AN ANALOG-TO-DIGITAL CONVERTER WITH TIME-VARIANT WINDOW

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ABSTRACT

We present a first order analog-to-digital converter with time-variant window. If a DC input is applied, the converter outputs a binary sequence until the internal state enters into the window. It is clarified analytically that the converter can realize higher resolution and lower error variation with shorter code length than previous converters. We also present a simple implementation method using a continuous-time circuit, where the digital output is represented by an impulse-train.

1. INTRODUCTION

Sigma-delta modulator-based analog-to-digital converters (ab. ADCs) are interesting systems especially in low-frequency high-precision applications [1, 2]. The ADCs are usually required to convert an analog input signal to a binary sequence that can approximate the input as precise as possible. In order to improve accuracy and efficiency of the conversion, various interesting systems have been published, among them: the correlation-based decoder [3], the time-referenced multi-bit ADC [4], the noise-reducing loop in a multi-bit ADC [5], and the adaptive ADC [6]. However, the quantization is a complex nonlinear operation and thus general analysis of the system performance is hard. Detailed analysis of basic systems is important as theoretical background for various applications [1, 2].

In this paper, we present a novel first-order ADC with time-variant window (ab. TWADC). The TWADC consists of an integrator, a 1-bit quantizer, and a window whose size varies as time goes. If a DC input is applied, the TWADC outputs a binary output sequence until the internal state enters into the window. If the window design is suitable, the TWADC can exhibit efficient performance. In fact it can be clarified analytically that the TWADC can realize higher resolution and lower error variation with shorter code length than previous first-order ADCs [7]. Note that we apply dynamical system theory to provide the analytical results: neither linear approximation nor statistical assumption is used. Next, we present a simple implementation method of the TWADC using a continuous-time circuit [8, 9] with integrate-and-fire dynamics [10], where the digital output is represented by an impulse-train. The basic performance is verified using a breadboard prototype. Such a circuit design has advantage in lower power consumption, lower voltage design and simpler filtering requirement.

2. SYSTEM

Fig.1 shows a block diagram of the analog-to-digital converter with time-variable window (ab. TWADC). In the figure, n is the discrete time, x_n is the state, u is a DC analog input, and y_n is the digital output. For convenience, let

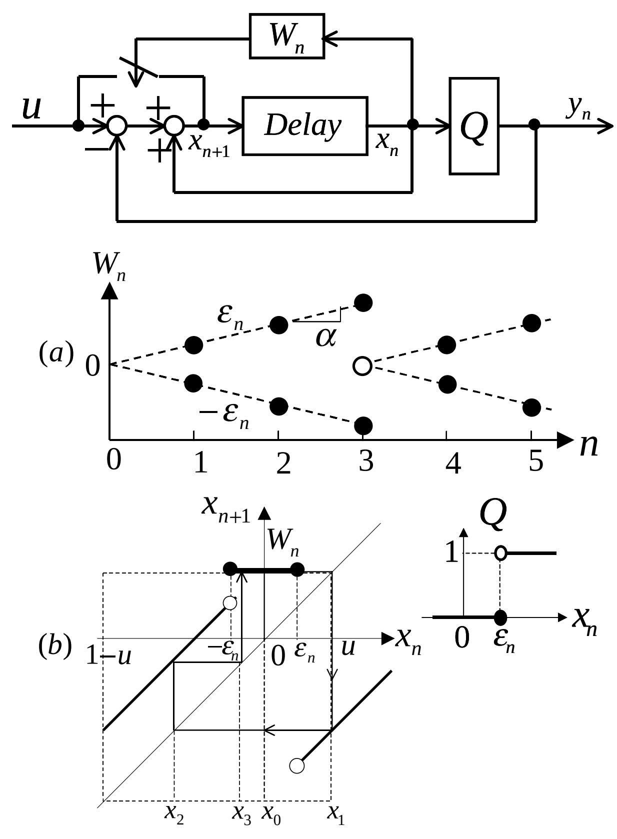

Fig. 1. An Analog-to-digital converter with time variant-window ($x_0 = 0$, $x_3 \in W_3$), (a) The time-variant window. (b) The dynamics of the TWADC.

$x_0 = 0$ and let $u \in [\frac{1}{l}, \frac{l-1}{l}]$, where l is an integer parameter that controls the code length. The dynamics is described by Equation (1).

$$x_{n+1} = F(x_n) \equiv \begin{cases} x_n - Q(x_n) + u & \text{for } x_n \notin W_n, \\ u & \text{for } x_n \in W_n, \end{cases}$$
$$W_n = [-\epsilon_n, \epsilon_n], \quad \epsilon_n = \alpha n, \quad \alpha > 0,$$
$$y_n = Q(x_n) = \begin{cases} 1, & \text{for } x_n > \epsilon_n, \\ 0, & \text{for } x_n \leq \epsilon_n, \end{cases} \tag{1}$$

The size of the window W_n varies as shown in Fig.1(a) and the dynamics is shown in the return map in Fig.1(b). The parameter α is fixed in Section 3. TWADC generates the output until the state x_n enters into the window W_n. If the state enters into the window at time $n^* : x_{n^*} \in W_{n^*}$, the TWADC is reset. In this case n is restricted by $0 \leq n \leq n^*$. For the obtained digital output sequence $\{y_0, y_1, \cdots, y_{l^*-1}\}$, the input can be approximated by

$$\tilde{u} = \frac{1}{n^*}\sum_{n=0}^{n^*-1} y_n, \quad |u - \tilde{u}| = \delta_T \leq \frac{\epsilon_{n^*}}{n^*} \tag{2}$$

Ref [7] studies the case of $\alpha = 0$, where $\epsilon_n =$ constant and the window is time-invariant. Also, a basic analog-to-digital converter (ab. BADC) has no window and resets the operation compulsorily at time $n = l$, where δ_W and δ_B denote quantization error of WADC and BADC, respectively. Fig.2 shows quantization and error characteristics. In the left figures, we can see that TWADC and WADC exhibits much better performance than BADC. In the right figures, we can see that TWADC controls variation of quantization error and maximum quantization error better than WADC. More detailed analysis results are shown in Section 3.

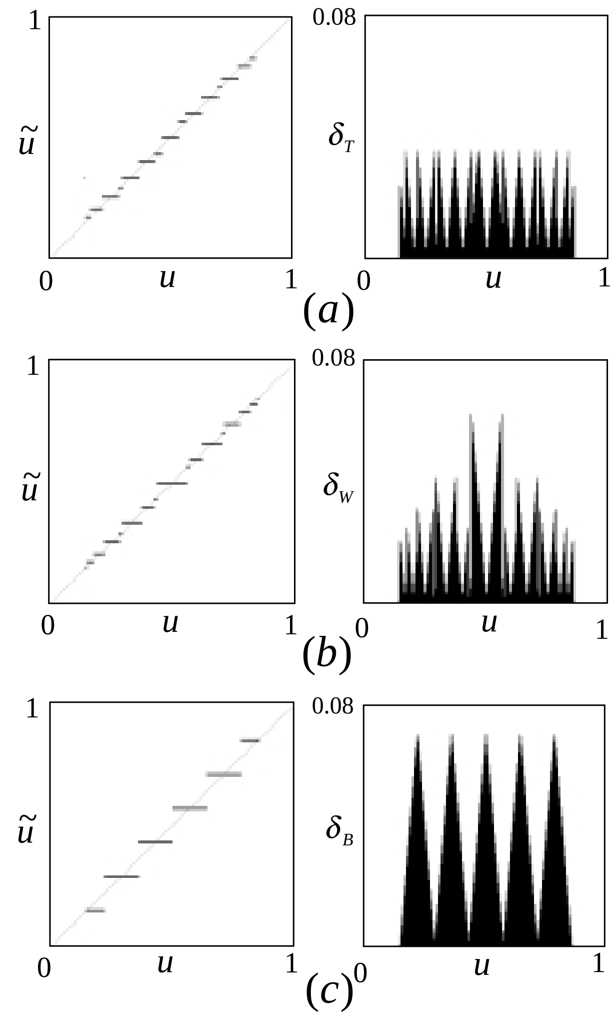

Fig. 2. Quantization characteristics (left) and Quantization errors (right), (l =7) (a) TWADC, (b) TWADC, (c) BADC.

3. ANALYSIS

In order to consider the error characteristics of the TWADC, we have two basic properties.

(Property 1) The orbit must enter into the window by time l: $x_{n^*} \in W_{n^*}$, $n^* \leq l$; if

$$\epsilon_n = \alpha n, \quad \alpha = \begin{cases} \dfrac{1}{4l} & \text{for odd } l \\[2ex] \dfrac{1}{4(l-1)} & \text{for even } l \end{cases} \tag{3}$$

We assume Equation(3) hereafter.

(Property 2) We assume Equation (3). The quantization error δ_T can be limited to

$$\begin{aligned} \delta_T &\leq \frac{1}{4l} \equiv \delta_{TMAX} && \text{for odd } l \\ \delta_T &\leq \frac{1}{4(l-1)} \equiv \delta_{TMAX} && \text{for even } l \end{aligned} \tag{4}$$

Note that the maximum quantization error δ_{TMAX} depends only on the parameter l. We then define the average error E_T is described by

$$E_T = \frac{l}{l-2}\int_{\frac{1}{l}}^{1-\frac{1}{l}} \delta_T du, \tag{5}$$

For the WADC, the orbit must enter the window $W_n = [-\epsilon_n, \epsilon_n]$ by l if $\epsilon_n = \frac{1}{l+1}$. In this case, the quantization error is given by

$$\delta_W \le \frac{1}{2(l+1)} = \delta_{WMAX} \tag{6}$$

The BADC has no window and the code length l is fixed. The quantization error is given by

$$\delta_B \le \frac{1}{2l} = \delta_{BMAX} \tag{7}$$

For the WADC and BADC, the averaged errors E_W and E_B are described respectively by

$$E_W = \frac{l}{l-2}\int_{\frac{1}{l}}^{1-\frac{1}{l}} \delta_W du, \tag{8}$$

$$E_B = (l-2)(\frac{1}{2l})^2 \tag{9}$$

Fig.3 (a) shows average code lengths of the output for l : the TWADC exhibits the best performance. Fig.3 (b) shows the average errors calculated by equations (5), (8), and (9). The WADC gives the smallest errors. Fig.3 (c) shows the maximum quantization errors. The TWADC exhibits the smallest errors.

4. IMPLEMENTATION

In order to realize the TWADC, we propose a continuous-time circuit with integrate-and-fire dynamics [10] as shown Fig.4. In this circuit, the VCCS is realized using the OTA. The circuit dynamics and switching rule are described by Equation (10).

$$\begin{aligned}
&C\frac{dv}{dt} = J_u(V_u) \quad \text{if} \quad S_R = \text{OFF} \\
&v = 0 \quad \text{if} \quad S_R = \text{ON} \\
&Y = \begin{cases} V & \text{at} \quad t = nT \quad \text{if} \quad , v(nT) < V_u \\ -V & \text{at} \quad t = nT \quad \text{if} \quad , v(nT) \ge V_u \end{cases} \\
&S_R = \begin{cases} \text{ON} & \text{if} \quad v(t) = V_{Th} \quad \text{or} \quad v(nT) \in W(t) \\ \text{OFF} & \text{otherwise} \end{cases} \\
&W(t) = [0, v_\epsilon(t)] \cup [V_{Th} - v_\epsilon(t), V_{Th}]
\end{aligned} \tag{10}$$

As an analog DC input voltage V_u is applied to the circuit via VCCS J_u. Y is the digital output. The capacitor voltage v corresponds to the state. S_R sets the initial state : $v = 0$ at $t = 0$. v is checked periodically with period T. If the v enters into the window at $t = mT$ then S_R resets the state : $v(mT) = 0$, and ADC is terminated. Fig. 5 illustrates the circuit dynamics. The window $W(t)$ is controlled by $v_\epsilon(t)$:

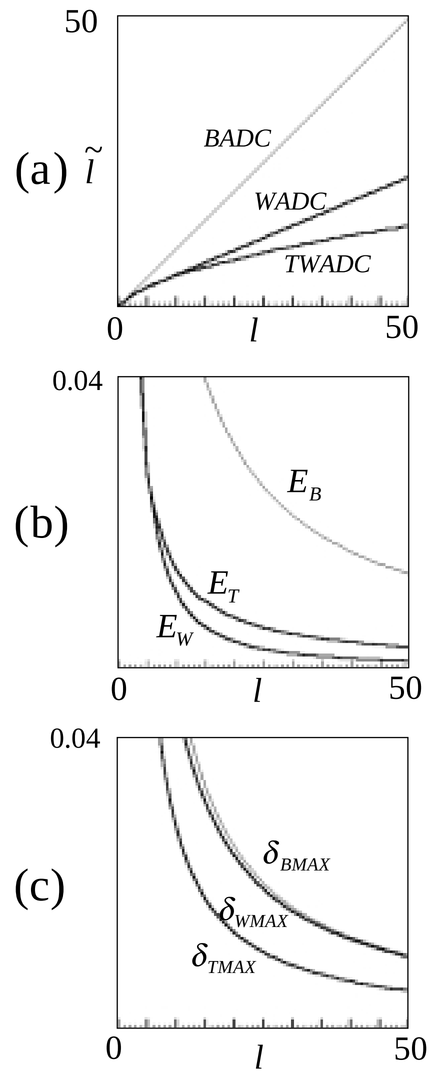

Fig. 3. Performances. (a) Average code lengths, (b) Average errors, (c) Maximum errors.

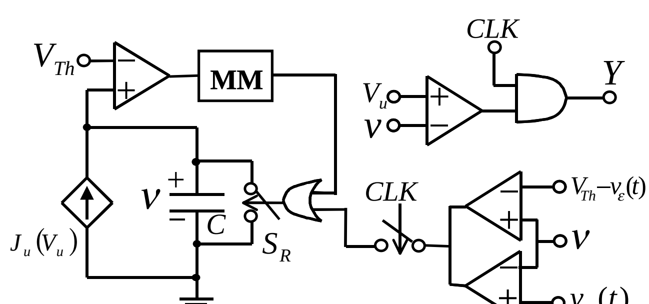

Fig. 4. Implementation of the TWADC

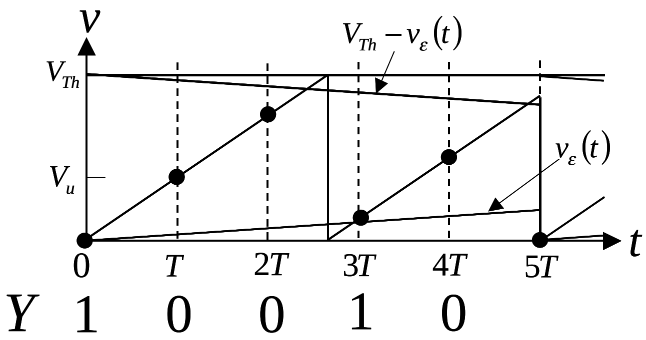

Fig. 5. Illustration : $u = 0.38, \tilde{u} = \dfrac{2}{5} = 0.4$

$v_\epsilon(0) = 0$. $v_\epsilon(t) = \alpha V_{Th} t$ for $0 < t < mT$. If v enters into the window, $v_\epsilon(t) = 0$. The output Y is given only at $t = nT$. Such an impulse-train output has an advantage in low power consumption. Fig. 6 shows the typical time domain waveform. Using the following dimensionless variables and parameters , the circuit dynamics can be reduced into Equation (1).

$$x_n = \frac{v(nT)}{V_{Th}}, u = \frac{J_u T}{C V_{Th}}, y_n = \frac{Y(nT) + V}{2V}, \epsilon_n = \frac{v_\epsilon(nT)}{V_{Th}}$$

5. CONCLUSION

We have considered an ADC with time-variant window (ab. TWADC). It is clarified analytically that the TWADC can realize higher resolution and lower error variation with shorter code length than previous ones. A simple implementation example is also presented where the output is represented by impulse-train. Now we are considering effects of non-idealities, a multi-bit version of the ADC and DAC for the variable code length.

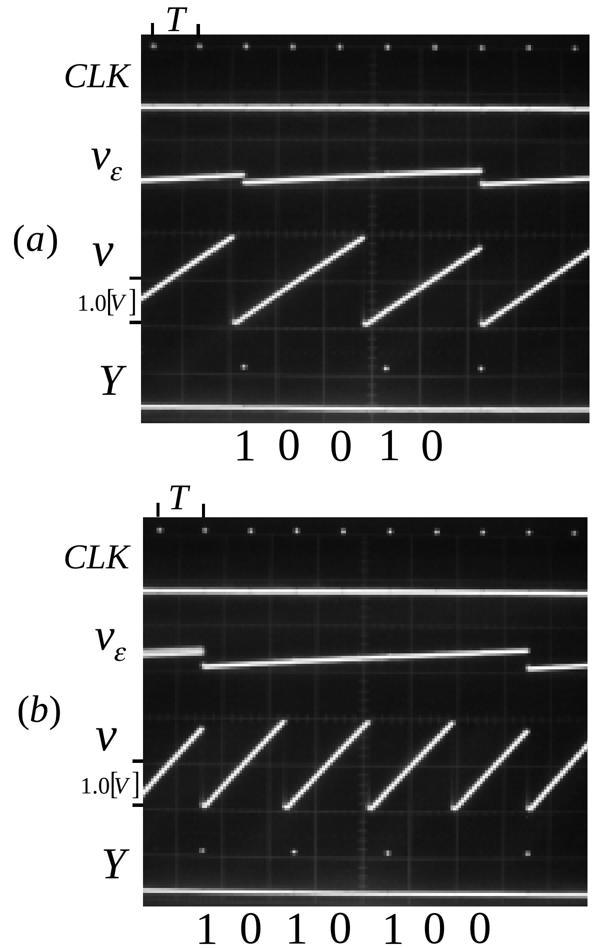

Fig. 6. Observed waveforms. $C \doteq 3.3[nF], T \doteq 1.0[ms]$, $l = 7$, $\alpha \doteq 3.5 \times 10^{-2}$. (a) $u \doteq 0.38$, output : 10010, $\tilde{u} = \dfrac{2}{5}$. (b) $u \doteq 0.43$, output : 1010100, $\tilde{u} = \dfrac{3}{7}$.

6. REFERENCES

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