Pedestrian travel behavior modeling

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Abstract

This paper presents a dynamic mixed discrete-continuous choice approach to modeling pedestrian travel and activity choice behavior in public facilities. The approach views revealed behavior as a manifestation of pedestrians' preferences by assuming that pedestrians choose the alternative that maximizes expected (subjective) utility, while taking into account the uncertainty in expected traffic conditions. The choice dimensions are trajectories between origin and subsequent destinations, areas where activities are performed (multiple vs. fixed destination), execution of discretionary activities, and finally activities completion times and order.

The disutility of a trajectory determines the trajectory choice of the traveler. Destination area choice is included in the modeling by determining time-dependent and destination-specific arrival cost. Furthermore, penalties for not executing a planned activity are introduced into the modeling framework. The resulting modeling approach has a clear analogy with stochastic control theory and dynamic programming in continuous time and space.

The main innovations presented here is the relaxation of the assumption that routes are discrete sets of travel links. The approach relaxes the need to build a discrete network, while routes (trajectories) are continuous functions in time and space. At the same time, destination choice is included in the modeling framework.

Keywords

Route choice, activity scheduling, pedestrians, International Conference on Travel Behavior Research, IATBR

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Frequently used symbols

$\Omega \subset I\!R^2$	:	Walking area in two-dimensional space
$O_p \subset \Omega$	:	Origin area p
$A_{ij} \subset \Omega$	:	Activity area j for activity I
t_0	:	Departure time of traveler
t_1	:	Fixed end time of traveler's planning period
$B_m \subset \Omega$	:	Obstacle m
Σ	:	Non-ordered set of activities i the traveler aims to perform
Ξ	:	Set of feasible activity schedules, respecting compulsory activities and ordering constraints
$S \in \Xi$	:	Activity schedule, i.e. ordered set of activities traveler aims to perform
I_j	:	Service set (set of activities that can be performed at activity area j)
ϑ_i	:	Activity area set (set of activity areas where activity i can be performed)
$x_{[t_0,T)}$	:	Planned / expected continuous trajectory of traveler from $x(t_0)$ to $x(T)$
$\xi_{[t_0,T)}$	:	Realized continuous trajectory of traveler from $\xi(t_0)$ to $\xi(T)$
$v_{[t_0,T)}$	:	Planned / expected continuous velocity trajectory of traveler
Υ	:	Set of feasible velocity trajectories
T_i	:	Expected completion time of activity I
T	:	Set of feasible activity completion times
C	:	Expected costs of choice for specific schedule S, velocity trajectory $v_{[t_0,T)}$, and activity complete times $\{T_i\}$
J_i	:	Expected costs of sub trajectory (including cost of waiting and performing activity) between activity i-1 and I
L	:	Running costs (cost along trajectory)
ϕ_i	:	Terminal costs (cost / utility of performing activity)
σ	:	Standard deviation, reflecting the level-of-uncertainty (probability that trajectory will be realized)
$W_i(t,x)$	:	Minimal expected costs of traveling towards and performing activity i, starting from instant t at location $x(t) = x$
$\Gamma(t,x)$	:	Admissible velocities at instant t and location x

1. Introduction

Planning and design of walking facilities, such as multi-modal transfer points, airports, and shopping malls are generally aimed at enabling efficient, comfortable and safe walking operations. This holds equally for timetable design for public transport services, or the planning of flight schedules for transferring airline travelers serving such transport interchanges. To this end, tools supporting architects, planners, and designers alike are needed.

The differences between pedestrians and other modes of travel are numerous, and thus require development of dedicated theories and modeling tools for travel behavior in walking facilities. For instance, traveler flows in vehicle-based transportation systems are generally restricted to one-dimensional flows with a discrete number of decision points (nodes), travelers can only choose between a finite number of routes, justifying the application of network-based approaches. On the contrary, pedestrian's freedom of movement in public spaces provides an infinite number of route alternatives through the facility. Consequently, network-based approaches are generally less applicable to pedestrian route choice modeling.

Pedestrians generally have opportunities to perform discretionary activities (buying a newspaper or a cup of coffee), and have more choice alternatives regarding the location, the time, and the order of compulsory activities (buying a train ticket, checking in luggage, etc.). As a result, activity-based approaches are preferred. Traditional theories and models of activity planning and travel choice are only applicable to discrete networks, are thus not appropriate to general pedestrian modeling.

1.1 Pedestrian route choice modeling

Several pedestrian modeling approaches have been proposed and tested (Helbing,1997). Gipps (1986), and Hamacher and Tjandra (2001) describe pedestrian route choice through the walking facility by determining a finite number of routes through the walking infrastructure and applying basic *discrete choice modeling*. Their models are choice-based: the main theoretical assumption is that pedestrians make a *subjective rational choice* between alternatives. The number of choice options is assumed finite. Verlander (1997) estimates discrete route choice models using household-based diary data.

Notwithstanding the practicality of assuming only a limited number of route alternatives, in real life pedestrians can choose between *an infinite (and in fact, non-countable) number of paths in the given space*. Hughes (2002) accounts for this aspect, by describing the optimal walking direction to the destination (in terms of travel time) as a function of the current location x of the pedestrian by using *potential functions*. The approach prohibits including general path attributes, such as traveled distance or stimulation of the environment, as well as uncertainty in the traffic conditions expected by the travelers. Beckmann and Puu (1985), and Puu and Beckmann (1999) have described continuous space modeling (only static case is considered); Yang *et al.* (1994) and Yang and Wong (2000) extended and applied the continuous space equilibrium approach for traffic assignment and determination of market areas of competitive facilities.

1.2 Activity scheduling approaches

Activity-based approaches are more and more used as a basis for travel demand analysis. In general, these approaches aim at generating travel patterns of individual travelers and households. The main unit of analysis in these approaches is thus an activity schedule and the associated trip chain; correctly modeling both choice processes is key to the success of activity-based modeling.

Roughly speaking, three major modeling streams can be identified (Arentze and Timmermans,2000): choice-based model approaches, constraints-based models, and computational process models. In this contribution, the focus will be on the *choice-based approaches*, in which observed behavior is typically viewed as a manifestation of people's preferences, and individuals are assumed to choose the alternative that maximizes their utility.

Abdelghany and Mahmassani (2003) present an activity-based approach to micro assignment for motorized traffic, where drivers simultaneously determine their departure time, route, and sequence of their intermediate activities to minimize their perceived travel cost. Here, trip chain patterns are defined by the respective locations of activity areas in the chain, arrival times and these activity areas, and the activity duration at the intermediate destinations.

1.3 Approach overview

This contribution presents a *choice-based model*, conceptually similar to the model of Abdelghany and Mahmassani (2003). In the approach, travelers simultaneously choose their trajectory, the areas where activities are performed, whether discretionary activities will be performed, and finally the order and instants (departure times for leaving activity area) at which these activities are executed.

The paper generalizes the utility optimization model in line with the potential function approach of Hughes (2002). Pedestrians are assumed to optimize their decisions given constraints from his or her activity agenda and risks involved in their decisions, while taking into account the uncertainty in the expected traffic conditions. This uncertainty reflects to among other things lack of experience, observability and randomness of future conditions, and thus pertains to *non-deterministic route characteristics*. For discrete networks, only a few examples of these so-called *risk models* have been proposed. An example is the work of Mirchandani and Soroush (1987); also see Bovy and Stern (1990).

In the considered approach, a route – or rather a trajectory – is defined by a *continuous function in time and space*. The trajectory determines the route through the walking facility, defined by the projection of the trajectory on the *xy*-plane, the departure and arrival times of the pedestrians, and the walking time spend to traverse a certain distance. The trajectory choice is determined by a number of factors, such as the following:

1. Temporal and spatial constraints of both compulsory and discretionary activities (departure of a train or an airplane, location of ticket offices).
2. Presence of physical obstructions and special pedestrian infrastructure (stairs, escalators).
3. Physical limitations, and preferences of the traveler; trip-purpose.

Destination area choice is included in the modeling by determining time-dependent and destination-specific arrival cost. Furthermore, penalties for not executing a planned activity are introduced into the modeling framework. These penalties describe the disutility of not performing an activity in time, and may be relatively moderate (in case of discretionary activities) or very high (in case of compulsory activities).

1.4 Limitations

The contribution mainly considers dynamic activity scheduling and trajectory choice in two-dimensional space. We assume that pedestrians will schedule their activities from prior activity sets; determination of the latter is not discussed here. Moreover, pedestrian walking behavior is also out of the scope of this paper (cf. (Hoogendoorn and Bovy,2002)).

2. Notation and definitions

Figure 1 shows an overview of the most important concepts described in this paper. In the remainder of this section, these concepts will be introduced.

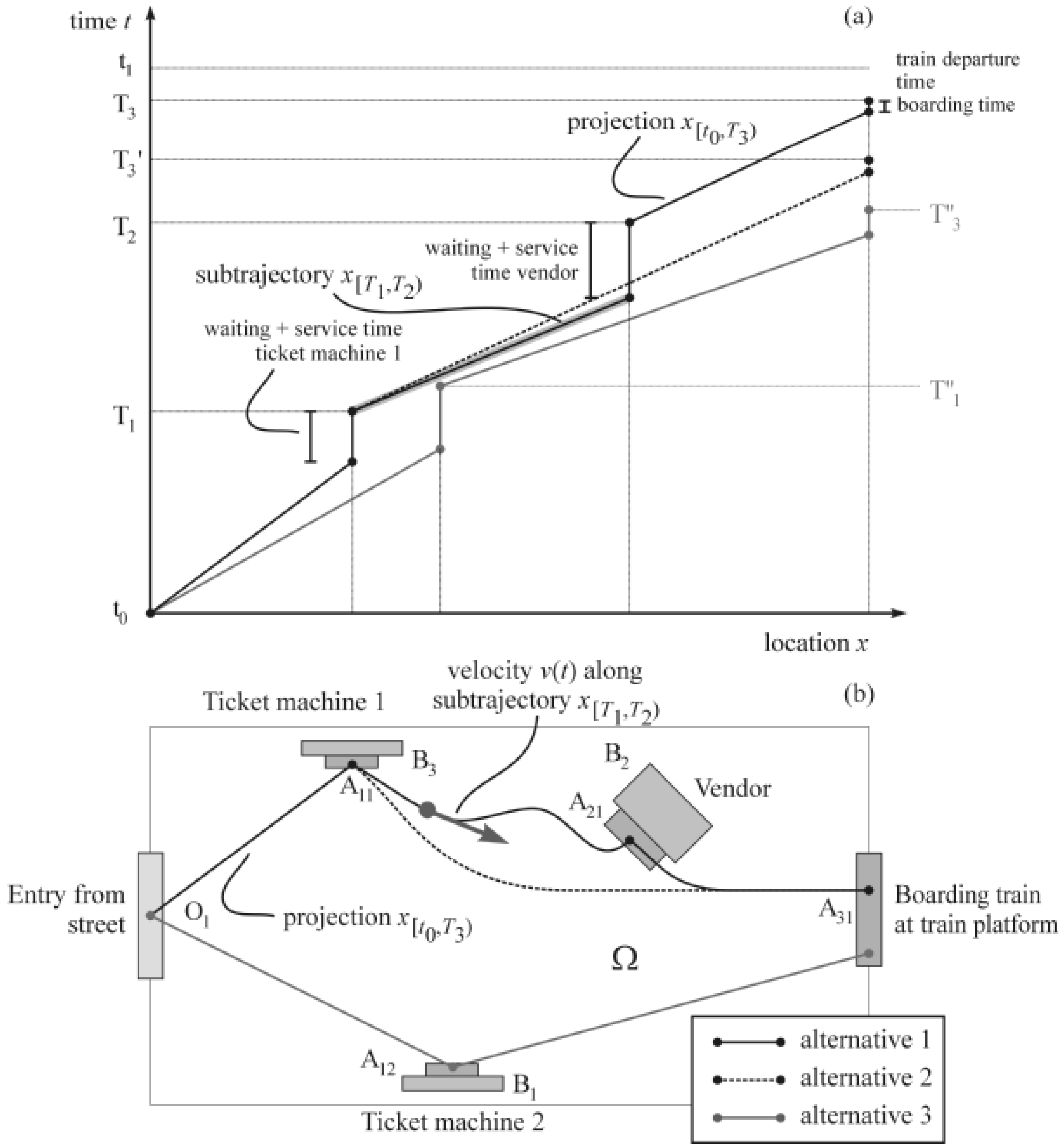
2.1 Description of infrastructure

Infrastructure is described by an area $\Omega \subset I\!R^2$ in which the travelers move. The travelers enter the infrastructure at the origin areas $O_p \subset \Omega$, and perform activities at activity areas $A_{ij} \subset \Omega$. Both the origin and activity areas are described by closed sets. We assume that the time t_0 the traveler enters the facility is fixed.

Obstacles $B_m \subset \Omega$ reflect physical obstructions, around which travelers will have to move while travelling. Figure 1(b) shows an origin area O_1, some activity areas A_{ij}, and several obstacles (ticket machines B_1 and B_3, and vendor B_2).

Besides obstacles, 'special' walking infrastructure needs to be considered. These among other things consist of stairs, escalators, ramps, etc. In general, walking infrastructure is also described by areas for which special travelling conditions hold. These conditions may either reflect restricted walking directions, increased or decreased walking speeds, etc.

Figure 1 Hypothetical station layout (b) and three trajectory choice alternatives (a). Alternative 1 includes activities 1 (buying ticket), 2 (buying newspaper) and 3 (boarding train at platform) and takes the longest time to complete, leaving least time for boarding. Alternatives 2 and 3 include only the mandatory activities 1 and 3, but activity 1 may be performed at different locations.



2.2 Activity sets and schedules

An activity set Σ is a *non-ordered set of activities i* an individual aims to perform. These activities can either be *compulsory* or *discretionary*, yielding 1) the set of compulsory activities Σ_0, that need to be performed before the end of the planning period, and 2) the discretionary activity set Σ_1 of activities that need not be performed, but will in general give some benefit to the traveler. Furthermore, some activities can only be performed in a certain order (e.g. “get money from ATM”, “buy newspaper at newspaper stand”).

An *activity schedule S* is an *ordered set of activities* from the activity set Σ; a *feasible activity schedule* is an activity schedule that 1) contains all activities in the compulsory activity set *and* 2) respects aforementioned ordering constraints. The set of feasible activity schedules is described by Ξ.

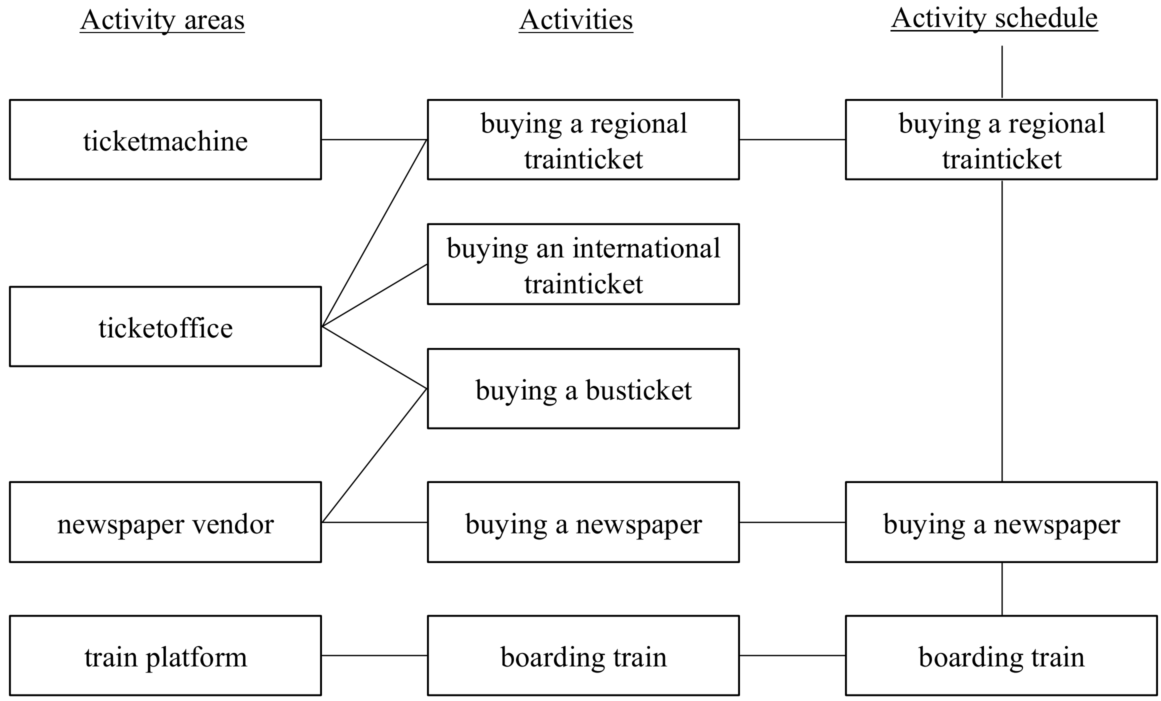
2.3 Activity areas

Activities i can be performed at designated activity areas j. These activity areas are described by regions $A_{ij} \subset \Omega$ in the facility at which specific activities may be performed (see Figure 1 for some examples). Each area j can offer one or more services to the travelers, enabling the traveler to perform (type of) activity i. For each area j we can thus determine a set of activities $I_j = \{i\}$, referred to as the *service set* of activity area j. At the same time, some activities may be performed at multiple activity areas. This is reflected by the *activity area set* $\vartheta_i = \{j\}$, describing all activity areas j at which activity i can be performed.

Figure 2 shows an example of an activity schedule of a traveler, consisting of a number of activities that can be performed at multiple activity areas. In illustration: the activity “buying a regional trainticket” can be performed at a ticket machine and at a ticket office, together constituting the activity area set. At a ticket office, a traveler can perform other activities. More specifically, the service set of a ticket office consists of the activities “buying a regional trainticket”, “buying an international trainticket”, and “buying a bus ticket”.

In general, performing activities will take time, depending on the service time. Service times (e.g. boarding time, time needed to buy a ticket) are dependent on the type of activity area. Furthermore, in over saturated conditions, waiting time may be incurred.

Figure 2 Example of activity schedule, activities and activity areas.



2.4 Trajectories, velocities along trajectories, and paths

An *admissible trajectory* is any possible movement through continuous time and space, mathematically defined by a parameterized curve

$$x_{[t_0,T)} = \{x(s) \in \Omega \mid t_0 \leq s \leq T, x(s) \notin B_m\} \tag{1}$$

where t_0 denotes the fixed departure time and T denotes the free terminal time. The admissible trajectory does not necessarily comply with the location constraints that are put of the trajectory given an activity schedule S.

Feasible trajectories respect the location constraints inferred by the (feasible) activity schedule S (a formal definition is given in section 2.5). Figure 1(a) and (b) depict an example of a feasible trajectory for schedule $S = \{1,2,3\}$, by showing two different projections.

A physical requirement is that a trajectory is a differentiable function of t. In other words, the derivative of x to t exists and is finite. Note that a trajectory is not necessarily loop-less: under specific circumstances, a traveler may decide to come back to a location visited at an earlier time. Consider for instance a traveler waiting for a while at a certain location that provides certain benefits, such as comfort, etc. If the traveler can wait there, while still reaching his / her destination in time, loops in the trajectory are possible.

Rather than the trajectories, the velocities $v_{[t_0,T)}$ along the trajectories $x_{[t_0,T)}$ will be used as the main decision variable of the travelers, for mathematical convenience mostly. These velocity trajectories are defined by

$$v_{[t_0,T)} = \{v(s) \in \Gamma(s, x(s)) \mid t_0 \leq s < T\} \tag{2}$$

where $\Gamma(t,x(t))$ denotes the set of admissible velocities at instant t and location $x(t)$. The set of admissible velocities describes the constraints both caused by the infrastructure (travelers cannot walk into an obstacle B_m, or in the direction opposite in the moving direction of an escalator), and by the flow conditions (traveler speeds are less or equal to a density-dependent speed limit). Clearly, the trajectory $x_{[t_0,T)}$ is determined uniquely by the velocity trajectory $v_{[t_0,T)}$ (and vice versa). In section 3.2 a further distinction is made between planned and realized trajectories and velocity trajectories.

2.5 Activity completion times and locations; feasible trajectories

Activities are performed *along the trajectory* $x_{[t_0,T)}$. Given a feasible activity schedule $S \in \Xi$, the activity completion times T_i are defined by the expected instant at which activity i will be completed; the difference $T_i - T_{i-1}$ equals the sum of the expected walking time and the expected time needed to perform activity i. Since activity i can only be performed at an activity area in the activity area set ϑ_i, a feasible trajectory $x_{[t_0,T)}$ must satisfy:

$$\exists j \in \vartheta_i \text{ such that } x(T_i) \in A_{ij} \tag{3}$$

for all i in the activity schedule, subject to $t_0 < T_1 < T_2 < ... < T_n = T$ (see Figure 1(a) for examples). This implies that revealed activity area choices can be described by the feasible trajectory $x_{[t_0,T)}$ and the activity completion times $\{T_i\}$. Formally, a feasible trajectory $x_{[t_0,T)}$ is thus defined for a feasible activity schedule $S \in \Xi$ using the following constraints: 1) the trajectory is physically admissible and 2) we can find a set of completion times T_i, such that expression (3) holds. In the remainder of the paper, the set of feasible trajectories is denoted by ϑ; Υ denotes the set of feasible velocity paths. Both sets are defined for a specific activity schedule $S \in \Xi$. The set of feasible activity completion times T is defined by the set of all feasible activity times $\{T_1,\ldots,T_n\}$, given the pair $\{S, v_{[t_0,T)}\} \in \Xi \times \Upsilon$.

Let us finally note that the activity completion times T_i partition the trajectory into a number of sub trajectories $x_{[T_{i-1},T_i)}$, describing the trajectory from activity i-1 to activity i (see Figure 1(a) for an example).

3. Activity-based choice theory

The approach asserts that pedestrians choose the alternative that maximizes expected (subjective) utility, within the constraints of time, money, public transit schedules, etc. It is well known that normative choice theory will not fully cover real-life human choice behavior. It does however provide a very convenient and flexible framework for modeling human decision-making. Several empirical studies have shown the applicability of utility-based approaches to pedestrian route choice (Hill,1982), (Bovy and Stern,1990). Hence, its use in pedestrian behavior theory is considered.

3.1 Choice dimensions

It was stated that travelers need to make a decision regarding which activities are performed and in which order, where and when to perform these activities (activity area choice), and how to get from one activity area to the next (trajectory choice). Note that the location of activity performance is determined uniquely by the instant at which the activity is performed, and the total trajectory of the traveler through the walking facility (the inverse is not necessarily true!). This is why the activity area choice is not included explicitly in the decision variables.

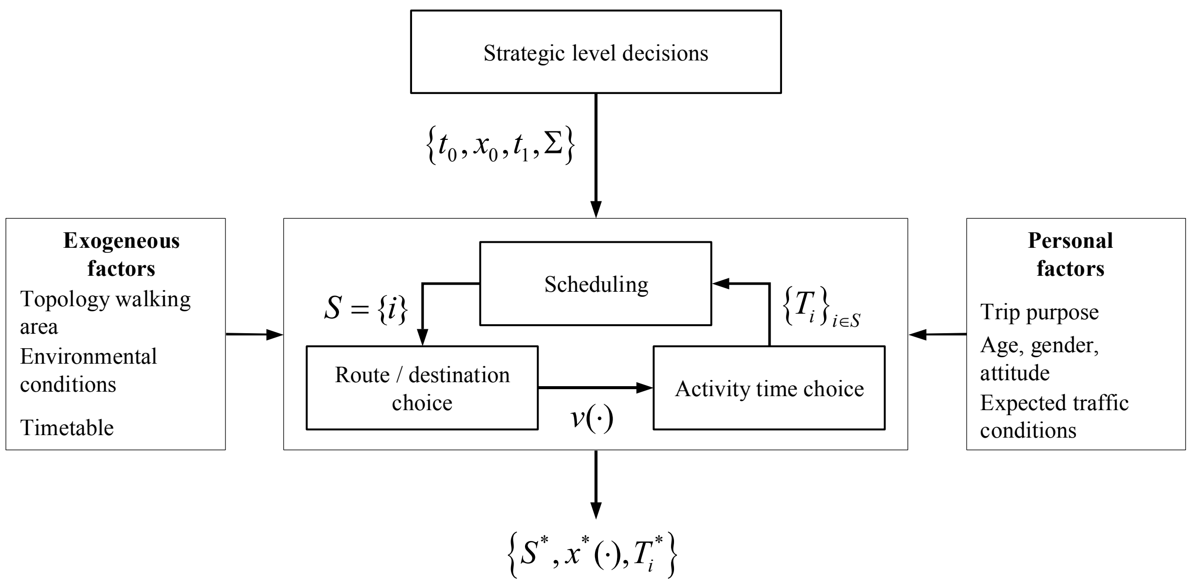
The considered choice dimensions thus pertain to the following aspects (choice of activity areas stem from combining aspects 2 and 3):

1. Choice of the activity schedule $S = \{i\}$ from the set of feasible activity schedules Ξ; recall that S describes *which* activities $i \in \Sigma$ are performed and in *which order*.
2. Choice of the velocity trajectory $v_{[t_0,T)}$ from the set of feasible velocity trajectories (or the trajectory $x_{[t_0,T)}$ through the facility).
3. Choice of the activity completion instants T_i from the set of feasible activity completion times T at which activities $i \in S$ are performed, such that constraint (3) is satisfied.

Figure 3 depicts an overview of these choice variables and their interrelations for a single traveler. In the figure, we consider an individual arriving at instant t_0 at location x_0 in the walking facility $\Omega \subset I\!R^2$, while aiming to perform activities from the subjective activity choice set Σ. This set consists of possible activities to perform in the facility, e.g. “buy a newspaper”, “buy a train ticket”, “wait at the train platform”, or “access the train”. The activities are generic, and reflect the purposes of the individual in the facility.

The choice process is influenced by both external and internal factors. External factors typically reflect influences of the infrastructure, traffic (both pedestrians and other), weather and ambient conditions, etc. Internal or *personal* factors reflect the characteristics of the pedestrians, such as gender, age, time-pressure and purpose of overall trip (commuting, shopping, etc.), attitudes, etc.

Figure 3 Route choice process for individual traveler moving though walking facility.



3.2 Modeling uncertainty

The proposed theory entails that a pedestrian chooses the *expected cost-minimizing triple* {schedule, velocity path, activity completion times} = $\left\{S, v_{[t_0,T)}, \{T_i\}_{i\in S}\right\}$ from the set of feasible triples. Uncertainty may however also affect the traveler’s decisions.

It is assumed that travelers predict the *expected outcomes of chosen options*. In doing so, a pedestrian is aware of the risk that the *sub trajectories* $\xi_{[T_{i-1},T_i)}$ *that are actually realized* may be different from the *planned sub trajectory* $x_{[T_{i-1},T_i)}$, i.e. pedestrians use a mental prediction model to estimate the probability that a certain sub trajectory $\xi_{[T_{i-1},T_i)}$ will occur, given the planned sub trajectory $x_{[T_{i-1},T_i)}$. These probabilities are used to determine *expected* route costs.

This concept can be formalized by the following *stochastic differential equation*:

$$d\xi = dx + \sigma dw = vdt + \sigma dw, \text{ for } s \geq T_{i-1} \tag{4}$$

Eqn. (4) states that if a traveler has arrived at $\xi(t)$ at instant t, and *intends* to apply the velocity $v(t)$, the location $\xi(t+dt)$ of the pedestrian at $t + dt$ cannot be precisely predicted, and will be affected by random term. The white noise w reflects this *uncertainty in the expected traffic conditions* and the resulting effects on the traveler's kinematics. The uncertainty reflects among other things lack of experience, observability and randomness of future conditions.

3.3 Expected subjective cost minimization

It is hypothesized that pedestrians act rationally, and choose the schedule, velocity trajectory, and activity completion times that will minimize the subjective expected costs C. These costs include costs of applying a velocity trajectory, performing an activity at a certain location and time, and performing the activities in a certain order. Without loss of generality, we can express the choice variables discussed in the previous sections:

$$C = C\left(S, v_{[t_0,T)}, \{T_i\}_{i\in S} \mid t_0, x_0\right) \tag{5}$$

Subjective utility optimization yields that the traveler makes the following simultaneous choice

$$\left(S^*, v^*_{[t_0,T)}, \{T_i\}^*_{i\in S}\right) = \arg\min C\left(S, v_{[t_0,T)}, \{T_i\}_{i\in S} \mid t_0, x_0\right) \tag{6}$$

When determining costs, it is assumed that pedestrians incorporate the uncertainty of realizing a certain trajectory given a certain planned velocity trajectory (see section 3.2), which will become clear when specifying the cost function in the following section.

3.4 En-route choice adaptation

So far, the theory pertains to the pre-trip scheduling and planning, based on the expected waiting times, service times, and traffic conditions. Without going into detail, it is noted that the approach may be extended in a rolling horizon framework, where travelers update their expectations based on experienced travel times, waiting and service times. This extension is straightforward and left to the reader.

4. Modeling subjective expected costs

By assuming that travelers base their decisions by optimizing the subjective expected costs of the alternatives, specification of these costs becomes a very important issue. This section discusses the way in which these costs may be modeled.

4.1 General costs expressions

The expected disutility of the combined choice of a pedestrian entering the walking facility at instant t_0 at location x_0 stem from performing specific activities at certain locations, the order in which the activities are performed, and expected cost of walking between activity areas. We assume that the total disutility can be written as the sum of the sub trajectory costs J_i and the scheduling costs $\psi(S)$:

$$C\left(S, v_{[t_0,T)}, \{T_i\}_{i\in S} \mid t_0, x_0\right) := \sum_{i\in S} J_i\left(v_{[T_{i-1},T_i)}, T_i \mid T_{i-1}, x(T_{i-1})\right) + \psi(S) \tag{7}$$

In eqn. (7), the sub trajectory costs J_i is the weighted sum of costs due to:

1. Walking from location $x(T_{i-1}) \in A_{i-1j}$ to $x(T_i) \in A_{ij}$ where activity i is performed, and
2. Waiting for and performing activity i at activity area A_{ij}, reflected by utilities U_{ij}

Activity i may be performed at *multiple activity areas* A_{ij}, yielding different utilities U_{ij}. These differences reflect personal preferences / expectations for using a certain area (e.g. describing that pedestrians expect different waiting and service times at different ticket offices), and the objective differences in service levels provided at the areas.

In the disutility framework described here, the difference between discretionary and compulsory activities is described by high penalties ϕ_i experienced when compulsory activity i is *not* performed, e.g. due to time-constraints. The order in which activities are performed generally depends on the *directness* (Helbing,1997). Directness is reflected by the cost stemming from the trajectory; $\psi(S)$ is generally only used to describe that the activity order is restricted, implying that some activities can be performed only once others are completed.

4.2 Sub trajectory cost factors

In the remainder of the paper, a distinction is made between so-called *running costs* and *terminal costs* (see eq. (8)). Whilst the former describes the cost along the sub trajectory, the latter pertains to costs or benefits that are incurred when waiting or performing an activity. The running cost part of disutility J_i of velocity sub trajectory $v_{[T_{i-1},T_i)}$ depends on (Chiolek,1978), (Gipps,1986):

1. Distance or travel time between origin and destination.
2. Proximity of obstacles or other physical obstructions; closeness to walls.
3. Energy expenditure due to walking at a certain speed for a certain period of time.
4. Expected number of interactions with other pedestrians (*level-of-service*).
5. Stimulation of environment, and attractiveness (e.g. ambience conditions, shopping windows, shelter in case of poor weather conditions, walking on special infrastructure).

Empirical studies (Bovy and Stern,1990), (Guy,1987), (Senevarante and Morall,1986) have shown that these factors are mutually dependent, while their importance in route choice will vary between different (homogeneous) groups of pedestrians, depending on the purpose of their trips, time-pressure, gender, age, etc. Note that besides choosing the shortest route, pedestrians can influence their travel time by increasing their walking speed. It is well known that the speed also depends on personal characteristics of the pedestrians; e.g. commuting pedestrians on average walk at a speed of 1.49 m/s, shopping pedestrians have an average speed of 1.16 m/s (Weidmann,1993). For leisure related trips, stimulation of the environment is far more important. The speed at which pedestrians walk is a trade-off between to the optimal energy expenditure per unit distance walked (at around 1.32 m/s; see Weidmann (1993)) and time pressure. This also explains the preference for using the escalator instead of the stairs.

Note that the effect of special walking infrastructure on route choice will be modeled by 1) changes in the travel time due to the use of the special infrastructure; 2) changes in the energy expenditure, and 3) infrastructure specific constant.

4.3 Sub trajectory cost modeling

Consider a pedestrian who has just finished activity i-1 at instant T_{i-1}. Consider the sub trajectory $x_{[T_{i-1},T_i)}$ or equivalently the velocity sub trajectory $v_{[T_{i-1},T_i)}$. The *expected subjective generalized sub trajectory cost* J_i is described as follows:

$$J_i(T_{i-1}, x(T_{i-1}) \mid v_{[T_{i-1},T_i)}) = E\left[\int_{T_{i-1}}^{T_i} L(s,\xi(s),v(s))ds + \phi_i(T_i,\xi(T_i))\right] \tag{8}$$

s.t. eq. (4). In eq. (8) L and ϕ_i denote the so-called *running cost* and the *terminal cost* respectively for pedestrians aiming to perform activity i at some area A_{ij}. Note that this is a stochastic model, the activity completion times are in fact also random variables. At this point, it is assumed that the activity completion times reflect the expected activity completion times and can thus be considered deterministic.

Eq. (8) shows how the costs of a sub trajectory is determined by expected value of the cost, given the uncertainty in the realized sub trajectory. The level-of-uncertainty thus influences the cost via eq. (4).

4.3.1 Running costs

The *running cost* $L(s,\xi(s),v(s))$ reflects the costs incurred during a very small time period $[s,s + ds)$, given that the traveler is at $\xi(s)$ and is applying velocity $v(s)$ to change his / her position. L express impacts of various attributes of the trajectory and the velocity needed to realize it. We assume that L is *linear-in-parameters*, i.e.

$$L(t,\xi,v) = \sum_k c_k L_k(t,\xi,v) \tag{9}$$

where L_k denote the contribution of different route attributes k, and c_k denote the relative weights (importance of the attributes). However, linearity is *not required for application of the approach described in the remainder of this article*. Note that not all weights can be uniquely determined from real-life observed behavior, since only the relative importance of the weights is relevant. The parameters c_k will be different for different homogeneous groups, e.g. groups having different travel purposes, but also reflect differences amongst pedestrians according to their age, gender, physical health, etc. The different cost factors L_k are described in the ensuing sections; the numbers refer to the cost factors described in section 4.2.

Expected travel time (1)

Expected travel time is included in the expected route cost by defining L_1 as follows

$$L_1(t,\xi,v) = 1 \tag{10}$$

Substitution of running cost (10) component yields the following contribution to (9)

$$E\left[\int_{T_{i-1}}^{T_i} L_1\left(s,\xi(s),v(s)\right)ds\right] = E\left[\int_{T_{i-1}}^{T_i} c_1 ds\right] = c_1 E\left[T_i - T_{i-1}\right] \tag{11}$$

Eqn. (11) shows that the contribution of (10) equals the expected travel time $T_i - T_{i-1}$, multiplied by the weight c_1, expressing *time-pressure*, depending on for instance the trip purpose.

Discomfort due to walking too close to obstacles and walls (2)

L_2 is a *monotonically decreasing function* g of the distance $d(\xi,B_m)$ between the location ξ of the pedestrian and the obstacle, e.g.

$$L_2(t,\xi,v) = g_m\left(d(B_m,\xi)\right) = a_m \exp\left(-d(B_m,\xi)/b_m\right) \tag{12}$$

In Eqn. (12), $a_m > 0$ and $b_m > 0$ are scaling parameters, describing the *region of influence* of obstacle m. Both a_m and b_m are dependent on the type of obstacle that is considered, e.g. they are different for building faces with and without a window, regular walls, trees, newsstands, etc. (HCM,2000).

Walking at a certain speed (3)

To describe that the planned walking speed $||v||$ is a trade-off between the time remaining to get to the activity area in time and the energy use due to walking at a particular speed, we assume that this kinetic *energy consumption* is a quadratic function of the pedestrian speed

$$L_3(t,\xi,v) = \tfrac{1}{2}\|v\|^2 = \tfrac{1}{2}v'v \tag{13}$$

Discomfort due to crowding and level-of-service (4)

In including the *cost of the expected pedestrian interactions*, we consider the function $\zeta = \zeta(t,\xi)$, describing the expected number of interactions with other pedestrians at (t,ξ). Note that for pedestrian flow operations, frequency and severity of interactions (or rather, physical contact) relates directly to the definition of the *level-of-service* (HCM,2000). In the remainder, it is assumed that $\zeta(t,\xi)$ is some (nonlinear) function of the expected density $k(t,\xi)$

$$L_4(t,\xi,v) = \zeta\left(k(t,\xi)\right) \tag{14}$$

Stimulation of the environment (5)

Stimulating effects of the environment can be described by considering the benefit (or cost) $\gamma(t,\xi)$ of walking at a certain location x at instant t. Note that these benefits has a negative sign (negative cost). We have

$$L_5(t,\xi,v) = \gamma(t,\xi) \tag{15}$$

In substituting the contributions (10)-(15) into the running cost (9), we get

$$L(t,\xi,v) = c_1 + c_2 \sum_m a_m e^{-d(\xi,B_m)/b_m} + \tfrac{1}{2} c_3 v'v + c_4 \zeta(t,\xi) + c_5 \gamma(t,\xi) \tag{16}$$

The approach is easily adapted when considering a heterogeneous population with *taste-variation* and *differences in the physical abilities*. In fact, these differences are the main reasons for including the different cost factors: empirical research has indicated large differences in how different types of pedestrians value route attributes (Bovy and Stern,1990), (Hill,1982), (Senevarante and Morall,1986), and (Guy,1987). The pedestrian population is stratified into homogeneous groups, and route-choice and activity scheduling is solved for each group. The characteristics of each homogeneous group is characterized by the utilities U_{ij} gained when performing activity i at A_{ij}, the weights c_k, and the maximum walking speed $v_0(t,x)$. For instance, when travel time is the dominant factor (e.g. for commuting pedestrians), c_1 will be relatively large compared to c_2, c_4 and c_5; when shopping, stimulation of the environment will be a relatively important attribute, which is expressed by c_5. Furthermore, the weights c_k also depend on the situation: in case of an emergency, travel time (or distance) will be the dominantly important attribute. In the latter case however, the applicability of utility optimization approaches may be limited, depending on the evacuation situation.

4.3.2 Terminal costs

The *terminal cost* $\phi_{ij}(T_i,\xi(T_i))$ reflects the cost incurred by the traveler ending up at position $\xi(T_i)$ at the end time T_i. It includes the following cost factors:

1. Penalty that may be incurred when the traveler does not complete the activity in time (e.g. between arrival and departure of a train), for $j \in J_i$.

2. Utility $U_{ij}(T_i)$ of performing the activity at a certain area and time instant, including expected costs due to a) expected waiting time T_{ij}^w and b) expected service time T_{ij}^s, and c) penalty when not performing the activity at the preferred time T_i^p (early or late arrival).

In mathematical terms, point 1 is described by

$$\phi_{ij}(t_1, x) = \Phi_i \tag{17}$$

where t_1 denotes the fixed end time of the planning period[1] (e.g. the departure time of the train), and where Φ_i denotes the penalty (e.g. of having missed the train).

Point 2 is expressed mathematically by

$$\phi_{ij}(T_i, x) = -U_{ij}(T_i) \text{ for } \xi(T_i) \in A_{ij} \text{ and } T_i < t_1 \tag{18}$$

where $U_j(T)$ denotes the utility of arriving at destination A_{ij} at time T_i. Note that this mathematical conduct allows also *penalizing early arrival*, e.g. arrival before the arrival time T_i^p of the train, e.g.

$$U_{ij}(T_i) = U_{ij} - \alpha_0 \max\left(0, T_a - T_i\right) \tag{19}$$

where

$$U_{ij} = U_{ij}^0 - \alpha_1 T_{ij}^w - \alpha_2 T_{ij}^s \tag{20}$$

denotes the utility, minus the costs due to waiting and being served.

5. Operationalization of pedestrian behavior theory

In the remainder, we will operationalize the outlined theory. We derive models by applying dynamic programming theory for dynamic stochastic control models. The operationalization is achieved by subsequently solving the following problems:

1. Optimal (velocity) sub trajectory and activity completion time choice for single activity i, for given and fixed initial time T_{i-1} and location $\xi(T_{i-1})$, including activity area choice.

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[1] Note that the planning period reflects the period within which the traveller plans his or her trip. The end of the planning period is generally not equal to the pre-specified or actual arrival time.

2. Optimal velocity trajectory and activity completion time choice for fixed schedule S (multiple activities, fixed order), based on the outcomes of 1 and constraints.

3. Optimal velocity trajectory, activity completion time choice, and scheduling (multiple activities, free order), based on the outcomes of 2 and constraints.

Step 1 will be most involved (section 5.1); step 2 (section 5.2), and 3 (section 5.3) are very basic extensions of step 1. In all steps, uncertainty will be included in the modeling approach. The results of the different steps will be illustrated by several examples. Note that all involved factors are in principle time-dependent.

5.1 Optimal sub trajectory choice model for single activity

Let us first consider a pedestrian who has arrived at location $\xi(t)$, $t > T_{i-1}$ at time instant t and is wants to perform activity i at one of the activity areas A_{ij} with $j \in \vartheta_\iota$. We aim to determine the planned velocity trajectory as well as the activity completion time.

5.1.1 Modeling principle and problem formulation

To derive the model, let us consider a pedestrian who at time $t \geq T_{i-1}$ has arrived at location $\xi(t) = z$, and aims to reach either of the activity areas with minimal costs from that point onward. The subjective utility optimization paradigm implies that the pedestrian will choose the velocity sub trajectory $v_{[t,T_i)}$ yielding the predicted route and used activity area that minimize the subjective expected cost for the remainder of the trip s.t eq. (4), i.e.

$$v^*_{[t,T_i)} = \arg\min J_i\left(v_{[t,T_i)}, T_i \mid t, z\right) = \arg\min E\left[\int_t^{T_i} L\left(s, \xi(s), v(s)\right) ds + \phi_i\left(T_i, \xi(T_i)\right)\right] \quad (21)$$

To solve the sub trajectory choice problem, let us define the so-called *expected minimum perceived disutility function W*(t,z) (often referred to as the *value function* in optimal control theory) by the expected value of the costs upon *applying the optimal velocity* $v^*_{[t,T_i)}$

$$W(t,z) := E\left[\int_t^{T_i} L\left(s, \xi^*(s), v^*(s)\right) ds + \phi_i\left(T_i, \xi^*(T_i)\right)\right] \quad (22)$$

subject to

$$d\xi^* = v^* dt + \sigma dw \text{ subject to } \xi^*(t) = z \quad (23)$$

To derive the dynamic programming equation, consider a small period $[t,t+h)$. According to Bellman's optimization principle (Bellman,1957), we have

$$W(t,z) = E\left[\int_t^{t+h} L\left(s,\xi^*(s),v^*(s)\right)ds + W\left(t+h,\xi^*(t+h)\right)\right] \tag{24}$$

Eqn. (24) describes that the expected minimal cost of walking from $(t,\hat{x})$ to A_{ij} equals the minimal expected cost of both walking from (t,z) to $(t+h,\xi^*(t+h))$ and walking from $(t+h,\xi^*(t+h))$ to A_{ij}. For small h, the following approximation is valid

$$E\left[\int_t^{t+h} L\left(s,\xi(s),v(s)\right)\right] = L\left(t,\xi(t),v(t)\right)h + O\left(h^2\right) \tag{25}$$

The random variate $\xi(t+h)$ describing the predicted location at instant $t + h$ subject to eqn. (4) can be expanded using a Taylor series

$$\xi(t+h) = z + hv(t) + \sigma\sqrt{h}w + O\left(h^{3/2}\right) \tag{26}$$

where $\sigma h^{1/2}w$ is a $N(0,h\sigma\sigma')$ distributed random variate. We can rewrite the expected value of the second term of the right-hand-side of eqn. (24)

$$E\left[W\left(t+h,\xi(t+h)\right)\right] = W\left(t+h,z+hv\right) + \frac{h}{2}\sum_{kl}\Theta_{kl}(z,v)\frac{\partial^2 W(t,z)}{\partial x_k \partial x_l} + O\left(h^{3/2}\right) \tag{27}$$

where $\Theta(z,v) := \sigma(z,v)\sigma'(z,v)$. Substitution of eqns. (25) and (27) into eqn. (24), using the appropriate Taylor series expansions, and taking the limit $h \to 0$ yields the so-called *Hamilton-Jacobi-Bellman* (HJB) or dynamic programming equation for decision making in continuous time and space *under uncertainty*

$$-\frac{\partial}{\partial t}W(t,z) = H\left(t,z,\nabla W,\Delta W\right) \text{ where } \left\{\Delta W\right\}_{kl} := \left\{\frac{\partial^2 W}{\partial z_k \partial z_l}\right\} \tag{28}$$

for $z \in \Omega$ and $z \notin \bigcup_m B_m$ with terminal conditions (describing the minimal cost when a pedestrian has arrived at the end of the planning period at time $t = t_1$)

$$W(t_1,z) = \Phi_i \tag{29}$$

and boundary conditions, describing the cost when the pedestrian has arrived at either of the activity areas A_{ij} before the end of the planning period:

$$W(T_i, z) = -U_{ij}(T_i) \text{ for } z \in A_{ij} \text{ and } T_i < t_1 \tag{30}$$

The *Hamilton function H* is an auxiliary function, defined by

$$H(t,z,\nabla W,\Delta W) := \min_{v\in\Gamma(t,z)} \left\{ L(t,z,v) + \sum_k v_k \frac{\partial W}{\partial z_k} + \frac{1}{2}\sum_{kl} \Theta_{kl}(z,v) \frac{\partial^2 W}{\partial z_k \partial z_l} \right\} \tag{31}$$

Using terminal conditions (29) at t_1, the HJB equation can be solved backwards in time, subject to boundary conditions (30). The solution $W(t,z)$ describes the minimum expected cost to either of the activity areas for a pedestrian located at $\hat{x}$ at instant t. From $W(t,z)$, the optimal route can be determined easily, as is shown in the following section.

5.1.2 Optimal speed and direction

Considering a pedestrian who has arrived at location z at instant t, the optimal velocity v^* at that instant t satisfies

$$v^*(t,z) = \arg\min \left\{ L(t,z,v) + \sum_k v_k \frac{\partial W}{\partial z_k} + \frac{1}{2}\sum_{kl} \Theta_{kl}(z,v) \frac{\partial^2 W}{\partial z_k \partial z_l} \right\} \tag{32}$$

subject to $v^*(t,z) \in \Gamma(t,z)$. Assume that the uncertainty level does not explicitly depend on v, i.e. $\Theta_{ij}(z,v) = \Theta_{ij}(z)$. It can then be shown that for the running cost definition (16), we find

$$v^*(t,z) = V^*(t,z)e^*(t,z) \tag{33}$$

where the optimal speed V^* and optimal direction e^* are defined by

$$V^*(t,z) := \min\left\{ \frac{\|\nabla W(t,z)\|}{c_3}, v_0(t,z) \right\} \text{ and } e^*(t,z) := -\frac{\nabla W(t,z)}{\|\nabla W(t,z)\|} \tag{34}$$

The partial derivatives ∇W are the *marginal cost* of the pedestrian location z: if z changes by a small amount δz, the change in the total minimal cost $W(t,z)$ equals $\nabla W(t,z)\cdot\delta z$. Eqn. (34) shows how the optimal direction $e^*(t,z)$ points in the *direction in which the optimal cost decreases most rapidly*. Upon walking into this direction, eqn. (34) shows that the optimum speed $V^*(t,z)$ depends on the rate $\|\nabla W\|$ at which the minimum cost W function decreases in the optimal direction e^*, the relative cost c_3 of walking at high speeds, and the maximum admissible speed described by $\Gamma(t,z)$: when the expected minimum perceived disutility function W decreases very rapidly, the pedestrian will walk at the maximum speed. When either $W(t,z)$ decreases very slowly in the optimal walking direction, pedestrians tend to walk at a lower speed. This may be the case when the time pressure is low: in line with empirical observations, where walking speed for pedestrians having higher time pressure (e.g. commuters) are farther from the 'energy consumption-optimal' walking speeds than in case the time pressure is lower (e.g. shopping pedestrians).

Note that $W(t,z)$ describes the optimal direction and speed of a pedestrian that has arrived at z at instant t, irrespective of his or her origin. This formulation hence also provide the means to simply redetermine the optimal directions for pedestrians that have strayed from their optimal trajectory, for instance due conflicting pedestrian flows. This renders the approach especially useful in a micro simulation setting where the positions of pedestrians are determined not only by the optimal trajectory, but also by interactions with other pedestrians in the flow.

5.1.3 Numerical solution approach

The dynamic programming equation (28) can be solved by discretizing the area Ω into small $\delta\times\delta$-cells, and considering approximate solutions on this lattice at fixed time instants $t_k = hk$ (i.e. δ is the spatial step size, and h is the temporal step size). We can show that the resulting problem is a *Markov diffusion process in two dimensions* with nearest-neighbor transitions that are determined by the stochastic differential eqn. (4) (Fleming,1993). Solving this (discrete) stochastic dynamic programming problem is related to solving eqn. (28) by replacing the partial derivatives with the appropriate finite differences. For details, we refer to Hoogendoorn and Bovy (2003).

Example 1 (*Route choice and activity area choice for free-flow conditions in Schiphol Plaza*) This example considers Schiphol Plaza, which is a multi-purpose multi-modal transfer station. Figure 4 shows a snapshot of the microscopic simulation model NOMAD described in (Hoogendoorn and Bovy,2002). In this figure, exits E1-E5 indicate exits from Schiphol Plaza; escalators E6 and E7 indicate exits to the train platforms. V1 and V2 depict the locations of the newspaper vendors.

Figure 4 Snapshot from Schiphol Plaza simulation using the NOMAD model (Hoogendoorn and Bovy,2002).

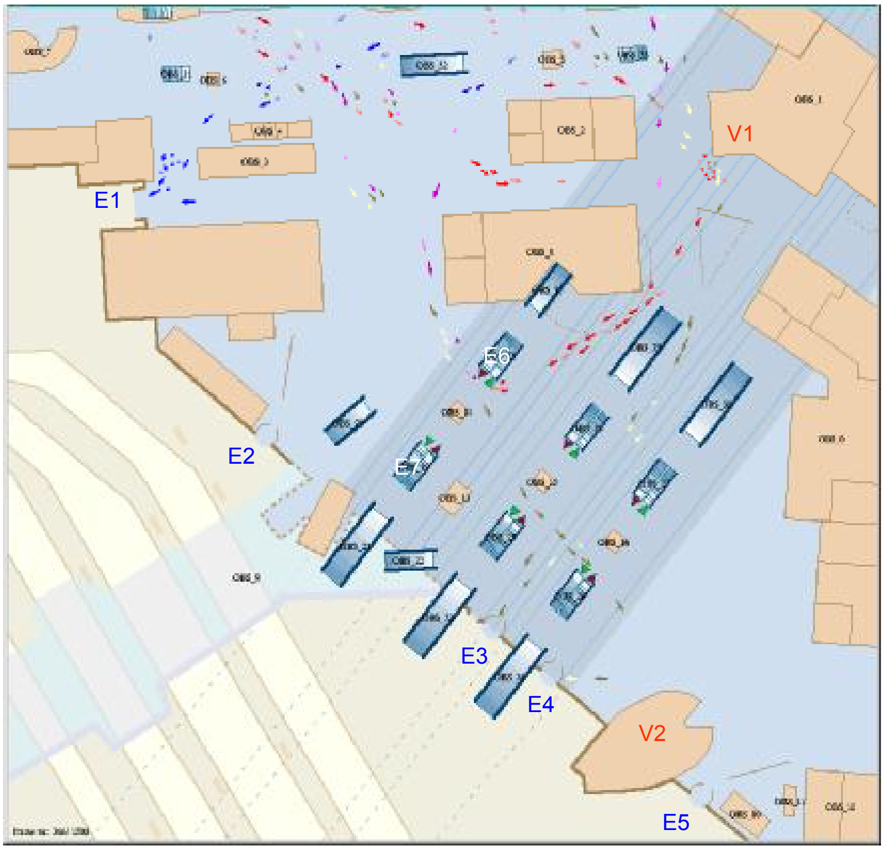


Figure 5 Expected minimum perceived disutility functions W and example routes describing combined route-choice and activity area choice for a) leaving Schiphol Plaza via either of the escalators E6 and E7 to the train platforms and b) leaving Schiphol Plaza via either of the exits E1-E5. The numbers indicate the generalized walking time (in seconds).

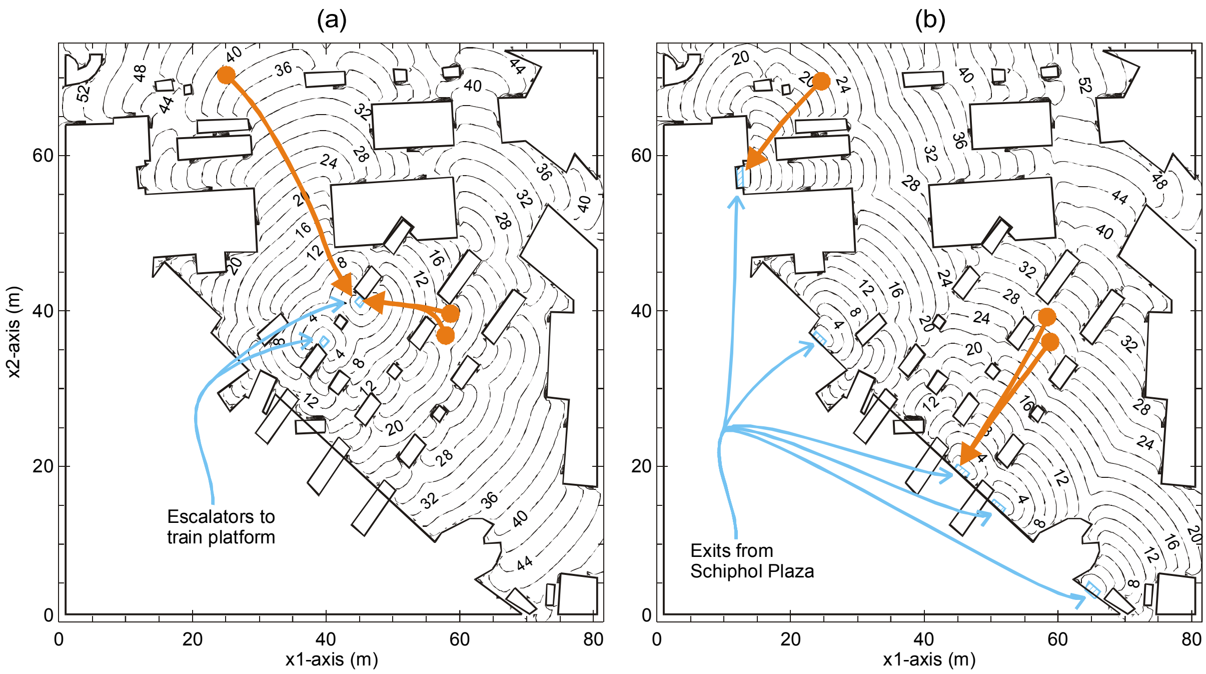
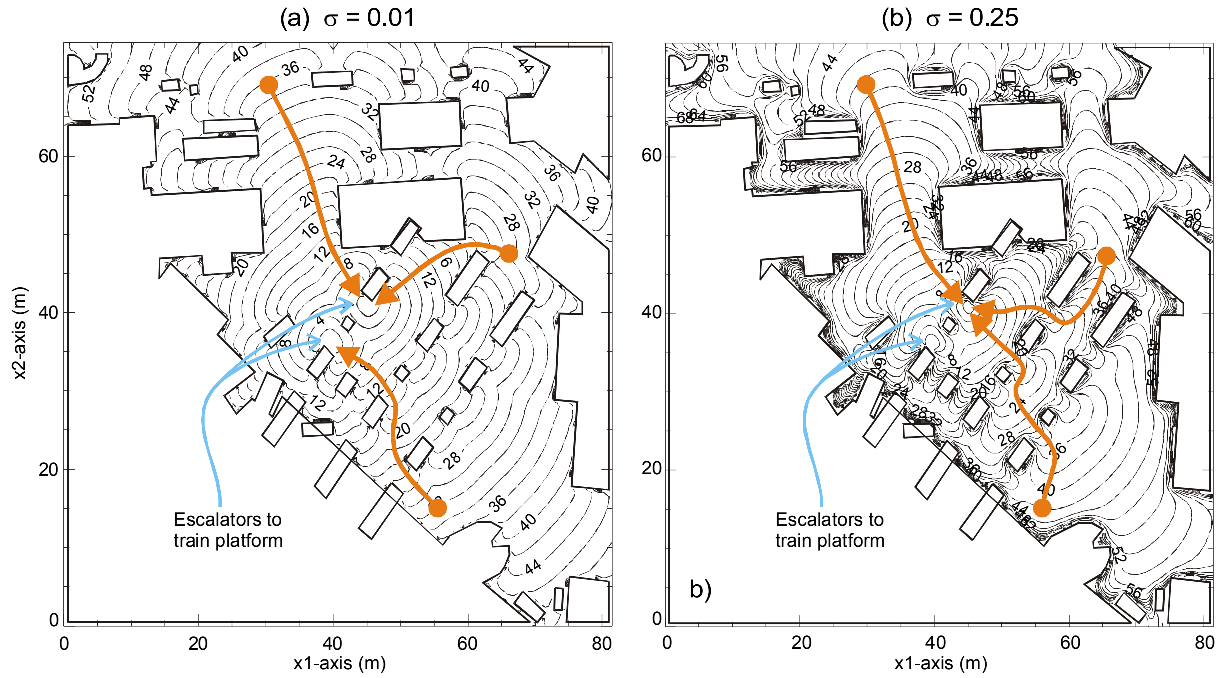


Figure 5a and Figure 5b respectively show the expected minimum perceived disutility functions W and example routes for pedestrians using the escalators E6 or E7 to get to the train platform and pedestrians using either of the exits E1-E5 to get outside. The expected minimum perceived disutility functions have been determined using the numerical solution approach described in section 5.1.3, with running cost factor weights $c_1 = 1$, $c_2 = 10$, $c_3 = 1.5$, $c_4 = c_5 = 0$, and $a_m = 1$ and $b_m = 0.1$ (for all obstacles m). Eqn. (33) shows that the optimal routes are perpendicular to the iso-expected minimum perceived disutility function curves depicted in the figures. Figure 5a shows three exemplar routes that all lead to the escalator E6. In this case, combined route-choice / activity area choice yield different routes but the same activity area. Figure 5b shows three exemplar routes leading to the exits. Not all routes lead to the same exit in this case. □

Example 2 (*route choice under uncertainty*) Let us reconsider the Schiphol Plaza case for pedestrians aiming to walk to either of the escalators E6 or E7. For this particular example, the level-of-uncertainty was constant, i.e. $\sigma(x,v) = \sigma_0$. Figure 6a and Figure 6b show the combined route choice and activity area choice behavior for different uncertainty levels. Figure 6b clearly shows that when future conditions are less certain, pedestrians are inclined to avoid narrow passageways or walking close to obstacles, yielding different route / activity area choices. □

Figure 6 Expected minimum perceived disutility functions for leaving Schiphol Plaza via escalators for uncertainty levels a) $\sigma_0 = 0.01$ and b) $\sigma_0 = 0.25$. Optimal paths are perpendicular to iso-expected minimum perceived disutility function curves.



5.2 Optimal choice behavior model for fixed schedule *S*

The approach can be easily extended to a *fixed-order activity schedule* considering activity schedules $S = \{1,\ldots,I\}$, where for each activity $i \in S$, several activity areas A_{ij} may be considered. In assuming that pedestrians make a simultaneous trajectory-choice and activity area choice decision (compare to (Abdelghany and Mahmassani,2003)), in the modeling we need to consider the next activity $i + 1$ (and the respective activity areas A_{i+1j} for $j \in \vartheta_{i+1}$) into the path and activity area choice associated with activity *i*. This is achieved by application of the *dynamic programming principle* due to Bellman (1957), claiming that *at each moment of the control interval, the remaining optimal velocity trajectory* $v^*_{[s,T_{i+1})}$, *with* $s \geq T_i$ *of an optimal velocity trajectory* $v^*_{[T_i,T_{i+1})}$, *is optimal with respect to the current state determined by the preceding control actions*. This implies that to determine the optimal velocity trajectory, we need to solve the problem backwards in time, starting by computing the expected minimum perceived disutility function $W_I(t,x)$ for the final activity I in the fixed schedule S. Having determined $W_{i+1}(t,x)$, we compute $W_i(t,x)$ for activity i by solving the HJB equation

$$-\frac{\partial}{\partial t}W_i(t,x) = H\left(t, x, \nabla W_i, \Delta W_i\right) \tag{35}$$

using boundary conditions

$$W_i(T_i, x) = W_{i+1}(T_i, x) - U_{ij}(T_i) \text{ for } x \in A_{ij} \tag{36}$$

Note that since we have assumed that activity $i + 1$ is completed, we need not impose an additional penalty for not being able to reach any of the activity areas A_{ij}.

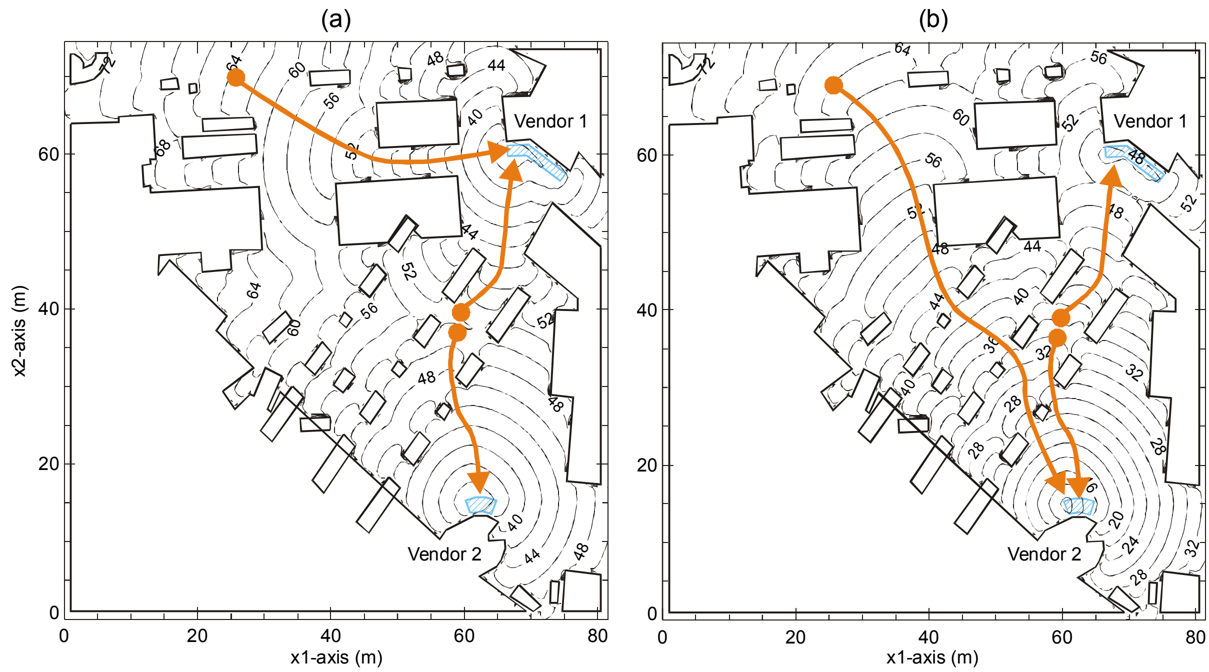
Example 4 (Route choice in Schiphol Plaza for fixed activity schedules). Let us consider the following pedestrian groups:

1. Pedestrian buying an item at either vendor V1 or V2 (see Figure 4) before using the escalators E6 or E7 to get to the train platform.

2. Pedestrians buying an item at either vendor V1 or V2 before exiting Schiphol Plaza using one of the exits E1-E5.

Figure 7a and Figure 7b show that vendor 1 is more attractive to pedestrians continuing their trip by train than to pedestrians leaving Schiphol Plaza by foot. When pedestrians leave using either of the exit, they will be more inclined to buy a newspaper at vendor 2.

Figure 7 Combined route-choice and activity area choice for a) pedestrians buying an item before heading towards escalators E6, E7; b) pedestrians buying an item before leaving via either of the exits



5.3 Optimal choice behavior and scheduling model

Consider a sequence of two *fixed-order activities*, where activity 2 follows activity 1. Let A_{1j} and A_{2j} denote the activity areas for activities 1 and 2 respectively. We hypothesize that a pedestrian will take into account both activities upon planning the route, i.e. in planning activity 1, he will consider that activity 2 will need to be performed afterwards. To this end, we first solve the route choice problem for activity 2, yielding the expected minimum perceived disutility function $W_2(t,x)$. Secondly, the pedestrian plans the primer activity by determining the path that is stipulated by $W_{12}(t,x)$, which is a solution of the HJB equation:

$$-\frac{\partial}{\partial t}W_{12}(t,x) = H\left(t,x,\nabla W_{12},\Delta W_{12}\right) \tag{37}$$

with boundary / terminal conditions

$$W_{12}(t_1,x)=\phi_{12} \text{ and } W_{12}(T_{12},x)=W_2(T_{12},x)-U_{1j}(T_{12}) \text{ for } x\in A_{1j} \text{ and } t<t_1 \tag{38}$$

Clearly, the terminal conditions describe how the (optimal) cost W_2 of walking to the second activity area are considered by the pedestrian when walking from any location x to the first activity area A_{1j}. The approach may be easily extended when a sequence of more than two activities have to be considered. Note that in most practical situations, the number of activities that pedestrians take into account during planning is generally limited.

The case, where the order of the activities *is not fixed* is equivalent to solving *two fixed activity order problems* and determining the minimum of $W_{12}(t,x)$ and $W_{21}(t,x)$. At any location $x(t)$, $\min\{W_{12}(t,x),W_{21}(t,x)\}$ determines both the optimal direction and speed, as well as the optimal order of the activities 1 and 2. In case of three or more activities, we need to consider the minimum of all combinations of activity sequences. This implies that, although the approach is conceptually very straightforward, in practice the number of combinations can become very large.

Conceptually, the inclusion of discretionary activities is equally simple. In illustration, consider the situation where activity 1 is mandatory, while activity 2 is discretionary. To determine the order of the activities as well as whether activity 2 will be performed or not, the pedestrian at $x(t)$ will determine the minimum of W_{12}, W_{21}, and W_1. If it turns out that if W_1 is optimal, activity 2 is skipped. Note that it is also possible that from a certain starting location $x(t_0)$, a discretionary activity is performed, while for other starting positions, pedestrians will not consider that performing the activity is worthwhile.

6. Applications of the model

The approach describes pedestrian activity scheduling and route choice for different types of pedestrians with distinct perceptions of route attributes. Contrary to network-based approaches, routes are continuous in time and space. Applications of the model are manifold. For one, the approach is used to model choice behavior in the pedestrian micro simulation model NOMAD (Hoogendoorn and Bovy,2002). Stand-alone applications of the model are however also possible to predict route choice in infrastructure facilities, such as transfer stations and shopping malls. For planning purposes, dynamic user-equilibrium solutions of the pedestrian assignment problem can be used to forecast pedestrian flows. Such predictions are valuable to reveal bottlenecks in infrastructure design, to predict average transfer times (walking time from egress to access locations, given expected traffic conditions), or the optimal location of a ticket machine or newspaper stand. For instance, the Schiphol Plaza example shows which vendor location is preferable from the viewpoint of passing pedestrian flows, given pedestrian origin-activity demands. This way, infrastructure design, platform allocation, timetables, etc. can be optimized. This pertains to regular circumstances, as well as to emergency conditions (albeit different models are required to describe walking operations). Practical application of the models will require calibration (and validation) of the model. This can be done by considering previous empirical studies showing the relative importance of different route attributes for varying trip purposes, and subsequently estimating the relevant weights. Comparing (qualitatively) the resulting path flows and speeds with observations reveals whether changes in the weights are needed. Given the fact that distance (or travel time) is the most important route attribute, we expect that model calibration is relatively straightforward, at least for distinct groups of pedestrians (e.g. commuters).

7. Summary and future research

This article puts forward a new dynamic mixed discrete-continuous theory for activity scheduling and trajectory choice, based on the assumption that pedestrians are *subjective utility maximizers*: they schedule their activities, the activity areas, and the trajectories between the activities (which, on the contrary to other transportation networks, are continuous functions in time and space) simultaneously to maximize the predicted utility of their efforts and walking. The utility reflects a trade-off between the utility of completing an activity, and the cost of walking towards the activity areas. In turn, the latter results from different factors, such as the travel time, discomfort of walking too close to obstacles and walls, stimulation of the environment, etc. Uncertainty pertaining to the predictability of the future conditions is included by assuming that the predicted routes are realizations of random processes. The effect of prevailing traffic conditions on pedestrian choice behavior have been considered as well. To operationalize the theory, different techniques from stochastic mathematical optimal control theory have been applied successfully. The different concepts have been illustrated by examples.

The main contribution of the article is the joint description of activity scheduling and route-choice behavior under uncertainty, hypothesizing that the pedestrian can choose between an infinite number of candidate routes, which are continuous paths in time and space. In doing so, no (discrete) network needs to be defined.

Future research is directed towards developing methods to efficiently solve the dynamic pedestrian assignment problem. Moreover, different applications of the approach will be considered, such as the routing control of Automated Guided Vehicles for container handling in terminals, and partially autonomous drones (e.g. for garbage collection).

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8. References

Abdelghany, A.F. and H. S. Mahmassani (2003). Temporal-Spatial Micro-Assignment and Sequencing of Travel Demand with Activity/Trip Chains. Paper presented at the *82nd Annual Meeting of the Transportation Research Board 2003*, Washing D.C.

Arentze, T. and H. Timmermans (2000). *ALBATROS: A Learning Based Transportation Oriented Simulation System*. European Institute of Retailing and Services Studies.

Beckmann, M.J., and T. Puu (1985). *Spatial Economics: Density, Potential and Flow. Studies in Regional Science and Urban Economics*, North-Holland, Amsterdam, The Netherlands

Bellman (1957). *Dynamic Programming*. Princeton University Press, Princeton.

Bovy, P.H.L. and E. Stern (1990). *Route Choice: Wayfinding in Transport Networks.* Kluwer Academic Publishers.

Chiolek, M.T. (1978) Spatial Behaviour in Pedestrian Areas. *Ekistics 268*, 120-121.

Gipps, P.G. (1986). Simulation of Pedestrian Traffic in Buildings. *Schriftenreihe des Instituts fuer Verkehrswesen 35*, University of Karlsruhe.

Guy, Y. (1987). *Pedestrian Route Choice in Central Jerusalem.* Department of Geography, Ben-Gurion University of The Negev, Beer Sheva (In Hebrew).

Hamacher, H.W., and S.A. Tjandra (2001). Mathematical Modelling of Evacuation Problems: A State of the Art. In: *Pedestrian and Evacutation Dynamics*, Springer, 59-74.

Helbing, D. (1997) *Traffic Dynamics: New Physical Modeling Concepts (in German)*. Springer-Verlag.

Hill, M.R. (1982). *Spatial Structure and Decision-Making of Pedestrian Route Selection Through An Urban Environment*. Phd Thesis. University Microfilms International.

Hoogendoorn, S.P. (2002). Wayfinding under Uncertainty in Continuous Time and Space by Dynamic Programming. *In: Proceedings of the 13th Mini-EURO Conference "Handling Uncertainty in the Analysis of Traffic and Transportation Systems"*, 71-76

[1] Panic. *Nature 407*, 487-490.

Hoogendoorn, S.P., And P.H.L. Bovy (2002). Normative Pedestrian Behaviour Theory And Modelling. *Proceedings Of The 15th International Symposium On Transportation And Traffic Theory*, Adelaide, Pp. 219-245.

Hughes, R. (2002). A continuum theory for the flow of pedestrians, *Transportation Research Part B: Methodological,* Vol. 36(6), 507-535.

Mirchandani, P. and H. Soroush (1987). Generalized traffic equilibrium with probabilistic travel times and perceptions. *Transportation Science*, 21(3), 133-152.

Puu, T. and M. Beckmann (1999) Continuous space modeling, in R.W. Hall (ed.) *Handbook of transportation science*, pp. 269-310

Senevarante, P.N. and J.F. Morall (1986). Analysis of Factors Affecting The Choice of Route of Pedestrians. *Transportation Planning and Technology*, 10, Pp. 147-159.

Verlander, N.Q. (1997). *Pedestrian Route Choice: An Empirical Study*. Proceedings of seminar F of the PTRC European Transport Forum, P415, 39-49.

Weidmann, U. (1993) Transporttechnik der Fussgaenger. *Eth, Schriftenreihe Ivt-Berichte 90, Zuerich* (In German).

Yang, H. and S.C. Wong (2000). A Continuous Equilibrium Model for Estimating Market Areas of Competitive Facilities with Elastic Demand and Market Externatility. *Transportation Science 34 (2)*, 216-227

Yang, H., S. Yagar, and Y. Iida (1994). Traffic Assignment in a Congested Discrete Continuous Transportation System. *Transportation Research 28B* (2), 161-174.