Power Distribution Law and Its Impact on the Capacity of Multimedia Multirate Wideband CDMA Systems*

Shihua Zhu and Jon W. Mark
Centre for Wireless Communications
Department of Electrical and Computer Engineering
University of Waterloo
Waterloo, Ontario, Canada
N2L 3G1
(szhu, jwmark)@bbcr.uwaterloo.ca

Abstract

The system capacity in an asynchronous direct-sequence code division multiple access (DS-CDMA) system is dominated by multiple access interference (MAI) which is proportional to the transmit power strength of other users. The transmit power therefore becomes a major resource and requires careful planning and control if optimal system performance and maximal user capacity are to be achieved. This work investigates the power distribution law and studies its convergence condition. The problem is formulated in the context of third generation multimedia multirate wideband CDMA (WCDMA) mobile communications. It shows that the convergence condition for any power distribution or power control algorithm is a function of the spread bandwidth (resource), user data rates, and QoS requirements of connections (traffic demands). The closer the demand is to the resource, the higher are the required transmit powers. If the demand exceeds the resource, no algorithm can converge, which means some transmitters may reach their saturation power level resulting in unsatisfactory quality of service (QoS). A new power control algorithm based on the convergence criterion is proposed. Numerical results are presented to validate the analytical results and the proposed power control algorithm.

*This work has been supported by the Natural Scieces and Engineering Research council of Canada under grant no. RGPIN 7779

1 Introduction

Code division multiple access (CDMA) has been widely accepted as the major multiple access scheme for third generation mobile communications [1, 2]. Ideal operation of a CDMA system requires orthogonal spreading codes among different users and perfect synchronization among all the spreading codes arriving at each cell-site receiver. This latter requirement is particularly stringent, especially when communication terminals are in motion. When synchronization cannot be maintained, orthogonal codes such as Hadamard code may no longer be orthogonal and, consequently, cannot be used to separate channels.

In third generation WCDMA systems such as UMTS/IMT-2000 (Universal Mobile Telecommunications System/International Mobile Telecommunications by the year 2000)[2, 3], two layered spreading is adopted: whenever synchronization can be easily maintained, orthogonal codes are used; otherwise, scramble codes are used to make other-user signals appear as random disturbances. This randomized other-user disturbance is called multiple access interference (MAI).

As a result, each user in the system is interfered by both thermal noise and MAI. The latter can become dominant when the number of users in the system is large. Obviously, the user performance is determined by the number of users using the same spreading band and the power levels at which each user transmits. For a given MAI, a user can always increase its transmit power (within saturation limit) to achieve the desired signal-to-interference ratio (SIR). But this will result in higher MAI to other users that may in turn have to increase their power to maintain their original SIR. This can lead to cyclic increase of power until some or all of the users reach their saturation power levels, resulting in unsatisfactory SIR or QoS. Therefore, the transmit power in WCDMA must be carefully planned and controlled as a system resource if the desired system performance and maximal user capacity are to be achieved.

The objectives of power control or distribution are two folds: 1) to assign transmit power within the saturation limit for each user in a cell such that all transmissions meet their QoS requirements under a given amount of thermal noise, intracell- and intercell-MAI; and 2) to minimize the total transmitted power from users to save handset battery. The first is mandatory and the second is desirable.

Power control has always been a critical issue for wireless mobile communications. Nettleton and Alavi [4] studied the minimum transmit power achievable for minimum co-channel interference. The problem turns out to be that of solving eigenvalues of a propagation-loss matrix equation under the assumption that thermal noise is negligible and all users require equal SIR. Zander [5] derived a similar matrix inequality and showed that the optimum power allocation must assign zero transmit power to those that cannot meet the required SIR. Dziong, Jia and Mermelstein [9] adopted a different approach: a new call is admitted to the system only if it will not deteriorate other existing connections to their minimum QoS level. The effectiveness of this algorithm is heavily dependent on the estimation accuracy of new call demand, transmission channel characteristics, and neighboring traffic information.

Hanly [6] and Yates [8] studied the problem in a more general setting. Hanly [6] studied maximization of system capacity using combined power control and cell-site selection. He derived the condition on the eigenvalue of the path-gain matrix for positive solutions of the equation. Among many results, the most interesting are that (1) if the power control matrix equation has solutions, there must exist a minimum solution which optimizes the system capacity, and (2) less QoS level requires less transmitted power. Yates and Huang [7] showed that the solution set of the feasible power vectors is typically not a convex set, and within all the vertices, there is a vertex at which all users reach their minimum transmitted power — the unique solution to the MTP (Minimum Transmitted Power) problem. In a later paper, Yates [8] presented a general interference constraint inequality and showed that the interference function is positive, monotonically increasing and that if a user has an acceptable connection under a power vector, then this user will have a more than acceptable connection when all powers are scaled up uniformly.

Yao and Geraniotis [10] formulated the bit error probability for multimedia multirate CDMA, which is used to serve as the constraint for maximization of the number of users and minimization of the total transmitted power. Dynamic programming (DP) method was used to obtain the optimal power control law. Hu and Liu [11, 12] also analyzed the signal-to-noise ratio (SNR) of a multirate CDMA system and modified the result in [10] by introducing a traffic exponent to the service rate so that the difference in processing gains for various information rates can be included in a unified power control algorithm.

All of the above works assumed the existence of an optimal solution and studied the various aspects of the behavior of power control algorithm when it converges to the solution. Hanly [6] addressed the existence problem. But the determination of the eigenvalue of the matrix, which is crucial to the solution, and its physical interpretation are still unknown. From the information theory point of view, the existence of solution should be governed by resource and demand, that is, if traffic demand is too high compared to system resources, no power control algorithm can converge for satisfactory QoS performance. In this paper, we will show that the existence of solution for any power control or power distribution law is indeed a function of the spread bandwidth (resource), user data rates, and QoS requirements of the connections (traffic demands); the closer is the demand to the resource, the higher is the transmit power required; if the demand exceeds the resource, no algorithm can converge.

The remainder of the paper is organized as follows. Section 2 describes the system environment in which the power distribution law is to operate, and makes the necessary assumptions for the ensuing work. The condition that any power distribution law must obey in order to achieve the desired QoS requirements for all connections is developed in Section 3. The convergence problem of the power distribution inequality derived in Section 3 is studied in Section 4, which shows that the maximum traffic load which can be carried in each cell is bounded by the spread bandwidth. Section 5 proposes a new power control/call admission algorithm based on the new criterion derived in the previous sections. Numerical results are shown in Section 6. Conclusions for the work are given in Section 7.

2 System model

The two-layered spreading adopted in UMTS/IMT-2000 is illustrated in Figure 1 where $\{C_{ok}\}_{k=1}^{K}$ denote orthogonal codes and C_S is the scramble code. K channels of diverse information rates and QoS requirements are generated from a single transmitter. Since these channels can be readily synchronized, orthogonal codes, C_{ok}, $k = 1, 2, \cdots, K$, are used to separate the channels. These channels are then linearly combined (summed) and spread (multiplied) by a transmitter-specific scramble code $C_{\rm S}$. In UMTS/IMT-2000, OVSF (orthogonal variable spreading factor) codes [13] are used as orthogonal codes which allow a constant spread bandwidth for various information rates, and Gold codes or large Kasami code sets are used as scramble codes.

For downlink, every base station uses the same set of OVSF codes as its channelization codes, but each base station uses a cell-specific Gold code as its scramble code. For uplink, each active mobile user equipment (UE) establishes a connection with its base station. The connection may consist of one or several channels. Every UE or connection uses the same set of OVSF codes as its channelization codes, while the connections are distinguished from each other by user-specific scramble Gold code or Kasami code.

Obviously, the number of downlink channels from a single base station is much larger than the number of uplink channels from a single UE. On the other hand, the transmission condition is severer for the uplink channels than for the downlink channels. This asymmetry has led to different structures for the uplink and downlink physical channels [2].

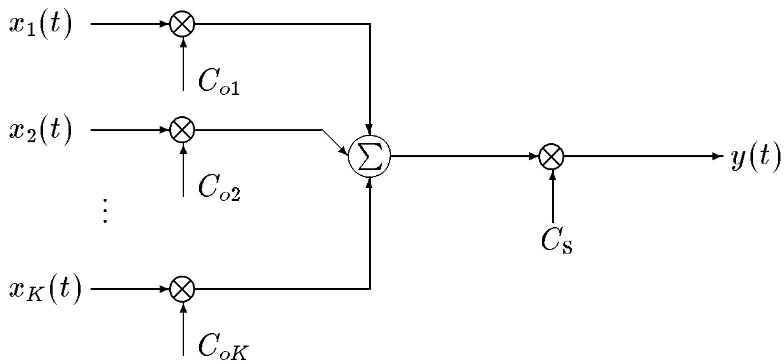

Figure 1: Multiple spreading

Since the number of interfering UEs to a receiving cell-site station is normally larger than the number of interfering cell-site stations to a receiving UE, the system capacity is essentially determined by the uplink transmission capacity. For this reason, we limit our discussion to the uplink case. We also make the following assumptions:

- The system consists of an arbitrary number of base stations among which we take one as our target base station.
- The target base station serves M UEs and the jth UE has K_j channels, $j = 1, 2, \cdots, M$. These channels can carry different media with different information rates and QoS requirements.
- The MAIs from other UEs at the cell-site receiver synchronized to the reference UE is truly random and has normal distribution.
- The background noise is additive white Gaussian noise (AWGN) with two-sided power spectral density $N_0/2$.
- Power control mechanism is sophisticated enough to guarantee that the signal power arrived at the target base station receiver from each UE is accurately controlled to the desired level.
- The QoS requirement by each service has taken shadowing and fading into account and has left enough margin for shadowing and fading.

3 Formulation of power distribution law

We assume that the QoS required by each service or medium is solely specified by the BER which can be maintained by specifying an appropriate signal-to-interference ratio (SIR), or E_b/I_0, at the receiver, where E_b is the information bit energy and I_0 is the interference power spectral density. For direct sequence CDMA, it can be written as [14, 15, 16]

$$\frac{E_b}{I_0} = \frac{S/R_b}{I_t/W} == \frac{S}{I_t}G \tag{1}$$

where S is the signal power, R_b is the information bit rate, I_t is the total interference power, W is the spread bandwidth, and G is the processing gain. The signal power at the receiver needed to achieve the required E_b/I_0 is therefore

$$S = \frac{1}{W} R_b \frac{E_b}{I_0} I_t. \tag{2}$$

Denote the minimum required E_b/I_0 by α, then for satisfactory performance, we must guarantee

$$S \geq \frac{1}{W} R_b \alpha I_t. \tag{3}$$

In a multimedia, multirate system, W is kept constant for all media from any transmitter due to OVSF spreading, but R_b, α and I_t can be different.

Since the channels carrying the multirate media from a single transmitter are spread by orthogonal codes, the total signal power received at the base station receiver from the jth UE is a simple summation of all the signal powers of these channels:

$$\begin{aligned} S_j &= \sum_{i=1}^{K_j} \frac{1}{W} R_{bji} \left(\frac{E_b}{I_0}\right)_{ji} I_{tj} \geq \frac{1}{W} \left[\sum_{i=1}^{K_j} R_{bji}\alpha_{ji}\right] I_{tj} \\ &= \frac{1}{W} \mathbf{R}_{bj} \mathbf{A}_j' I_{tj} = \ , {}_j I_{tj} \end{aligned} \tag{4}$$

where K_j is the number of channels the jth user occupies, R_{bji} is the data rate of the jth user's ith channel, I_{jt} is the total interference to the jth user's signals, and α_{ji} is the minimum required bit-energy-to-interference ratio for medium i of UE j. The vector

$$\mathbf{R}_{bj} = [R_{bj1}, R_{bj2}, \cdots, R_{bjK_j}] \tag{5}$$

is the rate vector for the K_j channels of the jth UE,

$$\mathbf{A}_j = [\alpha_{j1}, \alpha_{j2}, \cdots, \alpha_{jK_j}] \tag{6}$$

is the QoS requirement vector for the K_j channels of the jth UE, $\mathbf{A}'_j$ is the transpose of $\mathbf{A}_j$, and

$$, _j = \frac{1}{W}\mathbf{R}_{bj}\mathbf{A}'_j = \frac{1}{W}\gamma_j \tag{7}$$

is the normalized traffic demand of UE j, where

$$\gamma_j = \mathbf{R}_{bj}\mathbf{A}'_j \tag{8}$$

is the traffic demand of UE j.

The required transmit powers of all users in the cell can be written in matrix form as

$$\begin{aligned} \mathbf{S} &\equiv [S_1, S_2, \cdots, S_M]' \\ &\geq [, _1 I_{t1}, , _2 I_{t2}, \cdots, , _M I_{tM}]' \\ &= \mathbf{\Gamma}_D \mathbf{I}_t \end{aligned} \tag{9}$$

where M is the total number of UEs connected to the target base station,

$$\mathbf{\Gamma}_D \equiv \text{diag}[, _1, , _2, \cdots, , _M] \tag{10}$$

is the normalized traffic demand matrix, and

$$\mathbf{I}_t = [I_{t1}, I_{t2}, \cdots, I_{tM}]' \tag{11}$$

is the interference vector. Here we adopt the convention that the vector inequality is an inequality in all components, that is, the matrix inequality in (9) is componentwise.

The total interference to the jth user's signals is caused by the signals from other UEs in the system (MAI) and thermal noise, and can be represented by

$$I_{tj} = \sum_{l=1, l\neq j}^{M} S_l + n_j = \sum_{l=1}^{M} S_l - S_j + n_j \tag{12}$$

where $\sum_{l=1,l\neq j}^{M} S_l$ is the intracell MAI and n_j is the aggregate disturbance consisting of additive white Gaussian noise and intercell MAI. Substituting (12) into (4) gives

$$S_j \geq , _j \left[\sum_{i=1, i\neq j}^{M} S_i + n_j \right] \tag{13}$$

from which we can derive a system of inequalities for the M users in the cell:

$$\begin{aligned} S_1 - , _1 S_2 - \cdots - , _1 S_M &\geq , _1 n_1 \\ -, _2 S_1 + S_2 - \cdots - , _2 S_M &\geq , _2 n_2 \\ \cdots \qquad \cdots & \\ -, _M S_1 - , _M S_2 - \cdots + S_M &\geq , _M n_M \end{aligned} \tag{14}$$

or in matrix form,

$$\mathbf{\Gamma_S S} \geq \mathbf{\Gamma_D n} \tag{15}$$

where

$$\mathbf{\Gamma}_S = \begin{bmatrix} 1 & -{,}_1 & \cdots & -{,}_1 \\ -{,}_2 & 1 & \cdots & -{,}_2 \\ \vdots & \vdots & & \vdots \\ -{,}_M & -{,}_M & \cdots & 1 \end{bmatrix} \tag{16}$$

and

$$\mathbf{n} = [n_1, n_2, \cdots, n_M]' \tag{17}$$

is the noise vector.

The objectives of power control are 1) to find a solution of (15) for $\mathbf{S}$ which specifies the power distribution among the M users in the cell, and 2) to minimize the power transmitted by each mobile UE.

4 Convergence of power distribution law

To find solutions for $\mathbf{S}$ in (15), note that

$$\begin{aligned} \mathbf{\Gamma}_S &= \begin{bmatrix} 1 & -{,}_1 & \cdots & -{,}_1 \\ -{,}_2 & 1 & \cdots & -{,}_2 \\ \vdots & \vdots & & \vdots \\ -{,}_M & -{,}_M & \cdots & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & \ddots & \\ & & & 1 \end{bmatrix} - \begin{bmatrix} {,}_1 & & & \\ & {,}_2 & & \\ & & \ddots & \\ & & & {,}_M \end{bmatrix} \begin{bmatrix} 0 & 1 & \cdots & 1 \\ 1 & 0 & \cdots & 1 \\ \vdots & \vdots & & \vdots \\ 1 & 1 & \cdots & 0 \end{bmatrix} \\ &= \mathbf{I}_M - \mathbf{\Gamma}_D(\mathbf{J}_M - \mathbf{I}_M) \\ &= \mathbf{I}_M - \mathbf{\Gamma}_P \end{aligned} \tag{18}$$

where $\mathbf{I}_M$ is the $M \times M$ identity matrix, $\mathbf{J}_M$ denotes an $M \times M$ matrix of all 1's, and

$$\mathbf{\Gamma}_P \equiv \mathbf{\Gamma}_D(\mathbf{J}_M - \mathbf{I}_M) = \begin{bmatrix} 0 & {,}_1 & \cdots & {,}_1 \\ {,}_2 & 0 & \cdots & {,}_2 \\ \vdots & \vdots & & \vdots \\ {,}_M & {,}_M & \cdots & 0 \end{bmatrix}. \tag{19}$$

Substituting (18) into (15) yields

$$(\mathbf{I}_M - \mathbf{\Gamma}_P)\mathbf{S} \geq \mathbf{\Gamma}_{\mathbf{D}}\mathbf{n}. \tag{20}$$

Since $\mathbf{\Gamma}_P$ is a nonnegative, primitive matrix, by the Perron-Frobenius theorem [17, 6], we have

Theorem 1: $\mathbf{\Gamma}_P$ has a positive eigenvalue λ equal to the spectral radius of $\mathbf{\Gamma}_P$, and if $\lambda < 1$, the power vector in (20) has non-negative solution

$$\mathbf{S} \geq (\mathbf{I}_M - \mathbf{\Gamma}_P)^{-1}\mathbf{\Gamma}_{\mathbf{D}}\mathbf{n}. \tag{21}$$

4.1 An example

When $M = 2$, the eigenvalues of $\mathbf{\Gamma}_P$ can be derived by solving the characteristic polynomial equation

$$f(\lambda) = \det[\mathbf{\Gamma}_P - \lambda\mathbf{I}] = 0. \tag{22}$$

The two eigenvalues are found to be

$$\lambda_{1,2} = \pm\sqrt{,_1 ,_2}. \tag{23}$$

Applying Theorem 1 requires

$$,_1 ,_2 < 1. \tag{24}$$

Substituting for $,_j$ from (7), we have

$$\sqrt{\gamma_1\gamma_2} < W. \tag{25}$$

This is the bound imposed by the spread bandwidth on the traffic demands of the users in the cell for the problem to be solvable or for the power distribution law to converge. The powers needed to achieve the required QoS's are

$$\begin{aligned}
\begin{bmatrix} S_1 \\ S_2 \end{bmatrix} &\geq \begin{bmatrix} 1 & -,_1 \\ -,_2 & 1 \end{bmatrix}^{-1} \begin{bmatrix} ,_1 n_1 \\ ,_2 n_2 \end{bmatrix} \\
&= \begin{bmatrix} \frac{\Gamma_1(n_1+\Gamma_2 n_2)}{1-\Gamma_1\Gamma_2} \\ \frac{\Gamma_2(\Gamma_1 n_1+n_2)}{1-\Gamma_1\Gamma_2} \end{bmatrix}, \qquad ,_1 ,_2 < 1 \\
&= \begin{bmatrix} \frac{W\gamma_1 n_1+\gamma_1\gamma_2 n_2}{W^2-\gamma_1\gamma_2} \\ \frac{\gamma_1\gamma_2 n_1+W\gamma_2 n_2}{W^2-\gamma_1\gamma_2} \end{bmatrix}, \qquad \sqrt{\gamma_1\gamma_2} < W.
\end{aligned} \tag{26}$$

It is easy to verify that this is indeed the solution of (15).

Assume $\gamma_1 = \gamma_2 = \gamma$ and $n_1 = n_2 = n$, then

$$S_i \geq \frac{\gamma^2 n + W\gamma n}{W^2 - \gamma^2} = \frac{\gamma n}{W - \gamma}, \qquad \gamma < W, \qquad i = 1, 2. \tag{27}$$

Several conclusions can be drawn from this example:

1. For any power distribution or power control algorithm to converge, the traffic demands, which describe the information rates and QoS requirements of each user as in (8), is upper bounded by the spread bandwidth W, i.e., $\sqrt{\gamma_1\gamma_2} < W$.

2. The higher is the interference level and the closer are the traffic demands to the spread bandwidth, the higher is the transmit power required. When $\sqrt{\gamma_1\gamma_2} \to W$, the signal power of each user tends to infinity: $S_i \to \infty$, $i = 1, 2$.

3. Since $\sqrt{\gamma_1\gamma_2} \leq (\gamma_1 + \gamma_2)/2$, the uniform traffic achieves the minimum system capacity and, therefore, can serve as the sufficient condition for convergent systems.

4.2 A limiting case

For an arbitrary number of M users, we first consider a simple case in which thermal noise and the MAI from other cells are ignored (This is equivalent to the single cell case where the intracell MAI is the dominant interference.) and the signal powers are minimized. Under these conditions, inequality (15) becomes an equality given by

$$\mathbf{\Gamma_S S} = \begin{bmatrix} 1 & -,_1 & \cdots & -,_1 \\ -,_2 & 1 & \cdots & -,_2 \\ \vdots & \vdots & & \vdots \\ -,_M & -,_M & \cdots & 1 \end{bmatrix} \begin{bmatrix} S_1 \\ S_2 \\ \vdots \\ S_M \end{bmatrix} = 0. \tag{28}$$

Then we need to solve the homogeneous system of equations (28) for a non-trivial solution set of $\mathbf{S}$. Let $\mathbf{V}$ denote this set of solutions, then $\mathbf{V} = \mathbf{N}(L_{\Gamma_S})$, where $\mathbf{N}(L_{\Gamma_S})$ is the null space of transformation $\mathbf{L}_{\Gamma_S}$ performed by matrix $\mathbf{\Gamma}_S$. Hence $\mathbf{V}$ is a subspace of $\mathbf{F}^M$ of dimension $M - \text{rank}(\mathbf{L}_{\Gamma_S}) = M - \text{rank}(\mathbf{\Gamma}_S)$ where $\mathbf{F}^M$ is a vector space of M-tuples from field F. For (28) to have a non-trivial solution, we must have

$$\text{rank}(\mathbf{\Gamma}_S) < M. \tag{29}$$

For $M = 2$ and $M = 3$, by elementary matrix operations, $\mathbf{\Gamma}_S$ can be reduced to

$$\begin{bmatrix} 1 & -,_1 \\ 0 & 1 - ,_1 ,_2 \end{bmatrix} \tag{30}$$

and

$$\begin{bmatrix} 1 & -\Gamma_1 & -\Gamma_1 \\ 0 & 1-\Gamma_1\Gamma_2 & -\Gamma_2-\Gamma_1\Gamma_2 \\ 0 & 0 & 1-\Gamma_1\Gamma_3-\frac{(\Gamma_2+\Gamma_1\Gamma_2)(\Gamma_3+\Gamma_1\Gamma_3)}{1-\Gamma_1\Gamma_2} \end{bmatrix}, \tag{31}$$

respectively. Satisfaction of inequality (29) requires respectively

$$1-\Gamma_1\Gamma_2=0 \tag{32}$$

and

$$(1-\Gamma_1\Gamma_3)(1-\Gamma_1\Gamma_2)-\Gamma_2\Gamma_3(1+\Gamma_1)^2=0. \tag{33}$$

Again assuming $\Gamma_1=\Gamma_2=\Gamma_3=\Gamma$, we have

$$\Gamma=1 \tag{34}$$

for $M=2$ and

$$\Gamma=1/2 \tag{35}$$

for $M=3$; or from (7),

$$\gamma=W \tag{36}$$

for $M=2$ and

$$\gamma=W/2 \tag{37}$$

for $M=3$.

Now we show that for the homogeneous case where all the mobile users have the same traffic demands, in order for $\mathbf{S}$ in (28) to have a non-trivial solution, the traffic demand of every user should be equal to

$$\Gamma=1/(M-1) \tag{38}$$

or

$$\gamma=W/(M-1). \tag{39}$$

Substituting (38) into (16) and performing elementary matrix operations, $\mathbf{\Gamma}_S$ can be reduced to

$$\begin{bmatrix} 1 & \frac{-1}{M-1} & \frac{-1}{M-1} & \frac{-1}{M-1} & \cdots & \frac{-1}{M-1} & \frac{-1}{M-1} \\ 0 & \frac{M(M-2)}{(M-1)^2} & -\frac{M}{(M-1)^2} & -\frac{M}{(M-1)^2} & \cdots & -\frac{M}{(M-1)^2} & -\frac{M}{(M-1)^2} \\ 0 & 0 & \frac{M(M-3)}{(M-1)(M-2)} & -\frac{M}{(M-1)(M-2)} & \cdots & -\frac{M}{(M-1)(M-2)} & -\frac{M}{(M-1)(M-2)} \\ 0 & 0 & 0 & \frac{M(M-4)}{(M-1)(M-3)} & \cdots & -\frac{M}{(M-1)(M-3)} & -\frac{M}{(M-1)(M-3)} \\ \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & \frac{M}{2(M-1)} & -\frac{M}{2(M-1)} \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}. \tag{40}$$

It is indeed a matrix of $\text{rank}(\mathbf{\Gamma}_S) < M$.

The transmit power for the jth UE given in (4) is now

$$S_j = \frac{I_{tj}}{M-1} = \frac{1}{M}\sum_{i=1}^{M} S_i, \qquad j = 1, 2, \cdots, M. \tag{41}$$

Hence, all the transmit powers should be equal and, if taking interference into account, be raised to such a level that the interference can comparatively be ignored.

For the same reason as in the example of the last section, (38) or (39) can serve as the sufficient condition of system convergence.

4.3 The general case

For the general case of (15), if the power received from any single UE is constrained to be no higher than a fraction η of the total received power, i.e.,

$$S_j \leq \eta \sum_{i=1}^{M} S_i, \qquad j = 1, 2, \cdots, M, \tag{42}$$

then from (13) we have

$$,_j \leq \frac{S_j}{\sum_{i=1, i\neq j}^{M} S_i + n_j} < \frac{\eta}{1-\eta}, \qquad j = 1, 2, \cdots, M. \tag{43}$$

Consequently,

$$\prod_{j=1}^{j=M} ,_j < \left(\frac{\eta}{1-\eta}\right)^M. \tag{44}$$

From (7), inequality (44) is equivalent to

$$\left(\prod_{j=1}^{j=M} \gamma_j\right)^{\frac{1}{M}} < \frac{\eta W}{1-\eta}. \tag{45}$$

When $M = 2$ and $\eta = 1/2$, which implies $S_1 = S_2$, (45) becomes

$$\sqrt{\gamma_1\gamma_2} < W, \tag{46}$$

which is the same as (25).

Assume $\gamma_j = \gamma,\ j = 1, 2, \cdots, M$, and all the received signal strengths to be equal, then (45) reduces to

$$\gamma < W/(M-1), \tag{47}$$

which is similar to (39).

The upper bound on traffic demands in (45) can serve as the necessary condition for a power distribution algorithm to converge; the larger is η, the looser will be the bound.

5 A new power control/call admission algorithm

From the results in the above sections, an algorithm for power distribution among users, or call admission in a cell, which converges reliably and meets traffic demands of all the users is proposed as follows:

1. Suppose there are K possible services each occupying a channel with transmission rate $R_b \in \{R_i : \ i = 1, 2, \cdots, K\}$ and QoS requirement $\alpha \in \{\alpha_i : \ i = 1, 2, \cdots, K\}$. Each call or connection can consist of a maximum of L channels which can be any combination (even repeat) of these channels. Calculate the normalized traffic demand , according to (5), (6) and (8) for each possible combination. Find the mean $E(,)$ and variance $Var(,)$.

2. A connection can be established only after, say, UE j sends a request with anticipated traffic demand γ_j to the mobile switching center (MSC) and receives a permit.

3. After receiving a connection request and its anticipated traffic demand, the call admission controller at the MSC determines the admissibility of the call based on the following rule:

   If
   $$\sum_{i=1}^{M} \gamma_i < \frac{MW}{M-1}, \tag{48}$$
   the call is admitted; else if
   $$\left(\prod_{j=1}^{j=M} \gamma_j \right)^{\frac{1}{M}} \geq \frac{\eta W}{1-\eta} \tag{49}$$
   the call is rejected; and if
   $$\sum_{i=1}^{M} \gamma_i \geq \frac{MW}{M-1} \quad \text{and} \quad \left(\prod_{j=1}^{j=M} \gamma_j \right)^{\frac{1}{M}} < \frac{\eta W}{1-\eta} \tag{50}$$
   only a call with light traffic demand can be admitted.

   In the above admission rule, M is the total number of UEs within the cell including the new requesting one, γ_i, $i = 1, 2, \cdots, M$, are the traffic demands of the existing calls in the cell plus the new impending call, and η is determined by
   $$\eta = \frac{3Var(,)}{ME(,)}. \tag{51}$$
   Here we assume η covers 98% of possible service combinations in a connection or call.

4. The power controller instructs the newly admitted call to transmit at such a power level that the desired SIR at the cell-site receiver is achieved. Since the new call can cause extra MAI to other existing calls, this would result in all existing users to raise power (under the control of the power controller) until their original SIRs before the new admission is regained. So admission of a new call can potentially lead to an upscale of the transmit powers of the existing calls.

6 Numerical results

To verify Theorem 1, we first numerically calculate the eigenvalue λ of , $_P$. Figure 2 shows λ versus average normalized traffic demand per user, $E($, $)$, for different deviations, $\sigma = Var^{\frac{1}{2}}($, $)$. It can be seen that all traffic types satisfy $\lambda < 1$ when $E($, $) < 1/(M-1)$. If the average normalized traffic demand causing $\lambda = 1$ denotes the *critical traffic demand point*, then the uniform traffic (with $\sigma = 0$) gives the smallest critical traffic demand point. These results agree with conclusion 3 in Section 4.1.

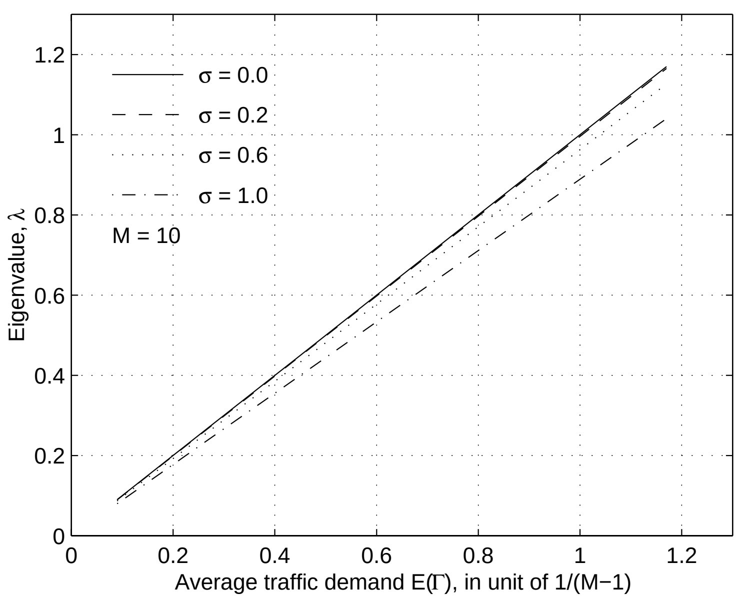

Figure 2: Eigenvalue *vs* traffic demand

The number of users in the cell is $M = 10$. For other values of M, the curves show the same trend but become indistinguishable when M becomes large. This implies that the convergent condition for the uniform case provides quite an accurate sufficient condition in practical systems where the number of users per cell is usually around 100 or even more.

Figure 2 shows that the traffic with higher variation has a larger critical point which corresponds to higher traffic capacity. This is confirmed by our numerical results in Figure 3. The trend shows that when the number of users per cell is large ($M \to \infty$), the system capacity is almost constant against traffic variation and approaches unity ($M/(M-1) \to 1$). This agrees with (47).

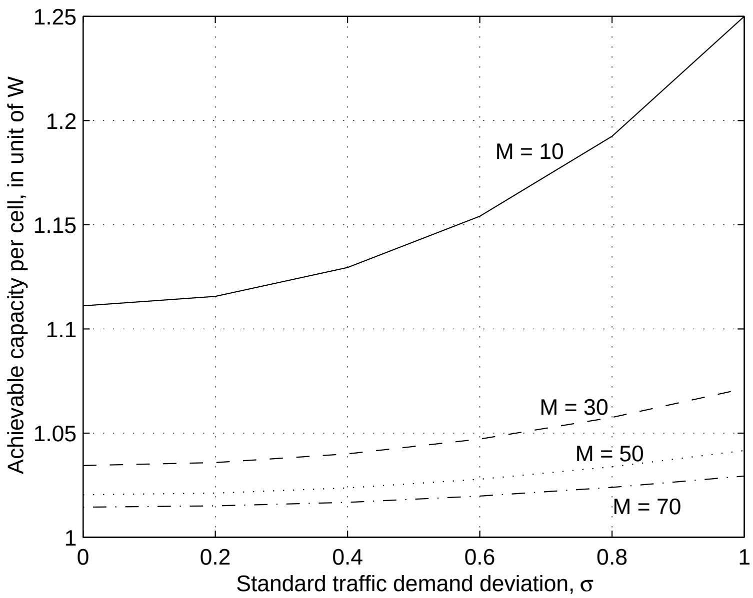

Figure 3: System capacity *vs* traffic demand variation

Finally, we use (21) to determine the power distribution among the M users in the cell. We found that the solved power vector contains negative elements when $\lambda > 1$; but is indeed a non-negative vector for $\lambda < 1$. Figure 4 shows the average transmit power required against the total input traffic demands within a cell. Although not shown, the curves for $M > 50$ are not so distinguishable. It can be seen from the figure that the transmit power increases dramatically when the total traffic demand approaches the spread bandwidth W. It is therefore suggested to control the aggregate traffic load per cell below $0.8W$ in order for the transmit power to be at practical levels.

From Figures 2–4, we see that, although non-uniform traffic gives better system capacity, the required average transmit powers are similar for different traffic variations when the number of users per cell is large. This implies that a system with larger traffic variation may transmit at much higher power levels by some transmitters which have higher traffic demands.

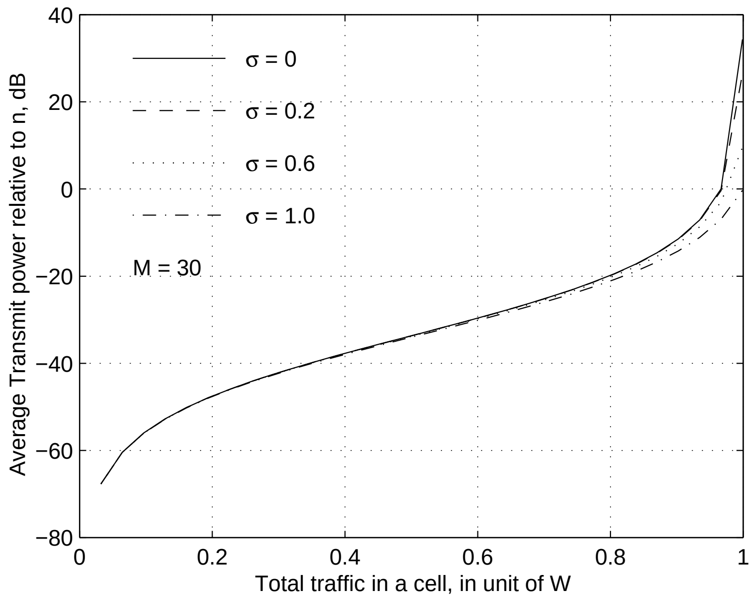

Figure 4: Average transmit power *vs* traffic demand with various deviation σ

7 Conclusions

Transmit power is a major resource in third generation WCDMA mobile communications systems, and must be carefully planned and controlled in order to achieve optimal system performance and user capacity. This work studies the power distribution problem and its convergence condition. A matrix inequality for the power vector of M UEs in a cell in terms of traffic demands and external disturbances is established. The solution of this inequality requires that the total traffic load in a cell should not exceed the spread bandwidth. It also shows that the closer the total traffic demands per cell is to the spread bandwidth, the higher are the transmit powers required. A new power control algorithm based on these results is proposed. Numerical results show that, for practical systems where the number of users is around 100 or more, (39) and (21) provide quite accurate traffic load bound and power distribution among users, respectively.

References

[1] A. Baier, U.-C. Fiebig, W. Granzow, W. Koch, P. Teder and J. Thielecke, "Design study for a CDMA-based third-generation mobile radio system," IEEE J-SAC, vol.12, no.4, pp.733-743, May 1994.

[2] E. Dahlman, P. Beming, J. Knutsson, F. Ovesjo, M. Persson, and C. Roobol, "WCDMA–The radio interface for future mobile multimedia communications," IEEE Trans. on Vehicul. Technol., vol.47, no.4, pp.1105-1118, Nov. 1998.

[3] E. Dahlman, B. Gudmundson, M. Nilsson, and J. Skold, "UMTS/IMT-2000 Based on Wideband CDMA," IEEE Com. Mag., pp.70-80, Sept. 1998.

[4] R.W. Nettleton and H. Alavi, "Power control for a spread spectrum cellular mobile radio system," in Proc. of IEEE VTC'83, pp.242–246, 1983.

[5] J. Zander, "Performance of Optimum transmitter power control in cellular radio systems," IEEE J-SAC, vol.41, no.1, Feb. 1992, pp.57–62.

[6] S.V. Hanly, "An Algorithm for Combined Cell-Site Selection and Power Control to Maximize Cellular Spread Spectrum Capacity," IEEE J-SAC, vol.13, no.7, pp.1332–1340, Sept. 1995

[7] R.D. Yates and C.Y. Huang, "Integrated power control and base station assignment," IEEE Trans. Vehicul. Technol., vol.44, no.3, pp.638–644, Aug. 1995.

[8] R.D. Yates, "A framework for uplink power control in cellular radio systems," IEEE J-SAC, vol.13, no.7, pp.1341–1347, Sept. 1995.

[9] Z. Dziong, M. Jia, and P. Mermelstein, "Adaptive traffic admission for integrated services in CDMA wireless-access networks," IEEE J-SAC, vol.14, no.8, Dec. 1996, pp.1737–1747.

[10] S. Yao, E. Geraniotis, "Optimal power control law for multi-media multi-rate CDMA systems," in Proc. of IEEE VTC'96, pp.392–396, 1996.

[11] T.H. Hu and M.M.K. Liu, "Power control for wireless multimedia CDMA systems," Electron. Lett., vol.33, pp.660–662, Apr. 1997.

[12] T.H. Hu and M.M.K. Liu, "DS-CDMA system with variable-rate traffic," IEEE Commun. Lett., vol.2, pp.64–67, Mar. 1998.

[13] E.H. Dinan and B. Jabbari, "Spreading Codes for Direct Sequence CDMA and Wideband CDMA Cellular Networks," IEEE Com. Mag., pp.48–54, Sept. 1998.

[14] R.L. Pickholtz, D.L.Schilling and L.B.Milstein, "Theory of Spread-Spectrum communications — A tutorial," IEEE Trans. on Communications, Vol. COM-30, pp.855–884, May 1982.

[15] A.J. Viterbi and A.M. Viterbi, "Erlang Capacity of power controlled CDMA system," IEEE J. Select. Areas Commun., vol.11, pp.892–900, Aug. 1993.

[16] K.S. Gilhousen, I.M. Jacobs, R. Padovani, A.J. Viterbi, L.A. Weaver, Jr., and C.E. Wheatley III, "On the Capacity of a Cellular CDMA System," IEEE Trans. on Vechic. Technol., pp.303–312, May 1991.

[17] P. Lancaster and M. Tismenetsky, "The Theory of Matrices: with Applications," 2nd ed., Academic Press, 1985.