *acceleration* (index *r* in Eq. (17) is now the *main system inertial acceleration*):

$$OF_d = \frac{\sigma_{Y_S}}{\sigma_{Y_S^0}} \tag{26}$$

$$OF_a = \frac{\sigma_{\ddot{X}_S}}{\sigma_{\ddot{X}_S^0}} \tag{27}$$

The evaluation of $\sigma_{\ddot{X}_s}$ requires the determination of the covariance matrix and is reported in Appendix. The two OFs in Eqs. (26) and (27) represent a stochastic *index of efficiency for vibration protection* whose effectiveness is good when its value is smaller than one. At the same time, a value close to one of the OF indicates practically negligible effects in vibration control. The optimization is performed assuming that all parameters involved in the problem, except the excitation are deterministic. In addition, an unconstrained optimization is considered in the case of a complete approach which considers also the mass ratio as design variable, and then Eqs. (22) and (23) become

find the optimum design vector

$$\mathbf{b} = (\omega_T, \xi_T, \gamma_T)^T \tag{28}$$

that minimizes

$$\frac{\sigma_{Y_S}(\mathbf{b})}{\sigma_{Y_S^0}} \text{(displacement criterion)} \tag{29}$$

or that minimizes

$$\frac{\sigma_{\ddot{X}_S}(\mathbf{b})}{\sigma_{\ddot{X}_S^0}} \text{(acceleration criterion)} \tag{30}$$

3. Numerical sensitivity analysis

In this section, the results of a sensitivity analysis of optimal solutions obtained by reduced and complete formulations are discussed. The optimization has been developed by means of Genetic Algorithm strategy; the standard Matlab GA is used with a hybrid strategy which adopts a first sub-optimal solution, obtained by a simple GA approach with a limited generation numbers, as the starting point of further optimization procedures. Optimal solutions are obtained for different input frequency content and damping. The first sensitivity analysis, which considers input frequency, is of primary importance because it is related to the input-main system resonance effects. In order to explore the sensitivity of optimal solutions versus this parameter a filter frequency variation has been assumed in the range of 0.5–5 with reference to the main system–input frequency ratio:

$$\Omega = \frac{\omega_0}{\omega_f} \tag{31}$$

Also the parameter ξ_f plays also a central role in optimal design because it defines the input frequency content. In the analysis it is assumed ξ_f=0.3. Moreover, three main system damping ratios are fixed, due to its great influence on TMD efficiency, which consider a moderate or small damping condition (ξ_0=0.02), a second medium damping (ξ_0=0.06) and a strong damping (ξ_0=0.1).

Optimum solutions are expressed in terms of optimal OF and DV elements, i.e. dimensionless frequency ratio, damping and mass ratio:

$$\rho_{\mathrm{TMD}}^{\mathrm{opt}} = \frac{\omega_{\mathrm{TMD}}^{\mathrm{opt}}}{\omega_0} \tag{32}$$

$$\xi_{\mathrm{TMD}}^{\mathrm{opt}} \tag{33}$$

$$\gamma_{\mathrm{TMD}}^{\mathrm{opt}}$$

The optimal solutions obtained by performing the complete approach defined by Eq. (28), are shown in Figs. 2 and 3, both considering the acceleration and the displacement criteria.

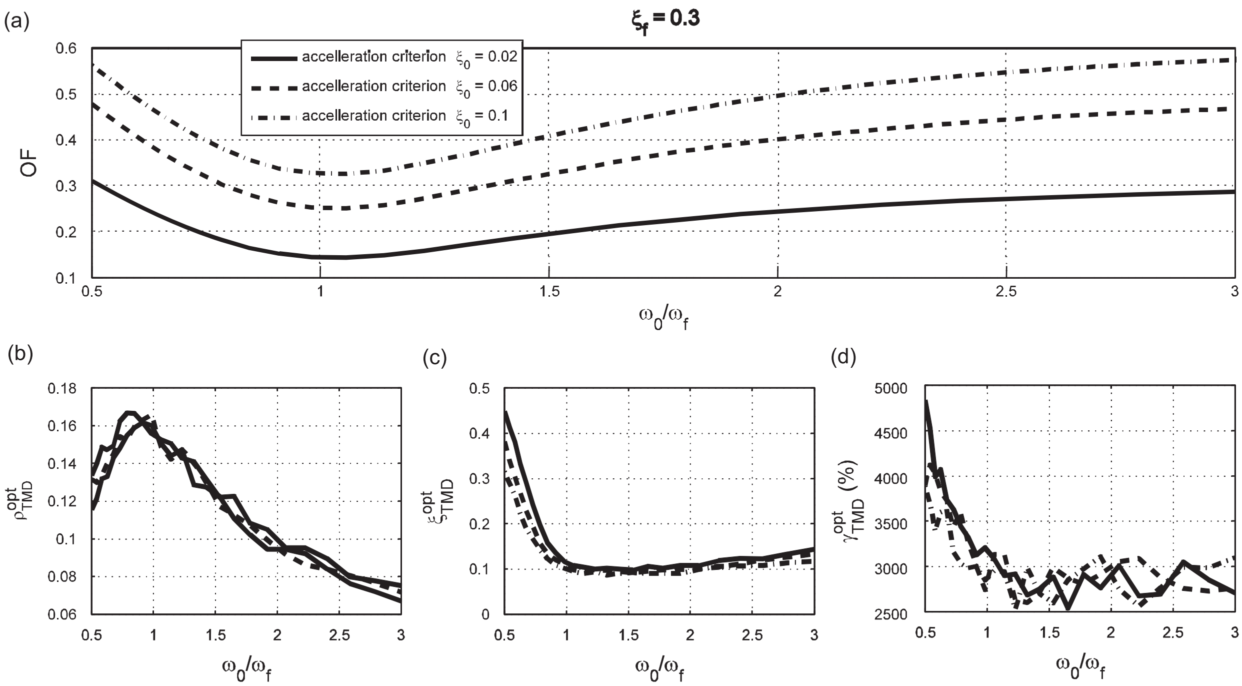

Fig. 2. Optimal solutions for the acceleration criterion.

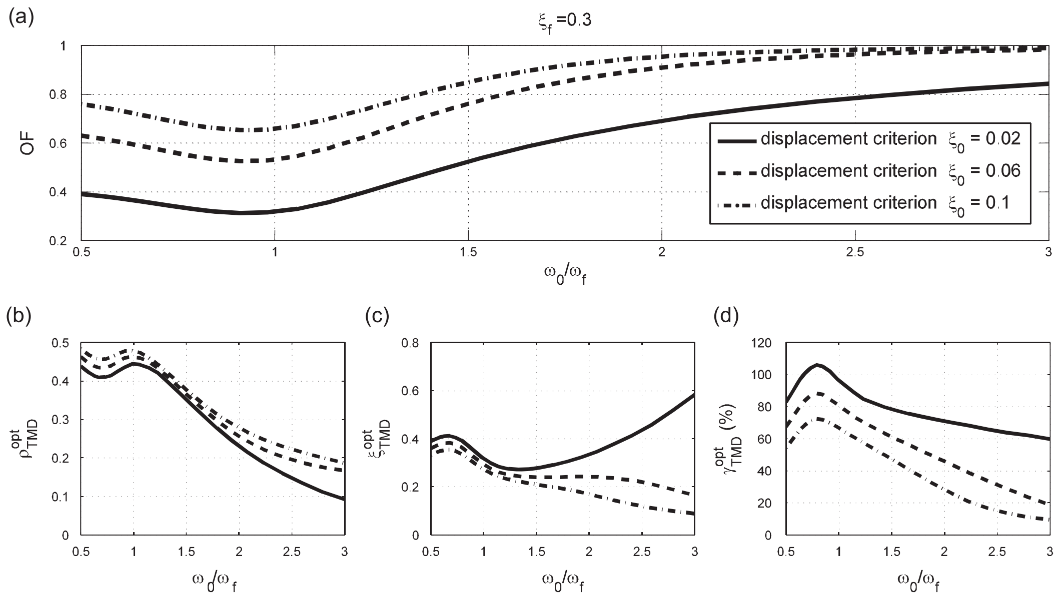

Fig. 3. Optimal solutions for the displacement criterion.

On the *x*-axis is reported the frequency ratio Ω, whereas on the *y*-axis of Figs. 2a, 2b, 2c, 2d, 3a, 3b, 3c, 3d, the optimized OF and the optimum values of design variables are given for the two different criteria adopted, respectively, acceleration criterion and displacement criterion. A value ξ_f=0.3 is considered for the filter damping ratio.

By analyzing plots of Fig. 2 one can observe that the complete optimization problem is able to produce optimal solutions whose performances are interesting, being greater than those obtained by the reduced approach, how one can see for example in [16]. In addition, also the sensitivity of the OF versus the ratio shows a similar trend observed in the reduced approach [16] showing this plot a minimum which means the maximum performance and that corresponds to the resonance condition of main system with input frequency. Moreover, it must be noticed that the tuning frequency ratio assumes values lower than those attained by reduced approach and this is true also if the system is in resonance condition with the input frequency. In this last situation in the reduced criterion, the tuning frequency ratio approaches to unit. In effect in [16] one can notice that when the mass ratio increases, the tuning frequency ratio decreases, and therefore the complete optimization approach which optimizes also the TMD mass ratio furnishes a lower tuning frequency ratio because optimization gives high values of mass ratio. However, the tuning frequency ratio shows a peak in resonance condition and this is quite obvious because the tuning in resonance condition must be the largest. Analyzing the sensitivity of optimum TMD damping ratio, one can observe first of all that the request of damping is greater in the complete approach with respect to the reduced one and that it presents a minimum in resonance condition. This aspect has been also examined in [16] for some values of mass ratio. This means that in resonance condition the TMD works principally by transferring the energy to the added device and then by dissipating it.

Concerning the other aspects of optimum solutions, the general results discussed in [16] are confirmed, i.e. the effectiveness of TMD strategy is greater for system with a low dissipative capacity, and that TMD strategy is more effective if the main system is in resonance with the input. In addition, one can deduce that the optimum design variables do not show important variability when the structural damping varies. In spite of this, it must be noticed, before every other considerations, that the TMD mass required from this approach in the acceleration criterion is extremely larger than admissible values for any practical engineering or technical application (i.e. one hundred times the mass of main system). This consequence is a strong limitation, making at the moment that results shown in Fig. 2 must be considered as purely academic study; but it cannot be excluded that these solutions could be adopted in a future for specific emerging applications (for example *micro* or *nano* mechanics), because they are able to reduce in some cases of 90% the inertial accelerations acting on main system.

The effectiveness of optimum design of a TMD with large mass ratio has been also examined by Hoang et al. [21]. It is known that mass ratio is an important parameter in TMD design. In fact, conventional TMD with a few percentage of mass ratio acts controlling the response via resonance, implying a fairly large movement relative to the primary structure. On the contrary, the control mechanism for a sufficiently large TMD mass differs from that for a small mass ratio, and this physical meaning can be illustrated by frequency analysis.

Fig. 3 shows the optimum solution developed by displacement criterion. First of all, one can notice that performances attained are lower with respect to the acceleration criterion; in addition the variability of the optimized OF versus Ω and main system damping ratio is the same observed in the previous criterion, being more effective the TMD strategy for system with a low damping. Concerning the tuning frequency ratio, optimum values achieved by this criterion are larger. The required optimum damping is of the same magnitude observed in acceleration criterion. Moreover, optimum design variables are more sensible to system damping ratio with respect to the acceleration criterion.

The displacement criterion shows optimal values of mass ratio significantly smaller than those obtained from the acceleration criterion. The maximum required TMD mass is at least of the same magnitude of the main system one, in resonance condition. Far from this situation $\gamma_{\mathrm{TMD}}^{\mathrm{opt}}$ decreases, up to values smaller than 20–30% for medium and high structural damping ratio. Thus this approach should be compatible with some practical applications in different engineering fields.

In Figs. 4–7 the results of a sensitivity analysis developed under different input characteristics are presented in case of displacement criterion.

In detail, two main system damping ratios and three filter damping ratios are considered in the analysis.

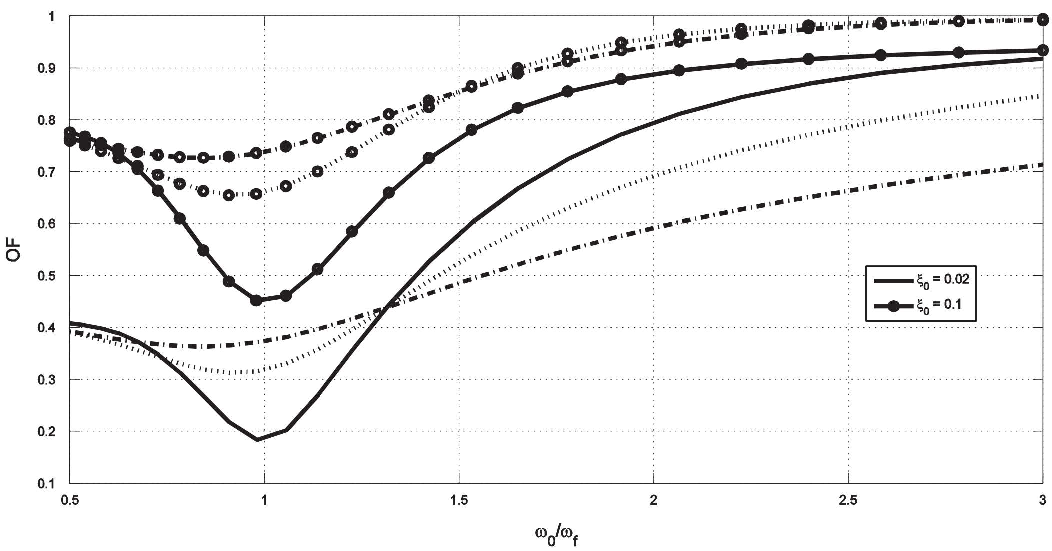

Fig. 4. Optimal displacement based OF (ξ_f=0.1 for continuous lines, ξ_f=0.3 for dotted lines and ξ_f=0.5 for dash-dotted lines).

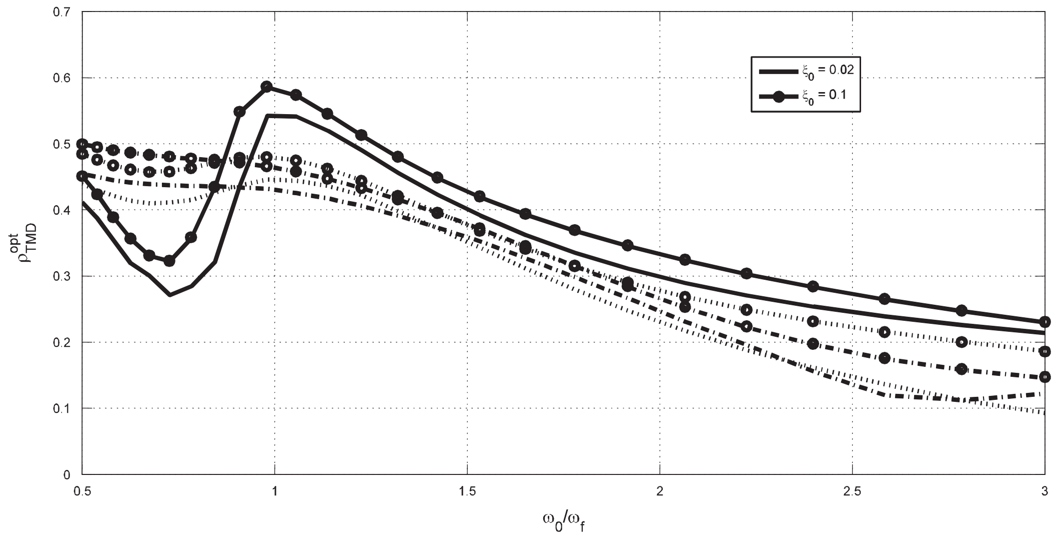

Fig. 5. Optimal displacement based TMD frequency ratio; (ξ_f=0.1 for continuous lines, ξ_f=0.3 for dotted lines and ξ_f=0.5 for dash-dotted lines).

Observing these figures, it is firstly clear that the optimal values of design variables are different, sometimes strongly, from corresponding values obtained by standard two dimensional optimization (invariable mass ratio with limited value). TMD frequency ratio is around 0.5 and less, anyway lower than those obtained from standard reduced optimization (see for example [11]). TMD damping ratio is always greater than 0.1–0.2, and reaches values of 0.5–0.6 for many cases, contrary to results obtained from bi-dimensional optimization. At the same time it must be noticed that both two optimal parameters but also all the other solutions, as will be detailed in the following, are strongly sensible to ξ_f. This result disagrees with the observation in [21].

By observing the sensitivity of the optimized OF, in addition to previous considerations concerning the effectiveness of TMD strategy versus input frequency and system damping ratio, one can deduce that the TMD works better in case of narrow band input, obtaining in this situation a lower OF. This outcome has been also pointed out in [21]. In effect, this result is true only in resonance condition whereas far from this situation the TMD strategy becomes more effective in case of broad band excitation. From the bandwidth of the input it depends also the sensitivity of the effectiveness against the frequency ratio Ω, being this more marked in case of narrow band. In fact, for ξ_f=0.5 the optimum OF shows a little variability when Ω varies.

Also with regard to the optimum tuning frequency ratio one can observe the same variability of solution when the bandwidth of the input varies. In general, in resonance condition the tuning frequency ratio has a peak, but this happens only for narrow band input. When the bandwidth of the input increases, the optimum tuning frequency ratio do not show a peak in resonance condition, anyway it decreases when Ω increases.

Also with reference to optimum TMD damping ratio a sensible variability can be observed with respect to input bandwidth. In effect, for narrow band input the required TMD damping is 50% higher than values obtained for the other conditions, and it presents a more marked variability versus Ω. Far from resonance conditions the behaviour is opposite and the required optimum TMD damping ratio is larger for broad band excitation. One can observe a similar tendency in the variability of optimum mass ratio. In fact when the input is of a narrow band kind, the required optimum TMD mass ratio becomes larger; it diminishes when the bandwidth of the input grows.

One should remember that the main aim of equipping a primary system with a TMD is to split and to reduce the higher-frequency resonant peak into two smaller peaks. This physical meaning becomes visible if a frequency analysis of the response of the combined primary–TMD system is carried out. These aspect have been well analysed by authors in a previous study [20] where it appers clear that the optimum TMD parameters aim to optimize the shape of the combined system transfer function $H_{x_s}(\omega)$, also in relation to the predominant peak of the excitation, whose spectral content is described by the input power Spectral Density Function $S_{x_g}(\omega)$. The optimum parameters, therefore, make minimum the area under the curve of the spectral density function of the response $S_{x_s}(\omega)=\left|H_{x_s}(\omega)\right|^2 S_{x_g}(\omega)$ (this relation is valid in stationary conditions). In this way, all results attained can be physically better understood and justified, and this approach as previous pointed out, has been analysed in a previous work.

Moreover, results regarding optimal mass ratio are the most interesting and present some and innovative meanings. Immediately, it must be noticed that the required mass ratio γ_{TMD}^{opt} is sensible to input main frequency. For $\Omega<1$ a peak is observed, about at Ω=0.6, but generally values are too greater for actual practical applications ($>$50%).

In addition, it is interesting to observe that some values attained for $\Omega>1.5$ are in the range of 20–40%, so that it is possible to be applied in some specific technical contests.

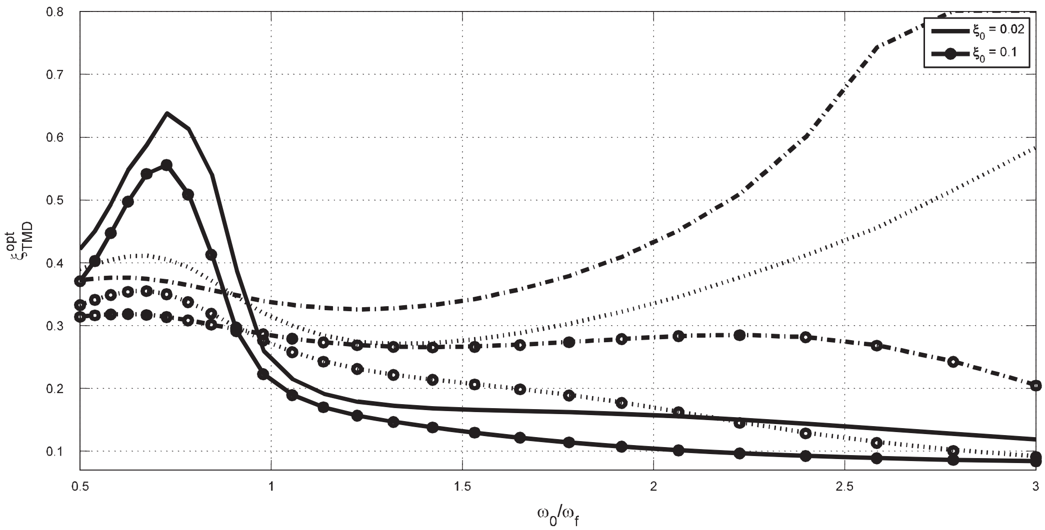

Fig. 6. Optimal displacement based TMD damping ratio; (ξ_f=0.1 for continuous lines, ξ_f=0.3 for dotted lines and ξ_f=0.5 for dash-dotted lines).

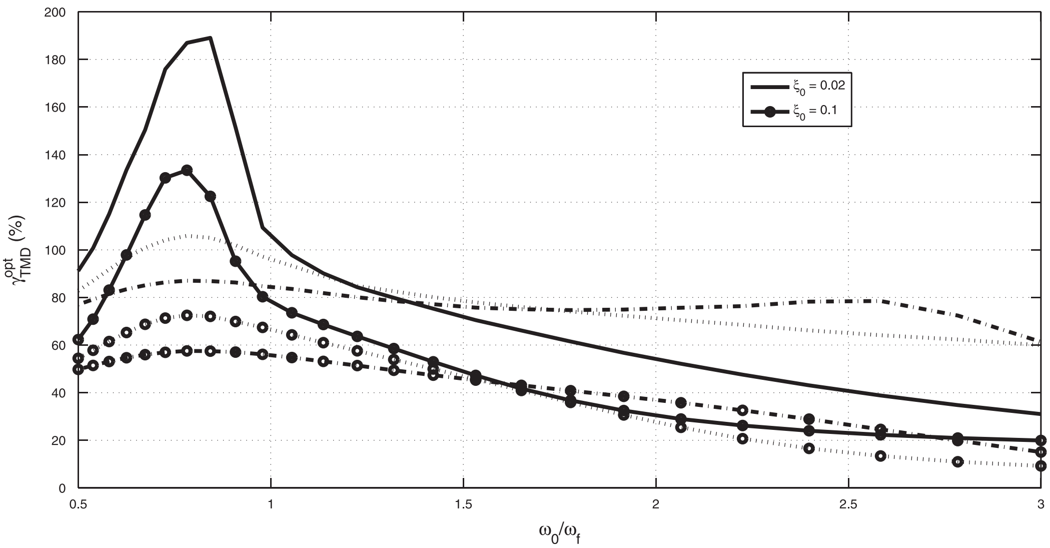

Fig. 7. Optimal displacement based TMD mass ratio; (ξ_f=0.1 for continuous lines, ξ_f=0.3 for dotted lines and ξ_f=0.5 for dash-dotted lines).

As pointed out, TMD mass ratio values depend also on frequency spectrum and are greater for narrow band inputs (for which also efficiency is greater). On the contrary, when input is characterised by a broad band process and the main system presents a small damping, γ_{TMD}^{opt} has small values (compatible with practical applications). Those cases are interesting because anyway the OF is in the range 0.6–0.7, that means a reduction of vibration effects close to 30% or 40%.

4. Conclusions

In this work the optimal solutions for linear TMD mechanical parameters have been evaluated considering also device mass as a design vector element. The case analysed deals with a single degree of freedom system subject to a random acceleration at the support, modelled as a stationary filtered white noise process to properly represent dynamic frequency content of real loading phenomena.

Two different optimization criteria have been investigated. They are based on limitation of main system displacement or inertial acceleration. The two optimization criteria have been considered under different input frequency contents and main system damping ratios. With reference to optimization obtained by complete approach, results show that they are more performable than those obtained by a reduced approach. Nevertheless, complete solutions are commonly inapplicable using acceleration criteria because optimal TMD mass is larger than admissible values in common technical applications. However, there are some limited but interesting situations where, adopting the displacement optimization criterion, complete DV results are compatible with some civil or mechanical applications. For instance further development can be expected in the field of nanomechanics.

Appendix

By defining the tuned and main system inertial acceleration vector as $\overline{\ddot{X}} = \left(\ddot{X}_S, \ddot{X}_T\right)^T$, where

$$\ddot{X}_T = \ddot{Y}_T + \ddot{Y}_b \tag{34}$$

$$\ddot{X}_S = \ddot{Y}_S + \ddot{Y}_b, \tag{35}$$

the system inertial acceleration covariance matrix is given by

$$\mathbf{R}_{\ddot{X}\ddot{X}} = \left\langle \overline{\ddot{X}\ddot{X}}^T \right\rangle = \begin{pmatrix} \sigma^2_{\ddot{X}_T} & E\left[\ddot{X}_T \ddot{X}_S\right] \\ E\left[\ddot{X}_S \ddot{X}_T\right] & \sigma^2_{\ddot{X}_S} \end{pmatrix} \tag{36}$$

and can be obtained as

$$\mathbf{R}_{\ddot{X}\ddot{X}} = \mathbf{D}\mathbf{R}_{XX}\mathbf{D}^{\mathrm{T}} \tag{37}$$

where

$$\mathbf{D} = \left(\mathbf{H}^1_S, \mathbf{H}^2_S\right) \tag{38}$$

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