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Vibration Control Using Semi-Active Force Generators

A type of force generator which can respond to general feedback signals from a vibrating system in order to control the vibration but which does not require the power supply of a servomechanism is described. Computer simulation studies show that performance comparable to that of fully active vibration control systems can be achieved with the semi-active type of device. Physical embodiments of the concept are discussed and compared to hardware used in active and passive vibration control systems.

Introduction

There is a large body of theoretical knowledge and practical experience in the design and construction of vibration isolators, absorbers, and damping treatments. In the vast majority of cases, vibration control is achieved using *passive elements* such as springs, dampers and masses in the form of metallic, pneumatic, hydraulic or rubber devices. The elements are passive in the sense that no power source is required, i.e., the vibration control elements only store or dissipate the energy associated with the vibratory motion.

Although many vibration problems are solved in an inexpensive, reliable, and satisfactory way with passive devices, it is clear that there are distinct performance limitations when only passive devices are used. In the past, many attempts were made to improve vibration control devices, typically by providing some adjustable parameters which could be varied to suit changing excitation or response characteristics. Early automobiles, for example, were fitted with manually adjustable shock absorbers, and some modern vehicle suspensions and isolation systems contain automatic leveling systems which adjust static deflection when a suspended load varies, [1].[1] Such variable parameter systems can have better vibration control performance than fixed passive systems and do represent a simple form of control loop (sometimes using a human operator) used to improve performance.

The idea of varying vibration control system parameters rapidly has a long history. As early as the 1920's, patents were issued for shock absorbers in which a seismic mass was supposed to activate hydraulic valving directly or through the use of electrical contacts and a solenoid valve. The latter means of valve actuation may be regarded as a fore-runner of modern active vibration control schemes since an electrical power source was required. It is doubtful, however, that many of the early inventions could be successful in practice since most of the devices were inherently nonlinear and the means for analysing and understanding the dynamic response of systems using the devices were not well developed.

It was not until the 1950's and 60's that sophisticated active systems for controlling vibrations were developed. By this time, there was considerable experience in the construction of high-performance servomechanisms, and analysis and computer simulation methods for automatic control systems had become well developed. Typical problems that were attacked using active controllers were the helicopter rotor isolation [2], [3], flexible aerospace vehicle bending mode control [4], [5], and isolation of fighter pilots from aircraft motion [6], [7]. In each of the cases listed above, it was not possible to do a very good job at vibration control with passive means. With the interest in optimization in control theory in the 1960's, general studies on the optimization of vibration control systems were accomplished in which the systems were not assumed to be passive [8], [9], [10]. A survey of optimization techniques for vibration isolation appears in Ref. [11].

Although an active vibration control system can be constructed which shows better performance than the best possible passive systems (or which accomplishes a task not even possible with passive means), it must be admitted that active systems in general are more costly, more complex and therefore often less reliable than passive systems. To date, therefore, the use of active means of vibration control has been limited to cases in which performance gains outweigh the disadvantages of increased cost, complexity and weight. It is the purpose of this paper to present a modern approach to semi-active vibration control in which some of the active system performance gains are realized with com-

[1] Numbers in brackets designate References at end of paper.

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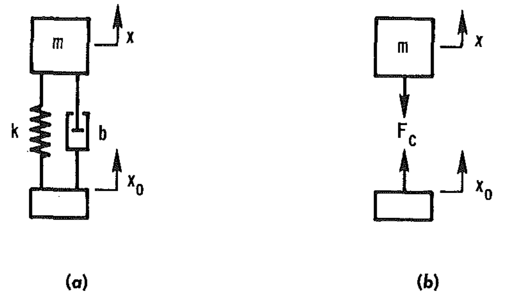

Fig. 1 Isolation of a mass from ground motion: (a) Passive linear isolator and (b) Active isolator: Force F is supplied by a servomechanism

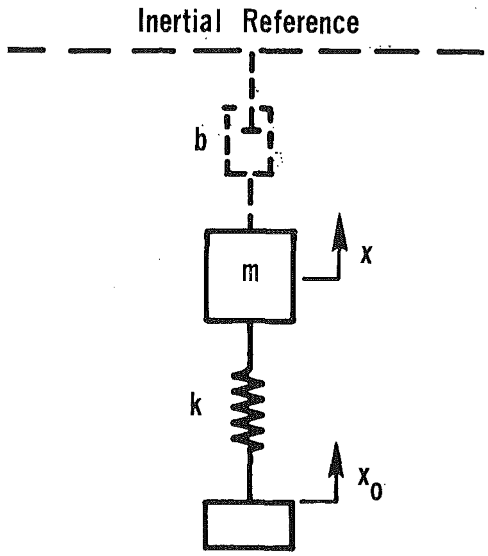

Fig. 2 The "Skyhook" damper system

ponents close to passive components in terms of cost and complexity.

Isolation of a Mass from Ground Motion

The simplest system which can be used to illustrate the performance limitations of conventional passive vibration control systems consists of a rigid mass to be supported on a moving surface. In Fig. 1a, a conventional linearized model of a passive suspension is shown in which the spring of constant k and damper of constant b might exist physically and separately as in an automotive suspension or aircraft landing gear, or might represent a lumped parameter representation of a visco-elastic support structure.

The essential problem in designing the suspension for this system may be stated thus: 1 the relative motion between the mass and the ground, $x - x_0$, must be controlled due to space and other limitations. This could be achieved by making the suspension very stiff. 2 The suspension should isolate the motion of the mass from the ground motion. In the linear case, with simple harmonic excitation, this is often expressed by stating that the transmissibility $|x/x|_0 = |\dot{x}/\dot{x}_0| = |\ddot{x}/\ddot{x}_0|$ should be less than unity for all but very low frequencies. This can be achieved by making the suspension very soft. Clearly, for the system of Fig. 1*a*, there will exist optimum values for k and b when the performance criteria 1 and 2 above are converted to precise mathematical forms and the input $x_0(t)$ is specified in some precise deterministic or statistical sense. See [8], for instance. What is also clear is that since only the parameters b and k are available to be varied, if the performance of the optimized passive system is still not acceptable, some new way of controlling the mass motion must be sought.

In Fig. 1*b*, the isolation problem is posed in a more general context. The control force F_c is studied without regard initially as to whether it can be supplied by a combination of passive elements or whether it must be supplied by some active system such as an hydraulic piston with suitable valving. It is conceptually easy to specify F_c as a function of time for a given $x_0(t)$ when the criterion for optimizing the suspension has been set out. But such an open loop system is practically much less useful than a closed loop solution in which F_c is found as a function of system variables such as $x(t)$, $\dot{x}(t)$ and $x_0(t)$ and $\dot{x}_0(t)$. In this context, the passive system may be conceived of as a special closed loop feedback system which generates a force which is a weighted sum of forces proportional to relative position and relative velocity. That is,

$$F_c = +b(\dot{x} - \dot{x}_0) + k(\dot{x} - \dot{x}_0) \tag{1}$$

for the passive linear system.

Such a force could be supplied by a servo, but passive elements supply a simpler means of generating the force. On the other hand, however, the active system is not restricted to generating forces of the form of equation (1).

The question of how F_c should be determined depends on just how the performance criteria for the system are stated. For most practical criteria of performance, it is not known just how F_c should be determined, but valuable clues may be found from linear optimal control theory. When quadratic criteria are used, then the optimal control policy involves feedback of all state variables using a linear weighing coefficient scheme. In vehicle context, for example, if we assume a roadway generates $\dot{x}_0(t)$ histories which are sample functions of a white noise process, and we minimize $c_1E(\dot{x}^2) + c_2E(x - x_0)^2$ where c_1 and c_2 are given constants and $E(\quad)$ stands for "expected value," then

$$F_c = b\dot{x} + k(x - x_0) \tag{2}$$

where b and k depend on c_1 and c_2, [9], [10]. A similar result is obtained for a transient response case in which the time integral of a weighted sum of $\dot{x}^2$ and $(x - x_0)^2$ is minimized.

The feedback law of Eq. (2) can, in fact, sometimes be realized by passive elements, as shown in Fig. 2. In many practical cases, however, it is not possible to connect a damper from the isolated mass to an inertial reference so that the damper force is proportional to absolute mass velocity. This is obviously the case for vehicle suspensions, for example, so this configuration is called the "skyhook damper" scheme.

The force law of equation (2), which arises naturally when Wiener Filter theory or a state-space optimal control theory is applied to the vibration control problem, can be explained readily using transmissibility plots for the conventional system of Fig. 1*a*, and the skyhook system of Fig. 2 or its active equivalent of Fig. 1*b*. These plots are shown in Fig. 3.

The value of spring constant k sets the natural frequency $\omega_n = (k/m)^{1/2}$ for both suspensions. For sinusoidal inputs below ω_n, $x \cong x_0$ and $x - x_0 \rightarrow 0$. Thus, for low frequencies, the spring is primarily responsible for maintaining small relative displacements. Near the natural frequency, the damper controls the resonance of the system. Fig. 3 shows several plots for various values of the damping ratio ζ,

$$\zeta = \frac{b}{2}\left(\frac{m}{k}\right)^{1/2}$$

For both suspensions, small values of b (or ζ) result in undesirably high values of response for input frequencies near ω_n.

For inputs with frequencies greater than the system natural frequency both suspension systems begin to isolate the mass from the base motions. There is, however, a major difference between the two schemes as the damping parameter is varied. When b is increased in the skyhook configuration, the response near the resonance frequency decreases and also the high frequency response decreases somewhat. In the conventional system, however, decreases in the resonant response are purchased only at the cost of increased response, (or degraded isolation) for high frequencies. That this difference should exist is intuitively evident. The skyhook damper exerts a force tending to reduce the *absolute* velocity of the mass while the conventional damper exerts a force tending to reduce *relative* velocity, $\dot{x} - \dot{x}_0$. For high frequency inputs, the conventional damper acts to stiffen the suspension, when a soft suspension is desired. An active system programmed to simulate the skyhook damper can therefore achieve a better combination of resonance damping and high frequency isolation than a conventional passive spring damper combination.

While the above considerations apply strictly only to linear systems, it seems clear that the same qualitative effects would be found in comparing nonlinear passive and active suspensions. The comparison presented above between the capabilities of active and passive systems, although limited to a single very elementary case, will serve to introduce the idea of a semi-active suspension which has some of the simplicity of a passive system with most of the performance advantages of the active systems. Later in the paper some uses of active and semi-active suspensions for more complex problems of vibration control will be discussed.

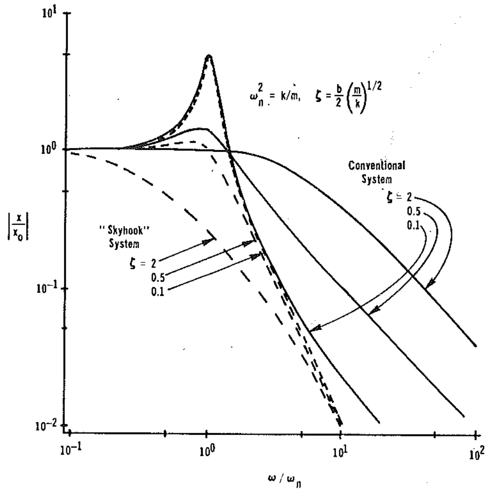

Fig. 3 Comparison of conventional and "Skyhook" vibration isolation scheme transmissibilities

A Single Degree-of-Freedom Semi-Active Isolator

The linear control law for an active controller of equation (2) can be partly realized using an ordinary spring, but the term $b\dot{x}$ cannot be realized by a passive element in the position of the conventional damper in Fig. 1*a*. The force component $b\dot{x}$ in F_c can be supplied by a servomechanism capable of either supplying or absorbing energy. Suppose now that a device is installed in place of the conventional damper, and that the device is passive, but that the force across the device is controllable. A system incorporating such a device is shown in Fig. 4. The symbol used implies that the device is a damper in which the damper coefficient is variable but what is meant is more general. We suppose the device is capable of generating essentially any force F_d such that the power

$$F_d \cdot (\dot{x} - \dot{x}_0) \geq 0 \tag{3}$$

i.e., such that the power associated with F_d is always dissipated. Thus, if the relative displacement is increasing, $(\dot{x} - \dot{x}_0) > 0$, F_d must be tensile, and if $(\dot{x} - \dot{x}_0) < 0$, F_d must be compressive. Such a device could be made from a manually adjustable hydraulic shock absorber, for instance, except that we intend the adjustments to occur so rapidly that F_d can be changed drastically during a single cycle of vibration. Before discussing some hardware realizations of this type of semi-active element, let us see how closely such a device can simulate the behavior of an active system programmed to use the skyhook control law of equation (2).

The desired value of F_d is $b\dot{x}$, but the semi-active device will only be able to generate this force if the sign of $\dot{x} - \dot{x}_0$ is proper since the device cannot supply power to the system. Thus

$$F_d = b\dot{x}, \text{ if } \dot{x}(\dot{x} - \dot{x}_0) > 0 \tag{4}$$

When $\dot{x}$ and $\dot{x} - \dot{x}_0$ are of opposite sign, the device could only supply a force with a sign opposite to the desired force $b\dot{x}$. The best the device can do to approximate the desired force $b\dot{x}$ then is to supply no force at all, so we specify

$$F_d = 0, \text{ if } \dot{x}(\dot{x} - \dot{x}_0) < 0 \tag{5}$$

If the expression $\dot{x}(\dot{x} - \dot{x}_0)$ vanishes exactly, two special cases may arise. In the first case, $\dot{x}$ may vanish, in which case, we desire $F_d = 0$. In the second case, $\dot{x} \neq 0$ but $(\dot{x} - \dot{x}_0) = 0$, and in this case the device can attempt to apply the force $b\dot{x}$. Depending on the subsequent time history of $\dot{x}_0$, the quantities $\dot{x}$ and $(\dot{x} - \dot{x}_0)$ either change so that the criteria of equations (4) or (5) apply in the succeeding instants or the force $b\dot{x}$ may be large enough to

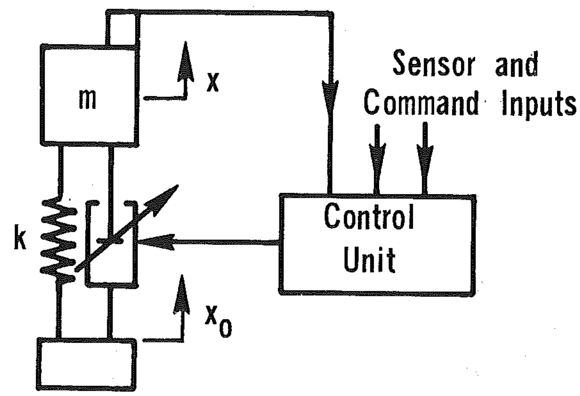

Fig. 4 A semi-active isolator system

lock up the system so that $\dot{x} - \dot{x}_0 = 0$ for a finite time. During lock up, $\ddot{x} = \ddot{x}_0$, and the spring force is $k(x - x_0)_l$ where $(x - x_0)_l$ is the spring deflection at lock up. The damper force is then

$$F_d = -m\ddot{x}_0 - k(x - x_0)_l \tag{6}$$

Thus we may say that the semi-active device will lock up when, (*a*) $(\dot{x} - \dot{x}_0) = 0$ and (*b*) the desired force $b\dot{x}$ is larger in magnitude than the expression of eq. (6).[2]

There are then three possible values for the force F_d, 1. The desired force $b\dot{x}$, 2. zero force, or 3. the lock up force. The device will switch among these three possible force values depending on the input $x_0(t)$, and the system response. Obviously, the semi-active device with its power limitations is inherently a non-linear element even when it is programmed to simulate a linear skyhook damper. Prediction of the dynamic response of a vibration control system using the semi-active force generator is very tedious by hand, but it is a straightforward if time-consuming job using digital simulation techniques.

Figs. 5, 6, and 7 show a set of typical computer simulation results. The base motion $x_0(t)$ in Fig. 4 was sinusoidal with a

[2] In both computer simulations and in hardware realizations, true lock up often does not occur. What happens is that the device begins to apply a large F_d, the relative velocity is driven through zero so that F_d drops to zero, the relative velocity again passes through zero, F_d begins to build up again, and so on. A limit cycle oscillation can exist with $\dot{x} - \dot{x}_0$ nearly zero, and the time average value of F_d is near the theoretical lock up force of equation (6). Eventually, the variables $\dot{x}$ and $\dot{x}_0$ change enough so that the system breaks out of the lock up condition and remains a finite length of time in the state described by either equation (4) or equation (5).

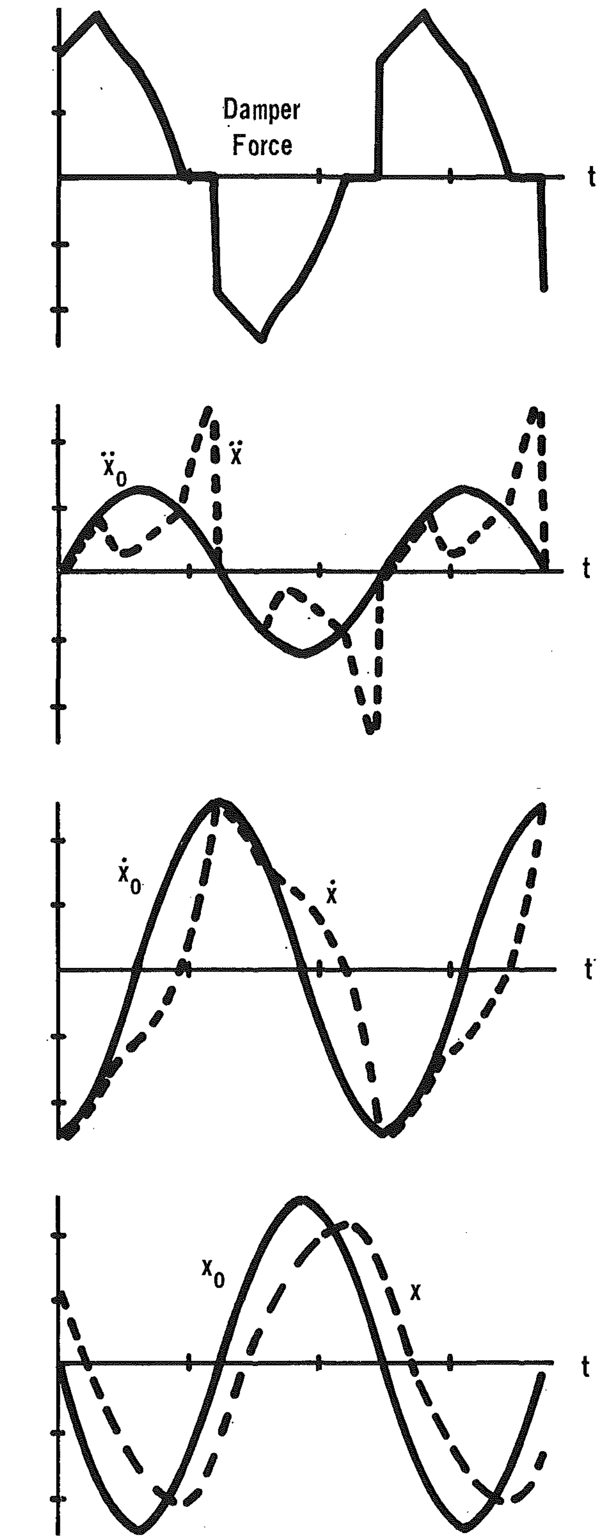

Fig. 5 Simulation results for steady-state response of semi-active system; sine wave input at one-half system natural frequency, damper programmed to simulate "skyhook" damper with $\zeta = 1.0$

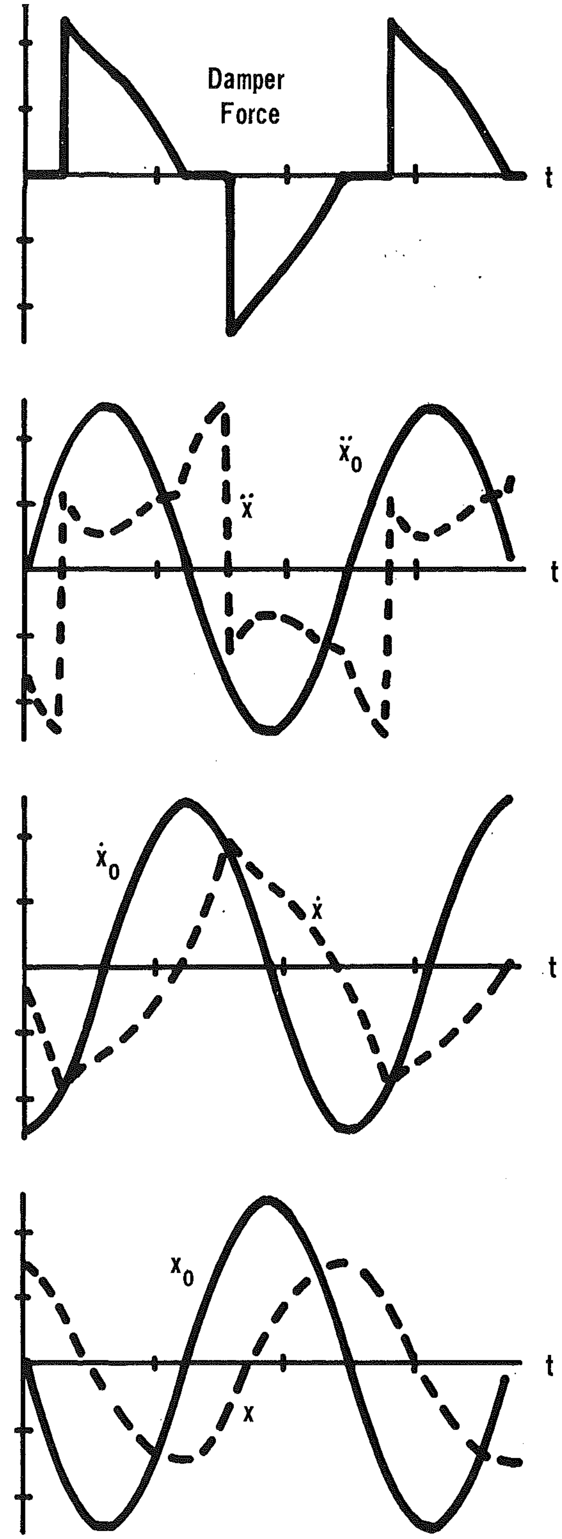

Fig. 6 Simulation results for steady-state response of semi-active system; sine wave input at system natural frequency, damper programmed to simulate "skyhook" damper with $\zeta = 1.0$

frequency equal to 0.5, 1.0, and 3 times $\omega_n = (k/m)^{1/2}$ in the three cases shown. In each case, the controllable damper was programmed according to equations (4), (5), and (6) with a value of b in equation (4) which corresponded to a unit damping ratio. (The semi-active system thus was attempting to reproduce the performance of a unit damping ratio "skyhook" system but with the damper force generator in the position of a conventional damper.)

The response plots were obtained by integrating the equations of motion directly. The system was allowed to run until a steady state was closely approached although only the last cycle and a half are plotted in the figures. In the low frequency

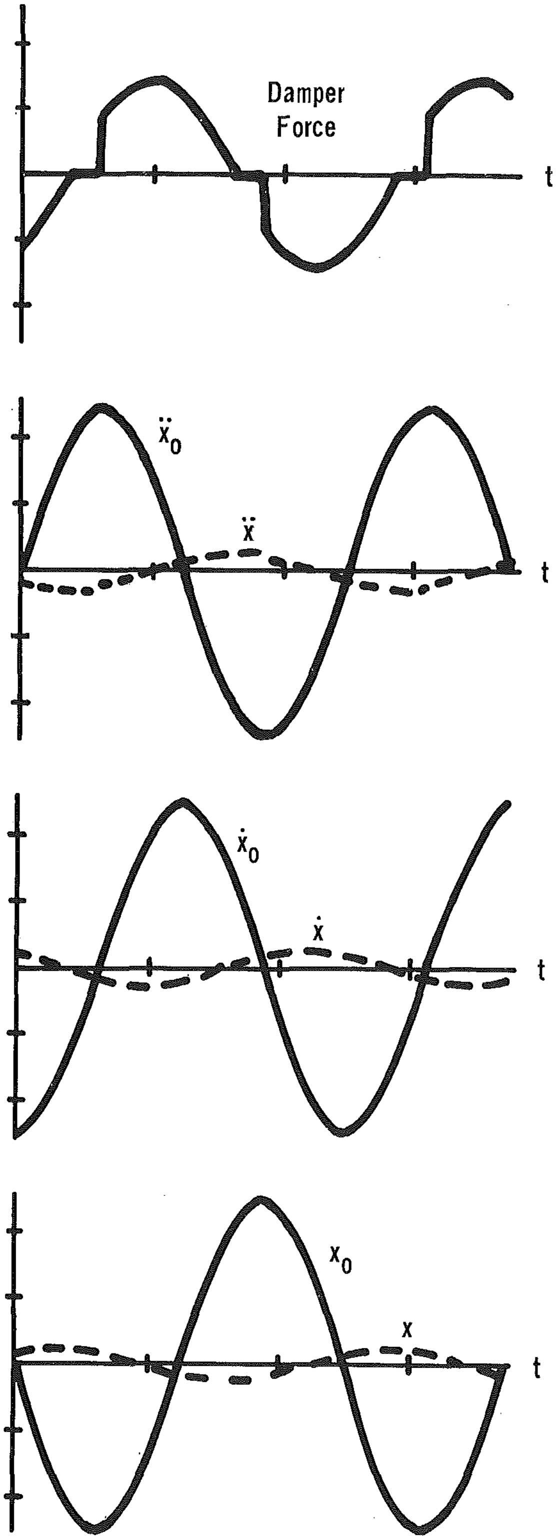

Fig. 7 Simulation results for steady-state response of semi-active system; single sine wave input at three times system natural frequency, damper programmed to simulate Skyhook damper with ζ = 1.0

input case (Fig. 5) one may readily identify portions of the cycle in which the force is $b\dot{x}$, or is zero, or in which the damper has locked up ($\dot{x} \equiv \dot{x}_0$, $\ddot{x}_0 \equiv \ddot{x}_0$). When the input frequency is at the system natural frequency (Fig. 6), no lock up occurs, but the damper force still does switch to zero during portions of the cycle. At still higher frequencies (Fig. 7) the force generator continues to switch between zero and the skyhook force. Note that the

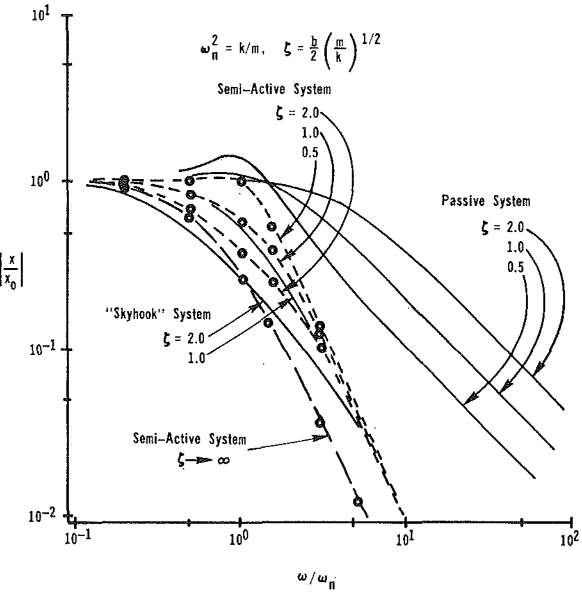

Fig. 8 Frequency response comparison

damper force plots in the three cases are not to the same scale. The damper force in Fig. 7 is proportional to $\dot{x}$ and hence is quite small compared to the damper force in Figs. 5 and 6.

Although the semi-active system is obviously nonlinear even when it is programmed to simulate a linear system such as the "skyhook" damper, the time histories of x and $\dot{x}$ resemble those from a linear system. In Fig. 8, an attempt is made to compare the frequency response of the semi-active system with both the conventional passive system and the "skyhook" system. For the semi-active system, $|x|$ represents the amplitude of the first harmonic of $x(t)$. (In most cases, the second and higher harmonic amplitudes amounted to only a few per cent of the first harmonic amplitude.) The three curves for ζ = 0.5, 1.0, and 2.0 show that the semi-active system has a performance intermediate between the skyhook system (which could be achieved with a fully active system) and a conventional system. At high frequencies, the isolation of the semi-active system approaches that of the skyhook system while near the system natural frequency the semi-active system cannot control the resonance quite as well as the skyhook system.[3]

One might imagine that by increasine the value of b (or ζ) in equation (4) for the semi-active system, one could effectively simulate any skyhook damping desired, but such is not the case. As Fig. 8 shows, there is a limit response when the gain b or ζ is increased indefinitely. In this situation, $\dot{x}$ is held to zero during part of a cycle, but, because of power constraints, the damper must still turn off during part of the cycle. The net result is that when $b \to \infty$ in equation (4), the response resembles that of a skyhook system with ζ = 1.0 and a natural frequency of about 0.6 $(k/m)^{1/2}$. The apparent lowering of the system natural frequency with high values of b is attained at the cost of a very

[3] The definition of transmissibility given above is meaningful for the semiactive system but since the system is not linear, some of the implications which a transmissibility would carry for a linear system do not apply to the semi-active system. For a single sine wave input, a change in the input amplitude does not affect the switching points for the damper, and since the system is linear between switches, the response amplitude is proportional to the input amplitude. However, for multiple sine wave inputs, the switch points are affected by both the amplitude and the phase of the inputs so that one cannot predict the response exactly using the results for a single sine wave input. Also, if one were interested in transmissibilities based on velocity amplitude ratio or acceleration amplitude ratio, these would not be exactly the same as those given in Fig. 8 since $|x|/|x_0| = |\dot{x}|/|\dot{x}_0| = |\ddot{x}|/|\ddot{x}_0|$ only for linear systems subjected to harmonic excitation.

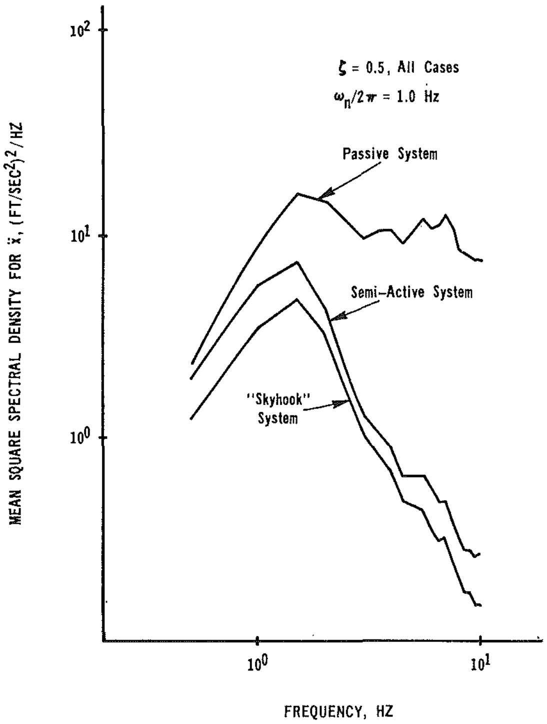

Fig. 9 Typical random input response plot

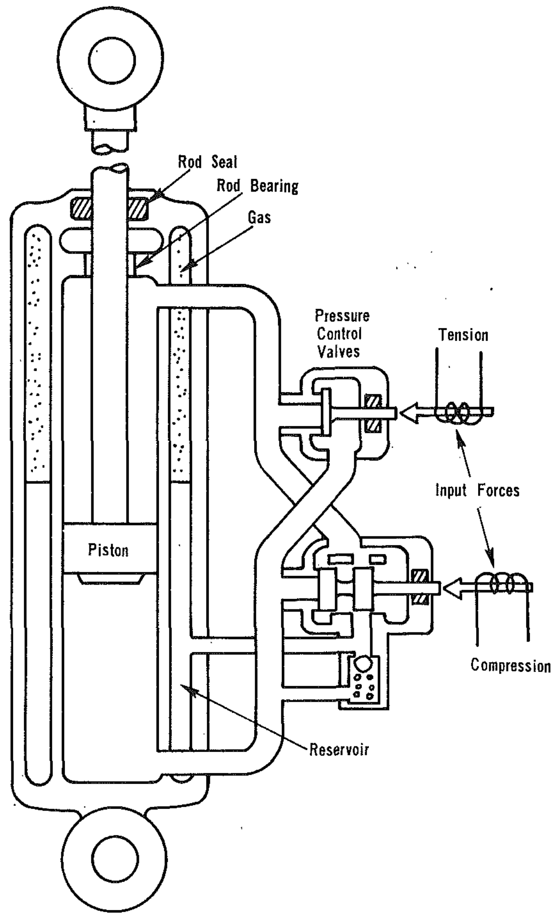

Fig. 10 Electro-hydraulic active damper schematic

discontinuous damper force. Practically it doesn't seem worthwhile to use such high values of gain since the semi-active system achieves significant performance gains over a passive system even when the damping ratio is near unity.

The frequency response plot of Fig. 8 cannot rigorously predict the response of the semi-active system to a sum of sinusoids as an input in the way which would be possible for a truly linear system. However, simulation studies using multiple sinusoidal inputs and random inputs indicate that the frequency response plot does give a fairly good indication of the behavior of the semiactive system. An example of a random input response is given in Fig. 9. The input $x_0(t)$ had a power spectral density approximating the roadway unevenness input to a vehicle wheel. The system had a natural frequency of 1.0 Hz and the input $x_0(t)$ was formed by passing white noise through a first order lag filter with a break frequency of 0.8 Hz. Thus $\dot{x}_0$ had a power spectral density which was approximately white for frequencies above 0.8 Hz up to a cutoff frequency of 15 Hz. The white noise was generated by selecting gaussian random numbers and using the numbers as constant input amplitudes over sampling times of 0.005 sec. The digital simulation was run for 50 sec and the power spectral densities were computed for frequency bands of 0.5 Hz up to 20 Hz and were plotted to 10 Hz. The linear systems and the semi-active systems were analysed in this manner, and as a check, transfer function predictions for the linear systems' power spectral densities were compared with the computed results.

The spectral density which resulted when the response of the semi-active system was analysed confirms that the semi-active system does have a performance intermediate between that of a conventional and an active isolator. Also, the shape of the spectral density could have been predicted fairly accurately using the input spectral density and the frequency response plot of Fig. 8.

The Active Damper as a Semi-Active Force Generator

There are several ways in which a semi-active force generator might be realized in practice. Fig. 10 shows a schematic diagram of an active damper which resembles a conventional direct acting hydraulic shock absorber except that the hydraulic pressure and hence the force F_d is controlled by a pair of poppet valves. The force required to open the valves is set by a torque motor or a staged force amplifier. As inspection of Fig. 10 will demonstrate, one valve sets the compressive force and the other the tensile force. When a compressive force is commanded, the compression valve sets the force F_d when the damper is being compressed and the tensile force valve is unloaded and acts like a check valve assuring that when the damper is extending, the tension force will be virtually nil. Similarly, when a tensile force is commanded, the desired force will be set by the tension valve force when the damper is extending and the compression valve is unloaded so virtually no compression force will arise if the damper is compressed.

The torque motor or other force amplifier does require a small power supply, but the larger damper force is generated in a passive manner. Hence the device is called semi-active and it functions as a force and power amplifier. The result is that a small amount of active power controls the vibration through the modulation of a larger amount of dissipative power.

The semi-active system does require sensors such as accelerometers and relative velocity transducers and a control unit in order to command the valve force actuator. However, the sensors operate at signal power level and the actuator at low power. A fully active system would require in addition a high power actuator with good frequency response which entails a relatively large power supply and the complication which goes into constructing a high power, fast servomechanism. Clearly, any dissipation device which can be modulated can form the basis for a semi-active system. Further examples include electrically

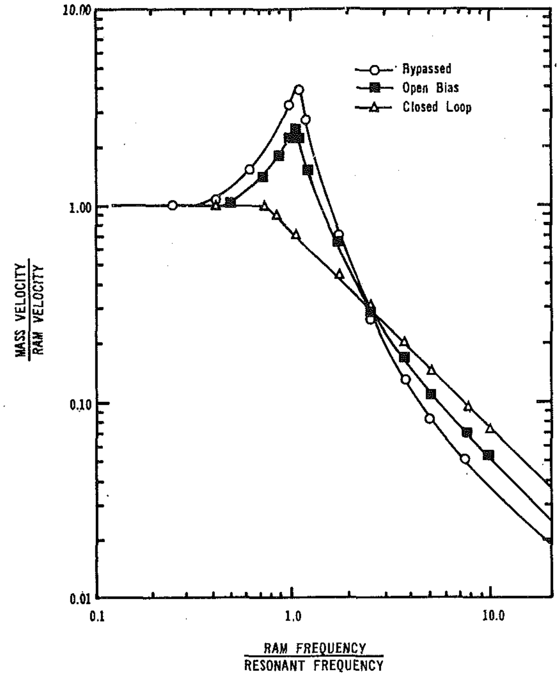

Fig. 11 Experimental results for breadboard semi-active isolator

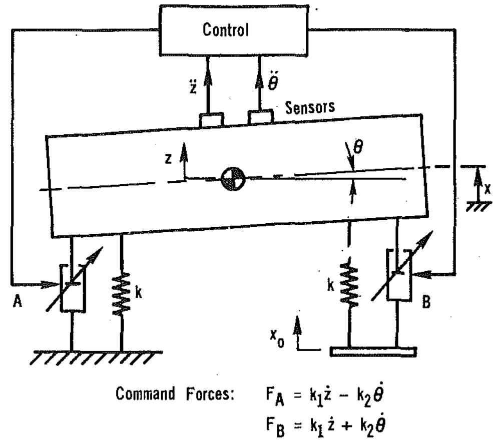

Fig. 12 Semi-active control of two degree-of-freedom system

actuated brakes and clutches, and elements using magnetic or electro-viscous fluids.

Two active damper test systems have been built and tested to prove the computer generated results. At the University of California, Davis, a single degree-of-freedom rotary system was constructed using an electrically activated drum brake as the semi-active element. Tests on this system showed that the semi-active isolator performed better than the passive system in which the brake functioned as a coulomb friction damper. Since the dry friction damper characteristics involved in both the active and passive versions of this isolation system rendered the system response amplitude dependent even for single sine wave inputs, the results are hard to compare directly with the computer simulation results. Qualitatively, however, it was clear that the semi-active system was capable of controlling the resonance effectively without compromising the high frequency isolation.

A translational system using controllable hydraulic fluid damping has been tested at the Lord Corporation and provides a more direct comparison with the computer studies since it was designed to provide linear absolute velocity damping of the isolated mass. This system did not use a device such as that shown in Fig. 10 but rather used standard pressure control valves to achieve the desired semi-active damper characteristics. Because of the "breadboard" nature of the configuration, and because industrial valves were used, the hydraulic circuits were longer and more restrictive than would be necessary in a prototype design. The result was that when the damper was turned off, the system had more damping than would be desirable. Experimental results shown in Fig. 11 indicate that the major influence of these effects was to compromise high frequency isolation somewhat. The curves labeled "bypassed" and "open bias", were obtained by eliminating presssure forces due to the control valves entirely and by biasing the valves to minimum control pressure respectively. Ideally, both curves should have shown no damping in these cases, but clearly some residual damping was present. The curve labeled "closed loop" shows results when the semi-active system was operating. Note that there is some isolation at the resonant frequency and that the high frequency isolation is somewhat degraded from the case in which the damper is simply turned off.

As a comparison with Fig. 8 will show, the semi-active system is clearly not acting as a heavily damped passive system. The high frequency isolation for the semi-active system is significantly better than that achievable with a passive system which effectively damps the resonance. (In fact, no linear passive system of any damping ratio can have an amplitude ratio less than unity at the resonant frequency, but the semi-active system does achieve this.) On the other hand, the compromises necessary in the breadboard system prevented the achievement of the two-to-one slope of the high frequency response which are achieveable in the ideal case for the semi-active system. The two breadboard systems have demonstrated the feasibility of realizing the predicted characteristics of the semi-active isolator and it seems likely that with careful design of a prototype system, it should be possible to approximate closely the results predicted through computer simulation.

Semi-Active Vibration Control for Complex Systems

Although the concept of semi-active vibration control has been illustrated above using only a very elementary single degree-of-freedom system, it is important to note that the force generator can be made to react to signals originating at locations remote from the point of application of the force. This is in contrast to passive isolators which, be they linear or nonlinear, can only be influenced by local variables such as positions, velocities or accelerations. In a multi-degree-of-freedom system, a semiactive force generator can receive inputs based on sensed aspects of the motion throughout the structure. As an example, it would be possible to control the structural dynamic response of a taxiing aircraft with active dampers in the landing gear using techniques similar to those reported in reference [4].

As a simple example of the use of semi-active force generators for a multi-degree-of-freedom system, consider the system shown in Fig. 12. The rigid body to be suspended might represent the heave and pitch motion of a vehicle. Often a suspension designer has little control on the location of suspension points and the vehicle mass distribution. The result is that selection of suspension parameters to control both pitch and heave motion effectively is made difficult by the inherent geometry of the mass to be isolated. In the case of the present example, if conventional dampers are adjusted to provide critical damping of the pitch mode ($\zeta_\theta = 1.0$), the heave mode has a damping ratio of one quarter of critical damping ($\zeta_Z = 0.25$). If an input motion is applied at x_0 in Fig. 12 and the response is computed at the point described by x, the result is as shown in Fig. 13 for this case. The spring rates, mass distribution and geometry combine to yield a heave natural frequency of 1 Hz and a pitch natural frequency of 4 Hz.

Now, if a fully active system were employed, it would be possible to use accelerometers to measure the pitch and heave acceleration ($\ddot{\theta}, \ddot{z}$) and to command forces which would damp the pitch and heave motion independently. Such command forces are indicated in Fig. 12 and the resulting frequency response when both pitch and heave motions have been given critical damping is shown in Fig. 13. The active system has two advantages over the passive system: (*a*) The damping ratios of the two degrees-of-freedom are independently adjustable without varying the mass distribution or the suspension points; (*b*) The high frequency isolation is excellent even for high damping since the force generator reacts to absolute rather than relative motion.

The final curve in Fig. 13 represents simulation results when active dampers were used in place of the fully active force generators. Each damper has a command force as given in Fig. 12, but because of the fact that the dampers can supply the commanded force only over part of the cycle, the performance is not quite as good as the performance of the active system. As Fig. 13 shows, however, the semi-active system does achieve both of the performance improvements of the active system to a significant degree. Perhaps the most significant point is that no passive dampers could control the two resonances independently as the active damper system does.

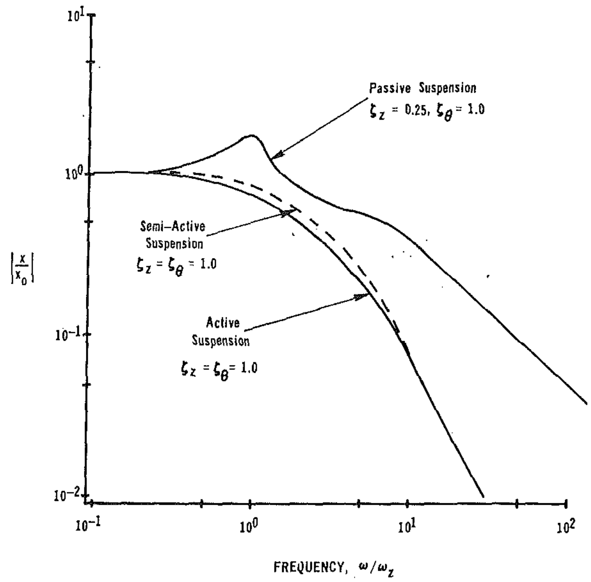

Fig. 13 Frequency response of system of Fig. 12

Conclusion

Active systems have proven advantages in vibration control over passive systems, but the increase in cost and complexity when active systems are substituted for passive systems can be justified only in cases in which performance is critical. Semi-active systems using modulated dissipation elements as the force generators can provide many of the performance gains of active systems. Since the semi-active systems require only signal processing and low level power supplies, the hardware for such systems should be significantly simpler and less costly than for active systems.

The semi-active systems are always nonlinear but simulations have shown that it is often possible to design such systems using linear control laws. It seems inevitable, however, that direct computer simulation will be required in order to discover just how closely a semi-active system can reproduce the performance of an active system. This approach also allows investigation of nonlinear feedback control schemes and the effect of hardware response limitations.

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