Optimization of Cooperative Sensing in Cognitive Radio Networks: A Sensing-Throughput Tradeoff View

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***Abstract*—In cognitive radio networks, the performance of the spectrum sensing depends on the sensing time and the fusion scheme that are used when cooperative sensing is applied. In this paper, we consider the case where the secondary users cooperatively sense a channel using the k-out-of-N fusion rule to determine the presence of the primary user. A sensing-throughput tradeoff problem under a cooperative sensing scenario is formulated to find a pair of sensing time and k value that maximize the secondary users' throughput subject to sufficient protection that is provided to the primary user. An iterative algorithm is proposed to obtain the optimal values for these two parameters. Computer simulations show that significant improvement in the throughput of the secondary users is achieved when the parameters for the fusion scheme and the sensing time are jointly optimized.**

***Index Terms*—Cognitive radio, cooperative sensing, sensing-throughput tradeoff.**

I. Introduction

Cognitive radio, which enables secondary users/networks to utilize the spectrum when primary users are not occupying it, has been proposed as a promising technology to improve spectrum utilization efficiency [1], [2]. Spectrum sensing to detect the presence of the primary users is, therefore, a fundamental requirement in cognitive radio networks. A longer sensing time will improve the sensing performance; however, with a fixed frame size, the longer sensing time will shorten the allowable data transmission time of the secondary users. Hence, a sensing-throughput tradeoff problem was formulated in [3] to find the optimal sensing time that maximizes the secondary users' throughput while providing adequate protection to the primary user.

Another technique to improve the spectrum sensing performance is cooperative sensing [4]–[10]. There are various cooperative schemes to combine the sensing information from the secondary users, such as the k-out-of-N fusion rule [11], soft decision based fusion [12], and weighted data based fusion [13]. Both the sensing time and the cooperative sensing scheme affect the spectrum sensing performance, such as the probabilities of detection and false alarm. These probabilities affect the throughput of the secondary users since they determine the reusability of frequency bands. In this paper, we propose joint optimization of the sensing time and the parameters of the cooperative sensing scheme that are used to maximize the throughput of the secondary users.

The main contributions of this paper are as follows. First, using the k-out-of-N fusion rule as the basis, we formulate an optimization problem using the sensing time and the fusion parameter k as the optimization variables to jointly maximize the throughput of the secondary users while giving adequate protection to the primary user. Second, we propose an iterative algorithm to obtain both the optimal sensing time and the k value for the optimization problem. In this paper, we prove the unimodal characteristics of the secondary users' throughput as a function of the sensing time when the k-out-of-N fusion rule is used. Last, using computer simulations, it is shown that optimizing both the sensing time and the fusion scheme together significantly increases the throughput of the secondary users.

In the literature, there are some studies on optimizing the k value of the k-out-of-N fusion rule [11], [14] that minimize the total decision error probability instead of maximizing the secondary users' throughput. Furthermore, sensing time was not considered in their optimization formulations. There are also studies on optimizing the total frame time if the primary users' traffic statistics are known to the secondary users [15], [16]. However, in this paper, the primary users' traffic statistics are assumed unknown, and hence, optimizing the total frame time is not considered.

The rest of this paper is organized as follows. In Section II, the system model is introduced. In Sections III and IV, the concept and the mathematical formulation of the sensing-throughput tradeoff problem with cooperative sensing are presented, respectively. An iterative algorithm to find both the optimal sensing time and the fusion parameter k is presented in Section V. Computer simulations are provided in Section VI to show the performance of the proposed algorithm. Finally, we draw our conclusions in Section VII.

II. System Model

We consider a cognitive radio network where there are $N-1$ secondary users and one secondary base station that act as sensor nodes to cooperatively detect the presence of the primary user. Denote $\mathcal{H}_0$ and $\mathcal{H}_1$ as the hypotheses of the absence and the presence of the primary user, respectively. The sampled signals that are received at the ith sensor node during the sensing period are given as $y_i(n) = u_i(n)$ and $y_i(n) = h_i(n)s(n) + u_i(n)$ at hypotheses $\mathcal{H}_0$ and $\mathcal{H}_1$, respectively, where $s(n)$ denotes the signal from the primary user, and each sample is assumed to be an independent identically distributed (i.i.d.) random process with zero mean and variance $\mathbf{E}[|s(n)|^2] = \sigma_s^2$. The noise $u_i(n)$ is assumed to be i.i.d. circularly symmetric complex Gaussian with zero mean and variance $\mathbf{E}[|u_i(n)|^2] = \sigma_u^2$. Similar to [14], we assume that the distances between any secondary users are small compared with the distance from any secondary user to the primary transmitter. Therefore, it is assumed that each channel gain $|h_i(n)|$ is Rayleigh-distributed with same variance $\mathbf{E}[|h_i(n)|^2] = \sigma_h^2$. Assume that $s(n)$, $h_i(n)$, and $u_i(n)$ are independent of each other, and the average received SNR at each sensor node is given as $\gamma = \sigma_h^2\sigma_s^2/\sigma_u^2$.

Consider that each of the sensor nodes employ an energy detector and measure their received powers during the sensing period. Then, their measured received powers are given as $V_i = (1/M)\sum_{n=1}^{M}|y_i(n)|^2$ for $i = 1,\ldots,N$. Denote M as the number of signal samples that are collected at each sensor node during the sensing period, which is the product of the sensing time τ and the sampling frequency f_s. Denote ε_i as the threshold parameter of the energy detector at the ith sensor node. When the primary user's

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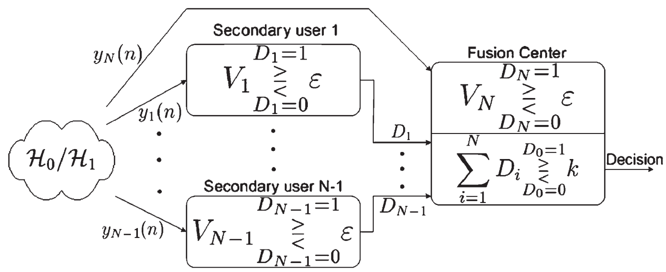

Fig. 1. Structure of cooperative sensing using the k-out-of-N fusion rule.

signal is a complex-valued phase-shift keying signal, the energy detector's probabilities of detection and false alarm at each sensor node are, respectively, approximated as

$$P_{di}(\tau, \varepsilon_i) = \mathcal{Q}\left(\left(\frac{\varepsilon_i}{\sigma_u^2(\gamma+1)} - 1\right)\sqrt{\tau f_s}\right), \qquad i = 1, \ldots, N \tag{1}$$

$$P_{fi}(\tau, \varepsilon_i) = \mathcal{Q}\left(\left(\frac{\varepsilon_i}{\sigma_u^2} - 1\right)\sqrt{\tau f_s}\right), \qquad i = 1, \ldots, N \tag{2}$$

where $\mathcal{Q}(\cdot)$ denotes the right-tail probability of a normalized Gaussian distribution.

After every secondary user makes its individual decision D_i, where $D_i = 1$ denotes that the primary user is detected and $D_i = 0$ denotes otherwise, their decisions are transmitted to the secondary base station, which acts as a fusion center. The base station combines the decisions it received together with its own decision to make a final decision D_0. Suppose that the k-out-of-N fusion rule is adopted as the fusion scheme. Then, the base station decides $D_0 = 1$ if $\sum_{i=1}^{N} D_i \geq k$ and $D_0 = 0$ otherwise. Note that k is an integer between 1 and N. By setting a common threshold ε for the energy detectors at the sensor nodes, the overall probabilities of detection and false alarm of the cognitive radio network when the k-out-of-N fusion rule is applied are, respectively, given as

$$\mathbb{P}_d(\tau, k, \varepsilon) = \sum_{i=k}^{N} \binom{N}{i} P_d(\tau, \varepsilon)^i \left(1 - P_d(\tau, \varepsilon)\right)^{N-i} \tag{3}$$

$$\mathbb{P}_f(\tau, k, \varepsilon) = \sum_{i=k}^{N} \binom{N}{i} P_f(\tau, \varepsilon)^i \left(1 - P_f(\tau, \varepsilon)\right)^{N-i}. \tag{4}$$

Fig. 1 summarizes the cooperative sensing process described above.

III. Sensing-Throughput Tradeoff With Cooperative Sensing

A basic frame structure of a cognitive radio network consists of, at least, a sensing slot and a transmission slot, and it will utilize a channel under two scenarios. The first scenario is when the fusion center manages to detect the primary user's absence, and the second scenario occurs when the fusion center fails to detect the primary user's presence. We denote C_0 and C_1 as the throughput of the secondary users if they are allowed to continuously operate in the absence and the presence of the primary user, respectively. Since a length of τ period out of the total frame time T is used for sensing, the achievable throughputs of the secondary users under these scenarios are, respectively, given as

$$R_0(\tau, k, \varepsilon) = C_0 P(\mathcal{H}_0)\left(1 - \frac{\tau}{T}\right)\left(1 - \mathbb{P}_f(\tau, k, \varepsilon)\right) \tag{5}$$

$$R_1(\tau, k, \varepsilon) = C_1 P(\mathcal{H}_1)\left(1 - \frac{\tau}{T}\right)\left(1 - \mathbb{P}_d(\tau, k, \varepsilon)\right) \tag{6}$$

where $P(\mathcal{H}_0)$ and $P(\mathcal{H}_1)$ are the probabilities of the primary user being absent and present in the channel, respectively. The average achievable throughput of the secondary users is given as $R(\tau, k, \varepsilon) = R_0(\tau, k, \varepsilon) + R_1(\tau, k, \varepsilon)$. It becomes obvious that the average throughput under the cooperative sensing scenario is dependent on the parameter of the fusion rule k; hence, we propose to include the parameter k as an optimization variable in the sensing-throughput tradeoff design when cooperative sensing is used.

IV. Formulation of the Optimization Problem

The sensing-throughput tradeoff problem with cooperative sensing is formulated to maximize the average throughput of the cognitive radio network using τ, k, and ε as the optimization variables subject to adequate protection given to the primary user, as shown by the following:

$$\max_{\tau, k, \varepsilon} : \quad R(\tau, k, \varepsilon) \tag{7a}$$

$$\text{s.t.} : \quad \mathbb{P}_d(\tau, k, \varepsilon) \geq \bar{\mathbb{P}}_d \tag{7b}$$

$$0 \leq \tau \leq T \tag{7c}$$

$$1 \leq k \leq N \tag{7d}$$

where $\bar{\mathbb{P}}_d$ is the minimum probability of detection that the fusion center needs to achieve to protect the primary user. The operation of the cognitive radio network during the second scenario experiences interference from the primary user, and hence, we have $C_0 > C_1$. From (1) and (2), we conclude that $P_f(\tau, \varepsilon) < P_d(\tau, \varepsilon)$, and, therefore, $(1 - \mathbb{P}_f(\tau, k, \varepsilon)) > (1 - \mathbb{P}_d(\tau, k, \varepsilon))$. Furthermore, in the cognitive radio network, we are interested in frequency bands that are underutilized, such as frequency bands with $P(\mathcal{H}_0) \geq 0.5$. Then, from these results, we have $R_0(\tau, k, \varepsilon) \gg R_1(\tau, k, \varepsilon)$. Hence, $R_0(\tau, k, \varepsilon)$ will be used as the objective function instead of $R(\tau, k, \varepsilon)$ in the rest of this paper.

The optimal solution occurs when constraint (7b) is at equality. The proof is similar to that in [3] without cooperative sensing, except that, in here, with the k-out-of-N fusion rule as the cooperative sensing scheme, we need to make use of the fact that $\mathbb{P}_d(\tau, k, \varepsilon)/\mathbb{P}_f(\tau, k, \varepsilon)$ is monotonically increasing in $P_d(\tau, \varepsilon)/P_f(\tau, \varepsilon)$ for a fixed k. Therefore, if a threshold ε_0 is able to satisfy $\mathbb{P}_d(\tau, k, \varepsilon_0) = \bar{\mathbb{P}}_d$, any other threshold, i.e., ε_1, which is smaller than ε_0, is able to satisfy constraint (7b) for the same τ and k since $\mathbb{P}_d(\tau, k, \varepsilon_1) > \mathbb{P}_d(\tau, k, \varepsilon_0)$. However, from (5) and (6), we deduce that $R_0(\tau, k, \varepsilon_1) < R_0(\tau, k, \varepsilon_0)$ and that $R(\tau, k, \varepsilon_1) < R(\tau, k, \varepsilon_0)$. This proves that $R_0(\tau, k, \varepsilon)$ or $R(\tau, k, \varepsilon)$ is maximized only if $\mathbb{P}_d(\tau, k, \varepsilon) = \bar{\mathbb{P}}_d$.

When constraint (7b) is at equality, for any given pair of τ and k, we are able to determine a threshold from (1) that is able to satisfy $\mathbb{P}_d(\tau, k, \varepsilon) = \bar{\mathbb{P}}_d$, which is given by

$$\varepsilon(\tau, k) = \sigma_u^2(\gamma+1)\left(\frac{1}{\sqrt{\tau f s}}\mathcal{Q}^{-1}\left(\bar{P}_d(k)\right) + 1\right). \tag{8}$$

Since constraint (7b) at equality can be satisfied by (8) for any given τ and k, the optimization problem (7) is reduced to

$$\max_{\tau,k}: \quad \hat{R}_0(\tau,k) \tag{9a}$$
$$\text{s.t.}: \quad 0 \le \tau \le T \tag{9b}$$
$$1 \le k \le N \tag{9c}$$

where $\hat{R}_0(\tau,k)$ denotes the value of $R_0(\tau,k,\varepsilon)$, with the threshold ε chosen by (8). The probabilities of false alarm at each sensor node and the fusion center with the threshold chosen by (8) are, respectively, given as $\hat{P}_f(\tau,k) = \mathcal{Q}(\alpha + \beta\sqrt{\tau})$ and

$$\hat{\mathbb{P}}_f(\tau,k) = \sum_{i=k}^{N} \binom{N}{i} \hat{P}_f(\tau,k)^i \left(1 - \hat{P}_f(\tau,k)\right)^{N-i} \tag{10}$$

where $\alpha = (\gamma+1)\mathcal{Q}^{-1}(\bar{P}_d(k))$, and $\beta = \gamma\sqrt{f_s}$. In Section V, we propose an iterative algorithm to find the solution of τ and k for the optimization problem (9).

V. Proposed Iterative Optimization Algorithm

Instead of directly solving the two-variable optimization problem (9), we propose an algorithm that solves (9) by decoupling it into two single-variable suboptimization problems.

A. Suboptimization Problem One

The first suboptimization problem is that, for a given $k = \tilde{k}$, we find the optimal value of τ that maximizes the throughput of the cognitive radio network subject to the total probability of detection meeting the target probability of detection $\bar{\mathbb{P}}_d$. The optimization problem is given as

$$\max_{\tau}: \quad \tilde{R}_0(\tau) \triangleq \hat{R}_0(\tau,k)|_{k=\tilde{k}} = C_0 P(\mathcal{H}_0)\left(1 - \frac{\tau}{T}\right)\left(1 - \tilde{\mathbb{P}}_f(\tau)\right) \tag{11a}$$
$$\text{s.t.}: \quad 0 \le \tau \le T \tag{11b}$$

where $\tilde{\mathbb{P}}_f(\tau)$ is $\hat{\mathbb{P}}_f(\tau,k)$ for a given $k = \tilde{k}$. Next, we will prove that $\tilde{R}_0(\tau)$ is a unimodal function in the range $0 \le \tau \le T$ for any given $\tilde{k}$. Denote τ^* as the optimal sensing time for any given $\tilde{k}$. If $\tilde{R}_0(\tau)$ is a unimodal function, then $\tilde{R}_0(\tau)$ is monotonically increasing from $0 \le \tau < \tau^*$ and is monotonically decreasing from $\tau^* < \tau \le T$. Hence, $\tilde{R}_0(\tau^*)$ is the only local maximum in the entire range of $0 \le \tau \le T$.

Proposition: If $\tilde{R}_0(\tau)$ satisfies the three conditions listed below, then $\tilde{R}_0(\tau)$ must be a unimodal function in the range of $0 \le \tau \le T$ for any $\tilde{k}$ value between 1 and N:

1) $\nabla\tilde{R}_0(0) > 0, \forall\,\tilde{k} = 1,\ldots,N$.
2) $\nabla\tilde{R}_0(T) < 0, \forall\,\tilde{k} = 1,\ldots,N$.
3) There is a unique τ^* where $0 < \tau^* < T$ such that $\nabla\tilde{R}_0(\tau^*) = 0, \forall\,\tilde{k} = 1,\ldots,N$.

∇ denotes the differentiation of the function with respect to its argument.

Proof: The first two conditions imply that there must be at least a point in $0 < \tau < T$ that maximizes $\tilde{R}_0(\tau)$. The first and third conditions together infer that $\tilde{R}_0(\tau)$ is strictly increasing in the range $0 \le \tau < \tau^*$, whereas the second and third conditions together infer that $\tilde{R}_0(\tau)$ is strictly decreasing in the range $\tau^* < \tau \le T$ for any given $\tilde{k}$. The three conditions jointly imply that $\tilde{R}_0(\tau^*)$ must be the only local maximum in the entire range of $0 \le \tau \le T$. ■

To show that the function $\tilde{R}_0(\tau)$ does satisfy all the three conditions for it to be a unimodal function, we first derive the derivative of the function $\tilde{R}_0(\tau)$, which is given as

$$\nabla\tilde{R}_0(\tau) = -C_T\left(1 - \tilde{\mathbb{P}}_f(\tau) + (T-\tau)\nabla\tilde{\mathbb{P}}_f(\tau)\right) \tag{12}$$

where $C_T = (1/T)C_0 P(\mathcal{H}_0)$ is a positive constant, and

$$\nabla\tilde{\mathbb{P}}_f(\tau) = \tilde{k}\binom{N}{\tilde{k}}\tilde{P}_f(\tau)^{\tilde{k}-1}\left(1 - \tilde{P}_f(\tau)\right)^{N-\tilde{k}}\nabla\tilde{P}_f(\tau) \tag{13}$$

where $\tilde{P}_f(\tau)$ is $\hat{P}_f(\tau,\tilde{k})$, and

$$\nabla\tilde{P}_f(\tau) = -\frac{\beta}{\sqrt{8\pi\tau}}\exp\left(-\frac{(\alpha+\beta\sqrt{\tau})^2}{2}\right). \tag{14}$$

At $\tau = 0$, from (14), we have $\nabla\tilde{P}_f(0) = -\infty$, and hence, $\nabla\tilde{\mathbb{P}}_f(0) = -\infty$ for all possible $\tilde{k}$ values. Therefore, $\nabla\tilde{R}_0(0) = \infty$, and the first condition is satisfied. At $\tau = T$, from (12), we obtain

$$\nabla\tilde{R}_0(T) = -C_T\left(1 - \sum_{i=\tilde{k}}^{N}\binom{N}{i}\tilde{P}_f(T)^i\left(1 - \tilde{P}_f(T)\right)^{N-i}\right). \tag{15}$$

Since $0 \le \tilde{P}_f(T) \le 1$, we have $\sum_{i=\tilde{k}}^{N}\binom{N}{i}\tilde{P}_f(T)^i(1 - \tilde{P}_f(T))^{N-i} < 1$ for all possible $\tilde{k}$. Therefore, $\nabla\tilde{R}_0(T)$ must be a negative value for all the possible $\tilde{k}$ values, and the second condition is satisfied. To prove that $\tilde{R}_0(\tau)$ satisfies the third condition, let $\nabla\tilde{R}_0(\tau) = 0$ to obtain $g(\tau) = h(\tau,\tilde{k})$, where $g(\tau) = (\alpha + \beta\sqrt{\tau})^2$ and

$$h(\tau,\tilde{k}) = -2\ln\left[\left(\frac{\sqrt{8\pi\tau}}{\beta\tilde{k}\binom{N}{\tilde{k}}(T-\tau)}\right)\phi(\tau,\tilde{k})\right] \tag{16}$$

where

$$\phi(\tau,\tilde{k}) = \sum_{i=0}^{\tilde{k}-1}\binom{N}{i}\tilde{P}_f(\tau)^{i-\tilde{k}+1}\left(1 - \tilde{P}_f(\tau)\right)^{\tilde{k}-i}. \tag{17}$$

The third condition is satisfied if the two functions $g(\tau)$ and $h(\tau,\tilde{k})$ intersect each other only once in the entire range of $0 \le \tau \le T$ for all $\tilde{k} = 1,\ldots,N$. First, we partition the region of τ into two regions, where the lower and higher regions of τ are defined as $\mathbb{R}_L = \{\tau|(\alpha+\beta\sqrt{\tau}) \le 0, 0 \le \tau \le T\}$ and $\mathbb{R}_H = \{\tau|(\alpha+\beta\sqrt{\tau}) > 0, 0 \le \tau \le T\}$, respectively. At the lower region of τ, since $\beta > 0$, the function $g(\tau)$ is monotonically decreasing in τ; however, at the higher region, it is monotonically increasing in τ. For the function $h(\tau,\tilde{k})$, it is always monotonically decreasing in τ for all $\tilde{k} = 1,\ldots,N$, as shown in Appendix A, where we have proved that $(\partial/\partial\tau)h(\tau,\tilde{k}) < 0$ for $0 \le \tau \le T$ and all $\tilde{k} = 1,\ldots,N$.

If $g(\tau)$ and $h(\tau,\tilde{k})$ intersect each other in the lower region, it is impossible for them to intersect more than once because $h(\tau,\tilde{k})$ is always decreasing at a faster rate than $g(\tau)$ at the lower region $\mathbb{R}_L$. The proof of the property $(\partial/\partial\tau)h(\tau,\tilde{k}) < (\partial/\partial\tau)g(\tau)$ for all possible $\tilde{k}$ values at $\tau \in \mathbb{R}_L$ is deferred to Appendix B. If $g(\tau)$ and $h(\tau,\tilde{k})$ have intersected in the lower region, then $g(\tau)$ must be greater than $h(\tau,\tilde{k})$ at the higher region $\mathbb{R}_H$. Since in $\mathbb{R}_H$, $g(\tau)$ is a monotonically increasing function while $h(\tau,\tilde{k})$ is a monotonically decreasing function, it is impossible for them to intersect in the higher region. Hence, in this scenario, only one intersection occurs in the range of $0 \le \tau \le T$, and it occurs at the lower region. In another scenario, when the two functions do not intersect in the lower region, then they must intersect at the higher region. This is true as the first two conditions infer that there must be at least one intersection in the entire range $0 \leq \tau \leq T$. In region $\mathbb{R}_H$, $g(\tau)$ is monotonically increasing, and $h(\tau, \tilde{k})$ is monotonically decreasing; therefore, they can only intersect at most once. In either scenario, there is only one intersection between the two functions in the entire range of τ, and therefore, $\tilde{R}_0(\tau)$ satisfies the third condition. ■

We have proven that $\tilde{R}_0(\tau)$ satisfies all the three conditions, which show that it is a unimodal function; therefore, the optimization problem (11) can easily be solved by algorithms such as the bisection search method, the Golden section method, and Newton's method [17] for any given $\tilde{k}$.

B. Suboptimization Problem Two

The second suboptimization problem is decoupled from the optimization problem (9) by treating τ to be a constant. Removing all constant terms, the optimization problem reduces to

$$\min_{k}: \quad \check{\mathbb{P}}_f(k) \triangleq \hat{\mathbb{P}}_f(\tau, k)|_{\tau=\check{\tau}}$$

$$= \sum_{i=k}^{N} \binom{N}{i} \check{P}_f(k)^i \left(1 - \check{P}_f(k)\right)^{N-i} \tag{18a}$$

$$\text{s.t.}: \quad 0 \leq k \leq N \tag{18b}$$

where $\check{P}_f(k)$ is $\hat{P}_f(\tau, k)$ for a given $\tau = \check{\tau}$. No closed-form solution for k is available for this subproblem, and a search over all possible k values is required. However, since k is an integer and ranges from 1 to N, it is not computationally expensive to search for the optimal k^*.

Combining the steps for solving the two suboptimization problems, the proposed iterative algorithm is summarized in Algorithm 1. Next, we analyze the properties of the proposed iterative algorithm. The objective function of the optimization problem, i.e., $\hat{R}_0(\tau, k)$, is nondecreasing at every iteration, as one can easily conclude from Algorithm 1 that

$$\hat{R}_0\left(\tau^{(j)}, k^{(j)}\right) \leq \hat{R}_0\left(\tau^{(j)}, k^{(j+1)}\right) \leq \hat{R}_0\left(\tau^{(j+1)}, k^{(j+1)}\right) \qquad \forall\, j. \tag{19}$$

Algorithm 1 : Find the τ and k-out-of-N fusion rule that maximize $\hat{R}_0(\tau)$.

Input: $\tau^{(0)}$ {Any value of τ in between 0 and T}
Initialization: $j \Leftarrow 0$; $k^{(0)} \Leftarrow 1$
repeat
1) Given $\tau^{(j)}$, find the k^* that solves (18) by computing $\check{\mathbb{P}}_f(k)$ from $k = 1$ to N.
2) $k^{(j+1)} \Leftarrow k^*$
3) Given $k^{(j+1)}$, find τ^* using Golden section method.
4) $\tau^{(j+1)} \Leftarrow \tau^*$
5) $j \Leftarrow (j+1)$
until $\tau^{(j)} == \tau^{(j-1)}$ and $k^{(j)} == k^{(j-1)}$
Output: $\tau^{(j)}$; $k^{(j)}$

The proposed iterative algorithm iterates until $\hat{R}_0(\tau, k)$ converges to a maximum point in the 2-D space since one can conclude that, at the converged solutions τ^* and k^*, $\hat{R}_0(\tau^*, k^*) \geq \hat{R}_0(\tau^*, k)$ for all $k = 1, \ldots, N$ and $\hat{R}_0(\tau^*, k^*) \geq \hat{R}_0(\tau, k^*)$ for $0 \geq \tau \geq T$. At τ^*, $\hat{R}_0(\tau^*, k^*)$ is the largest across the k dimension, and at k^*, $\hat{R}_0(\tau^*, k^*)$ is the largest across the τ dimension. Although the proposed iterative algorithm may converge to a solution of a possible local maximum point, this point is actually the global maximum point, as we have found from the simulations described in Section VI. This is true for any initial value of $\tau^{(0)}$ that is chosen in between zero and T, which suggests that the proposed iterative algorithm is not sensitive to the initial value of $\tau^{(0)}$ and always converges to the same maximum point.

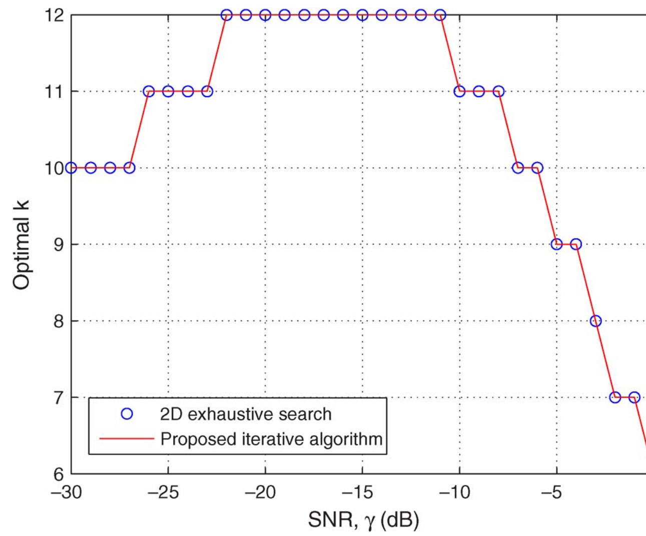

Fig. 2. Optimal k-out-of-N fusion rule that maximizes the throughput for $N = 25$.

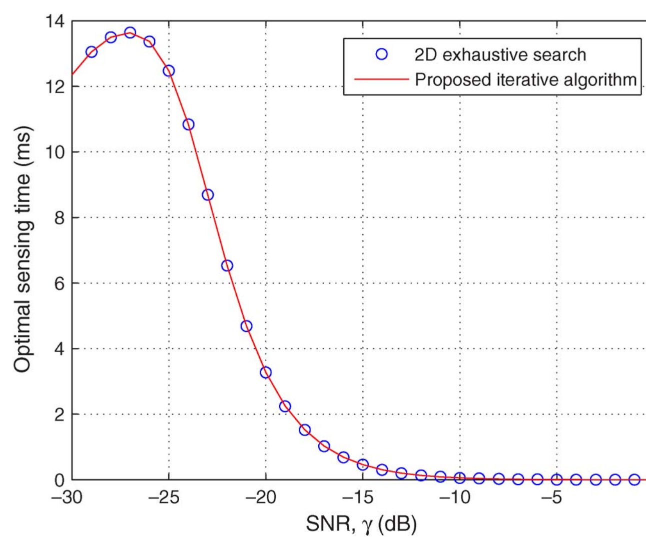

Fig. 3. Optimal sensing time that maximizes the throughput.

VI. Computer Simulations

In the following simulations, we set the number of secondary users to be $N = 25$ and the frame duration to be $T = 20$ ms. The sampling frequency of the received signal is assumed to be 6 MHz, and $\bar{\mathbb{P}}_d$ is set at 99.99%. The SNR γ of the primary user's signal that is received at the secondary users is varied from -30 to 0 dB.

Figs. 2 and 3 show the optimal k and τ at different γ values, respectively. The optimization problem is solved using the proposed iterative algorithm, and the obtained solutions are compared with those determined by a 2-D exhaustive search over k and τ. It can be seen from Figs. 2 and 3 that the proposed iterative algorithm does obtain the global optimal values for k and τ in all cases. From Fig. 2, it shows that there is no single k value that is optimal for all cases, and hence, there is a need to optimize the k value to enhance the throughput of the cognitive radio network. Fig. 3 shows an interesting result of the optimal sensing time with different SNRs. The simulation results suggest that, at low SNRs, i.e., 0 to -27 dB, allocating a larger portion of the total frame time for sensing to lower $\hat{\mathbb{P}}_f(\tau, k)$ and spending less time for transmission, in fact, increase the throughput of the cognitive radio network. However, at an extremely low SNR, i.e., below -27 dB,

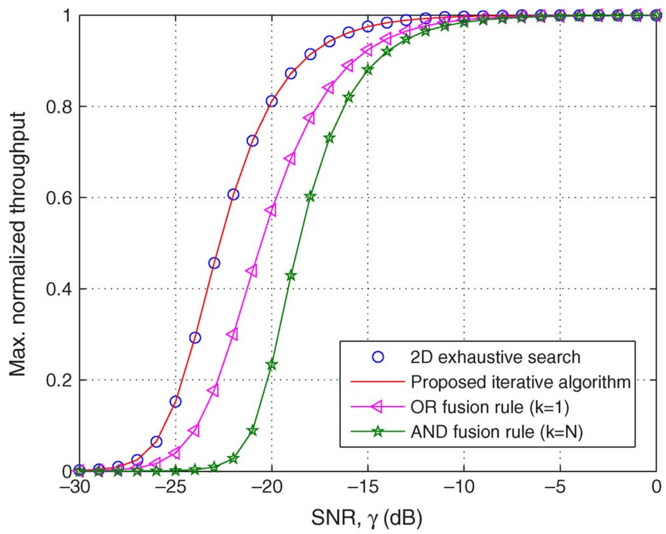

Fig. 4. Maximum achievable throughput by various k-out-of-N fusion rules.

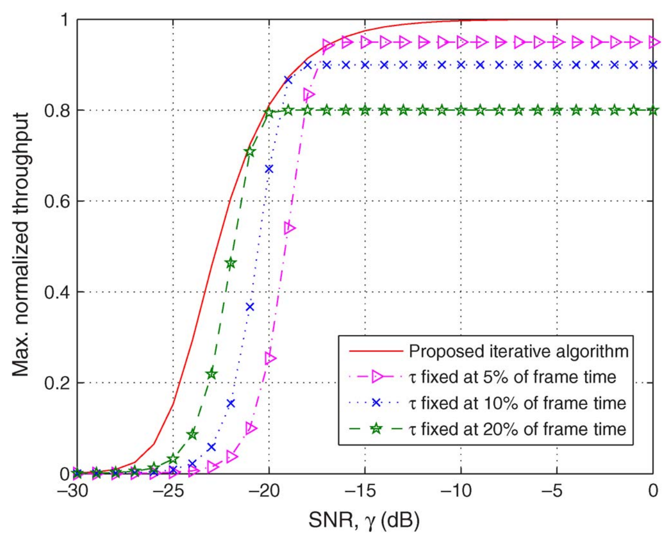

Fig. 5. Maximum achievable throughput for various sensing time.

$\hat{\mathbb{P}}_f(\tau, k)$ is very close to 1; although a large amount of the total frame time has been allocated to sensing, as from Fig. 3, nearly 13.7 ms out of the total 20 ms is spent on sensing at -27 dB. This suggests that, at extremely low SNRs, it is wiser to allocate more time for transmission than for sensing to improve the throughput.

Fig. 4 compares the maximum normalized throughput when k is optimized to cases where k is fixed to 1 (OR fusion rule), and k is fixed to N (AND fusion rule). The normalized throughput is defined as $(1-(\tau/T))(1-\hat{\mathbb{P}}_f(\tau,k))$. Optimal sensing time is used in each of the fusion rules, and it is seen that the AND rule always performs the worst for all SNR values. In particular, at SNR $= -20$ dB, when k is optimized, its throughput is almost four times than that when using the AND fusion rule. Fig. 5 compares the maximum normalized throughput when τ is optimized, as opposed to those with τ fixed at 5%, 10%, and 20% of the total frame time. Optimal k values are used for each fixed τ. It is clear that the disadvantage with a fixed sensing time is that, at high SNR levels where the primary user can be easily detected, the throughput of the cognitive radio network is bounded by the percentage of the total frame time that is spent for sensing. Finally, in Fig. 6, the effect of the parameter k on the sensing-throughput tradeoff performance is illustrated. The normalized throughput curves are plotted at SNR $= -20$ dB. It is clear that optimizing k maximizes the achievable throughput for a given sensing time. The interrelationships between the throughput, the sensing time, and the k value are presented, and we have shown the significant performance gains by jointly optimizing k and sensing time, as proposed.

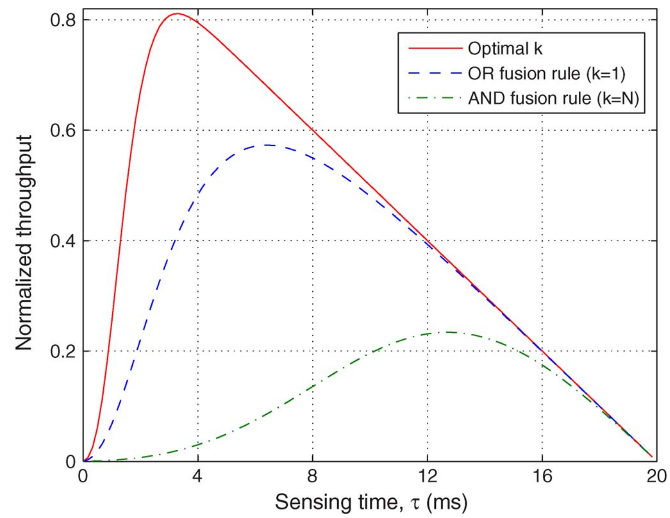

Fig. 6. Sensing-throughput tradeoff with various k-out-of-N fusion rules.

VII. Conclusion

In this paper, we have proposed an iterative algorithm to obtain the sensing time and the k value of the parameter of the fusion scheme that maximizes the throughput of the secondary users, subject to adequate protection to the primary user. The results of the proposed iterative algorithm have been verified to be optimum by comparing them with exhaustive search results. We have shown that significant improvement in the throughput of the secondary users has been achieved when both the parameters for the fusion scheme and the sensing time are jointly optimized.

Appendix C
Proof of $h(\tau, \tilde{k})$ Is Monotonically Decreasing in τ

We prove that $(\partial/\partial\tau)h(\tau,\tilde{k}) < 0$ for the range $0 \le \tau \le T$ and for all $\tilde{k} = 1, \ldots, N$. From (16)

$$\frac{\partial}{\partial\tau}h(\tau,\tilde{k}) = -\frac{1}{\tau} - \frac{2}{T-\tau} + \frac{2(A+B+C)}{\phi(\tau,\tilde{k})}\nabla\tilde{P}_f(\tau) \qquad (20)$$

where

$$A = \sum_{i=0}^{\tilde{k}-1}\binom{N}{i}\tilde{P}_f(\tau)^{i-\tilde{k}+1}\left(1-\tilde{P}_f(\tau)\right)^{\tilde{k}-i-1} \qquad (21)$$

$$B = \sum_{i=1}^{\tilde{k}-1}\left[(i-1)\binom{N}{\tilde{k}-i} + i\binom{N}{\tilde{k}-i-1}\right]\left(\frac{1-\tilde{P}_f(\tau)}{\tilde{P}_f(\tau)}\right)^i \qquad (22)$$

$$C = (\tilde{k}-1)\left(\frac{1-\tilde{P}_f(\tau)}{\tilde{P}_f(\tau)}\right)^{\tilde{k}}. \qquad (23)$$

From (14), we have $\nabla\tilde{P}_f(\tau) < 0$ for $0 \le \tau \le T$, and from (21)–(23), we have A, B, and $C \ge 0$ for all $\tilde{k} = 1, \ldots, N$. Hence, from (20), we obtain $(\partial/\partial\tau)h(\tau,\tilde{k}) < 0$ for $0 \le \tau \le T$ and all possible $\tilde{k}$.

APPENDIX D
PROOF OF $(\partial/\partial\tau)h(\tau,\tilde{k}) < (\partial/\partial\tau)g(\tau)$

We first prove, for the case when $\tilde{k}=1$, that $(\partial/\partial\tau)h(\tau,\tilde{k}) < (\partial/\partial\tau)g(\tau)$ in the lower region of τ, i.e., $\mathbb{R}_L$. From (21)–(23), when $\tilde{k}=1$, we have $A=1$, and B and $C=0$. Therefore, from (20), we have

$$\frac{\partial}{\partial\tau}h(\tau,1) = -\frac{1}{\tau} - \frac{2}{T-\tau} - \frac{\beta}{\sqrt{2\pi\tau}\,(1-\mathcal{Q}(\alpha+\beta\sqrt{\tau}))}\exp\left(-\frac{(\alpha+\beta\sqrt{\tau})^2}{2}\right). \quad (24)$$

The derivative of the function $g(\tau)$ is given as $(\partial/\partial\tau)g(\tau) = (\beta/\sqrt{\tau})(\alpha+\beta\sqrt{\tau})$. Therefore, the detailed expression of $(\partial/\partial\tau)h(\tau,1) < (\partial/\partial\tau)g(\tau)$ is given as

$$-\frac{1}{\tau} - \frac{2}{T-\tau} - \frac{\beta}{\sqrt{2\pi\tau}\,(1-\mathcal{Q}(\alpha+\beta\sqrt{\tau}))} \times\exp\left(-\frac{(\alpha+\beta\sqrt{\tau})^2}{2}\right) < \frac{\beta}{\sqrt{\tau}}(\alpha+\beta\sqrt{\tau}). \quad (25)$$

Let $x=-(\alpha+\beta\sqrt{\tau})$, and since in the lower region of τ, i.e., $\mathbb{R}_L$, we have $(\alpha+\beta\sqrt{\tau}) \le 0$, and therefore, $x \ge 0$. Substituting x into (25), we obtain

$$-\frac{1}{\beta\sqrt{\tau}} - \frac{2\sqrt{\tau}}{\beta(T-\tau)} - \frac{1}{\sqrt{2\pi}\mathcal{Q}(x)}\exp\left(-\frac{x^2}{2}\right) < -x \qquad \forall\, x \ge 0. \quad (26)$$

From [18], it is shown that $(1/\sqrt{2\pi}\mathcal{Q}(x))\exp(-x^2/2) > x$ for all $x \ge 0$. Therefore, the inequality at (26) is verified, and we have proven $(\partial/\partial\tau)h(\tau,1) < (\partial/\partial\tau)g(\tau)$ at the lower region of τ. Next, we will prove $(\partial/\partial\tau)h(\tau,\tilde{k}) < (\partial/\partial\tau)h(\tau,1)$ for $\tilde{k}=2,\ldots,N$. From (20) and since $\nabla\tilde{P}_f(\tau) < 0$ for $0 \le \tau \le T$, we have

$$\frac{2(A+B+C)}{\phi(\tau,\tilde{k})} > \frac{2}{1-\tilde{P}_f(\tau)} \qquad \forall\,\tilde{k}=2,\ldots,N \quad (27)$$

and after some algebraic manipulations, we have $1+((B+C)/A) > 1$ for all $\tilde{k}=2,\ldots,N$. Since A, B, and $C>0$ for all $\tilde{k}=2,\ldots,N$, the inequality $1+((B+C)/A) > 1$ is verified. This completes the proof that $(\partial/\partial\tau)h(\tau,\tilde{k}) < (\partial/\partial\tau)h(\tau,1) < (\partial/\partial\tau)g(\tau)$ when $\tau \in \mathbb{R}_L$ and $\tilde{k}=2,\ldots,N$.

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Characterizing Intra-Car Wireless Channels

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***Abstract*—This paper describes the methodology and results of a series of experiments performed to characterize *intra-car wireless channels*. Specifically, the experiments target parameters such as the coherence time, statistics of channel loss, and fade statistics. Based on previous experiments, *flat fading* is assumed; the methodology is developed, and the results are interpreted in this context. These efforts are motivated by the end goal of designing an *intra-car wireless sensor network*; therefore, some of the implications of results in practical design are discussed. It is found that although the in-vehicle channels exhibit a very large amount of power loss, robust system design can be achieved by utilizing the results of these experiments.**

***Index Terms*—Channel measurements, wireless communications, wireless sensor networks.**

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