Channel Holding Time in Hierarchical Cellular Systems

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***Abstract*—In this letter, we study the characteristics of the channel holding time in the multitier cellular systems supporting overflow and underflow schemes with the general call holding time and the general cell residence time. The comparison between our result, together with previous results, and the simulation shows that our result is more universal and more accurate.**

***Index Terms*—Call holding time, cell residence time, channel holding time (CHT), hierarchical cellular systems (HCS), overflow/underflow.**

I. Introduction

THE characteristics of the channel holding time (CHT) plays a pivotal role in the performance evaluation of the standalone wireless network, e.g., GSM, as well as the next generation multitier wireless multimedia network, such as the currently standardizing UMTS and WLAN interworking network [1]. Owing to the critical significance, the CHT derivation in standalone wireless network has attracted extensive studies (e.g., [2], [8]). However, in multitier wireless network, CHT still remains a significant research issue since it is not only dependent on the relationship between call holding time and cell residence time but reliant on the resource allocation strategy and the network architecture. In addition, the general call holding time and cell residence time further complicate CHT investigation. For the sake of analytical tractability, CHT is traditionally provided under the exponential call holding time and the exponential cell residence time [3], [4], [6] in hierarchical cellular systems (HCS). One recent work [5] calculated CHT with the hyper-erlang call holding time and general cell residence time.

In this letter, we present an approach for the CHT derivation in a two-layer HCS supporting slow and fast call overflow and underflow schemes. Both the call holding time and cell residence time follow general distribution functions. The lower microlayer is designed to provide service for the slow-mobility mobile station(MS) while the upper macro-layer for the fast-mobility MS. Overflow mechanism indicates that if there is insufficient resource in the appropriate service layer upon the moment of a call arrival, the call may transfer to another layer to avoid the connection from being dropped. With the underflow scheme, an overflowed call can return to its own service level upon crossing the microcell boundary. In the following, let $f_x(t), F_x(t), f_x^*(s), F_x^*(s)$ denote the probability density function (pdf) of the nonnegative random variable x, the cumulative distribution function (CDF) of x, Laplace–Stieltjes transform (LST) of the pdf, and the LST of the CDF.

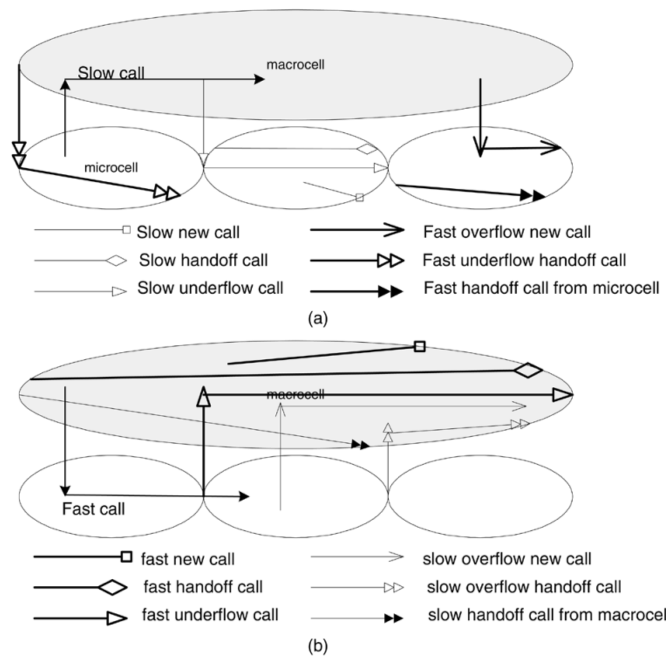

Fig. 1. The different call traffic and the corresponding typical behavior in: (a) microcell and (b) macrocell.

II. System Model and Analysis

Fig. 1 shows the typical call trajectory for different call traffics in microcell and macrocell. It is evident that the derivation of the slow call and fast call CHT in microcell, or fast call in macrocell, is exactly similar as the technique in standalone network, which has been already discussed extensively (e.g., [2]). Hence, we will neglect these types of call traffics and focus on the slow call CHT in macrocell.

Let t_c denote the call holding time with the average $1/\mu_c$. Denote by $X_M(X_{Mr})$ as the slow call cell residence time (residual cell residence time) in macrocell with expected value $1/\eta_M$. Using the residual life theorem, the pdf of X_{Mr} is given by $f_{X_{Mr}}(t) = \eta_M[1 - F_{X_M}(t)]$.

We will focus on the overflowed slow new call CHT T_{on}. Let $X_{m,k}(k = 1, 2, 3\ldots)$ represent the i.i.d. cell residence time in

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the kth underlying microcell an overflowed slow new call traversed with the generic form X_m and the average $1/\eta_m$. Accordingly, the residual cell residence time in the kth microcell is denoted as $X_{mr,k}$ with the generic form X_{mr}.

Denote by ξ_{sn} as the probability that an overflowed slow new call is moving out the coverage of the current serving macrocell under the condition that it is leaving the coverage of the corresponding underlying microcell. The equation to calculate this quantity is

$$\Phi_{sn} = \sum_{k=1}^{\infty} \mathcal{P}(X_{mr,1}+X_{m,2}+\cdots+X_{m,k} < t_c)(1-\xi_{sn})^{k-1}\xi_{sn} \tag{1}$$

where $\Phi_{sn} = \mathcal{P}(X_{Mr} < t_c)$ represents the probability that an overflowed slow new call moves out the serving macrocell when virtually no underflow mechanism is utilized. The right side is the summation of probabilities that the MS will eventually move out the macrocell after traversing several underlying microcells.

Denote $\chi_k = X_{mr,1} + \sum_{i=2}^{k} X_{m,i}$ with the LST of its pdf given by

$$f^*_{\chi_k}(s) = \frac{\eta_m[1-f^*_{X_m}(s)][f^*_{X_m}(s)]^{k-1}}{s}, \qquad k = 1, 2 \ldots. \tag{2}$$

Employing the similar technique in [2], the probability in (1) right-hand side becomes

$$\begin{aligned}
\mathcal{P}(\chi_k < t_c) &= \int_{t=0}^{\infty} \mathcal{P}(\chi_k < t) f_{t_c}(t)\, dt \\
&= \int_{t=0}^{\infty} \frac{1}{2\pi j} \int_{\sigma-j\infty}^{\sigma+j\infty} \frac{f^*_{\chi_k}(s)}{s} e^{st} ds f_{t_c}(t)\, dt \\
&= \frac{\eta_m}{2\pi j} \int_{\sigma-j\infty}^{\sigma+j\infty} \frac{\left[f^*_{X_m}(s)\right]^{k-1}\left[1-f^*_{X_m}(s)\right] f^*_{t_c}(-s)}{s^2}\, ds
\end{aligned} \tag{3}$$

where j is the imaginary unit and σ is a sufficiently small positive number. Analogously, the probability Φ_{sn} is expressed as

$$\Phi_{sn} = \frac{\eta_M}{2\pi j} \int_{\sigma-j\infty}^{\sigma+j\infty} \frac{\left[1-f^*_{X_M}(s)\right] f^*_{t_c}(-s)}{s^2}\, ds \tag{4}$$

If $f^*_{t_c}(s)$ has only finite possible isolated singular point in the left-half complex plane, we can apply the Residue Theorem [7] using a semicircular contour in the right-half plane [2]. In this case, we substitute (3) and (4) into (1) and apply the Residue Theorem

$$\begin{aligned}
&\eta_M \sum_{s_0\in\Omega} \mathrm{Res}_{s=s_0} \frac{[1-f^*_{X_M}(s)] f^*_{t_c}(-s)}{s^2} \\
&\quad = \xi_{sn}\eta_m \sum_{s_0\in\Omega} \mathrm{Res}_{s=s_0} \frac{\left[1-f^*_{X_m}(s)\right] f^*_{t_c}(-s)}{s^2\left[1-f^*_{X_m}(s)(1-\xi_{sn})\right]}
\end{aligned} \tag{5}$$

where Ω denotes the set of poles of $f^*_{t_c}(-s)$ in the right complex plane and $\mathrm{Res}_{s=s_0}$ represents the residue at poles $s = s_0$.

Based on (5), a recursive approach is developed to compute ξ_{sn}.

$$\xi_{sn}^{(l)} = C \frac{U_{\text{upper}}}{U_{\text{lower}}\left(\xi_{sn}^{(l-1)}\right)}, \qquad l = 1, 2, \ldots \tag{6}$$

with

$$C = \frac{\eta_M}{\eta_m}, U_{\text{upper}} = \sum_{s_0\in\Omega} \mathrm{Res}_{s=s_0} \frac{\left[1-f^*_{X_M}(s)\right] f^*_{t_c}(-s)}{s^2}$$

and

$$U_{\text{lower}}\left(\xi_{sn}^{(l-1)}\right) = \sum_{s_0\in\Omega} \mathrm{Res}_{s=s_0} \frac{\left[1-f^*_{X_m}(s)\right] f^*_{t_c}(-s)}{s^2\left[1-f^*_{X_m}(s)\left(1-\xi_{sn}^{(l-1)}\right)\right]}.$$

Next, we will compute the channel holding time of the overflowed slow new call T_{on}. For an accepted slow overflow new call in macrocell, its behavior can be broadly classified into two categories. On the one hand, the call connection can be: 1) normally completed in the coverage of the first microcell when $t_c < X_{mr,1}$; or 2) normally completed in the coverage of the second microcell after a failed underflow attempt when $X_{mr,1} < t_c \le X_{mr,1} + X_{m,2}$; or 3) normally completed in the coverage of the third microcell after two times failed underflow attempts in the case of $X_{mr,1} + X_{m,2} < t_c \le X_{mr,1} + X_{m,2} + X_{m,3}$, and so on. In this case, T_{on} is equal to t_c due to the call normal completion in the upper layer.

On the other hand, the CHT can be: 1) $X_{mr,1}$ due to the successful underflow upon crossing the first microcell boundary or leaving the macrocell coverage; 2) $X_{mr,1} + X_{m,2}$ due to the successful underflow upon crossing the second microcell boundary or leaving both the second microcell and the macrocell coverage provided that the first underflow is failed; and 3) $X_{mr,1} + X_{m,2} + X_{m,3}$ due to the successful underflow upon crossing the third microcell boundary or leaving the third microcell coverage provided that the first and second underflows are failed, and so on. As a result, the CDF of the overflowed slow new call CHT is expressed as

$$\begin{aligned}
\mathcal{P}(T_{\text{on}} < t) &= \sum_{k=1}^{\infty} \mathcal{P}(t_c < t, \chi_{k-1} < t_c \le \chi_k)[(1-\xi_{sn})P_{bu}]^{k-1} \\
&\quad + \sum_{k=1}^{\infty} \mathcal{P}(\chi_k < t, \chi_k < t_c)[(1-\xi_{sn})P_{bu}]^{k-1} \\
&\quad \cdot [\xi_{sn} + (1-\xi_{sn})(1-P_{bu})]
\end{aligned} \tag{7}$$

where P_{bu} is the underflow blocking probability. The item $[(1-\xi_{sn})P_{bu}]^{k-1}$ accounts for the probability that the overflowed slow call remains in the macrocell after $k-1$ times of failed underflow attempts. The term $[(1-\xi_{sn})P_{bu}]^{k-1}[\xi_{sn} + (1-\xi_{sn})(1-P_{bu})]$ represents the probability that the overflowed slow call remains in the macrocell after $k-1$ times of failed underflow attempts, but in the kth underflow, the call either moves out the coverage of the serving macrocell with probability ξ_{sn} or successfully underflows to the lower layer with probability $(1-\xi_{sn})(1-P_{bu})$.

After the mathematical manipulation, we obtain the pdf as

$$f_{T_{\mathrm{on}}}(t) = f_{t_c}(t)[1 - F_{\chi_1}(t)] + (1-a) f_{\chi_1}(t)\,[1 - F_{t_c}(t)] + \sum_{k=2}^{\infty} a^{k-1} \left\{ f_{t_c}(t) h_k(t) + (1-a) f_{\chi_k}(t)\,[1 - F_{t_c}(t)] \right\} \tag{8}$$

where $a = (1-\xi_{sn})P_{bu}$ and $h_k(t) = \int_0^t f_{\chi_{k-1}}(\tau)[1-F_{X_m}(t-\tau)]d\tau$ with its LST

$$h_k^*(s) = \frac{\eta_m \left[1 - f_{X_m}^*(s)\right]^2 f_{X_m}^*(s)^{k-2}}{s^2}, \qquad k = 2, 3 \ldots. \tag{9}$$

Consequently, the statistical moments of the CHT can be obtained on the basis of the pdf of T_{on}. In particular, the expected value is given by $E(T_{\mathrm{on}}) = \int_0^{\infty} t f_{T_{\mathrm{on}}}(t) dt$.

Suppose that the call holding time has an n-stage Erlang distribution with mean $1/\mu_c = n/\mu$, variance $V_c = n/\mu^2$, and the probability density function

$$f_{t_c}(t) = \frac{\mu^n t^{n-1}}{(n-1)!} e^{-\mu t}, \quad t > 0; n = 1, 2 \ldots. \tag{10}$$

In this case, the mean of T_{on} is given by

$$E(T_{\mathrm{on}}) = \frac{n}{\mu} - \frac{(-1)^n \mu^n}{(n-1)!} \left. \frac{d^n F_{\chi_1}^*(s)}{ds^n} \right|_{s=\mu} + \sum_{k=2}^{\infty} a^{k-1} \frac{(-1)^n \mu^n}{(n-1)!} \left. \frac{d^n h_k^*(s)}{ds^n} \right|_{s=\mu} + (1-a) \sum_{k=1}^{\infty} a^{k-1} \sum_{i=0}^{n-1} \frac{(-1)^{i+1} \mu^i}{i!} \left. \frac{d^{i+1} f_{\chi_k}^*(s)}{ds^{i+1}} \right|_{s=\mu}. \tag{11}$$

III. Numerical Results and Conclusions

In this section, we will compare our result with the previous results, and the simulation with the exponential call holding time and the exponential cell residence time. In such case, all the slow call CHT in macrocell follows the same exponential distribution due to the memoryless property. If not specified, the set of parameters are chosen as: $1/\mu_c = 180.0$ s, $\eta_m/\eta_M = 3.0$, and $P_{bu} = 0.1$. Hence, we denote CHT as T and drop the subscript.

$$1/E(T) = \mu_c + \eta_m(1 - P_{bu}) + \eta_M P_{bu}. \tag{12}$$

In contrast, the results in [3] (denoted as JF) and [4], [5] (denoted as BCF & YJ) are given by

$$1/E(T)_{\mathtt{JF}} = \mu_c + \eta_M \tag{13}$$

$$1/E(T)_{\mathtt{BCF\&\ YJ}} = \mu_c + \eta_m(1 - P_{bu}) + \eta_M. \tag{14}$$

We argue that JF result has ignored the reason for channel release due to successful underflow and that the studies BCF & YJ have disregarded the prerequisite for the validity of overflow slow call CHT equal to the cell residence time in macrocell.

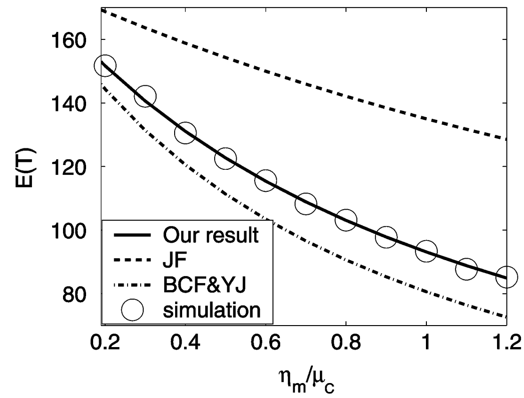

Fig. 2. $E(T)$ in terms of η_m/μ_c with exponential call holding time and exponential cell residence time.

Fig. 2 plots the mean CHT of slow call in macrocell in terms of η_m/μ_c. It is evident that our result matches the simulation result perfectly well while JF result exhibits a significant discrepancy from the simulation, and BCF&YJ is smaller than and only approximative to the actual value.

In contrast with the previous results, our result is superior in the following aspects: 1) more general in the sense of the absence of any specific distribution assumption for the call holding time and cell residence time, and the support of the overflow and underflow mechanisms in HCS; 2) more accurate and validated by the simulation result; and 3) simpler to compute due to the closed-form formula. An interesting future topic is to employ the obtained results to study the performance of HCS with generalized critical teletraffic parameters.

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