Relative Spatial Distance Matrix: a Novel and Invariant Data Structure for Representation and Retrieval of Exact Match Symbolic Images

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Abstract

This paper proposes a new data structure called relative spatial distance (RSD) matrix, useful for the representation of symbolic images. RSD matrix is constituted by the relative spatial distances between the iconic objects in the symbolic image. A set of triplets constituted by the pair-wise relative spatial distances between iconic objects forms the surrogate representation of the symbolic image. In order to reduce the space complexity of the representation scheme, a unique one-to-one mapping function is devised to map each triplet uniquely onto real space as key values. The mean and standard deviations of these unique keys in the real space along with number of iconic objects consisted by the symbolic image, forms the final representation vector of the symbolic image. A symbolic image database (SID) is created by storing the representative vectors of images under consideration in sorted order. An SID having 8372 instances of symbolic images is considered for experimentation, to validate the integrity and accuracy of the proposed work. Modified binary search [11] of complexity O(logn) is employed to retrieve the exact match images from the SID.

1. INTRODUCTION

Archival and retrieval of images has been an area of high research activity due to the onset of image databases and its allied areas of application. Storing the image directly into the database and foreseeing retrieval based on a query image is truly an intricate process to realize. Hence the components in the original image are iconified and assigned labels, and the centroid of the components are considered as the positions of the equivalent representative icons. Encoding each iconic object present in the physical image by the respective object label produces the corresponding symbolic image [7]. The problem of handling natural images is thereby reduced to the problem of handling a set of points with labels assigned to each of them. The natural image database is thereby replaced by symbolic image database (SID), for better and easier archival.

Various researchers [6], [3], [14], [12], [15], [16], [10] and [13] have impressed upon the importance of symbolic images. Some of the earlier representation schemes for symbolic images were 2D string representation [5],[2], nine directional lower triangular (9DLT) matrix [1], direction of reference [10] based representation and more recently, triangular spatial relationship (TSR[8]) based representation [13].

The first methodology for exact match retrieval of symbolic image was proposed by Chang and Wu [3] based on 9DLT-Matrix. It was followed by models proposed by Guru and Punitha [10], Punitha and Guru[13]. The 9DLT-Matrix based approach is efficient but may not be effective in many cases as this type of representation is not invariant to rotation. Models proposed by Guru and Punitha [10], Punitha and Guru [13] are more efficient, as well as effective in contrast to the contemporary works, as they are invariant to image transformations and also possess *O(logn)* retrieval time. Despite being an

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invariant and distinct representation scheme for symbolic images, Guru and Punitha's[10] model has a shortcoming in retrieval of exact match symbolic images, as more than one distinct symbolic images may be retrieved for a given query image as it employs principal component analysis (PCA) based retrieval scheme. Many distinct distributions may share the same principal components and the drawbacks of PCA have been enumerated in their work itself. Punitha and Guru's [13] model is far more consequential, as it has an invariant and unique representation scheme and also has a perfect exact match retrieval system. The representative tuples for the image in Punitha and Guru's [13] model are quadruples that are obtained on the basis of TSR[8]. An alternate representation, which does not require angle information and considerably have better computational efficiency, is envisaged in this paper.

In this paper, we propose a novel data structure called relative spatial distance (RSD) matrix, which is used for the representation of symbolic images. RSD matrix is constituted by the relative spatial distances between the iconic objects in the symbolic image and is also invariant to all the 2-dimension transformations viz. rotation, translation, scaling and composite transformations like flipping and reflection. A set of triplets constituted by the pair-wise relative spatial distance between iconic objects and their respective iconic labels, forms the surrogate representation of the symbolic image. The number of triplets increases with respect to the number of iconic objects present in the symbolic image. In order to reduce the space complexity of the representation scheme, a unique one-to-one mapping function is devised to map each of these triplets uniquely onto real space as key values. The mean and standard deviations of these unique keys in the real space along with number of iconic objects consisted by the symbolic image, forms the final representation vector of the symbolic image. A symbolic image database (SID) is created by storing the representative vectors of the given images under consideration in sorted order. Modified binary search [11] of complexity *O(logn)* is employed to retrieve the exact match images from the SID.

The paper is divided into sections as follows. Section 2 introduces the RSD matrix. Section 3 describes the representation of symbolic images in the symbolic image database (SID) using RSD matrix. Section 4 describes the creation of SID and the retrieval mechanism. Experimental results are stated in Section 5. Discussion and conclusion follows in Section 6.

2. RELATIVE SPATIAL DISTANCE MATRIX

Concept of relative spatial distance is illustrated with a simple example before a formal definition is stated for relative spatial distance (RSD) matrix. Consider the symbolic image shown in Figure.1.

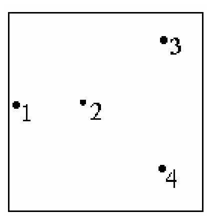

Fig. 1: A symbolic image taken from Chang and Wu [4]

The image consists of four components labelled 1, 2, 3 and 4. The pair-wise Euclidean distance is considered between all the components. Since each spatial relationship is represented by a single tuple (x,y,d), where x is the referenced component, y is the contrasted component and d is the Euclidean distance between x and y, the pair-wise distance between each component can be represented as lower-triangular matrix shown in Fig. 2.

	1	2	3	4
1	--	--	--	--
2	76.00	--	--	--
3	176.26	108.46	--	--
4	176.26	108.46	124	--

Fig. 2: Pair-wise distances between the components of image in Fig. 1

	1	2	3	4
1	--	--	--	--
2	0.4317	--	--	--
3	1	0.6153	--	--
4	1	0.6153	0.703	--

Fig. 3: The RSD matrix deduced from pair-wise distance matrix as in Fig. 2. for image in Fig. 1

The largest element in the pair-wise distance-matrix shown in Fig. 2, is chosen as the pivot element, and the pair-wise distance-matrix is scaled down by that pivot element, which results in the RSD matrix shown in Figure. 3. The RSD matrix consists of pair-wise relative distances between components and hence it is called so.

RSD matrix can be formally defined as follows. Let V = $\{v_1, v_2, \ldots, v_m\}$ be set of m distinct components of objects. Let Z consist of ordered distinct or non-distinct components $z_1, z_2, \ldots, z_s$ such that $\forall$ i = 1,…,s, $z_i \in V$. Let $0 < C \leq 1$, where C is the pair-wise relative spatial distance. So an RSD-Matrix T is an $s \times s$ matrix over C in which t_{ij} is the ith row and jth column element of T, is the relative spatial distance of Z_j to Z_i if i > j, and undefined otherwise because the relative-distance between any two components is symmetric. The matrix T is an RSD matrix according to the ordered set Z.

2.1 Invariant Properties

The RSD matrix described in Section. 2 has the following properties.

Property 1. RSD matrix for any symbolic image is invariant to translation.

Proof. Pair-wise relative distances between components of the image constitute RSD matrix. These relative distances remain unaltered in case of translation of the image.

Property 2. RSD matrix for any symbolic image is invariant to rotation.

Proof. Similar to that stated in property 1.

Property 3. RSD matrix for any symbolic image is invariant to scaling.

Proof. In the case of scaling, the inter-spatial distances between the components in the image are increased or decreased on a uniform scale. In a scaled image, the altered pair-wise distances between the components are scaled by an altered pivot element as described in Section. 2. The relative spatial distances remain unaltered.

Property 4. RSD matrix for any symbolic image is invariant to flipping.

Proof. Flipping transformation can be described in terms of rotation. A planar object is said to flip, if it revolves about a line in the 2D plane. As RSD matrix is invariant to rotation, transitively it is also invariant to flipping,

3. REPRESENTATION OF SYMBOLIC IMAGE IN SID USING RSD MATRIX

A set of triplets is obtained for a given symbolic image as explained in Section. 2. The triplets are of the form (x,y,rd), where x is the referenced component, y is the contrasted component and rd is the relative Euclidean distance between x and y. As the change in nomenclature between the referenced and contrasted components would not make any difference, because of the symmetrical property of the relative distance between given two iconic objects, the values of x and y are swapped if necessary, to maintain the condition x £.y. Hence each triplet is:

(x,y,rd) where x £.y and $0 < rd \leq 1$.

The set of triplets for the image given in Fig. 1. is P = { (1,2,0.4317), (1,3,1), (1,4,1), (2,3,0.6153), (2,4,0.6153), (3,4,0.703)}. The image has 4 components and hence has 6 triplets as the identification signature, because pair-wise relative distance is considered. Hence for an image of m components, ${}^mC_2$ triplets are obtained. Storing the entire triplets in the SID would adversely affect tractability of SID in terms of space. Therefore the triplets are mapped to real space on one-one basis as unique keys. If (x,y,rd) is a triplet then the key K is computed as

$$K = ((\lceil (x+(x/y)) \times c_1 \rceil + rd) \times c_2) \tag{1}$$

with c_1 and c_2 being scaling factors.

With this mapping function the set of ${}^mC_2$ triplets are mapped as ${}^mC_2$ keys. However storing those ${}^mC_2$ keys would still add to the space complexity. Therefore the mean (m) and standard deviation (s) of the ${}^mC_2$ keys, and the number of components (m) in the image, are stored as a triplet, which acts as the final representative vector of the image. The final representative vector is thus constituted as (m, m, s) which is stored in SID.

The set of triplets obtained for image given in Fig. 1. is mapped to real space by generating a key uniquely, with both the scaling factors c_1 and c_2 set at 100. The keys generated are {13474.71, 12553.74, 15100, 37593.53, 26769.63, 25075.39}. The mean and standard deviation for the following population is 21761.17 and 8986.526 respectively. The number of iconic objects in the symbolic image is 4. Therefore the final representative vector is (4, 21761.17, 8986.526). Like-wise representative vectors are computed for the given set of symbolic images, and are stored in the SID in a sorted order. The mechanism of creation and maintenance of SID is dealt in the next section to follow.

4. CREATION OF THE SID AND THE EXACT MATCH RETRIEVAL MECHANISM

In spite the method is theoretically claimed to be invariant, due to the limitations of the computing system in handling floating point numbers and also because of rotation errors, the components m and σ of the representative vector (m, m, σ) of a symbolic image cannot be expected to remain entirely invariant but to lie within a certain range. Also, since the representative vector has three values, each rotated instance of a symbolic image can be looked upon as a point in 3-dimensional Euclidean space R^3. Therefore, the set of all rotated instances of a symbolic image defines in this manner a subspace of R^3 and the centroid of that subspace is chosen as the actual representative vector of the symbolic image in SID.

The following algorithm has thus been devised to create a SID for a given set of symbolic images useful for exact match retrieval.

Algorithm : Creation_of_ SID
Input: Set of symbolic images
Output: SID, Symbolic Image Database
Method:
Step 1: For each symbolic image I to be archived in the SID do
For each rotated instance R_I of I do

i. Obtain relative spatial distances as explained in section 2 and obtain a set of triplets P_{R_I} preserving relative spatial distance among the components present in R_I.

ii. For each triplet in P_{R_I}, compute a unique key K using equation (1).

iii. Compute the vector $D = (m, m, \sigma)$ where m is the total number of components present in the symbolic image, m is the mean and σ is the standard deviation of the keys generated.

For end

Compute the representative vector C_I (for the symbolic image I) which is the centroid of all Ds computed for I.

For end

Step 2: Store the centroids obtained for all images in a sorted sequence.
Creation_of_ SID ends.

It should be noticed that creation of a SID is an offline process and consideration of several instances of the same symbolic image in different orientations helps in recording the possible variations in the components of its representative vector, so that the centroid can be chosen as the best representative vector of the image. Indeed, consideration of several instances at the time of SID creation does neither increase the storage requirement (still it takes O(3)) nor the retrieval time.

Exact match retrieval as stated earlier is an image retrieval process where a symbolic image I is retrieved as an exact match to a given query image Q, if and only if both I and Q are identical. Since each symbolic image I is represented in terms of a vector, which is the average vector of the vectors computed for all rotated instances of I, the retrieval process reduces to a problem of searching for, if not an exact, at least a nearest neighbor for the computed vector D_q of the query image Q, in the SID. Since the vectors in SID are stored in a sorted order, the desired image can be retrieved in O($log\ n$) search time, by employing the modified binary search [11] where, n is the number of images stored in the SID. The modified binary search [11] technique searches for two successive vectors which bound D_q in the SID. Once such two vectors are found, their distances to D_q are computed and the image corresponding to the vector which is nearer to D_q is retrieved from the SID as the desired image.

Algorithm: Exact match retrieval
Input: Q, a symbolic query image
SID, Symbolic Image Database
Output: Desired image
Method:
Step1: Preserve RSD existing among the components of Q by the use of triplets.
Step2: For each triplet, compute the corresponding key K using equation (1).
Step3: Compute the vector $D_q = (m, \mu, \sigma)$ as explained in section 3.
Step4: Employ the modified binary search to find out the two adjacent vectors D_i and D_{i+1} such that $D_i \leq D_q \leq D_{i+1}$.
Step5: Find out the distances, d_1 and d_2 of D_q to D_i and D_{i+1} respectively.
Step6: Retrieve the symbolic image corresponding to the index $\begin{cases} i & \text{if } d_1 < d_2 \\ i+1 & \text{otherwise} \end{cases}$
Exact match retrieval ends.

5. EXPERIMENTATION

Three different sets of experiments were conducted, out of which only one is presented here. Images are considered from [9] which is given in Fig. 4.

Table 1: Experimental results for the images given in Fig. 4.

		Span in μ		Span in σ		Average of μ and σ	
Image Index	**N**	**Min**	**Max**	**Min**	**Max**	**μ(Mean)**	**σ(Std. Deviation)**
S_1	4	21733.69	21763.92	8983.059	9001.946	21749.48	8990.065
S_2	5	25051.85	25078.76	11698.68	11717.54	25066.93	11705.55
S_3	5	27712.34	27738.52	16269.7	16293.02	27725.25	16277.64
S_4	6	35282.4	35303.87	18608.96	18615.37	35290.48	18610.68
S_5	5	34501.89	34527.79	22297.25	22309.4	34513.52	22302.36

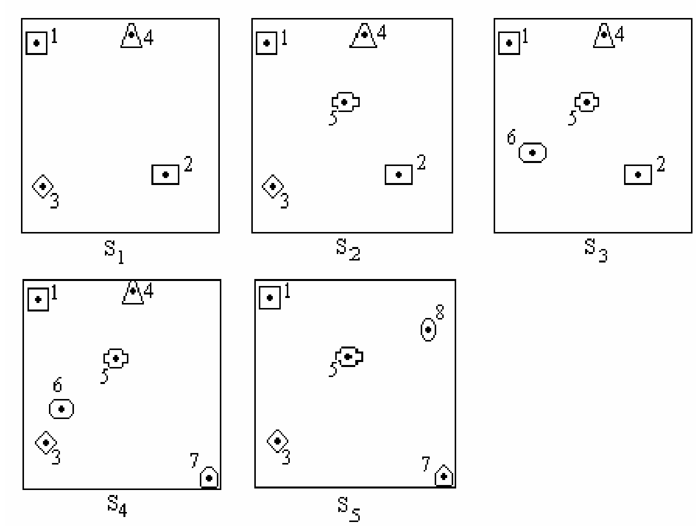

Fig. 4: Symbolic images of Guru et.al.,(2003)

Set of keys $G_1, G_2, \ldots G_{76}$ for 76 instances for each image in Fig. 4 out of which 72 are rotated instances and 4 are scaled instances are generated and corresponding representative vectors are $D_1, D_2, \ldots D_{76}$. As *m* (number of components) does not alter with transformations on the image, only spans due to the variations in μ and σ are taken into account. The lower and upper bounds of the parameter variances for the respective images are shown in Table. 1. The average of the respective spans, also called centriod (D_c) is considered as the final representative vector of the symbolic image. The SID is created by storing the centriods $D_{c1}, D_{c2}, \ldots D_{c5}$ for the images S_1 to S_5 given in Fig. 4 in non-decreasing order and replicates are eliminated. The SID created is given in Table. 2.

Table 2: SID for the images given in Fig. 4

Image Index	**N**	m	s
S_1	4	21749.48	8990.065
S_2	5	25066.93	11705.55
S_3	5	27725.25	16277.64
S_5	5	34513.52	22302.36
S_4	6	35290.48	18610.68

For a given query image *Q*, the representative vector D_q is computed. Exact match retrieval from the SID is conducted as stated in section. 4. For the given set of images considered here, many rotated and scaled instances of these images were given as query images to the SID in Table. 2. The retrieval process is perfect in this context.

6. DISCUSSION AND CONCLUSION

A remote probability exists that an image other than the exact match may be retrieved for a given query image. This is on account of the rounding-off errors introduced by the scaling factor c_1 and c_2 in the key mapping function. The RSD representation involves least mathematical computations and as a result it has become more simpler and less error prone. With the elimination of direction information and angle information as benchmarks for image representation, the RSD representation is more effective in practice as it is more inured to rotational errors. The probability of failure to retrieve an exact match is least as compared to the existing methodologies. Beyond the realms of symbolic image retrieval, we conclude that RSD can also be employed for spatial knowledge representation more conveniently.

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