The Application of Network Generation Methods in the Study of Multicast Routing Algorithms

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Abstract

Constructing multicast trees fulfil that the quality of service requirements be essential for next generation networks. The comparison and the analysis of the efficiency of these algorithms require the application of network structures that reflect the real topology of the Internet in the most accurate way. The article presents the initial phase of the research on the network topology influence on the efficiency of multicast heuristic routing algorithms. The influence of the methods generating network topology and the parameters of the networks on the quality of multicast trees constructed by the algorithms is studied and analysed.

1 Introduction

Multicasting is a way of transmission between a source node and specified destination nodes – the members of *multicast group*. In practice, however, there is a given group of nodes receiving the same data that run parallel in the same time. Multicasting requires efficient routing algorithms defining a tree with a minimal cost between the source node and the particular nodes representing the users. Such a solution prevents duplication of the same data (packets) in the links of the network. Routing of the sent data occurs only in those nodes of the network that lead directly to destination nodes.

To ensure accurate transmission of real time data, a particularly defined bandwidth is needed and, primarily, a situation in which the delay between the source and destination nodes can be maintained at the steady, unchanged level. The occurring jitter phenomenon is an undesirable phenomenon here.

To ensure reliable transmission, multimedia applications set up high standards for quality parameters (*Quality-of-Service* parameters). Quality requirements concerning, *inter alia*, steady and guaranteed delay values in relation to a transfer of a packet between the source and destination nodes along a defined path in the network still pose a great challenge for those designing the application in real time. Hence, the process of optimization includes the second metric of the network - the delay (d). With the constructions of multicast trees, the maximum delay between two end points in the network (Δ) is thus an applicable appropriate criterion. In [1, 2], it is proved that finding the tree poses an $\mathcal{NP}$-complete problem for one or more QoS parameters. Due to the complexity of the problem, the presented algorithms use the techniques approaching the solution – heuristics.

The analysis of the effectiveness of the algorithms known for the authors and the design of the new solutions apply the numerical simulation based on the abstract model of the existing network. These, in turn, need structures (network models) reflecting, in the best adequate way, the Internet network.

Zegura et al. [3] introduced a comprehensive model based on simply network topology models (e.g. Waxman generative method). They also defined basic topology parameters that allow to control network structures. Medina et al. [4] suggested adopting the power laws of the Internet topology to improve the generation of such structures. They have propose the BRITE tool as a tool for generation of realistic topologies.

Many researchers neglect the topology influence on mutlicast routing algorithm. They evaluate algorithms using only one generative method (usually Waxman method) [5].

If a communication network is presented as a graph, the result of the implementation of the routing algorithm will be a spanning tree rooted in the source node and including all destination nodes in the multicast group. Two kinds of trees can be distinguished in the process of optimization: MST – *Minimum Steiner Tree*, and the tree with the shortest paths between the source node and each of the destination nodes – SPT (*Shortest*

Path Tree). Finding the MST, which is a $\mathcal{NP}$-complete problem, effects in a structure with a minimal total cost. The relevant literature provides a wide range of heuristics solving the above problem in polynomial time [2, 6, 7]. From the point of view of the application in data transmission, the most commonly used is the KMB algorithm [2]. The other method minimalizes the cost of each of the paths between the sender and each of the members of the multicast group by forming a tree from the paths having the least costs. In general, it is first either Dijkstra algorithm [8] or Bellman-Ford algorithm [9] that is used, and the branches of the tree that do not have destination nodes are then cut off.

The article discusses the effectiveness of the most commonly used constrained heuristic algorithms as well as their comparative usefulness. Chapter 2 presents Steiner tree problem and its heuristics – constrained algorithms: KPP (*Kompella, Pasqualle, Polyzos*) [10], CSPT (*Constrained Shortest Path Tree*) [6], DCSP (*Delay Constrained Shortest Path*) and *least-delay* (LD) algorithm. Chapter 3 presents the methodology of generating structures representing the topology under scrutiny and important network parameters. Chapter 4 includes the results of the simulation of the implemented algorithms along with their interpretation.

2 Problem formulation

2.1 Network Model and Minimum Steiner Tree

Let us assume that a network is represented by a directed, connected graph $G = (V, E)$, where V is a set of nodes, and E is a set of links. The existence of the link $e = (u, v)$ between the node u and v entails the existence of the link $e' = (v, u)$ for any $u, v \in V$ (corresponding to two-way links in communication networks). With each link $e \in E$, two parameters are coupled: cost $C(e)$ and delay $D(e)$. The cost of a connection represents the usage of the link resources. $C(e)$ is then a function of the traffic volume in a given link and the capacity of the buffer needed for the traffic. A delay in the link is in turn the sum of the delays introduced by the propagation in a link, queuing and switching in the nodes of the network. The multicast group is a set of nodes that are receivers of the group traffic (identification is carried out according to a unique i address), $M = \{m_1, ..., m_m\} \subseteq V$, where $m = |M| \leq |V|$. The node $s \in V$ is the source for the multicast group M. Multicast tree $T(s, M) \subseteq E$ is a tree rooted in the source node s that includes all members of the group M and is called *Steiner tree*. The total cost of the tree $T(s, M)$ can be defined as $\sum_{t \in T(s,M)} C(t)$. The path $P(s, m_i) \subseteq T(s, M)$ is a set of links between s and $m_i \in M$. The cost of path $P(s, m_i)$ can be expressed as: $\sum_{p \in P(s,m_i)} C(p)$, while the delay measured between the beginning and the end of the path as: $\sum_{p \in P(s,M)} D(p)$. Thus, the maximum delay in the tree can be determined as: $\max_{m \in M}[\sum_{p \in P(s,M)} D(p)]$.

A Steiner tree is a good representation for solving the routing multicast problem. This approach becomes particularly important when we have to deal with only one active multicast group and the cost of the whole group has to be minimal. However, due to the computational complexity of this algorithm ($\mathcal{NP}$-complete problem) [1], heuristic algorithms are most preferable. If the set of the nodes of the minimal Steiner tree includes all nodes of a given network, then the problem comes down to finding the minimal spanning tree (this solution can be obtained in polynomial time).

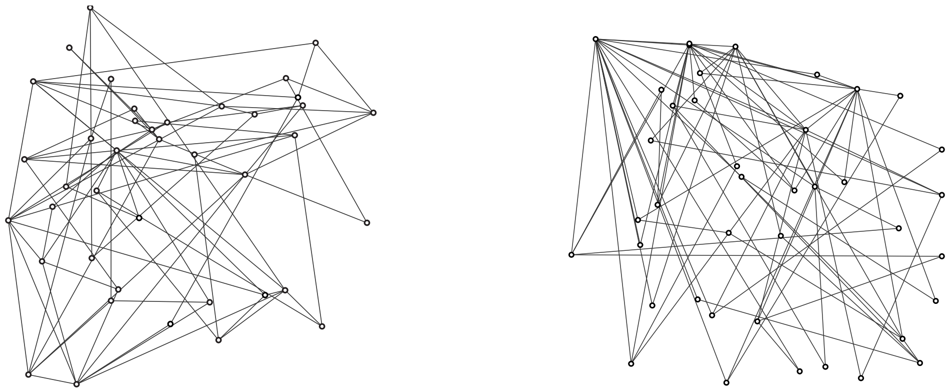
2.2 Representative heuristic algorithms

Constrained algorithms are heuristics that find minimal trees using an additional parameter – delay (Δ) for each path in the network.

KPP heuristics [10] finds a closure graph of the constrained shortest paths between all multicast nodes (including the source node). Each edge in this complete graph represents the cost of the shortest path between these nodes in the original graph G. Then, KPP finds a constrained spanning tree of the closure graph. Finally, it replaces the edges of the tree by the paths from the original graph G and removes loops using Prim's algorithm. The time complexity of this algorithm is $\mathcal{O}(\Delta|V|^2)$.

The *Constrained Shortest Path Tree* algorithm is an example of the minimum cost path tree heuristics [6]. If the delay on path to any destination exceeds the delay bound for this path, then the path will be replaced by a minimum delay path. Thus, if the least-cost tree (LC) exceeds the delay bound (Δ), it constructs a minimum delay tree (LD) and combines both trees. This algorithm finds a constrained multicast tree if one exists.

The *least-delay* algorithm (LD) was implemented as a Dijkstra's shortest path algorithm in which $C(u, v) = D(u, v)$. It guaratees minimum end-to-end delay from the source node to each destination nodes. The time complexity of Dijkstra's algorithm is $\mathcal{O}(|V|^2)$.



(a) (b)

Fig. 1. Visualization of network topologies obtained using the Waxman (a) and the Barabasi-Albert (b) method (D_{av}=4, n=40)

3 Network topology

The main purpose of this article is to analyze the efficiency of the presented algorithms in identical network conditions, i.e. the size of the network, adopted topology model, values of metrics for each of the edges, etc.

3.1 Generative methods

In the study, a flat random graph constructed according to the Waxman method was used [11]. This method defines the probability of an edge between node u and v as:

$$P(u,v) = \alpha e^{\frac{-d}{\beta L}} \tag{1}$$

where $0 < \alpha, \beta \leq 1$, d is Euclidian distance between the node u and v and L is the maximum distance between the two freely selected nodes. An increase in the parameter α effects in the increase in the number of edges in the graph, while a decrease in the parameter β increases the ratio of the long edges against the short ones.

Another method was proposed by Barabasi [12]. This model suggests two possible causes for the emergence of the power law in the frequency of outdegrees in network topologies: incremental growth and preferential connectivity. The network growth process consist in incremental addition of new nodes. The preferential connectivity refers to the tendency of a new node to connect to existing nodes that are highly connected or popular. When a node u connects to the network, the probability that it connects to a node v (already belonging to the network) is given by:

$$P(u,v) = \frac{d_v}{\sum_{k \in V} d_k} \tag{2}$$

where d_v is the degree of the node belonging to the network, V is the set of nodes connected to the network and $\sum_{k \in V} d_k$ is the sum of outdegrees of the nodes previously connected.

With the construction of the network models based on Waxman and Barabasi-Albert method, BRITE (*Boston university Representative Internet Topology gEnerator*) [4] was used as a tool for generation of realistic topologies. The application provides a range of network topology models and appropriate generative methods. Fig. 1 shows typical topologies generated with the application of the Waxman and Barabasi-Albert method.

A network model was adopted in which the nodes were arranged on a square grid with the size of 1000×1000 (Waxman parameters: $\alpha = 0,15$, $\beta = 0,2$). Onto the existing network of connections, the cost matrix $C(u,v)$ was applied (as a matrix of Euclidean distances between the nodes) and the delay $D(u,v)$ resulting from euclidean distance between the nodes.

It was an important element during the simulation process to maintain a steady average node degree of the graph (for each of the generated networks) defined as: $D_{av} = \frac{2k}{n}$ (where n is the number of the nodes of the network, k is the number of the edges) which, in practice, meant the necessity of maintaining a steady number of edges. It is generally accepted that for $D_{av} \geq 2$, the so-called *two-connected* network, that is a connected network can be constructed. Toronha and Tobagi [13] proved that the efficiency of the routing algorithm implemented in a real network was identical to the efficiency of the same algorithm in a random *two-connected* network. In the implementations, the adopted degree of the graph was within the 3 to 5 bracket.

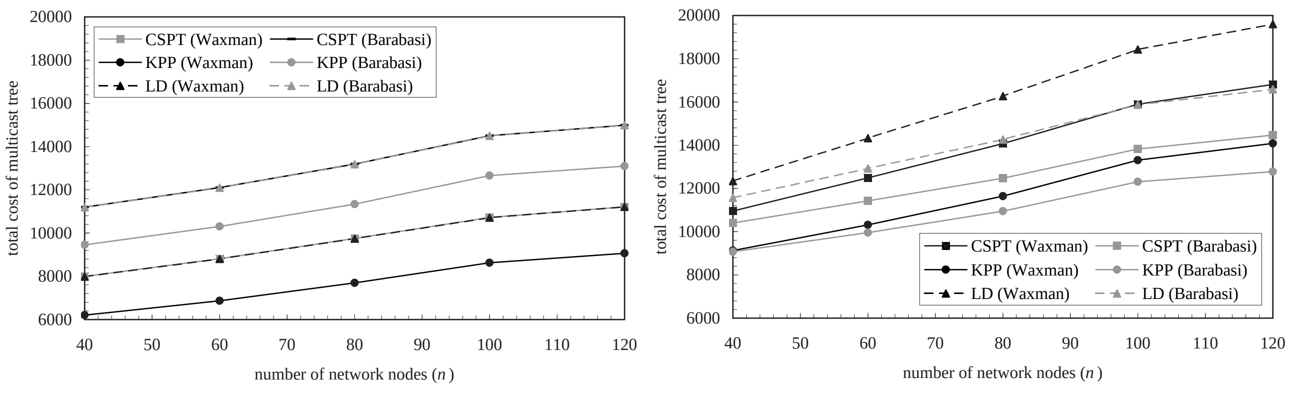
3.2 Network topology parameters

The efficiency of multicast algorithms depends on implemented network structure. Thus, it is important to define basic parameters describing network topology:

- *average node degree* :
$$D_{av} = \frac{2k}{n} \tag{3}$$
where n - number of nodes, k - number of links,
- *diameter* - is the length of the longest shortest path between any two nodes (a low diameter corresponds to shorter paths),
- *hop diameter* - is the length of the longest shortest path between any two nodes; the shortest paths are computed using *hop count* metric,
- *length diameter* - is the length of the longest shortest path between any two nodes; the shortest paths are computed using Euclidean distance metric,
- *clustering coefficient* (γ_v) of node v is the proportion of links between the vertices within its neighbourhood divided by the number of links that could possibly exist between them [14].
Let $\Gamma(v)$ be the neighbourhood of a vertex v consisting of the vertices adjacent to v (not including v itself). More precisely:
$$\gamma_v = \frac{|E(\Gamma(v))|}{\binom{k_v}{2}} = \frac{|E(\Gamma(v))|}{k_v(k_v-1)}, \tag{4}$$
where $|E(\Gamma(v))|$ is the number of edges in the neighborhood of v and $\binom{k_v}{2}$ is the total number of possible edges between neighbourhood nodes.
Let $V^{(1)} \subset V$ denotes the set of vertices of degree 1. Therefore [15, 16]:
$$\hat{\gamma} = \frac{1}{|V| - |V^{(1)}|} \sum_{v \in V} \gamma_v. \tag{5}$$

4 Simulation results

The study carried out by the authors focused on the network topology parameters influence on the efficiency of multicast heuristic algorithms.



(a) (b)

Fig. 2. Total cost of multicast tree versus the number of network nodes n when cost is an Euclidean distance (a) and randomly generated (b) ($m = 20$, $D_{av} = 4$, $\Delta = 10$)

The results presented in Figs. 2 and 3 show a comparison of KPP, CSPT and LD algorithms in relation to a number of nodes n in the network and the number of the members of the group m (destination nodes) with cost as a parameter. Two cases were considered: i. the cost of a link is an Euclidean distance between the nodes, ii. the cost of a link is randomly generated within the range of 200 to 500.

The analysis of the adopted assumptions shows an analogy between the adopted assumptions and the state of the network in which a multicast tree is created. Thus, the adoption of a metric which is an Euclidean distance, in which the cost of a link is proportional to the length of the transmission medium, can be interpreted as a tree creation in a network which is not overloaded or overloaded to a small degree. Random values of costs of particular links constituting the generated network can, in turn, be interpreted as a tree creation in the overloaded network. For simplicity, it is adopted that in both cases the delay metric $d(u, v)$ is a linear function of the distance between the nodes.

The graphs presented in Figs. 2a and 3a show that the KPP algorithm allows to construct multicast trees of the smallest cost with the application of network structures based on Waxman method and Euclidean metrics. As for the method based on the Barabasi model, the efficiency of the algorithm is worse than that of the simple CSPT algorithm that applies networks generated by the Waxman method (despite the fact that the implemented network has the same number of nodes and links).

The CSPT and LD algorithms generate trees of the same costs when the metric of the cost $c(u, v)$ is an Euclidean distance. This is a consequence of the assumptions adopted in BRITE application: the delay parameter $d(u, v)$ is proportional to the volume of the cost $c(u, v)$.

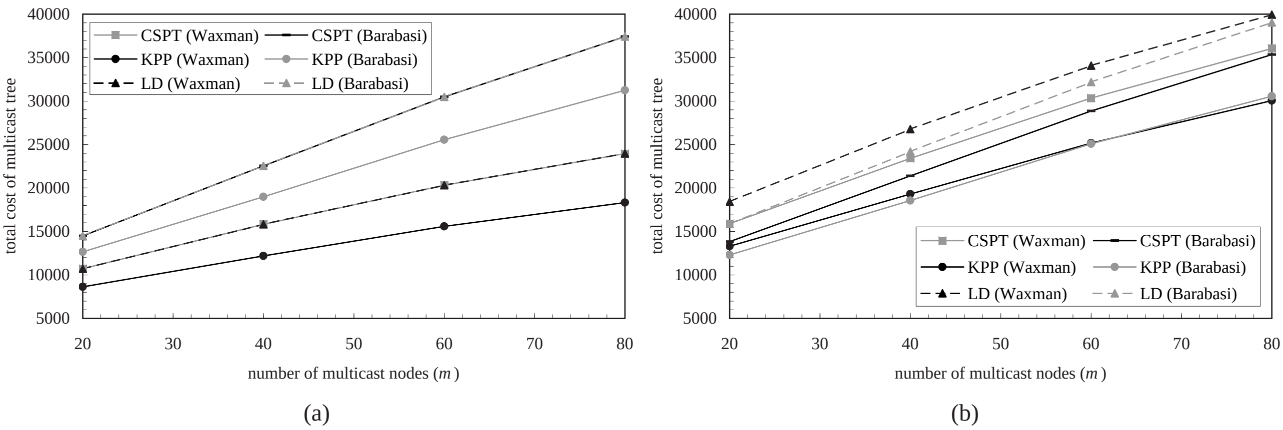


Fig. 3. Total cost of multicast tree versus the number of multicast nodes m when cost is an euclidean distance (a) and randomly generated (b) ($n = 100$, $D_{av} = 4$, $\Delta = 10$)

Literature [17–20] proved that the total cost of the trees generated with the help of KPP algorithm (when $\Delta \to \infty$) was, on average, only 5% worse than that obtained by the optimal method (*minimum Steiner tree*).

The authors also computed the average distance from the minimal solution (KPP algorithm after Waxman method – see Fig. 2a) normalized by the cost of the minimal solution for each algorithm (for both generative methods), denoted by δ:

$$\delta = \frac{C_h - C_{min}}{C_{min}} \cdot 100 \quad [\%] \tag{6}$$

where C_h is a cost of tree of given heuristic algorithm and C_{min} is a cost of minimal solution.

Table I is the implementation of formula (6) and allows to evaluate the efficiency of the algorithms under scrutiny in relation to the KPP algorithm (applying network structures based on Waxman method) that constructs minimal multicast trees. Relevant calculations show that the costs of the trees obtained for the same algorithm are over 50 per cent higher with the application of other generative methods. The study was carried out for networks with the same parameters.

The results of the presented algorithms were also dependent on other parameters of the network such as diameter and the clustering coefficient. The research was carried out for a constant number of the nodes and a constant node degree of the graph. These parameters are not included directly in the BRITE configuration file but are determined for each of the structure which has been generated by the BRITE.

n	Waxman method			Barabasi method		
	KPP	CSPT	LD	KPP	CSPT	LD
40		29%	29%	52%	80%	80%
60		28%	28%	50%	76%	76%
80	-	27%	27%	47%	71%	71%
100		24%	24%	46%	68%	68%
120		24%	24%	44%	65%	65%

Table 1. Average distance from the optimal solution for examined algorithms ($m = 20$, $D_{av} = 4$)

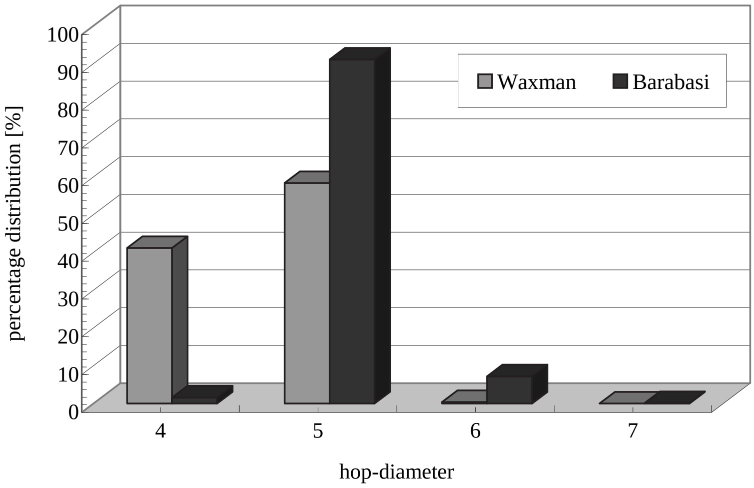


Fig. 4. Percentage distribution of the diameter of the generated graphs for different generative methods ($n = 40$, $D_{av} = 4$)

The results presented in Figs. 5 and 6 lead to a conclusion that the cost generated by these algorithms solutions does not depend on the hop-diameter and clustering coefficient of the network (for the same size of the network) [20]. However, the total cost of the multicast tree increases with the increase of the length-diameter. This research shows, however, that this algorithm generates solutions with lesser cost with the applications of the structures generated by the Waxman method, while the Barabasi-Albert method generated structures with the constant hop-diameter values.

Fig. 4 shows that 91% of the network topologies have hop-diameter equal to 5. The Waxman method generates the networks of greater hop-diameter values. With the same parameters, the latter method generates structures with hop-diameter equal to 4.

The values of the clustering coefficient $\hat{\gamma}$ (Fig. 5) are also smaller for the Waxman method. More adequate model of the topology of the Internet by the Barabasi-Albert method results in the increase in the cost of the path and the number of links in the path between given nodes of the network.

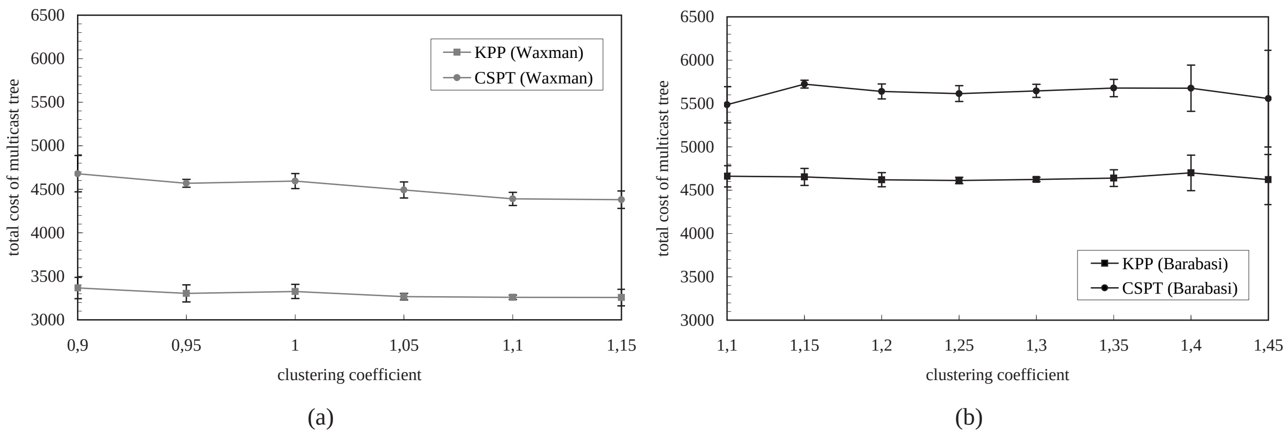
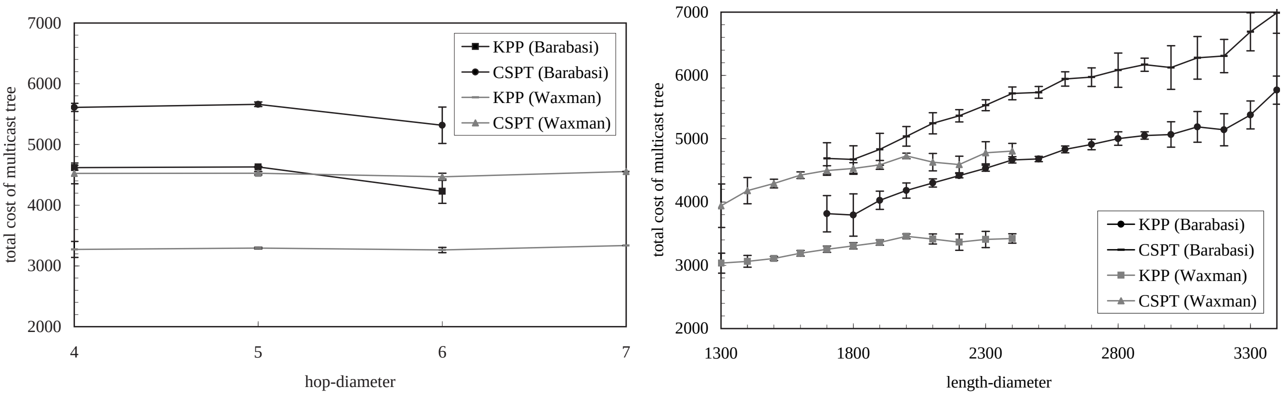


Fig. 5. Total cost of multicast tree versus the clustering coefficient $\hat{\gamma}$ obtained for the Waxman (a) and the Barabasi-Albert (b) method for constrained algorithms ($n = 40$, $m = 10$, $D_{av} = 4$)

The above results can be reflected in the cost of the trees obtained by the algorithms under scrutiny - both KPP algorithm and CSPT algorithm generate trees with lesser cost with the application of the Waxman method. This means that while doing research one can obtain trees with lesser cost also as the result of the application of the generative method, which allows for a greater variety of the structures of the network, and not only as the result of the application of the more efficient routing algorithm.

The results of the simulations are shown in the charts (Figs. 2–6) in the form of marks with 95% confidence intervals that were calculated after the *t-Student* distribution. 95% confidence intervals of the simulation are almost included within the marks plotted in the figures (Figs. 2–6).



(a) (b)

Fig. 6. Total cost of multicast tree versus the hop-diameter (a) and the length-diameter (b) obtained for the Waxman and the Barabasi-Albert method for constrained algorithms ($n = 40$*,* $m = 10$*,* $D_{av} = 4$*)*

5 Conclusions

The article presents and compares representative routing algorithms for multicast connections emphasizing the quality of the network model (the accuracy of the illustration of a real Internet topology). To this effect, topology generator BRITE was used.

While analyzing a variety of the diameters of the networks obtained by the application of different generative methods one can notice that any obtaining of the tree with the lesser cost can be the result of the application of the generative methods and not only the result of the application of a more efficient routing algorithm. However, while comparing the values of the clustering coefficient obtained for different methods of the network topology generation, it can be noticed that the Barabasi-Albert model, despite being more adequate model of the topology of the Internet, generates structures with greater values of clustering coefficient.

Recent research in the field proposes heuristic methods in generation realistic topologies. These methods are based on the information on the Internet structure gathered by routers. *Inet* [21] application is based on these sets of information to build realistic topologies. Further studies will focus on the exact comparison that apply heuristic topology generators.

This research study forms a basis for the development of a methodology of a study of routing algorithms. We believe that the inclusion of the methods of topology generation as well as basic parameters of the test network in the implemented algorithms is a necessary condition to have the existing routing algorithms compared unequivocally.

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