Peer-supported problem solving and mathematical knowledge (draft)

by

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Abstract

The thesis is about building an online, peer-produced, mathematics learning environment. By linking problems into planetmath.org's peer-produced mathematics encyclopedia and user community, we provide a rich "knowledge layer" and peer support system for learners. Conversely, the presence of problems and problem-solving interactions provide an important quality and completeness check on the encyclopedia. The ability to deliver learning interactions online in a peer-produced context represents an important innovation for mathematics education: but even more critical for this thesis is the opportunity this affords for understanding what sorts of interactions best support learning. Thus, the core part the thesis will deploy novel design and analysis methods to understand the factors that influnce learning outcomes in mathematics. The results have implications for other technical fields.

Turning and turning in the widening gyre
The falcon cannot hear the falconer;
Things fall apart; the centre cannot hold;
Mere anarchy is loosed upon the world,
The blood-dimmed tide is loosed, and everywhere
The ceremony of innocence is drowned;
The best lack all conviction, while the worst
Are full of passionate intensity.

Surely some revelation is at hand;
Surely the Second Coming is at hand.
The Second Coming! Hardly are those words out
When a vast image out of *Spiritus Mundi*
Troubles my sight; somewhere in sands of the desert
A shape with lion body and the head of a man,
A gaze blank and pitiless as the sun,
Is moving its slow thighs, while all about it
Reel shadows of indignant desert birds.
The darkness drops again; but now I know
That twenty centuries of stony sleep
Were vexed to nightmare by a rocking cradle,
And what rough beast, its hour come round at last,
Slouches towards Bethlehem to be born?

W. B. Yeats, 1920

Synopsis

Historical Background

☞ p. 7

Why this? Why now? Why me? There's an almost formal answer to these questions in an essay by Fabio Landini.[90] At least, that essay provides half of the answer. In addition to the "cultural subsidy" that Landini posits, I would argue that "free as in freedom" works also convey a "cultural surplus" to their developers, if they are successful (i.e. Marxian *surplus value*). How else can you ethically get hundreds of people working on your project for free?

Theoretical Background

☞ p. 17

I'm quite taken with Rheinberger's notion of the *epistemic object* or *epistemic thing* and a corresponding "*shift of perspective from the actors minds and interests to their objects of manipulation and desire*."[132] (p. 1) I endeavor to develop an epistemic framework for looking at a certain class of systems that are based inextricably on the human use of human beings[161]. I begin by examining the *social roles* that are found in education, and then consider the special case of peer produced peer learning.

Literature Review

☞ p. 34

The wide net that was cast in the previous section coalesces into a "grid" in which I examine and contextualize key work by Thurston[153], Stahl[144], McCalla[103], Krowne,[89] and Resnick et al.[130] on *mathematical thinking*, *collaborative knowledge building*, *the ecological approach in learning design*, *architectures for collaboration*, and *building online communities* (respectively).

Methodology

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We will try to detect evidence of learning when it is happening. One simple way to do this is to model "learning" as *learning vocabulary*. In a very simple model, if the vocabulary is new (to a given individual) then we say they have learned it. In a more sophisticated model, we're interested in the "depth" of the vocabulary, and we would say learning is happening when a person uses terms of progresively increasing depth. Of course, if we have some sort of check on correctness (e.g. if the terms are used in solutions and the solutions are marked as right or wrong) then we can say more. We will be particularly interested in understanding the circumstances that produce this sort of quantifiable "learning outcome". Of course, part of the methodology is *design* and implementation work. Where we cannot verify results quantitatively (and even where we can) we will seek to gain understanding of the user perspective through focus groups, interviews, and other channels for feedback.

Implementation

☞ p. 53

In short, we've rebuilt PlanetMath's old custom software ("Noösphere") using Drupal 7, incorporating a range of other mathematics-specific and semantic web tools.[85;83] In addition to improving the site's look and feel, we've provided a whole new set of interactions around *problems* and *solutions*, as well as *questions* and *answers*. This will give us more vectors along with to study learning-relevant factors, and also new form of ground-truthing. We anticipate that these changes will have a positive effect on learning, on the encyclopedia, and on the future development of the PlanetMath project as a whole.

Primary Research Strands

☞ p. 58

(1) *What factors influence learning outcomes in peer-produced mathematics?* For example, self-motivation has a lot to do with it – but so does teaching, mentoring, access to the right information at the right time. Using legacy data from PlanetMath, together with new statistical techniques for understanding time series data,[166] we would like to be able to say what sorts of interactions (and "interventions") are most likely to lead to high quality learning outcomes.

(2) *Specifically, what is the effect of problem solving, and its constituent factors, on learning outcomes in this space?* The "null hypothesis" in mathematics is that the best way to learn is to solve problems. Deploying new features on PlanetMath that will allow people to pose, solve, critique, and discuss problems, we expect to see an increase in outcomes associated with learning. Given time constraints, rather than technical ones, we will mainly rely on qualitative methods to understand how users see the effect of the changes to the software system.

Discussion

☞ p. 61

With the addition of problem solving interactions, "articles without attached problems" may be seen to have secondary utility; and, "problems that can't be solved using information in the attached articles" may point to flaws or omissions in the knowledge resource. These features make PlanetMath an interesting testbed for examining "knowledge building" theories of learning.

FAQ #1: "What if people use it cheat?" If the result of the above "innovations" is that mathematics problems become easier solve (including by looking up the answer), this may indeed have dramatic consequences. The argument I'm making here is that we actually don't know what conditions are most conducive to learning, and we're only just starting to be able to ascertain that now; but (as with logarithm tables, hand calculators, and computer algebra systems) one should expect that as some basic problems get easier to solve, people will move on to more difficult ones. Can we quantify/qualify this? I would argue yes, using the idea of the Zone of Proximal Development (see Zaretskii[168]), and generalizations and extensions of this idea (most particularly, through the work of Peter Sloterdijk[139;138]).

Future Work

☞ p. 64

Feature development There's a lot more we can do here! Check out the Future milestone in our issue tracker. Why does this matter? Think about it this way: Coursera et al. are sort of like the cool freeware apps you can download for Mac or Windows. But PlanetMath can be more like GNU/Linux – an entire free operating system.

Once we can detect similar/related problems, etc., we can make recommendations. "Give me an easier related problem" is the classic Pólya heuristic[120] – it would be nice to automate this. More general recommendations (for content and behavior) would come from analyses described above. Making simple recommendations ("MathWords") is easy. Making good recommendations is bound to be hard. Now we have techniques to evaluate the relevance of these recommendations for learning, which is an important advance.

The Free Technology Guild How can students and practitioners collaborate, learn, and deliver high-quality work, in, say, science and technology in general? Part of the answer could be a PlanetMath-like collaboration to build a shared knowledge resource about technology. But the broad view is the idea of "a several-sided market", serving the interests of consumers, service providers, mentors, learners, and the public at large.

Summary: Towards the instrumentation of learning efficacy

The main contribution in this thesis is a way to measure the effectiveness of a class of generalized learning strategies. At present, the common-place advice to "solve problems" is like a good home remedy. But what kind of problem solving is most conducive to learning? What can help problem solving work better? Are there any "helps" that are actually not so helpful? This thesis will deliver a method for answering questions like the above, using statistical techniques, and the basic components of a "lab" for testing claims about mathematics learning.

Acknowledgements

This wouldn't have been possible without: collaboration on the Planetary System by colleagues at KWARC, led by Michael Kohlhase; programming assistance from Lucas Anastasiou with support from KMi; statistical analysis and computation by Tim Teravainen at Columbia (to which I've aimed to contribute good questions) who has also been a good friend of mine for years; conversation with Howard Rheingold and others in his Cooperation course and in the Peeragogy project. Aaron Krowne and Raymond Puzio have been stalwart companions in the PlanetMath project for a decade now. Other key collaborators and discussants include: Charlie Danoff, Marisa Ponti, Fabrizio Terzi, Wouter Tebbens, Jamie Gabbay, Anesa Hosein, and Andrea Kohlhase (whose talk at the 2012 Conference on Intelligent Computer Mathematics introduced me to the work of Vilém Flusser and its use in the context of mathematical communication[80]). Finally, thanks go to Alex and Peter, my supervisors, and to my dear friends and family.

TODO

Status As of October 6, I think this can be a final TODO list for submission! I'm adding things to the *top* of this list and striking things ~~out~~ when they are really done.

- Draft questions for Edinburgh focus group
- semi-PRS: Get NNexus up and running
- semi-PRS: Get a new beta up (coordinate with Deyan)
- PRS: Outline the statistical methods that we will apply
- Which items that have been posted to the PlanetMath tracker can realistically be solved before the end of November?
- Look again at the code that transforms PlanetMath's version history into Git. Is it really doing things right? Do we have everything?
- Technically, what couldn't we get to this time, but what could still be useful? (SNA stuff)
- Talk about how Tim's techniques might apply in other online forum settings
- Use the new one-page FTG summary in the Future Work chapter
- Discussion: Make detailed outline of my response to William Thurston and Frank Quinn (Anyone else?)
- Get the analysis routines running locally
- Process the data
- PRS: Pull out all the "vectors" from the legacy data that seem reasonable
- PRS: Barabási's stuff versus the semi-parametric model
- Revise the literature review into a proper, readable, "survey"
- Include some broader ("macro") background on the historical context for mathematics education

Works used (c)

- Chapter 1 (Historical Background):
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  - Joseph Corneli and Mircea Alexandru Dumitru (2012). PlanetMath/Planetary, in *Proceedings of Works in Progress track, MathUI and OpenMath workshops at the Conferences on Intelligent Computer Mathematics 2012*. (http://metameso.org/~joe/docs/planetmath-wip.pdf)
- Conclusion:
  - A revised version of http://en.wikipedia.org/wiki/File:Dharma_Wheel.svg by User:Esteban.barahona (under the terms of the CC-By-SA license).

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☛ Introduction

This thesis describes my work to build a platform that provides social, semantic, mediated support for mathematical problem solving. An example from print media that may be familiar to many readers is the Schuams outline series, which provides succinct summaries of mathematical topics in a format suitable for self-study.* In my thesis project, I ask what happens if such a series of works is made collaboratively, with expository texts and problems contributed in real time, and with social and automated support for problem solving. My hunch is that this will provide an appealing tool for learners, and that both expository texts and the problem repository will improve over time. At a high level, this work has three parts.

At present, the PlanetMath encyclopedia defines many terms (over 15,000). A person (or room) that uses these terms accurately can be said to know the terms.

Nevertheless, demonstrating knowledge is different from learning. Even a traditional exam measures comprehension, rather than learning. So in order to detect *learning* instead of *knowledge*, we would want to look for things like mistakes, open statements of ignorance, requests for help, contestation and argument, and so on.

Sadly, we haven't had time to get into the sophisticated NLP work that would allow us to capture that information, so, for now, we use a blunter measure: for now, we simplify, and look at learning as a change in demonstrated knowledge, in other words, as the acquisition of new concepts. Nevertheless, by measuring learning (even in this simplified sense) *as it happens*, we have more agility than the traditional exam framework.

In order to make *research* work well, we consider the following refinements to the question (α):

1. *Which activities (strategies) have the biggest payoff in terms of our learning model?*
2. *How does the answer to this question change when we focus our attention on "peer supported problem solving in a knowledge-rich learning environment"?*
3. *Is the learning model ultimately a good model?*

The first question can be studied in a preliminary fashion using legacy data. In other words: we can take the legacy encyclopedia, forum posts, and things like *corrections*, and use a simple network model to describe vocabulary acquisition over time. We can then look for causal factors that impact vocabulary growth.

Encyclopedia authors will of course have collectively *covered* the encyclopedia – but it will be interested to look at their specializations (because of the historical "homesteading" feature, people do not have an equal chance to demonstrate knowledge about every single topic – it's first come, first served in that respect). And the relationship of concepts in forum posts to encyclopedia material is at present a complete unknown! These investigations will form the basis of the first study (see 5.1).

The ideas are then explored in greater depth and specificity in the context of *peer supported problem solving* (see 5.1). For this second study, we needed a more developed tool – one that could support both peer interactions and mathematical knowledge. Our hypothesis is that individual learning will flourish in this space together with new growth to the shared knowledge resource. The more sophisticated tool, together with the analytical framework we developed, addresses the second question.

*Other hoary examples that might fit in this context would be *Methoden der mathematischen Physik* by Courant and Hilbert or *Algèbra* by N. Bourbaki.

With regard to the third question: when we look at what the learning model actually *is*, we should guess that its quality will be determined by

(a) the quality of the encyclopedia (since, after all, this is where we get the words we're trying to detect); and

(b) the quality of our text- and data-mining tools (how well can we detect words, or other more general interaction patterns, whether they be grammatical or graphical?)

(c) the quality of our understanding of how behaviors link to outcomes.

One way to get at quality would be to see if we can replicate old measures of success: e.g. we could run an experiment where students do all of their coursework on PlanetMath, and then are evaluated by an (off-line) exam. Does our learning model successfully predict results on the exam? For an experiment like this to work, we would need a version of PlanetMath that can support coursework, and a teacher who is willing to run the course using this experimental software.

That would be nice, but, on the other hand, the "teaching function" is already embodied in the software and distributed across the community – and PlanetMath is now set up as a sort of continuous, unlimited exam. In this view, the "learning model" ultimately cannot be divorced from the implementation, and in an almost tautological way, the quality of the learning model comes down to *learning quality*.

To put this another way: mathematics cannot be understood as a "pure" object outside of some embodiment, e.g. the PlanetMath knowledge base and community, together with the Planetary system. We knew that already,[74] but the *power* of this idea was never so clear. Ultimately, it means that we can assess the learning model by assessing the system, or, in short, that user feedback is the key "diagnostic".

To summarize:

Theory

Basically should sum up the paragogy theory here.

We have been rebuilding the website PlanetMath.org, replacing custom code from 2001 with a mathematics- and semantic-web-enhanced Drupal 7 platform, including features like these:

- Metadata management: MSC as a SKOS ontology, improved features for adding links, etc.
- Problem sets and other problem-solving paraphernalia, e.g. activity tracking
- Easier extensibility for, e.g., multilingual support

Tool

Basically should sum up the Planetary System here.

At a certain level, the new system *is* the main intervention, and using the framework discussed above, we can look at the change in mathematical behavior that the new tool helps bring about. However, in light of the analytical and instrumentation power that this setup gives us, we can go further. Wherever we can detect some new learning-relevant feature (e.g. vocabulary, heuristics), we can give people feedback on the way they are using this feature, and attempt to discover the impact of the availability of *this feedback* on learning.

Test

Basically should sum up the statistical experiment, and the way it "confirms" the theory that provides grist for its mill...

At a high level, the question we're curious about is: *"what parameters are relevant to learning mathematics?"* (α)

If we can detect relevant parameters and behaviors, then we can potentially take action (e.g. making recommendations to students or their tutors). As a whole, the learning ecosystem should improve over time as a result of user engagement. Thus, we will also be interested to detect both actual learning by students, and improvements to the learning ecosystem.

We would like to be able to detect things like these:

- Vocabulary acquisition
- Styles of engagement
- Similar texts
- Related articles
- Easier problems
- Use of heuristics

And out of this, we would like to discover specific factors that drive (or at least encourage) learning outcomes. (Since it is the easiest to measure, we take "vocabulary acquisition" as our basic learning model.)

Mine from this into the above The approach underway takes the existing knowledge base as its core resource and extends it with a range of new materials and interactions focusing on problem solving – "the heart of mathematics."[62] In short, our aim is to build a knowledge-rich online learning environment for mathematics. A learning environment is made of human practices and material objects[156]. It is a part of the human ecology wherein learners work alone or with others, using resources to pursue learning goals. It hosts the interactions of different types of learners, who interact, not only with each other, but also with artefacts, technologies and content[111]. This "hosted interaction" feature is what distinguishes a learning environment from a learning object repository or digital library. Historically, PlanetMath has indeed hosted some interactions, namely those having to do with authoring and discussing mathematical content: however, its largest user population consists of non-logged-in visitors, for whom it serves as a reference resource. In the future, we aim to provide more active support for hosted interactions that help people learn mathematics.

We follow Conole in thinking of "learning design" as a way of creating and representing practice.[36]

The design we are using to transform PlanetMath from being primarily a reference resource into a new learning environment (Figure 1) will include all of the traditional activities associated with building and maintaining PlanetMath's encyclopaedia (e.g., writing and editing articles, forum discussions, corrections and requests), and a new "problem solving layer", which contains problems, solutions, links from the problems into the encyclopaedia (and vice versa), as well as discussions about problems (including evaluation or marking), and course packets that combine problems with expository material. Learning in this environment primarily entails solving problems, though numerous other activities will have learning relevance, e.g. asking a question, giving

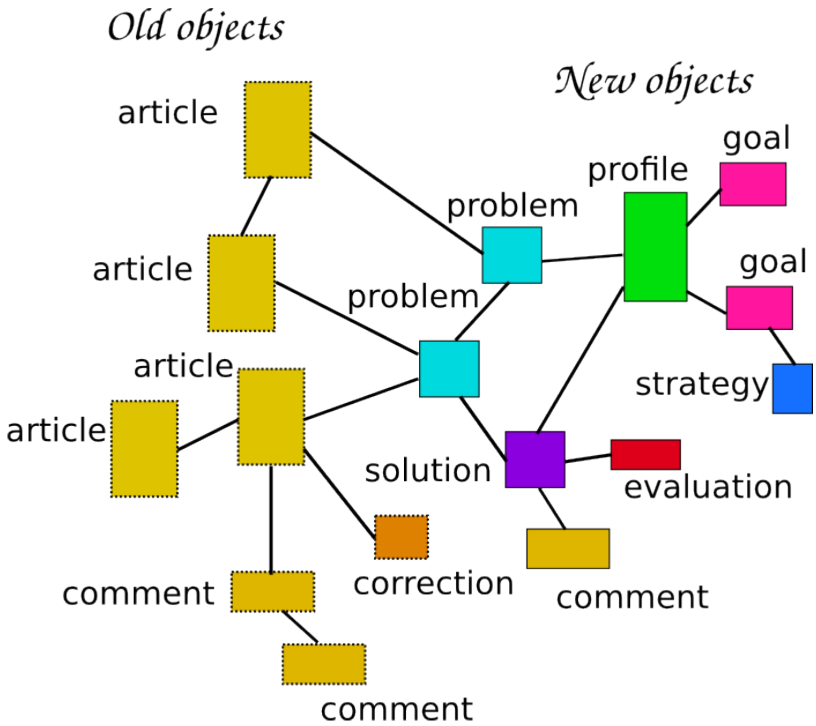

Figure 1: This is the sort of extension we'd like to develop

or receiving advice, or improving an expository text. From the user's perspective, the environment will offer some significant advantages over a standard textbook or problem archive. For instance, nothing is more daunting than being faced with a problem to solve and not knowing what the terms in that problem mean. On PlanetMath, we will be able to provide automatically-generated links to the definitions of technical terms in problem statements. Frequently, such definitions are not enough, and the user will be able to make annotations asking for hints about how to use the definitions. These requests will help improve the quality and relevance of encyclopaedia articles. Over time, we expect a large archive of hints and worked examples to accumulate, overcoming the cold-start problem for learners. In particular, as generations of learners interact with the site, sharing and reflecting on strategies for meeting their learning goals, a collection of efficient learning pathways should emerge. Our hope is this archive and guide will help learners connect to mathematics in a meaningful way: rather than having textbook problems serve as a daunting obstacle to application or research, they should, in this context, serve as stepping stones to relevant and meaningful engagement in mathematical practice.

The thesis is divided in Chapters as follows:

Chapter 0.1 deals with the history of the PlanetMath encyclopedia.

Chapter 1.1 presents a novel theoretical perspective on the analysis of learning in a space like PlanetMath.

Chapter 2.1 is a review of the literature on similar systems, approaches, and studies.

Chapter 3.1 details the methodology we are using to "detect learning".

Chapter 4.1 describes the implementation of both system-level and intervention-level features.

Chapter 5.1 describes two research studies that I have carried out in this space.

Chapter 7.1 discusses possible future work building on this thesis.

Chapter 8.1 summarizes the major conclusions and contributions of the work.

☛ Historical Background

The introduction to this chapter should make good on the "why me" summary I included in the synopsis. Also, after I've talked about that and discussed the PlanetMath encyclopedia, it would be good to broaden out and talk a little more about the "state of the world." I.e. I'd like to include some information on the number of graduates in various mathematics and science fields, some information about what becomes of them, and so forth. We don't need too much of this macro trend stuff, but since it's something the popular press talks about a lot, it's really something that we ought to include. "Why does this matter" in other words. What's the challenge?

Why this? Why now? Why me? There's an almost formal answer to these questions in an essay by Fabio Landini.[90] At least, that essay provides half of the answer. In addition to the "cultural subsidy" that Landini posits, I would argue that "free as in freedom" works also convey a "cultural surplus" to their developers, if they are successful (i.e. Marxian *surplus value*). How else can you ethically get hundreds of people working on your project for free?

> To simplify the analysis, the paper will present a simple model. In the model two (representative) agents are involved in the production of a composite information good and must choose how to organize production. [...] an increase in the degree of technical malleability (i.e. the diffusion of digital technologies) enlarges the set of parameters for which peer production is viable. When peer production is viable, two organizational equilibria exist in the economy, namely peer and firm-based production.[90]

> Some other factors must have necessarily played an important role. [...] In the case of peer production an *external subsidy* of this sort indeed existed and is related to the set of ethics that motivated the early adherents to the free software movement.[90]

He quotes RMS[147] (p. 24):

> By direct admission of RMS, in fact, the early applications of the GNU system 'had no technical advantage over Unix. [...] [Yet, they had] a social advantage, allowing users to cooperate, and an ethical advantage, respecting the user's freedom'.

This is the "subsidy" that drives the main example in this paper. But there's something more general here, related to community participation. First, of course, community members have to exist! Second, though, they have to see some value to be had from interacting – this value could be amplified by some "subsidy", but, again, the important thing is that it has to exist.

This is similar to the question about why people organize in "closed" firms versus organizing in "open" commons regimes. Remember, firms exist in the first place because it is sometimes cheaper to produce something "in house". But whenever you can't get what you need easily "in house", you have to reach out and either contract with other people, or collaborate with them informally, etc.

In particular, sometimes *cash* isn't the only thing relevant in the exchange. Sometimes it takes *time* as well – like in a learning project. So, commons regimes would apply when people get what they need better by interacting with others, AND when cash exchange isn't the answer.

That points to a sort of checklist of questions:

1. Do your contributions create something that *others value*?

2. Do others have to *interact* to get 'value' out of the resource, or can they just receive the value passively (or semi-passively, by paying for it, stealing it, etc.)?

3. Is there some added advantage that others get by interacting with the resource you're concerned with, *as opposed to some other resource*?

4. Do *their interactions* convey added value to the resource that you're interested in?

In particular, at the first stage we could ask:

- Do our actions help to break down the resource into small pieces where people can engage and get value without a huge investment? ("*technology diffusion*")
- Do we motivate people by highlighting that the resource and interactions we're talking about are "special" in some way, so that alternative options just don't look as good? ("*cultural subsidy*")
- What do we get out of it? ("*surplus value*")

The PlanetMath Encyclopedia

The history of PlanetMath.org is discussed, tracing its inception, stabilization, and some defining challenges. Research efforts by core contributors are reviewed, and the scope and reach of the resource are discussed. Recent developments are discussed briefly. Some remarks evaluating PlanetMath's trajectory and content conclude the paper.

Introduction

From PlanetMath.org's landing page*:

> *PlanetMath is a virtual community which aims to help make mathematical knowledge more accessible. PlanetMath's content is created collaboratively: the main feature is the mathematics encyclopedia with entries written and reviewed by members.*

This short paper describes the history of the PlanetMath encyclopedia. The history of this resource can not be easily separated from a history of the PlanetMath community and the technology behind the site, though the presentation here is not especially technical. The reader who is interested in a succinct overview of the *current* characteristics of the encyclopedia will find what they seek in Sections 0.1 (quantitative aspects) and 8.1 (qualitative aspects).

Beginnings

The early history of PlanetMath is wrapped up with that of the similarly-named website, MathWorld.† Eric Weisstein began collecting the material now found in MathWorld as a high school student, and continued the project as a college student in the late 1980s. "Eric's Treasure Trove of Mathematics," went online in 1995, when Weisstein was a graduate student in astronomy at the California Institute of Technology.‡ In November 1998, Weisstein made a deal with the CRC Press to publish his encyclopedia in book format, as the *CRC Concise Encyclopedia of Mathematics*. One year later, Weisstein accepted the position of encyclopedist at Wolfram Research,

*HTTP://PLANETMATH.ORG

†HTTP://MATHWORLD.WOLFRAM.COM

‡HTTP://WWW.ECHARCHA.COM/FORUM/ARCHIVE/INDEX.PHP/T-19516.HTML

Inc., and the renamed "MathWorld" site was unveiled in December 1999.* In March 2000, CRC Press sued Weisstein and Wolfram Research for copyright violation, forcing MathWorld off of the internet[125].

In the words of Eric Weisstein: "if you ever assemble a body of knowledge that you want to share with others, you don't want to go through what I have just gone through."† So it came to pass that in the Fall of 2000, Nathan Egge and Aaron Krowne, both undergraduates at Virginia Tech, came up with the idea for PlanetMath: a collaboratively created mathematics reference work that would have resistance to copyright threats built in, in the form of an open content license. By the summer of 2001, the basic infrastructure for creating an encyclopedia was complete, and a fledgling community had grown up around the resource.

The CRC lawsuit was settled for an undisclosed sum in late 2001, and on November 6, 2001, MathWorld returned to the internet.‡ But in the mean time, a new online community had been born – with some very different principles and practices. Whereas MathWorld's terms of use disallow archival copies, PlanetMath regularly publishes snapshots of the content for download. Moreover, users are permitted (and, indeed, encouraged) to copy, mirror, redistribute, print, remix, and reuse PlanetMath content for commercial or any other purpose – so long as all such works are published under the same license as PlanetMath, granting downstream users the same rights.

Stabilization

The key reference for PlanetMath is Aaron Krowne's 2003 Master's Thesis, written at Virginia Tech[89] under the supervision of Ed Fox. In this thesis, Krowne describes how the early design and development of the site benefited from continuous feedback in the #math IRC channel on Undernet.§ He also details the key technical and community features of the site as they developed in this period:

- A state-of-the-art system for displaying mathematical notation on the web, starting from LaTeX sources.
- A flexible authority model that can support both wiki-style articles (that anyone can edit), and a more academic style, where articles are owned by one person, who may, if they wish, grant co-authorship permissions to chosen others, and who must respond to separate commentary from peer reviewers[87].
- A discussion forum attached to every encyclopedia article, which helps give the resource a "pedagogical slant".
- An autolinking service that helps integrate content into the site, by enabling authors to focus on the contents of one article at a time.
- Workflow built around corrections and watches, including a feature whereby articles are "orphaned" if a correction is not responded to after a given period of time.
- A scoring feature that provides a rough estimate of how the value each user has contributed to the site.

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*HTTP://EN.WIKIPEDIA.ORG/WIKI/ERIC_W._WEISSTEIN

†HTTP://WWW.ECHARCHA.COM/FORUM/ARCHIVE/INDEX.PHP/T-19516.HTML

‡HTTP://MATHWORLD.WOLFRAM.COM/ABOUT/FAQ.HTML#HISTORY

§IRC://IRC.UNDERNET.ORG/MATH

In 2003, PlanetMath incorporated, and in 2005, obtained non-profit status, so that it could accept tax-deductible donations (in the US). Together with a small stream of ad revenue, this has covered hosting and other maintenance costs.

Pushing the limits

In 2003, the present author was enrolled as a graduate student in mathematics at the University of Texas in Austin, and in possession of a large and growing personal collection of very tersely-written definitions and proofs relevant to the department's prelim exams.* In fact, this work had as much to do with the tradition of computer mathematics in the air in Austin (QED[28], Maxima[70], ACL2[76], AM[97;98]) as it had to with exams. A representative example:

(lebesgue outer measure: fact: lebesgue outer measure is infimum of lebesgue outer measures of open supersets)

1: $X \subset \mathbf{R}^n$
2: $L = \{O \Subset \mathbf{R}^n : O \supset X\}$
3: $|X|_e = \inf_{O \in L}(\{|O|_e\})$

After discovering PlanetMath and striking up a correspondence with Krowne, one night we uploaded the contents of the "Austin Problems in Mathematics – Cross-Index" (styled APM-ξ) into the PlanetMath encyclopedia as world-editable "seed entries". This turned out to be a first-rate disaster.†

The primary complaints from community members were:

(1) the entries could not be understood without reading an accompanying FAQ;

(2) a casual visitor to the PlanetMath website might get the wrong impression about the nature of the encyclopedia when looking at "apmxi" entries; and,

(3) nearly 600 entries had been introduced into PlanetMath by the site's administrator in one big batch, circumventing, at least in outward appearances, the site's usual model of careful review and collaborative editing of entries.

Subsequent to a poll, it was decided that the apmxi entries would be "orphaned", and any that were not adopted by community members after a week would be deleted from the encyclopedia. This was the fate that befell most.

The event was a defining moment in the history of PlanetMath. In the first place, it was a testament to the strength of the community's norms. Secondly, it showed that the specific affordances of computers, e.g. for mass processing of data, or for dealing with hypertextual complexity associated with alternate related treatments of a given topic, needed to be tempered to work well for the *people* involved. These issues would set much of the research and development agenda around PlanetMath for the following decade.

Research, outreach, and some critiques

In 2005, several established PlanetMath contributors met in person at a Symposium on *Free Culture and the Digital Library* at Emory University, where Krowne was then based. Contributed papers looked at

*HTTP://METAMESO.ORG/~JOE/MATH/XI.PDF

†HTTP://WIKI.PLANETMATH.ORG/CGI-BIN/WIKI.PL/ONE_WEEK_IN_OCTOBER

- an adaptation of PlanetMath's software for collaborative creation of course notes in a graduate course on ordinary differential equations [106];
- experiments with a novel hypertext system based on the idea that everything is annotatable [38]; and,
- the dynamic tension between the non-copyrightability of ideas, and the necessity of conveying ideas in copyrightable expressions, and the ramifications for mathematics [124].

These reflections on copyright (and copyleft) were subsequently expanded in an article for *First Monday*, which looked at the drawbacks of current copyleft licenses, particularly "license lock" [88].

We presented talks about PlanetMath in the *Math on the web pavilion* at two Joint Mathematics Meetings (San Antonio, 2006; New Orleans, 2007)[*,†] and in a session on *The Role of Open Source Math Projects in the Mathematics Community* at MathFest (Madison, 2008)[‡]; and at more specialized workshops: *The Evolution of Mathematical Communication in the Age of Digital Libraries* (Minneapolis, 2006)[§], and *Mathematical Knowledge Management: Sustainability, Scalability and Interoperability* (Halifax, 2007)[¶].

We also made efforts to create a print version of the PlanetMath encyclopedia (retitled the "Free Encyclopedia of Mathematics"). The 2004 attempt, in two volumes[‖], and a 2005 attempt in one much nicer-looking volume[**], thanks to Ross Moore's contribution of `multinclude.sty` and other tweaks.[††] Still, the resulting 1971 page PDF was more a proof of concept than a printer's proof.

On the development side, PlanetMath was thrice supported by Google's Summer of Code (2006–2008). The best outcome of this was that PlanetMath's autolinking subsystem was turned into a modular piece of code, NNexus[‡‡], as written up in [58]. PlanetMath's software improved further under contract with Springer, pursuant to the creation of StatProb.com.[§§] PlanetMath's sister site PlanetPhysics.org[¶¶] is currently in the process of switching over to this platform, termed Noosphere 1.5.

However, the development effort wasn't particularly able to keep pace with the feature requests generated by the user community.[***] Nor did the Noosphere codebase present a particularly compelling resource for capable critics and developers like Claus Zinn [170] and Christoph Lange [91], to jump into and improve. Zinn wrote:

> *If we could harness the collaborative authoring process and encourage and guide wiki authors to continually provide content and metadata, then intelligent services could unleash their true potential, with immediate return and added value for authors and learners.*

[*] http://www.jointmathematicsmeetings.org/meetings/national/jmm/san-prog.pdf
[†] http://www.dessci.com/en/company/shows/jmm/mow2007.htm
[‡] http://www.maa.org/abstracts/mf2008-program.pdf
[§] http://www.ima.umn.edu/2006-2007/SW12.8-9.06/
[¶] http://projects.cs.dal.ca/ddrive/seminars/mkm.shtml
[‖] http://www.scribd.com/doc/9691966/, http://www.scribd.com/doc/9692058/
[**] http://metameso.org/~joe/docs/book.pdf
[††] http://metameso.org/~joe/math/fem-2005.tar.gz
[‡‡] http://code.google.com/p/nnexus/
[§§] http://statprob.com
[¶¶] http://planetphysics.org
[***] http://wiki.planetmath.org/cgi-bin/wiki.pl/Feature_Requests

Scope and reach

At the time of this writing, PlanetMath contains 8945 entries, dealing with 15655 concepts. 298 people have contributed an entry in the encyclopedia, and 2742 have contributed something (perhaps just one forum post).

Out of these, an exceptional group of 24 authors have produced more than 100 encyclopedia articles apiece. Their contributions comprise 74% of the total number of articles. About 71% of this core group joined before 2004 (in the "early days" for the site).

130 users have a score of 1000 points or more, which would correspond to contributing 10 or more new encyclopedia articles, but actually, a significant fraction of this value has been contributed through things like corrections, revisions to existing objects, and posting in the forums. All told, this group has contributed 96% of the total number of articles. About 58% of this group joined before 2004.

According to Alexa.com, PlanetMath.org is currently the 165,011$^{\text{th}}$ most popular website in the world, comparable to the website for the Mathematical Association of America[*] (119,267$^{\text{th}}$), or the relative newcomer MathOverflow.net[†] (184,818$^{\text{th}}$), and still dominating the relative outlier ProofWiki.org[‡] (481,380$^{\text{th}}$) and more specialized tricki.org[§] (2,299,888$^{\text{th}}$).

PlanetMath is a far cry from competing with Wikipedia, but note that at least 295 articles on Wikipedia incorporate text from PlanetMath (under terms of the shared CC-By-SA license).[¶] Wikipedia is mentioned (although not necessarily quoted) in a comparable number of PlanetMath articles, 267 to be precise.

A new era

In 2010, a new project to completely rebuild PlanetMath's software began. *Planetary* is based at Jacobs University, Bremen, and led by Michael Kohlhase, with major contributions from many members of his research group (along with the present author).[‖] The Planetary system is described in[46]. Planetary is considerably easier to extend than Noosphere: it is currently comprised of around 20 plugins for the popular open source platform, Vanilla Forums[**], many of which integrate sophisticated software tools previously developed by the KWARC group. Notably, the system now includes support for semantic authoring and flexible metadata interaction, addressing the critiques mentioned in Section 0.1.

2011 will see the publication of a book chapter discussing the future use of PlanetMath as the core of a problem-based learning system[39], the focus of the author's doctoral studies[41]. With this as a basis for a showcase of innovative uses for the PlanetMath content, and with a well-documented and easy to extend software platform supporting the system, we hope to see PlanetMath become a central integration platform for free software projects working with freely licensed math on the web.

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[*] HTTP://MAA.ORG

[†] HTTP://MATHOVERFLOW.NET

[‡] HTTP://PROOFWIKI.ORG

[§] HTTP://TRICKI.ORG

[¶] HTTP://EN.WIKIPEDIA.ORG/WIKI/CATEGORY:WIKIPEDIA_ARTICLES_INCORPORATING_TEXT_FROM_PLANETMATH

[‖] HTTP://TRAC.MATHWEB.ORG/PLANETARY

[**] HTTP://VANILLAFORUMS.ORG

Conclusion

PlanetMath been successful as an online community: the software stabilized early on and has required little upkeep, while the site has continued to grow. However, there has been a danger that with the software system running more or less on "autopilot", new features would not be developed. With any luck, this threat is in the process of being eliminated, heralding in the opportunity to build one or more "new" online communities in close relationship to PlanetMath (e.g. a developer community working on sophisticated tools for scientific communication; a learning community using the PlanetMath encyclopedia as part of a remix-driven interactive textbook). We should do a careful evaluation of what has worked and what could be improved for next time. There is not room in the current paper to conduct this discussion, but framework proposed by Resnick *et al.* would be an excellent place to begin[130].

For all of its potential as a software showcase and its possible future role as a player in a larger landscape of related interlinked online communities, PlanetMath should, at least for the moment, be evaluated first and foremost as an encyclopedia. This too would best be handled as an ongoing task. For now, a quick summary following the framework used by Emma Previato in her review[122] of the *CRC Concise* gives us a look at how PlanetMath measures up (Table 1).

In a philosophical sense, PlanetMath represents a new and distinct type of encyclopedia, perhaps poised to begin its final ascent to the literal classical meaning of a "complete instruction". Because the technology that supports the site is special-purpose, we have been able to hone in on what works best for commons-based peer production in mathematics. Features like the autolinker facilitate integration of content, and the corrections system helps avoid messy battles. There is much more work to be done, but at the close of PlanetMath's first decade of life, it shows amazing potential for the future.

At the same time, Table 2 shows that PlanetMath will need some help if it is going to continue to be a productive community! Note that Table 2 gives us the counts as they appear in the current database. To obtain numbers reflecting *actual contributions* in 2011, the numbers we should be rescaled by a factor of around $4/3$ to compensate for an unfortunate data loss:

$$\textbf{messages}: 710 \quad \textbf{articles}: 122 \quad \textbf{corrections}: 68 \quad \textbf{users}: 241$$

Thus, even though 2011 was still an all-time low for articles and corrections, and nearly an all-time low for messages, the numbers don't represent quite as dramatic a dropoff as it would appear in Table 2.)

Other related work

In Aaron Krowne's brief overview[86] of the principles that informed the design of PlanetMath in its original incarnation, he states:

> The basic, universal goals of digital libraries are to provide a logically organized, conveniently accessible, and (if possible) easily *actionable* collection of digitized *knowledge* in some field or fields for an audience of *learners*. (Emphasis in original.)

In moving from an encyclopedia (a reference work) to a learning environment, we extend each of these critical dimensions: the resource becomes more directly useful, it includes new kinds of knowledge, and it is accessible to a wider population of learners.

Work from the same era in the ARIADNE project[113] gives a loose typology of the kinds of interactions that can occur in a digital library: user-user, user-staff, staff-staff. Although PlanetMath does not currently have a staff per se, the addition of a problem solving focus does create

Coverage	The median entry would be an advanced undergraduate or beginning graduate topic. PlanetMath is generally considered to have more in-depth treatment of technical issues, e.g. of proofs, than that found in Wikipedia.
References	Present in many articles, although there is not yet a unified database of references or style of presenting them (this is planned).
History	315 items, mostly 20th Century or later (HTTP://PLANETMATH.ORG/BROWSE/OBJECTS/01AXX/)
Audience	Consistent with the coverage, there have been 4127 posts in the "Graduate/Advanced" forum, 4261 posts in the "University/Tertiary" forum, and 1199 posts in the "High School/Secondary" forum.
Clarity	There is no hard and fast rule. Some articles will tend to be minimalistic, but precise (particularly when the authors have English as a second language). Other articles even from native speakers may be verbose and vague. In any case, debates over clarity of presentation are intense, and a high standard is maintained.
Pictures	There are over 600 images, but since this is only about 7% of all entries, the prevalence could be improved. A unified database/gallery of pictures would help.
Accuracy	At the time of this writing there are 48 outstanding corrections; more than 14080 corrections have been filed since the site began, though 2337 are classified as "addenda", meaning that no mistake is implied, and 9182 are classified as "meta/minor", which suggests that in the entire history of PlanetMath some 2561 real errors have been found through the peer review process (and all but a few fixed!). Note that these numbers do not take into account changes initiated by authors without prompting, and would tend to underrepresent error fixing in world-editable articles.
Unusual	PlanetMath provides "math for the people, by the people".
Weight	PlanetMath can be comfortably edited from a lightweight laptop weighing about 1kg (and browsed from a mobile device weighing considerably less).

Table 1: Succinct review of the PlanetMath Encyclopedia

	messages	articles	corrections	users
2011	533	92	51	181
2010	1462	151	107	504
2009	2459	529	274	1976
2008	3620	1114	944	3345
2007	4529	1435	2176	3553
2006	3861	1119	3721	3627
2005	3651	829	1732	3605
2004	2933	1001	2314	3605
2003	1437	917	1450	2772
2002	746	1530	1235	1128
2001	79	314	98	102

Table 2: New content per category per year

an interesting mixed model, where tutorial sessions feed into the knowledge base. This avenue brings with it a business model that has been lacking in the past decade at PlanetMath. We're interested in exploring use of real-time technologies like Etherpad and Skype to support tutorials. (OpenStudy experimented with an in-house Etherpad derivative called StudyPad, though it is no longer actively used on their site.)

Contemporary approaches to building OER repositories of which we are aware (e.g. Rice University's Connexions project*, the Open University's OpenLearn†, or MIT's Open CourseWare project‡) tend not to emphasise learner-produced materials. Our proposed combination of a largely learner-produced OER and the orientation towards encyclopedic completeness in this project seem to be unprecedented, at least in the online space. In the print world, we might find some parallels in Springer's popular Graduate Texts in Mathematics series, or even Bourbaki's series of textbooks, but the strategy of peer-producing a comprehensive collection of learning materials seems novel, and affords quite a few new opportunities. The earlier effort in the Free High School Science Texts project§ may present an intermediate point in the spectrum, though, again, student- or learner-produced materials were not emphasised.

As the project matures, we expect to hear a variety of differing opinions about the sensibility and utility of this approach, similar to the voices in the now-classic debate over the use of graphing calculators or computers in mathematics classrooms. Questions about how to learn best in a knowledge-rich setting are not yet well understood.

Our project connects with another contemporary effort to understand and develop software support for dealing with massive amounts of knowledge, namely the Linked Data strategy. We plan to make all of our materials available as Linked Data, which will enable downstream users to remix the repository's contents with sources from external Linked Data repositories, like DB-pedia. Where it seems relevant, we may pursue content sharing arrangements with other other mathematics projects who use a compatible license (e.g. ProofWiki¶ and MathOverflow).

Once we have access to a wide range of mathematical resources, we will be able to make useful recommendations for both self-hosted and and externally-hosted materials, following the methods

*HTTP://CNX.ORG

†HTTP://OPENLEARN.OPEN.AC.UK

‡HTTP://OCW.MIT.EDU

§HTTP://WWW.FHSST.ORG

¶HTTP://PROOFWIKI.ORG

of e.g. the FolkSemantic widget for OER recommendations.*

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*HTTP://WWW.FOLKSEMANTIC.COM/WIDGETS

☛ Theoretical Background

Introduction

I will describe the "epistemic object" idea in detail here. This is the upholstery button that holds this chapter together (I hope).*

What's interesting to me in the big picture is how the work I'm doing with software relates to modelling mathematical behavior, including motivational aspects and so forth. How do you understand a given way of acting?

Miettinen and Virkkunen take up this question in the domain of organizational science:

> To crack open the previously hidden self-evidence and 'givenness' of ways of acting and to transform the activity, the routines themselves must be made into an object of enquiry, that is, into an epistemic object.[105]

Here, they are drawing on the theory of the epistemic object or epistemic thing developed by scientist/historian Hans-Jörg Rheinberger. Rheinberger focuses on a case study in traditional experimental science, but he also acknowledges the following:

> In following the development of epistemic things rather than that of concepts, topics, problems, disciplines, or institutions, boundaries have to be crossed, boundaries of representational techniques, of experimental system, of established academic disciplines, and of institutionalized programs and projects.[132] (p. 34)

To build a sufficient understanding of mathematical behavior will require some of the boundary-crossings mentioned above: the first of these being a transposition to experimental science, out of the realm of pure logic, ideation, and rigour (or whatever mathematics is *supposed* to be).

PlanetMath as an epistemic object

PlanetMath can be conceptualized as an epistemic object: a shared, never-complete map and instruction manual for understanding the mathematical terrain. It develops slowly, by critique, questioning, and dissent. Ultimately, it embodies the the question: *what can we do when we computers and mathematics together?*

Conceptualizing PlanetMath as an epistemic object connects us to the socio-cultural historical approach, which examines the foundations of learning not in "abstract rules", but in material artefacts.[105] In this view, PlanetMath can be seen as a shared artefact objectifying forms of expression of cognitive processes and patterns that exist outside of the minds of individual participants. The process of learning mathematics can be embodied in this artefact's constituent tools and signs. Drawing on Vygotsky's concept of mediated action[155], a participant can internalize these, by interacting with other users, with content, and with tools.

Planetary as an epistemic object

The development of PlanetMath as a platform is accompanied by a parallel process of tinkering on the software and infrastructure level. This is the epistemic object that has come to the fore in my thesis project.

*HTTP://WWW.LACAN.COM/ZIZEKCHRO1.HTM

Planetary is interesting because at any given moment, it embodies what we know about the way people communicate mathematics – to the extent of our ability as modellers and programmers (and as permitted by our time budget).

The model is by no means perfect. Planetary doesn't do much to capture or express kinesthetic sense, for example. But what it can do is transform mathematical behavior into *data*.

To summarize: we have underlying source of mathematical behavior, and a changing, shifting model of that behavior (which also constrains the exact form the behavior takes). Planetary allows us to "crack open" certain mathematical routines. But it is *itself* an evolving object, not an abstract mathometer.

> There is a life cycle to experimental systems. They are brought into being as research devices, become transformed into kits, and finally are replaced. But there is a symmetrical counterpart to this cycle. Kits can become destabilized and turn into research devices, either by transplantation or by the introduction of new representational techniques.[132] (p. 81)

The plan of for the rest of the chapter will be to follows the development of an idea, from early intuitions, to framework, through several generalizations and analogues of this framework, ultimately settling on some simple but powerful features to look for in interactions, and to bake into our designs where possible. The chapter is by necessity open-ended, more about generating possibilities than pinning things down. In it, we develop the shape and feel of our epistemic object.

Crowdsourcing Education: A Role-Based Analysis

We began by rethinking Nonaka and Takeuchi's well-known SECI model of knowledge creation[114], applying to study crowdsourced education. This required two revisions to the SECI model. First, what initially appeared to be a simple shorthand (obtained by mapping Nonaka and Takeuchi's Socialization/Externalization/Combination/Internalization framework onto Ken Wilber's I/We/Its/It[162]) ended up leading us to a very different way of thinking about things. And, second, Nishida's philosophy of basho (summarized in English by Masao Abe[1]), which inspired the creators of the SECI model, began to take on an even more central role in our version of the theory.

To put it somewhat colorfully, the "Golden Age" SECI is here updated to make it suitable to the analytical challenges present in our "Modern Age". These challenges include organizations that make significant use of Commons-Based Peer Production (CBPP)[16], organizations without a traditional management structure, and collaborations that cut across organizational boundaries. The focus in our analysis is on the various social roles taken on by the persons involved in such settings.

SECI is given a very useful critique in Engeström's "Innovative learning in work teams: Analyzing cycles of knowledge creation in practice"[51]. In particular, Engeström makes a convincing case that "SECI" doesn't adequately represent a *cycle*, despite the claims of its initial creators. Instead of knowledge building as a cycle, we use Wilber's terms to describe a given social role in a knowledge space terms of its constituent actions. So, for example, the role of "being a student" might be described as follows:

> "I go to class, we do a class project, the various aspects of which are things I can add to my portfolio or work-record; and fundamentally it's all about gaining a skill."

I. I *go to class*, we *do a class project*, the various aspects of which are *things I can add to my portfolio or work-record*; and fundamentally it's all about *gaining a skill*.
II. I *lead a class*, we *plan and implement the curriculum*, my work involves *giving lectures and feedback to students as well as meeting with my colleagues*; and fundamentally it's all about *helping people learn*.
III. I *ask a thought-provoking question*, we *discuss or experiment*, our *results are written up in papers*; and fundamentally it's all about *generating new knowledge*.
IV. I *transform ideas into code or policies*, we *manage a body of work*, the key pieces are *components of a functioning system*; and fundamentally it's all about *creating a workflow that works*.
V. I *engage in dialog, advocating for an interest group*, we *try to find common ground*, the *results are assembled into strategies*; and fundamentally it's all about *creating a distinctive organizational identity and strong partnerships*.
VI. I *endeavor to discern societal needs*, we *work to achieve a rough consensus with a larger body of stakeholders*, the *results describe a certain clearly-defined skill set*; and fundamentally it's all about *understanding the appropriateness and relevance of a certain training process*.

Table 3: Sketch of the social roles in a traditional university

This simple background story gives us a notion of role, persona, or identity: a role that is defined by its constituent actions, relative a given social context – a context that itself is conceived of, after Nishida, as a *shared context in motion*.

One observes that the story above doesn't have much to do with either "knowledge creation" or "epistemic action". Still, now that we have a convenient way to talk about roles, we can move on to talk about how roles can change over time, how new roles come into existence, how different roles can conflict, and so on.

Our concern here is with the way a given context creates and is in turn created by the social roles that are involved therein. One then asks how change plays out, given either contextual changes or changes to roles within a given context. In the process, we are free, of course, to invent imaginary roles and contexts in which to explore these questions; however, the primary usefulness of our approach is to generate real strategies for social and organizational change.

Educational Communities

We begin with a look at how educational communities are built. A traditional university, for example, is populated by students, teachers, researchers, administrators and staff, and possesses a certain legal status by maintaining a relationship with accreditation bodies and government. We get a sense of the dynamics of the university when we look at the actions that comprise these various roles (Table 1).

Such sketches are not definitive, but rather, are paradigmatic. It seems reasonable to say that any social setting that supports actions sufficiently like these is an "educational context". A setting

	Solo consumer	Mediated sociality	Social consumer
Highly modular	Library	Gutenberg.org 2.0	University
Integrated modules	Encyclopedia	**OER communities**	Course
Highly integrated	Book	Interactive Hypertext	Tutor

Table 4: We are in between several familiar institutions and a couple others that are less familiar.

that intersects only a few of them (e.g. an academic publishing house) nevertheless forms part of the broader social context in which education sits.

This leads to the idea that a given educational context can be distributed in space and time in various different ways. Many of the support functions related to infrastructure can be subcontracted or otherwise outsourced. For example, instead of giving lectures in person, an instructor may deliver lectures via podcast.

It is in this distributed setting that Open Educational Resources (OER) and Open Educational Communities arise and become relevant to the future of education. We are concerned in Table 1 with social roles, and various social contexts arise to support these as well, e.g. ranging from study groups, to tutoring services, to "virtual colleges" and so on.

In many cases it is not suitable to view such settings as only "ancillary", since they are significant communities in their own right. For example, Wikipedia is more than just an encyclopedia: for its creators and devotees, it is the flagship of a social movement.

Educational communities, online

Two well-known sources of educational content are Connexions and MIT OpenCourseWare. All educational resources are produced in a social way, but these two characterize the difference between open and closed production, or, more precisely, production after the model of the firm, or the model of the commons. Both Connexions and MIT OpenCourseWare contribute to the broader commons of OER, but as production communities, they are very different.

Indeed, the meaning of "community" is quite challenging to pin down. For example, at the time of this writing, although thousands of people visit the Connexions website each day, discussion forums are still on the way there. At present, it seems that the Connexions blog is the central "community" featurev; there is also an associated software development community.vi

In Table 2, we suggest that OER tend to sit in between the realms serving the "solo consumer" and the "social consumer", and also between the realms of "highly integrated" systems and "highly modular" systems. This helps explain why OER are often produced by online educational communities via the methods of CBPP.

For example:

> The Peer 2 Peer University (P2PU) is an online community of open study groups for short university-level courses. Think of it as online book clubs for open educational resources. The P2PU helps you navigate the wealth of open education materials that are out there, creates small groups of motivated learners, and supports the design and facilitation of courses. Students and tutors get recognition for their work, and we are building pathways to formal credit as well.

We can use the I/We/Its/It framework to consider the roles of various particiants in a setting like P2PU:

As we contemplate how these roles fit together (and what's missing relative to traditional educational communities), we start to see a picture of an alternate future emerge.

I	say what I want to study
We	talk about difficulties and successes
Its	discussions on a shared mailing list
It	helps me learn mathematics (and improve my skills at being a self-directed and peer-to-peer learner)

Table 5: The role of an an "ideal" participant in DIY Math course

I	try to figure out what to study
We	sometimes give or get advice that isn't always so helpful (and most of it isn't for me, anyway)
Its	a bunch of good intentions that lead nowhere
It	confirms my sense of the difficulty of learning anything in a self-directed fashion (and the difficulty of mathematics in particular)

Table 6: The role of a more realistically-conceptualized participant in the DIY Math course

I	come up with a course I'd like to facilitate, and then facilitate it
We	discuss ideas about how our courses might work and what "facilitation" means (e.g. as opposed to "teaching")
Its	discussions on a community mailing list and other settings (including discussions with participants in the course as it runs)
It	helps me improve my skills at a course designer and facilitator (and it's fun talking about and practicing this stuff!)

Table 7: The current role of a facilitator at P2PU

For instance: if P2PU was taking an ongoing survey of the "wished for" course topics, a future course organizer would presumably be able to create courses specifically tailored to the interests of pre-self-selected participants. A system could be set up so that a course would only run when sufficient interest had gathered.

The system could further be adapted to provide a way to set individual to specify their level of commitment in advance, building a sort of "contract" with the facilitator, and, more broadly with the community. Pre-planning in this manner could help ensure that people knew what they were signing for, and also help everyone feel confident that they and their peers were making appropriate commitments.

This is not to say that high-commitment engagement is inherently preferable. This approach is consistent with the idea that a peer-based community doesn't easily arrange itself into explicitly delineated and wholly "visible" structures, but may instead have a lot going on in the background.[52]

"Math for the people, by the people"

We can apply similar thinking to PlanetMath, considering several likely roles. For instance, a "casual browser" or someone who only posts a few questions in the forum will have a profile from

I	write about things I'm interested in, in the form of encyclopedia articles
We	give each other feedback on the things that have been written so far
Its	a collection of articles, forum posts, and metadata
It	helps me learn mathematics (by giving me the chance to practice expressing myself clearly)

Table 8: The role of a contributor to the PlanetMath encyclopedia

I	write or find and contribute problems that link together with other problems and with encyclopedia articles
We	help each other determine the context that is best-suited to a given problem
Its	a collection of problems and semantic links
It	helps me share mathematical understanding with others and helps me understand mathematics more deeply myself

Table 9: Possible future role of a contributor to the PlanetMath's collection of problems

I	solve problems online and get help from encyclopedia articles, peers, or, if I want, a tutor
We	turn to each other for help when we see we have common interests
Its	a collection of exercises, articles, solutions, and metadata that's intended to support independent and peer-based learning
It	helps me learn mathematics much as I would in a traditional classroom (but I can go at my own pace and pick my own topics)

Table 10: Possible future role of a person using PlanetMath as a place to solve problems

a dedicated contributor. In particular, a casual browser would be likely to have only a very weak sense of "We".

In light of this, it would be tempting to imagine that "producer" and "consumer" roles are quite distinct. But at least in some parts of PlanetMath, they will tend to be closely combined. For example, one must *actively engage* with problems in order to solve them, and if information about this process is captured by the site, a "consumer" of problems is also by definition a producer of problem-solutions.

And is a contributor of problem significantly different from a contributor to the encyclopedia? At the very least, different motivations are likely to be near the surface, e.g. "altruistic" motivations centered on helping others learn; or "self-serving" motivations associated with getting help with one's own problems. Both of these types of motivation are *explicitly* related to learning.

If there is one clear lesson from the past 10 years of PlanetMath, it is that there is no shortage of great ideas out there. Unfortunately, there is a relative shortage of idea-implementers. The difference between Table 12 and Table 13 is meant to provide a view on that. These two roles differ only rather subtly, but the main idea is that we could in theory have many "citizen programmers",

I	get paid to answer questions
We	work together to create the best collection of resources for helping tutors answer questions (and helping learners work on their own)
Its	a collection of previous tutoring sessions, augmented with annotations and links created by us or others
It	lets me use my mathematical proficiency to make money and help other people

Table 11: Possible future role of a tutor on PlanetMath

I	review usage data from the system, integrate this with feedback from the community, and implement systems that serve their interests
We	meet along the boundary between content and code
Its	a collection of policies and programs maintained on behalf of the user community
It	keeps the site alive (and, thanks to me, growing)

Table 12: Possible future role of a "benevolent technocrat" on PlanetMath

I	contribute to code development as part of my regular form of interaction with the site and the corpus
We	make decisions horizontally and have engineered out systems and policies so that no one person has much more power than any other (in particular, we've endeavored to widen out organizational bottlenecks)
Its	a collection of policies embodied in code
It	not only keeps our site running, but allows us to make further inroads into online education more generally

Table 13: Possible future role of a "citizen programmer/hacktivist" on PlanetMath

whereas at any given point in time, we are likely to have only a few "benevolent technocrats".

By building Planetary out of modular extensions to the popular open source Vanilla Forums, we hope to make it relatively easy to extend. To actually get people involved in the programming effort may require us to go quite a bit further, e.g. possibly creating a "PlanetComputing" site with the Planetary System's code in it.

People will not contribute to the development if they aren't motivated to do so. While motivation can come from a number of different places, one of the key factors is facility. People who sees themselves as capable of making effective changes to the codebase are much more likely to do so than people who sees that form of engagement as "beyond them".

We hope that by making the development process more visible within future implementations of PlanetMath, and by looking for ways to translate "ideas" into "incentives", that we will be able to channel user input to increasingly useful ends. Some things that are difficult or impossible to achieve now should be fairly easy to deal with once the development process itself becomes more social. This will require careful ongoing analysis and design of social roles and contexts.

How does PlanetMath stack up?

The “student” and “teacher” roles are well represented in the crowdsourcing model that we’ve developed with these tables. “Research” is not explicitly mentioned in any of our tables, but it certainly takes place, both research about crowdsourcing and peer-based education, and crowdsourced, peer-based, research, e.g. “Density Hales-Jewett and Moser numbers” by D.H.J. Polymath.[59] Building better support for this kind of massive collaboration – or even just better support for more tame computer-mediated research collaborations – is on PlanetMath’s equally massive todo list. “Developer/administrator” roles are represented here in Table 12, and an interesting blended “developer/advocate” role is described in Table 12.

In short, it seems that the only role from Table 1 that we haven’t encountered in our analysis of P2PU and PlanetMath is the “accreditation body” role. Of course, it would be somewhat rare to find such persons at a traditional university, since accreditation is importantly an “outside” role. P2PU has some innovative ideas about “peer assessment” and other strategies for measuring when learning is taking place, but these are quite a ways off from offering actual credits or diplomas.xiv In the case of PlanetMath, sufficiently rich and robust online problem sets should provide a clear sense of a learner’s current state of knowledge; such measurements may in some cases be substitutable for a degree. At least according to our sketch in Table 1, it all depends on “knowing the appropriateness and relevance” of the training process. In some cases, this could be measured by learner performance on outside exams (e.g. AP tests, GRE subject tests, or perhaps the Mathematical Tripos).

For example, we could inject P2PU-hosted or P2PU-like courses into PlanetMath (e.g. assignments could include “write or improve a PlanetMath article” or “post questions and answers in the forum”). PlanetMath could also draw more on other resources created elsewhere, as well as do more to share or mash up its resources and services with others (e.g. with Wikipedia and MathOverflow.net; also the popular mathematics preprint server, ArXiv).

The idea of a “a marketplace of interests, skills, and ideas” from Table 7 seems to have no ready correspondent in the world of traditional universities, except perhaps in a limited form, in the course catalog. At the same time, despite the general popularity of social networks and the integration of some social networking functionality in both P2PU and PlanetMath, the usefulness of the social networking tools employed on these sites still seems quite limited (simple questions like “find me a PlanetMath user who is interested in pedagogy” don’t seem to have easy answers).

Another difference is commitment. Whereas students in traditional universities are presumed to be motivated by the prospect of earning a degree or certification, in informal education, a learner’s prospects are entirely related to the skills they acquire, and to whatever enjoyment comes from the learning process itself.

If people need to be cajoled into committing, that isn’t a terribly good sign. It seems it would be better if the learning environment itself facilitated involvement from people in the ways they feel suits them best. Perhaps incentives don’t need to be created so much as “exposed” or “acknowledged”.

An educational community may or may not seek to become “sustainable” in some sense or another. The notion of sustainability could apply to features like growth, innovation, longevity, communication, and more, and it seems clear that a diversity of approaches are needed. One way the sort of analysis we have conducted is useful is that it can help suggest where to create new roles as other aspects of the context shift, or vice versa.

What's missing?

One outcome of thinking about how to improve on the DIY Math course while building on the strengths of the P2PU organization as a whole is a new theory of peer-based teaching-and-learning, termed "paragogy". This is the topic we take up in the following section.

Paragogy

Paragogy is a theory of peer learning: we endeavor to say how it works, and how it works best. This paper outlines paragogy's contemporary relevance and expounds its principles, showing their connections to other theories. We present an extended example of paragogy in practice, where we use it to evaluate our experiences working at the Peer 2 Peer University (P2PU).

We use the term *paragogy* to characterize the critical study and practice of peer learning (literally, "para-" alongside, "-gogy" leading, here adapting the classical concept of *pedagogy* and the recent notion of *andragogy*[79] to a peer learning context). The fact that paragogy is a word in Greek meaning "production" shall not dissuade us from this new usage in English. Indeed, along with J. Philipp Schmidt, executive director at the Peer–2-Peer University (P2PU)*, we believe that learning is frequently found at the heart of peer production processes[136]. In the case study that forms the heart of *this* work (Section 1.1), we will use paragogy to evaluate our experiences as course facilitators at P2PU.

Although peer learning has been the subject of various studies, it is typically given a secondary role, within a pedagogical framework. This rather staid definition, from a book that approaches peer learning from the perspective of cognitive psychology, illustrates our point[116]:

> *Peer learning is an educational practice in which students interact with other students to attain educational goals.*

Although this definition is not in itself unreasonable, we are fascinated by the growth and evolution of opportunities for learning outside of formal institutions. A recent article from Fast Company, an influential business magazine, gives an expanded view of peer learning:†

> *Just as more and more employees are expected to have basic multi-media skills – the ability to blog, for example, or to shoot images or videos on their smartphones – so will they be expected to have the basic ability to share knowledge with their peers.*

Thus, peer learning can of course take place between non-students, and it can concern productive, as well as educational, goals.

In addition to an increased emphasis on informal learning in the workplace, recent years have seen the rise of open, online spaces that serve the needs of learners via a commons-based approach. Here we cite Cormac Lawler's recent work on Wikiversity[95;94]. Lawler uses and advocates an *action research* approach, with thematic questions "*What does it take to change a given system? [...] and how does the process of changing a system develop our knowledge about that system?*" We have brought these questions to P2PU, and by extension to the education system that P2PU both sits in, and challenges.

Our aim is to develop a set of "good practices" around peer learning, suitable for use by everyone involved (individual learners, organizers, administrators). A model is provided by two related works, one from Crowston *et al.* concerning "open software success"[45] and the other

*HTTP://P2PU.ORG

†HTTP://WWW.FASTCOMPANY.COM/1746901/HOW-MINT-EXECS-NEW-COMPANY-IS-GOING-TO-MAKE-TEACHERS-OUT-OF-US-ALL

from Resnick *et al.* on "starting new online communities"[131]. We will, however, have to wait for a future work to bring these contributions into one coherent frame with paragogy.

The paragogical principles were conceived by turning Knowles's principles of andragogy[79] on their edge. In succinct form, these principles are:

1. **That adult learners are self-directed.**
2. **That they bring a wealth of experience to the educational setting.**
3. **That they enter educational settings ready to learn.**
4. **That they are problem-centered in their learning.**
5. **That they are best motivated by internal factors.**

Blondy[24] points out both uses and challenges for each of the Knowles principles, focusing on how they work in online learning environments. For instance, with reference to the first principle, "*Cheren stated that while learners may express a desire to be self-directed in their learning, most lack the required understanding of learning necessary to be self-directed and thus need guidance and encouragement in the learning process.*"

While our principles can be read as a critique of andragogy, it is largely a matter of point of view: thus, unlike andragogy (which takes the view of the adult educator) or pedagogy (which again studies teachers teaching learners), and unlike heutagogy[63] (which focuses on self-directed learners), as we have seen above, paragogy focuses on cases in which learners are actively engaged in co-creating their learning environments. In formulating our first principle, we drew on Nishida's notion of *basho* ("shared context in motion"), which looks at the way a context constrains or supports different types of (inter-)actions, and simultaneously at the ways in which we can (re-)shape the contexts we find ourselves in[1]. Thus, instead of asking whether or not learners are self-directed, we would follow Bingham[23], and assert that self-directedness is only meaningful within a relational context (e.g. within a social field). So much for the first principle, others are subject to a similar re-thinking.

Paragogy is not the only framework that has been used to study peer learning. We'll mention Scardamalia's 12-point framework for Knowledge Building[135] and Mwanza's 8-step process coming from Activity Theory[109;110]. Scardamalia's 12 "socio-cognitive and technological determinants of knowledge building" are framed by the idea of collective cognitive responsibility in the workplace. (Collective responsibility for creating a suitable learning context would be another way to describe our first principle.) Scardamalia's more extensive framework will in general support a more detailed analysis, but may be less intuitive to work with. Mwanza's eight steps map a given situation to Engeström's activity triangle[50], and are used to generate design requirements. This method is less normative than either Scardamalia or the present work, but also less specific. As with the work on software and community-building best practices mentioned in the introduction, we must defer the task of fully comparing and contrasting these approaches with our own.

A. T. Ariyaratne's essay on *Rural Self Help*[7], one of the foundational writings of the Sarvodaya Shramadana movement in Sri Lanka*, begins:

> *Nobody needs to teach rural communities about "group effort" and "self-help". [...] The real question, therefore, is to examine what are the constraints that exist inhibiting the expression of their group effort and self-help qualities designed to improve*

* HTTP://WWW.SARVODAYA.ORG/ABOUT/PHILOSOPHY/COLLECTED-WORKS-VOL-1

food and nutrition levels, clothing, shelter, health, sanitation, education and cultural life?

We approach peer learning in a similar spirit: it is something we all know how to do, but can't always do well. Intuitively, there are bound to be difficulties for a group of peers studying a subject together, outside a traditional classroom or without a teacher. Indeed, peer learning is different from other forms of group effort, the proverbial "barnraising" for example, in which the persons involved can be presumed to know how to build barns – or at least to know someone who knows, and stand ready to take orders. Typically, peers are not experts in learning, didactics, or in the subject they are studying, and are faced with multiple difficulties associated with putting together knowledge about the subject, assembling a suitable learning strategy, and communicating with one another.

We have five principles, with which we endeavor to both describe the phenomenon of effective peer learning, and to prescribe key aspects of its best practice.

1. **Changing context as a decentered center.**
2. **Meta-learning as a font of knowledge.**
3. **Peers provide feedback that wouldn't be there otherwise.**
4. **Learning is distributed and nonlinear.**
5. **Realize the dream if you can, then wake up!**

Generally the ideas embodied in these principles are not unique to paragogy, indeed, we will try to ground each of them in previously existing literature, while showing their relevance to peer learning. Note that adopting a "model with five principles" should be considered a somewhat ironic act, particularly after reading the paper by Lisewski and Joyce.[100]

Changing context as a decentered center.

In paragogy, we recognize that we are not merely teachers or learners, but are actually co-creating the learning context as a whole. The central role of environment is not unfamiliar in constructivist thinking about education:

Thinking of instruction as an environment gives emphasis to the 'place' or 'space' where learning occurs. At a minimum, a learning environment contains: (1) the learner; (2) a 'setting' or a 'space' wherein the learner acts, using tools and devices, collecting and interpreting information, interacting perhaps with others, etc.[163] (p. 4)

Again, in the paragogical view, the environment should not be taken as "given" but should instead be viewed as co-created by peers.

Meta-learning as a font of knowledge.

Here we are concerned both with efforts to "learn how to learn", and efforts to learn how to support others in their learning efforts[141]. Further, while it is a good idea for any organization to learn its business well[96], learning about learning is especially vital for those in the learning business. In peer learning, that is all of us.

Peers provide feedback that wouldn't be there otherwise.

Learners must not simply seek confirmation of what they already know, they must confront and make sense of difference as part of the learning experience. Clearly, differences pose challenges but these are worth grappling with. Firstly, for psychological reasons: in many domains feedback is only available from peers (but of course peer learning can be relevant in domains like rock climbing and computer programming, where automatic feedback does exist). Secondly, there are philosophical or political reasons to affirm difference. In a space like P2PU, which aims to provide "learning for everyone, by everyone, about almost anything", we can hardly avoid developing an "understanding of social relations without domination in which persons live together in relations of mediation among strangers"[167].

Learning is distributed and nonlinear.

Learning does not go in a straight line[56]. In particular, involvement in co-creating the learning context becomes an important "strand" in the paragogical understanding of peer learning.

Realize the dream if you can, then wake up!

Without clear goals, there will be be nothing to realize. Without critical thinking about goals (leading us to change them), learning is a mostly passive game. Paragogy calls for a strategy of "deliberate practice"[53].

A case study in paragogical evaluation

The paragogy principles provide guidelines on best practices for building successful peer learning experiences. In this section we will apply these principles to evaluate the lessons learned from our work at P2PU as facilitators in 2010–2011. For each of the principles we run through the steps of an After Action Review to look at how well the principle was implemented.

Implementing paragogy *Maybe we should fold the PAR ideas into this subsection.*

How to implement the principles? In this paper we will incorporate a strategy used in the US Army's training programmes: the *After Action Review* (AAR)[8]. As the name indicates, the AAR is used to review training exercises. It is important to note that while one person typically plays the role of evaluator in such a review (and despite the fact that military personnel are differently ranked), the review itself happens among peers, and examines the operations of the unit as a whole. The four steps in an AAR are:

1. Review what was supposed to happen (training plans).
2. Establish what happened.
3. Determine what was right or wrong with what happened.
4. Determine how the task should be done differently the next time.

The stated purpose of the AAR is to "identify strengths and shortcomings in unit planning, preparation, and execution, and guide leaders to accept responsibility for shortcomings and produce a fix." We note here the similarity of the AAR to the action research cycle[94].

Mapping system dynamics and semantics

Review what was supposed to happen. We both organized multiple courses where participants were supposed to interact and learn about the subject matter: Collaborative Lesson Planning Fall 2010 and Winter 2011 (co-organized with Dr. Majorie King); DIY Math; Math for Game Designers; Open Governance and Learning (co-organized with Marisa Ponti); and, in Spring 2011, Shaping P2PU*, which was an "intervention" based on a preliminary version of this section.

Establish what happened. Due to critically low participation, the mathematics courses did not run to completion. Participation in Collaborative Lesson Planning and in Open Governance and Learning was minimal, but sufficient for a conversation to be sustained for the entire 6 week session. The theory of paragogy was born in an effort to understand how to produce successful courses. Finally, as of the time of this writing, 32 people have signed up for Shaping P2PU, but so far participation has been very low.

Determine what was right or wrong with what happened. In the more active courses, there were nice examples of learning by course participants.† Low participation was common across P2PU, as illustrated by Dan Diebolt's graphical analysis, which showed that participation within courses was uneven and falling.‡

Determine how the task should be done differently the next time. Our best experiences as course organizers happened when we were committed to working through the material ourselves. Combining this with gently prompting peers to follow through on their commitments could go a long way towards keeping engagement at a reasonable level – but this only works when commitments are somewhat clear in the first place. The case of Shaping P2PU shows that organizer commitment is not enough. In this case, we feel that further clarification about the aims and intentions of those who are already highly involved in shaping the organization would improve things.

Looking at this another way, the P2PU ecology contains an implicit rubric for learning and engagement: from the time a member signs up for a course, to its completion, peers go through a cycle.§ As we understand this cycle better, it should be possible to evaluate it for quality. Then P2PU could implement more formal check points throughout the cycle, requiring participants to specify, reaffirm, or adapt their commitments in relationship to judgments about quality.

Transparency, accountability, and tone

Review what was supposed to happen. Support for community members was offered as a P2PU course (Course Design Orientation), in mailing lists, via weekly phone calls, in a Q&A issue tracker, and via other informal channels. Participants in courses were presumed to be ready and willing to contribute in a useful fashion.

Establish what happened. Core members do hold themselves accountable, but this behavior is not necessarily transferred or communicated to new members, for whom accountability is low. Course participants frequently disappeared.

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*http://new.p2pu.org/en/groups/p2pu-the-course/

†E.g. http://open-governance-and-learning.posterous.com/

‡http://bit.ly/lqPChA, http://bit.ly/kH89OP

§See https://wiki.mozilla.org/Drumbeat/SoW-engagement-ladder

Determine what was right or wrong with what happened. Core members are doing a lot of work, and the project is moving forward, with grant funding, incorporation, and several new staff positions. Apart from contractual agreements within the nonprofit, community members have little or no accountability to one another. Governance follows a "rough consensus" model (after David Clark's "We reject: kings, presidents and voting. We believe in: rough consensus and running code."*). As implemented at P2PU, the rough consensus model has its strengths, in particular, it helps avoid tyrannies of the minority in the mailing lists. However, there are a number of ways in which rough consensus seems incomplete.

Determine how the task should be done differently the next time. It is typical for online communities to have strictly enforced community norms. It would be helpful to have a concise discussion of these available, together with up to date information on "best practices" for organizers and participants. The current Course Design Handbook provides one starting point, but it falls short of being a complete guide to P2PU.† This sort of resource would be particularly useful for newcomers and people who cannot attend the community telephone calls.

Dealing with problems in a respectful way

Review what was supposed to happen. Discussions about P2PU happen in the community mailing list and other places mentioned above. Bug reports are supposed to go into the Lighthouse tracker.‡

Establish what happened. Discussions about P2PU happen in many places (e.g. in courses). Even within the mailing list, it can be difficult to keep track of the full range of ideas circulating at any given time. There has been some talk about using the Lighthouse tracker for organizational matters, but this hasn't taken off. Earlier experiments, like using a shared spreadsheet to keep track of organization-level tasks, appear to have been undersubscribed.

Determine what was right or wrong with what happened. Apart from development work, it can be hard to tell what's happening around P2PU. Presumably participants who have identified critical and unsolvable problems simply leave. The Q&A tracker and mailing list both provide ways to build factual knowledge, but seem less effective for building *strategic* knowledge.

Determine how the task should be done differently the next time. In a traditional university, there are typically a lot of ways to resolve problems without dropping out. P2PU's new "Help Desk" could, indeed, help with this issue – if people use it.§ The Help Desk and Q&A tracker will also function as a light-weight way to build certain kinds of organizational knowledge. However, there could be more clarity about how to contribute to the process of "shaping P2PU".

Design considerations

Review what was supposed to happen. People are supposed to choose and assemble suitable learning resources (blogs, OER, etc.) for their courses, in which everyone is supposed to learn something.

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*Cf. HTTP://WWW.WIRED.COM/WIRED/ARCHIVE/3.10/IETF_PR.HTML,

†HTTP://WIKI.P2PU.ORG/W/PAGE/27905271/COURSE-DESIGN-HANDBOOK

‡HTTP://P2PU.LIGHTHOUSEAPP.COM/DASHBOARD

§HTTP://NEW.P2PU.ORG/EN/GROUPS/P2PU-HELP-DESK/

Establish what happened. This is essentially what happened, but it is hard to measure when and whether knowledge was gained.

Determine what was right or wrong with what happened. The organization is striving to handle the complexity of life online, for example, by integrating RSS feeds into the site to allow learners to transparently draw in work that they are doing elsewhere. This system is explicitly in an experimental "beta" stage, and quality control has a somewhat precarious meaning in a beta or "eternal beta"; on the other hand, this makes life interesting.

Determine how the task should be done differently the next time. In terms of measuring learning, P2PU would have to work hard to use anything but "participation" as a proxy value. In terms of broader issues of quality control, one serious thought is for P2PU core members (including staff) to use the platform to organize their activities – entirely in the open.

High level roadmap

Review what was supposed to happen. At one time, the high-level vision was arguably a Declaration of Independence from Formal Education.[*] But arguably each participant has their own vision.[†]

Establish what happened. P2PU recently had its first board meeting, but, so far, documentation about the organization's vision and roadmap have not been presented to or affirmed by the user community (nor has the user community presented any stipulations to the organization).

Determine what was right or wrong with what happened. P2PU has made considerable progress (e.g. in the form of successful grant applications), but without more transparency about these efforts, the ability of non-core members to learn from organizational successes is limited. This, of course, limits the ability of volunteers to contribute to further successes of this sort, and may, to some extent, limit the ability of volunteers to "strike off on their own" to pursue alternative development goals.

Determine how the task should be done differently the next time. It is our firm belief that P2PU should work on a public roadmap that leads from now up to the point where the vision is achieved. Both vision and roadmap should be revised as appropriate.

A brief experiment in paragogical design

We recast the ideas above into a set of recommended design principles to use when creating peer learning environments (with specific reference to the PlanetMath case). Here, it is important to look at what each of the paragogical principles says about *interaction* and *change*. (We discussed in broad terms the kinds of interactions that take place within various educational roles in Section 1.1. What changes do such actors accomplish?)

[*] HTTP://WWW.YOUTUBE.COM/WATCH?V=T8WXUBU1W_0#T=12M11S
[†] HTTP://WWW.YOUTUBE.COM/WATCH?V=T8WXUBU1W_0#T=13M12S

Changing the nature of the space As technological facilities develop, new features will be rolled out regularly, moving from basic support for adding and discussing problems and solutions, to automatically linking the problems to related articles in the encyclopedia, all the way through to a recommender system that will mine previous user interactions to suggest useful problems or readings to try next. Although problems will be separate from the encyclopedia, their introduction will change the nature of the space: people will now be able to ask "Which encyclopedia articles are missing problems?" or "Are the encyclopedia articles that are connected to the current problem well-written?", for example. Looking at expository material as part of an ecosystem that contains problems and solutions adds a very useful check on quality. Small independent changes to encyclopedia articles that adjust them to serve the needs of learners are expected.

Changing what I know about myself As individual learners accumulate a track record of uploading and solving problems, asking for and offering help, giving feedback on and modifying encyclopedia articles to suit, etc., they should get a better sense of how they learn best. They should be able to ask for specific kinds of feedback and see how their progress improves (e.g. in formulating proofs or demonstrating an understanding of the concept of a limit). They should be able to keep track of particularly helpful and particularly non-helpful suggestions offered by peers or by the recommender system.

Changing my perspective Hopefully, peer mentors – and system developers – will be able to learn from learner feedback about what's helpful and what's confusing. Feedback should be particularly valuable to learners ("Wow, I didn't even know there was such a thing as spherical trigonometry!"). Ideally, giving and receiving feedback will be comfortable for all involved.

Changing content or connectivity In addition to peer-producing mathematical content, our hope is that learners and other contributors will be able to develop their own semantic queries. Such queries could be used to identify holes in the corpus, or interesting relationships between activity patterns. Not everyone needs to be able to build these queries to use them, e.g. to generate a feed showing all the latest additions of problems having to do with tori or klein bottles.

Changing objectives A shortcoming that was noted in the previous decade of PlanetMath's existence was that support for individual "projects" was not particularly strong. For example, a project to improve the entries about real numbers chose to base its operations on the organisational wiki rather than in PlanetMath itself.* Content quality in PlanetMath has so far been maintained using a "correction system" that points out places where individual articles are mistaken or could be improved. In order to support the production of educational content, it would be good to generalise the correction system to include ranges of content (sub-collections of the encyclopedia or sub-areas of mathematics).

Paragogical Praxis

Here, we can follow a similar procedure to establish recommendations for paragogical praxis itself. In other words, we take the "paragogical principles" as a set of problems to be solved in practice, and present some conjectural solutions. So far, we've developed an understanding of paragogy as a set of dimensions in which learners make changes:

1. *Changing context as a decentered center.* We interact by changing the space.

*HTTP://WIKI.PLANETMATH.ORG/CGI-BIN/WIKI.PL/REAL_NUMBERS_ON_PM

2. *Meta-learning as a font of knowledge.* We interact by changing what we know about ourselves.

3. *Peers provide feedback that wouldn't be there otherwise.* We interact by changing our perspective on things.

4. *Learning is distributed and nonlinear.* We interact by changing the way things connect.

5. *Realize the dream if you can, then wake up!* We interact by changing our objectives.

However, there is a tension between the whimsical, nonlinear, non-coercive modality of peer production, in which all of these thing can happen (in theory, or in a best-case scenario) as and when people like – and the nitty-gritty practical, action-oriented approach to learning and adaptation, in which we need *to get things done*. These are our points of advice on how to deal with this tension.

Develop empirical studies and a critical apparatus. The challenge is to find or create learning environments that we can analyze and critique along various relevant dimensions ("are people learning", "is the system growing and improving", etc.). In Chapter 3.1, we begin to establish a critical apparatus of this nature in the concrete case of mathematics learning. More generally, *language* and *metadata* are typically what are available for us to study.

Find companions for the journey. Not all peer learning experiences are created equal, particularly in terms of how deeply interested the participants are in understanding the process itself. Procedural investigations may be pejoratively deemed "navel-gazing" by those who are not interested in them. Sloterdijk considers non-pejorative variations on the theme, as part of his massive project to understand coexistence, beginning with life in the womb.[139] This project should give rise to a philosophical discourse that we can learn from as participants.

Work with real users. Some institutions are incorporating trendy networked learning techniques into their pedagogy, and students are generally far ahead of this trend. Students are intensely interested in working systems. On the other hand, proselytizing more staid institutional players will generally result in a clash, when the natural conservatism in extant pedagogical and business models senses something "new and different" in paragogy. From an economic perspective, it may be less important to convince institutions to do things "our way" than it is for us to create a new market.[35]

Study and build nonlinear interfaces. We need systems that support nonlinearity in writing, reading, and editing. Natural questions like "give me all of the problems in multivariable calculus that don't yet have solutions" should be easy to get answers to; and new queries should be equally easy to ask. Contemporary technologies like SPARQL, Git, and Etherpad, among others, can be brought to bear, but there will be further design problems to solve. We do not think of these tools in terms of "technological determinism" or even in terms of "provisionism" in the educational context[26], but rather, as part of a workshop or laboratory for open experimentation with nonlinear effects. A

Limit philosophizing. Philosophical talk is not going to solve our practical problems, but we may be better able to understand what the practical problems are through this discussion.

Let me then briefly summarize what we've established as a paragogical praxis within this thesis project. This quote from Young sums up our general sentiments:

> *If institutional change is possible at all, it must begin from intervening in the contradictions and tensions of existing society. No telos of the final society exists, moreover; society understood as a moving and contradictory process implies that change for the better is always possible and always necessary.*[167]

The rebuild, reworking an existing resource, opening up to another audience, working within P2PU vs PlanetMath vs Peeragogy project... (*say more here, probably I could spend a page or two on this*).

Reflecting on education-relevant potential of new media, Martin Weller writes: "*It is [...] no easy task to adopt a decentralised model, since it will require massive procedural, economic and professional change in higher education*"[158]. We would argue that what's new here is not simply a disruptive force in the traditional educational landscape: there is also a compelling chance to understand learning better. We hope that further developments in paragogy can contribute to this process in a practical way.

Finally, it is an elegant and intuitive idea to integrate user input into a rich interactive learning environment, but students, and those who care about them, will typically be less moved by some potential future peer production goal than by tangible learning outcomes in the present. Reuse of contributor materials should be taught and practiced as a creative and immediate art form (the art of remix). Knowledge artifacts and learning environments may indeed be built through such a process, but in order to qualify as *paragogy*, they should be built by users, not for them.

☛ Literature Review

I should describe briefly the "conceit" I use here, dividing the literature review into five sections based on ideas from Nowak, paragogy, Flusser, etc.

Mathematical thinking

How do people understand mathematics? This is the key issue for Thurston's "On Proof and Progess in Mathematics".[153] He had this admonishment for the mathematics community:

> We need to focus far more energy on understanding and explaining the basic mental infrastructure of mathematics – with consequently less energy on the most recent results.

human language
vision, spatial sense, kinesthetic sense
logic and deduction
intuition, association, and metaphor
stimulus/response
process and time

Table 14: Critera from William Thurston

Note the emphasis here on the *human experience* of mathematics. While it is possible to mechanize certain aspects of a "formal theory of patterns", people, by contrast, make use of a wide, embodied channel. Thurston explains that human insights in mathematics draw upon these dimensions, among others (Table 14).

He also emphasizes the importance of a social dimension in building a successful research programme. The key to Thurston's argument is that the way people think about mathematics is vastly different from what we see in the formal, narrow, channel that typifies *papers* on mathematics. In general, communicating technical details in a manner that relies on specialized knowledge is difficult for humans.

There are thus certain *ergonomic* challenges to consider when building a system for mathematical communication. These tend to apply *a fortiori* in computer programming, since programs are designed, in the first instance, to be interpreted by a machine, and only secondarily by another human. Programming language design, and cybernetics, more broadly, are fields that seek to make it easier for humans to express (and communicate) formal notions.

PlanetMath has historically been constrained largely (but by no means entirely) to linguistic features of mathematical communication. We are certainly not constrained to follow the same level of rigor as formal mathematics or programming. As the new software system evolves (with its own requirements for "correctness" at the syntax level if nothing else!), we have the opportunity to build an increasingly sophisticated model of mathematical behavior, broadly construed.

The heart of mathematics *Problem solving* is generally considered to be "the heart of mathematics".[62;120] This speaks a dual role for problem solving activities: they typically cross back and forth between the informal, experiental, human ways of thinking about a problem, and a more "formal" result (solution, proof, etc).

Thus, the transition from a *reference resource* to a *learning environment* should bring into play a new set of social, spacial, and motivational patterns, among a new cohort of contributors/participants. Considering that more people study (and use) school- and university-level mathematics than research-level mathematics, we may well expect that this new cohort of contributors will be larger than the old one, with a generally beneficial effect on both the quantity and quality of materials hosted on the site. Support for problem solving interactions is a key "ergonomic" B
step for the PlanetMath platform, since it brings it into closer alignment with what people *do* when they do mathematics.

Learning, Doing, and Researching Mathematics, online Online environments are often as *feature*-specific as they are *content*-specific. Two very well-known contemporary models are *forums* and *wikis*. These models can be further hybridized and specialized. For example, YouTube, for example, is ultimately an online forum for sharing videos. *Q&A sites* are essentially forums, often with some additionaly wiki-like features, for sharing factual knowledge.

For school-level mathematics and other subjects, a well known online learning environment is provided by Khan Academy, which is built around instructional videos on YouTube, and which suplements these with work-book style exercises. And, yet despite much play among bloggers and in the popular press, research is limited. In the trade press, we see comments like "My concern is that if you're still relying on lecture as your primary mode of getting content across, you haven't done anything to shift the type of learning that's occurring."[9] And, importantly, "It's a thing you do in the context of an overarching pedagogy, not the pedagogy itself." Nevertheless, there are signs that a flipped-classroom pedagogy is indeed evolving.[22;77]

In the context of another, more inherently collaborative, online mathematics learning environment (the rather *well*-researched Virtual Math Teams project), Gerry Stahl notes that people studying online collaboration are in an interesting position, since they have access to the same information that participants in these environments have.[146] This is of course not the case for persons researching *offline* interaction, who typically need to apply instruments like talk-aloud protocols to find out what people are thinking. And yet, we see in the nascent field of *learning* *analytics* that the system "in the large" may contain contain more information than any individual participant sees.

One example of such research in the mathematics context is work by Ursula Martin and Alison Pease. Martin and Pease wrote a paper called "Seventy four minutes of mathematics" that talks about *Polymath III*, the third segment of an collaborative project in online research mathematics.[118] They categorized comments into *concepts*, *examples*, *conjectures*, and *proofs*: a nice concise way to think about mathematical communication.

On a whole, Polymath does seem to have had one flaw:

> In the end, far fewer individuals than hoped, almost all experienced mathematicians, actually made substantial contributions.[151;59]

Part of the idea with PlanetMath is to take the best of both worlds (or, better, all worlds), and integrate them into one platform. The legacy platform has always been a combination of forum and wiki. One of the key questions here is how to make participation more meaningful to more people.

Making mathematical meanings There is a paper by Constant Leung[99] that summarizes some key points about learning mathematical language.

- When you learn mathematics, you have to learn both *formal* and *semantic* features ("core" and "non-core")
- You have to *think through* the concepts involved.
- Learning it is incremental, in that meanings expand and develop by building on one another.

This will be quite important for my work, since it says what learning vocabulary *does*. In particular, Leung's work suggests that it would not be entirely unreasonable to use "vocabulary learning" as a model for "learning".

However, before we consider such a reduction, we should also look at least briefly the other way, and consider Halliday's writing on learning, itself, as "language learning".[60]

Halliday's conception of language The idea that a language contains terminology *and* a theory of human experience *and* an enactment of interpersonal relationships[61] (p. 50) is consistent with Leung's view, above. It is also consistent with Thurston's perspectives on the mathematical experience.

Writing on students learning science, Halliday said:

> Where children are most likely to be put off is in the early years of secondary school, when they first come face to face with the language of their "subjects" – the disciplines. Here they meet with unfamiliar forms of discourse; and since these often contain numbers of technical terms, when we first reflect on scientific language we usually think of these as the main, perhaps the only, source of the difficulty. [...] But they are not the whole story. The distinctive quality of scientific language lies in the lexicogrammar (the "wording") as a whole, and any response it engenders in the reader is a response to the total patterns of the discourse.[61] (pp. 200–201)

This view heakens forward to critique of mathematics education made by Marvin Minsky that we will discuss in detail in Chapter 3.1. But the power of this critique is amplified when we consider Halliday's perspective that, for humans, learning how to relate to others, and learning how to think are deeply and intristically linguistic processes.[60] We shall see how this applies, also, to learning mathematics.

Learning mathematics from examples (and by doing) Halliday's notion of learning as language learning has can in some sense be taken further *language learning as pattern recognition*.[164;128] Indeed, if we accept that claim, then we've essentially come "full-circle" and can assert that mathematics (as a theory of patterns) is more or less essential for successful learning. But this then begs the question: *how do people learn mathematics?*

A fairly classic paper by Zhu and Simon demonstrates the feasibility of a learning-by-example approach (in this case, the examples are spelled out in texts).[169] This may not come as a surprise for persons steeped in the pedagogies of distance learning. The underlying model is also fairly simple, that is to say, good for rote learning.[157]

But where in the process of following examples does one learn to be creative? We might try and contrast the idea of learning from examples with the so-called "Moore" (named after mathematician* R. L. Moore) or "discovery" method. In such a course of study, there is indeed a preselected array of "examples" (theorems to prove), but the student is meant to supply all of the proofs by him or herself. We can easily see that to prove (non-trivial) theorems, one would need "a reasonable degree of sophistication concerning the logic of verification and concept formation".[148] Let's assume that one attains that skill by practice, for the moment, without further differentiation.

The effects of problem solving One might (eventually) compare evidence which suggests that a shift to a "community oriented problem-based curriculum" was beneficial to medical students[150]. For me to do this, I'll need an *environment* where some realistic and relevant problem solving is possible, and I need to see some *activity* there, so that I can gather some *data*, and then I need to *analyse* it. With this, the ground would be nicely prepared for a "contribution to knowledge" – but at the moment (October 10, 2012) that particular epistemic process hasn't run to full completion! I do however have some clues about relevant/associated *meanings* (as per Rheinberger) that are coming into focus now.

Collaborative knowledge-building

Tacit preunderstanding
Personal belief (∗)
Personal comprehension (∗∗)

Personal belief
Public statements
Argumentation and rationale
Shared understanding
Collaborative knowledge
Cultural artifacts

Table 15: Critera from Gerry Stahl

"A model of collaborative knowledge building" is a short paper by Gerry Stahl, in which he talks about "learning as a social process of knowledge building."[144] The basic social perspective on how people create knowledge (and how this can inform educational praxis) had been developed earlier by Bereiter and Scardamalia.[17;18;21] The aim of Stahl's paper is to add a cognitive dimension to this theory. In particular, the paper aims to mesh "personal understanding" (considered as a cycle) with "social knowledge building" (considered as another cycle). Table 15 is a minimal sketch of Stahl's diagram.

We saw in Section 1.1 that there are often convincing reasons to disaggregate cycles in order to recontextualize their elements as dimensions. For example, we saw how breaking down a social interaction scenario into roles using an "experiential" formula helped us to "segment" or "facetize" the education sector. C

*racist

Stahl's perspective on learning and knowledge building is useful, whether or not we accept the strong claim that there are two interlocking cycles whereby personal belief ($*$) becomes personal comprehension ($**$) through engagement in a social knowledge building cycle – or simply two sets of important related dimensions associated with learning and knowledge creation.

In short, viewing PlanetMath as a learning environment that encompasses interactions among humans, technology and domain-specific artifacts[19] (mathematical terms, articles, problems and solutions, etc.), we join the process of learning to a social context, more particularly, to a knowledge building process in which learner-produced materials have an important role to play. For example, questions can help the community advance its collective understanding, as particpants engage in direct problem solving and/or seek to understand problems at deeper levels.[20]

Modeling the process of learning "Modeling the process, not the product of learning" is the first chapter in the second volume of *Computers as cognitive tools*[3] (pp. 3–28), published around the same time as Stahl's short paper. In this chapter, Akhras and Self look for ways to model learning within a *constructivist mindset*. The authors, somewhat improbably, position their work as complementary, not contradictory, to the classic approach to student modeling in intelligent tutoring system. The classic approach seeks to model domain knowledge in certain objective representations, which the student is then supposed to learn (e.g. Wenger[159]). The constructivist view is, rather, that knowledge, as an inherently process-based phenomenon, "cannot be objectively defined and statically represented." Akhras and Self are therefor more concerned with modeling the *context*, *activity*, and *temporal factors* of learning than with what is "actually" in the learner's mind. The dimensions mentioned by Stahl and quoted above would be likely candidates to examine through this lens.

The basic constructivist notion that knowledge builds on itself suggests some useful directions for learner modeling, particularly in a knowledge-rich environment like PlanetMath (or to a lesser extent, Stahl's VMTs). In a fairly sophisticated and delicate compromise between the classic approach to learner modeling and the constructive view, we might think of each learner's knowledge as an evolving *weighted subgraph* of the community's collective knowledge, also viewed as an evolving network structure – itself the sum of participants' knowledge structures. A simpler version of the same idea would just give increasing weight to individual concepts used by a learner, corresponding to the constructive notion that the more places a person has reason to use a given idea, the more meaningful it becomes to them.

Tools for thought A key part of the argument I'm making in this thesis is that we don't actually know as yet which conditions are the most conducive to learning. We can certainly learn something from the "calculator debate" about how to do research on technology-based classroom interventions.[67]

But perhaps we do not yet have a clear perspective on the epistemic history of mathematical artifacts. In modern times, mathematicians, engineers, and students have collectively moved from slide rules and logarithmic tables[43] to calculators to computer algebra systems.[165] Meta-analyses show an almost across-the-board *positive* effect on mathematics learners associated with classroom calculator use[49;66] – in despite of which, the calculator debate continues, in a political context.[*]

The (multidimensional) Zone of Proximal Development One of the interesting things about the "tools for thought" mentioned above is that none of them are (explicitly) tools for collaboration. Nevertheless, if we follow Stahl, then as *cultural artifacts* that resulted from a *collaborative*

[*] HTTP://WWW.THEYWORKFORYOU.COM/WHALL/?ID=2011-11-30A.329.0

knowledge building process, they are poised to feed back into further knowledge building "cycles".

These tools also point to a very interesting and broad pattern, and indeed technologies for collaboration (from Mersenne onwards[11]) illustrate the same effect. *As certain basic problems get easier to solve, people will move on to more difficult ones.*

Can we in qualify (or quantify) this statement? One tack would use Vygotsky's notion of the *Zone of Proximal Development* (see Zaretskii[168]), initially developed, and most frequently applied, within the field of child psychology.

> The ZPD can [...] be seen not as a plane comprised of the ways in which a child and adult work together on the former's assimilation of subject-specific content, but as a sphere formed by the aggregate of vectors that pass through a "point" of difficulty and that delineate a child's diverse possible areas of development (the zones of potential personality and cognitive changes, among others).[168] ❖

As the learner moves within this multi-dimensional "zone", he or she may overcome particular difficulties with the help of competant others. Once a given challenge can be satisfactorily D met with help, it can often later be managed independently. It is indeed compelling to consider generalizations and extensions of this idea through the philosophical work of Peter Sloterdijk on co-existence, beginning with life in the womb[138;139]). For our current purposes it is sufficient to note that the conjoined "cycles" from Table 15 are a fairly convincing example of Sloterdijk's *microsphere*:

> We are in a microsphere whenever we are
>
> – firstly in the intercordial space
> – secondly in the interfacial sphere
> – thirdly in the field of "magical" binding forces and hypnotic effects of closeness
> – fourthly in immanence, that is to say in the interior of the absolute mother and its postnatal metaphorizations
> – fifthy in the co-dyad, or the placental doubling and its successors
> – sixthly in the care of the irremovable companion and its metamorphoses
> – seventhly in the resonant space of the welcoming maternal voice and its messianic-evangelistic-artistic duplications[139] (p. 540)

This reading, should, of course, underscore the theoretical and practical benefits of disaggregation, and the move to what Sloterdijk terms "foams", or, in our context, clusters of collaborating peers.

Peer instruction *Peer instruction* is a teaching practice developed by Eric Mazur and colleagues for use in their physics classrooms[44]. The basic idea is that discussion amongst peers – in short, a more active form of engagement than that usually found in the traditional lecture/homework/test format – *improves performance* (and, presumably, learning). This is useful but not unexpected because it shows that this particular form of peer learning *works*. The basic model is "turn to your neighbor" and discuss. Similar techniques have been successfully applied with an explicit focus on problem solving.[64;65]

What else is like this? Howard Rheingold's notion of "smart mobs"[133] ("people who are able to act in concert even if they don't know each other") and Richard P. Gabriel's related notion of "mob software"[57] go considerably further in the direction of disaggregation, while retaining the need for some sort of "glue" to hold things together. This is the direction we turn next.

The Ecological Approach in learning design

"The Ecological Approach to the Design of E-Learning Environment" by Gordon McCalla, 2004, talks about using 'stigmergic' metadata as part of a learning environment. It has been followed up with some experimental work by Champaign and Cohen.

In some sens, with more of a *data* than *metadata* driven approach), this is exactly the route we've taken with PlanetMath/Planetary. It is particularly interesting to think about how/why this relates to *learning*, and not only to general Computer Mediated Communication (CMC) and Computer Supported Collaborative Work.

Probably worth reviewing Nardi and O'Day briefly here.[111]

And, if I'm going to talk about "the ecological approach" to anything, I should probably include some Gregory Bateson.

Words as markers One popular website that uses the Regressive Imagery Dictionary[102] which has had fairly significant uptake in epirical study of arts and humanities subjects[15], along with the Linguistic Inquiry and Word Count system[119] as a way to diagnose things like "mood while writing". There are some obvious questions about the validity of this approach[72], which should apply with a few changes to our work as well.

Simulations and experiments Work by John Champaign at the University of Waterloo has developed the idea of peer-tutoring "in the large"[32;31]. So far his proofs of concept have mainly used simulated students, but his work shows the relevance and feasibility of large-scale "ecological" approaches in building learning environments, in the sense of McCalla. One can compare Hummel *et al.*'s work on sequencing learning activities[68]. Another project with a major partner at OU is looking at how learning materials can be assembled using a light-weight "peer-production" model based on link-sharing and use-tracking[27].

Predicting the perceived value of quality of online mathematics contributions "*Reputation variables* where better predictors of perceived quality in questions than answers." (my emphasis)

"The difference between a good and bad answer may have more to do with a signle insightful idea than overall expertise and reputation." (Note the Black Swan / Killing Joke effect.)

I'm perhaps more interested in quality metrics that apply to content than to users, but in general quality metrics that allow the system to make good recommendations are always relevant (at least to future work and direction, if nothing else).

(Note the "othering" of the mathematical population in the introduction to this paper – the idea is that assessing mathematical quality is something that only mathematicians can do – as if their "passionate ethic identification" (Žižek) was all that could allow this...)

Discovering value from community activity This is a 2012 paper by Anderson and Huttenlocher et al.

Their idea: questions *together with the various answers* is what brings lasting value to a Q&A site. This is the interesting idea in the paper – it comes together with some concrete points that work incredibly well for prediction (number of answers, sum of scores, number of previous questions, etc).

There are some interesting patterns in this literature. If PlanetMath is going to be, in part, a question answering site, then we will have to deal with them. Topics like:

- motivations (studied)
- ranking of experts (an obsession)
- "information retrieval"

In general we should have a look at Benkler's ideas about *cohesion*, which are also similar to e.g. Taleb on narration. People have information needs that aren't strictly procedural ("What should I do in this case?").

Note that Stack Overflow is sort of like *bullet time IRC*. One hour for an answer: not bad.

Detecting learning moment by moment This is cited in the "It's about time" bibliography, and provides a survey of ideas in this area. E.g. Beck et al, 2008 looks interesting. They talk about *Bayesian knowledge tracing* (as if it is common knowledge).

The idea seems to be about looking for a way to see *when* something is learned within a given workflow.

"Spikyness" per concept and per learner... likeliness for a success with a given concept or problem would indeed be useful for recommendation generation. They cite other papers that talk about how you can use the "quantity learned" as a predictor.

2008: Does help help? How who should practice. Proc. 9th International Conf. on ITS.

The analysis is interesting in that it uses *future* interactions as well as past ones. It is indeed useful to try and give a "valence" to a given usage of a term, since in general it may be incorrect or unknowing (e.g. when asking a question – some of the stuff by Claudia Wagner et al just looks at whether a question mark appears!).

The tool they used is "RapidMiner" from Mierswa, 2006.

They talk about "regression trees" (Weka), which, although interesting, seem to require something stronger than zero/one data.

Question: how did they ground truth their model?

The PlanetMath workplan This describes some upcoming efforts for people involved with the PlanetMath organization. It's interesting for me because I can check which, if any, of the activities listed there map into my decomposition figures. Something like this:

Context $\sim$ Software (which ends up creating the context where we work, and the actions that are available within it)

Engagement $\sim$ Community, since clearly there have to be people to engage in order for this kind of stuff to grow

Quality $\sim$ Organization, noting that "inreach" and "outreach" tend to yeild two different kinds of content

Structure $\sim$ Catalog, which shows what links to what – not just locally, but in a cross-cutting sense, using the PM Xi idea.

Heuristic $\sim$ Content, which basically defines the topics of interest, the kinds of things you can do, and the ways of thinking about them

Feature development I've recorded a bunch of tickets that talk about the things *someone* would like to see appearing within the PlanetMath software. Many of these won't be finished, but *these too* can be analysed to see whether they fit into the Nowak/Flusser/etc. framework. (See comments about software in the discussion of Thurston's paper.)

Clearly, the accretion of requests like this is an example of stigmergy in action.

What did we learn or change? The website is *nearly* ready, and could in theory be deployed "any time". THe process of building the website has been interesting and involving, and I think I would likely want to go about it a slightly different way next time, focusing more on the "must haves" of data and basic interactions from the start. On the other hand: as it turned out, this is definitely an example of "bricolage", so that partly means that we can't fully understand how it works...

IF all the pieces come together, we'll have a very good way to understand better the *factors of production* in the knowledge domain.

Dictionary versus Encyclopedia Umberto Eco (001.51 ECO), also critiqued by Andrei Cornea talks about the *Porphyry tree* versus the *rhizome/labyrinth*. What's interesting here is the idea of weaker thought and integrations thereof – whether by narrative or by something else.

The "question" is whether there are fixed things that have to be learned – or, if, instead, "knowledge" is all free-flowing and subvertible.

The idea of an encyclopedia as "just local knowledge" is reminiscent of E. Ostrom's work (as opposed to essential/Platonic forms).

Blending in *genealogical* (cf. Kin selection) and *historical* approaches... is interesting.

The relevance: there are very clearly gaps in the language that we use (e.g. Drupal) which can nevertheless be filled in slowly using a modeling approach (my five categories, my own tree imposed on the network).

One cannot unwind a net: compare Flusser. But also: look at Sloterdijk's *foams* as being very "post-net".

But systematic knowledge bases are different, right? Things like Free Software, Open Source intelligence databases, and just about any shared knowledge base would tend to have some similar aspects (without necessarily having an explicit "learning" or "problem solving" aspect, though paragogy would argue that one or the other is probably taking place below the surface).

The idea of connecting open heterogeneous contributions to one another via a meticulously managed core corpus isn't 100% new, but it is in many ways different from MathOverflow and Wikipedia – where, in particular, *you don't see the links*. In other words, PlanetMath has the opportunity to be more "rhizomatic", even if that idea is used (as we saw in the case of d'Alembert) together with treelike structures.

Architectures for collaboration

"An architecture for collaborative math and science digital libraries" was described by Aaron Krowne in his 2003 thesis (AKA "the PlanetMath thesis"). It describes where things were at with the system in 2003, and how they got that way.

What have we learned since then? One interesting point is that Drupal has "taken off" and become usable as a generic basis for websites (cf. "The Definitive Guide to Drupal 7").

Also, various aspects of our learning orientation and corresponding research have become more clear – in particular, we see how some focused attention to "learning" can help with PlanetMath's mission.

The PlanetMath Encyclopedia I wrote a paper about this. The encyclopedia is one of the main sources of data for my analysis. In particular, since it is a *network structure*, one can apply standard techniques like PageRank and various SNA stuff (cf. datamining approaches mentioned already) to use the network itself as an an analysis tool, e.g. to give a measure of *depth*.

Note that there is already a huge body of literature about another crowdsourced encyclopedia – namely Wikipedia. When is it appropriate to turn to this literature? (E.g.: "Have people applied SNA techniques to Wikipedia data, and if so, how?")

The Planetary System The main idea was to build a new and extensible "modern" system for PlanetMath, making it into a setting where we can do experiments. In other words, we want a platform that can support and adapt to a range of interactions, providing a "model of behavior".

The basic system is just about ready to use, so I can start gathering data in a large-scale field trial, and hopefully some smaller scale studies as well.

Note: The Planetary project has underscored the huge amount of (often pioneering) work that Aaron Krowne did with Noösphere.

The *exciting idea* is to be able to adapt as we go. For instance, our current platform allows us to integrate with a decade worth of work at by the Knowledge and Reasoning for Content (KWARC) research group at Jacobs University, Bremen. (LaTeXML, JOBAD, TNTBase) Drupal 7 allows us to build custom features (like the Collections module) fairly easily, and serves as a basic layer or glue to hold these other parts together. (The Definitive Guide to Drupal 7, Pro Drupal 7 Development)

Detecting mathematics learning online The main idea in this paper was to detect mathematics learning as it occurs. The paper gave some ideas about how that might be done (vocabulary acquisition, but also modeling heuristics that would be interesting to detect/support). By now, this particular paper is mostly background for my thesis, but it is interesting to read other related papers and see what kinds of techniques they use.

Artifacts and Organizational Change Mietten and Virkkunen, 2005, look for ways for *new practices* to emerge, as opposed to theories based on "routine." The literature of epistemic objects goes through Knorr-Cetina 1997 and perhaps 2001 as well as Rheinberger. The main point is that the object embodies "the open ended."

So, in particular, "workflow" can be understood to be something that can be creatively reconfigured. For me, the *code* is very much an epistemic object in this sense. Furthermore, it's precisely because I'm thinking about routines and lack thereof that I was interested in these guys.

What produces knowledge in mathematics? What produces change in the associated software?

> To crack open the previously hidden self-evidence and 'givenness' of ways of acting and to transform the activity, the routines themselves must be made into an object of enquiry, that is, into an epistemic object.

(Just for example: I have frequently insisted on there being a *roadmap* in the projects I've participated in.)

Learning and epistemology The main idea in my paper (in progress) with Tim is to devise techniques to assess the learning relevance of features of platforms for collaboration.

This will hopefully turn into the first of two major "strands" in my thesis. When is it good to sit and think? When is it good to ask for help? What interventions are useful? The key is to have some mechanism that helps answer questions like this.

(Noting, for instance, that people say that you must have *failure* and *losses* for learning to happen.)

Tim's technique Tim's paper "Estimation of System Reliability Using a Semi-Parametric Model" is one of the "why now?" things.

Whereas this paper models rate of failure and its causes, my work is (at least ostensibly – see previous paragraph) more concerned with rate of "success" and its causes.

The paper indicates that the method applies to "any blip treatment", but it focuses on relating a failure event to future failures. In the PlanetMath case, we would want a variety of generalizations: relating one sequence of zero/one data to another (intervention vs learning outcome), or if we can manage it, one sequence of scaled event data to another (some parameterized intervention to a parameterized outcome, like depth).

Or we could go even further, with the idea of "error codes" so that machines could fail in possibly several distinct ways at each point of failure. That would correspond to possibly learning several different words with each post – which would pose significant advantages since we can then take geometric or landscape features into account (i.e. it is more likely to learn terms that are nearby to terms that you already know).

The tricky thing here is that all individuals are rather different from one another, so the model will at least need this extension. The *wrong* way to approach this issue is through binning and Markov models.

Time is precious This is mentioned in the "It's about time" bibliography, and provides a nice survey (with an appropirate title). Basically, it gives a guide to various literature that deals with temporal data.

It references Langley 1999 "Strategies for theorizing from process data" and Poole 2000, "Organizational change and innovation processes". Also Poole and De Sanctis 2004, "Structuraction theory in informaiton systems research".

The basic idea is to contrast the "variable" and "events-based" models. Some "semi-Markov" process model might be a competitor to Tim's approach?

The paper also reviews different kinds of causality. Details in Strijbos 2006 "Content analysis" and Wever 2006 "Content analysis schemes".

The paper also talks about "narrative explanations", citing Abell 2004 – an interesting topic, given how much NNT rants about narration.

"Patterns in sequence data" by Abbot 1990 is another survey, and a sort of primer on sequence methods. (But, again, a "sequence" is not the same as a "series".)

One interesting thought described here: using *kinship* style closeness instead of *temporal* closeness.

Note: our data has aspects of both recurrent and non-recurrent processes.

Tech notes I'm meaning to write up some notes that I made when talking with Lucas about our work on Planetary. These notes will cover things like setting up the *groups* mechanism, ideas about collections, planned improvements to the UI, and so forth. This is interesting because it can show how our "model" works at present.

What's new? Why is this suddenly possible? In my Synopsis I mentioned that there could be a good *formal* argument drawing on *Institutional Change and Information Production* by Fabio Landini – with a little bit of Marx thrown in.

Putting stuff out there where people can contribute opens up a chance for a lot of "end user contributions" (compare: tail recursion optimization!). If you want to build a big *knowledge base*, you might want to make it fun (or at least worthwhile) for people to contribute. With PlanetMath, there's that opportunity – to get more contributions by making contribution more meaningful for participants. So, we get a "surplus" in exchange for a "subsidy".

What works? In general, what balance between "communication" of research work and "production" thereof? How distinct should these things be?

If I am building a network as I write this literature survey (which I think I am, or anyway, the ingredients of one), could I also apply my analysis schemes to this network? I.e. to my own thesis? What would this sort of generalization take?

What else should we change? Deploying some aspect of the *pomodoro technique* will help me personally, I think. As for the community and the development team, we should stick with the roadmap development process: build up an increasingly large knowledge base of *dev* (and *sysadmin*) stuff, find more ways to reduce the impact of individual *bottlenecks*. Better understanding of all of the technical features in this space will help with that. Eventually, outreach to other people who might be interested, in some capacity, will be useful... especially if we can be clear about how they can usefully be involved.

Dynamic memory This is a book by R. Schank that provides a particular model of what "learning" is. Maybe it will fit in with my "distributed and nonlinear" idea.

Measuring learning efficacy The main idea was already in one of the papers by the Carnegie Mellon guys (with H. A. Simon, I think?) Can we validate claims about learning quality? (Perhaps ironic that the Cognitive Tutor receives *skepticism* on this point, at least in the popular press.)

This is relevant for all kinds of places that charge money for their teaching services (gauging student satisfaction, and learning too – they aren't always the same). And relevant to PlanetMath, if it is really going to deliver a good product.

In short, a major inspiration for me has been to build an environment that I'm confident will help *me* learn.

Building large knowledge bases (... By mass collaboration.) This is a paper by Richardson and Domingos, 2003. They have an interesting Figure 1 (an input output diagram). At least in hindsight, this paper seems a bit naive. (Were they aware of Wikipedia?) It's interesting, though, because of the motivation they describe, even if they were not successful at implementing it.

People *do* end up asing one another for help on the same topics over and over again. And it *would* be cool if that could be staved off through automation. (Google's Q&A services do this, by combining search with asking a new question – another good figure to insert.)

Their framework of six points (Quality, Consistency, Relevance, Scalability, and Motivation) seems good – even if their implementation failed to realize these, or if other points are needed.

Thus: interesting as a sort of *test suite*: we should make sure that what we're doing with PlanetMath is *no worse* than what these guys did.

The Hyperreal Dictionary of Mathematics, reconsidered Starting in 2003, I wrote about somewhat QED-like plans for a "Hyperreal Dictionary of Mathematics", i.e.

> a free and comprehensive database of mathematics together with tools that enable efficient interaction with this database [...] To introduce more colorful terms, the goal of the HDM project is to make a *simulation* of mathematics that surpasses 'real' mathematics in possibility.* (my$_{2012}$ emphasis)

While we can expect the current project to contribute significantly, in a material way, to this overall effort, it will of course not *directly* impact the formal, AI-oriented bottom line, so much as the more human-oriented one. Nevertheless, as we saw in Section 2.1, without accomodating and supporting the relevant human factors, there is little reason to believe a massive database- or AI-building project should be successful.

Building Online Communities

In the chapter "Starting New Online Communities" by Resnick, Konstant, et al., in *Building Successful Online Communities*, the authors provide a bunch of advince about things that can make an online community desirable. In fact, these tips are relevant not only for *new* communities.

For me, it seems especially interesting to try and cluster their large number of recommendations along the lines of the MathOverflow paper, or my paragogy categories, to see what the advice means in terms of *motivation*, *learning*, and so on. See also M. Rowe on the health of *existing* communities.

The state of the world I'm hoping I can find some macroeconomic or sociological reports of mathematical literacy rates and any other large-scale trends that are relevant in this area.

It would be very interesting to see what the associated power laws look like. Maybe they would give some clues, like "Scaling PlanetMath through introducing *X* feature would be likely to work!" In short, what's the demand?

(I have some preliminary results on this from my survey of PlanetMath users, but those are very localized.)

Zipf/Pareto This is the basic idea of the "trumpet" or the "long tail". I'm currently reading up on some related topics (fat tails!) in *The Black Swan*, but I also have my own ideas about efficient networks and systems. The 1949 book by Zipf is a good starting place[171], from which other contemporary works derive.[4]

Think about the way *this* network grows. There are a few central nodes and lots of peripheral ones. If there were lots of central nodes and only a few peripheral ones, that wouldn't make as much sense! (Cf. Tenenbaum on "The large-scale structure of semantic networks: stastical analyses and a model of semantic growth", 2005.)

Furthermore, the larger the network gets, the more paths there are that *don't need to go through the center*, but can get from "here" to "there" by shorter chains. Thus the greater efficiencies of larger networks – which hopefully the underlying corpus of PlanetMath *would be* after more different types of nodes and links get attached.

*HTTP://WIKI.PLANETMATH.ORG/CGI-BIN/WIKI.PL/ORIGINAL_HDM_ESSAY

Social Engineering "Users who did not identify themselves on the site displayed different behavior: they had lower online reputation and did not participate as much." – Tausczik and Pennebaker.

In general: small pieces and integration thereof (Benkler).

Participation in an online mathematics community This paper by Tausczik and Pennebaker uses a framework from Dholakia:

- Getting information
- Giving information
- Reputation building
- Relationship development
- Recreation
- Self-discovery
- and Constructive Feedback

It cites a range of work that tries to predict *participation*. This sort of stuff could dovetail nicely with the Nowak stuff. (And, remember, the spikiness of some people's participation in PM is precisely what makes the analysis of this data difficult or, anyway, different from the electric grid data.)

- Joyce and Kraut, "Predicting continued participation in newsgroups"
- Burke, Marlow, Lento, "Feed me", Proc. CHI 2009

Specific indicators were used to study how important these factors were (the LIWC, which is similar to 750words.com).

It in interesting to look at the relationship between 'learning' and 'participation', or 'learning' and 'motivation'. And, again, specifically thinking about the mathematics context.

Note that MathOverflow is strictly research level, but the authors mention math.stackexchange.com as a place for non-research math questions and answers. I'll have to check that out!

Code motivations Possibly some of the same motivations that apply to working on PlanetMath also apply to work on Planetary. For me: do I get motivated by thinking about success? Failure? Helping? Learning? Fame? Fortune? Getting to know other people better? Having fun? Learning more about myself? Getting useful feedback from others?

I guess the PAR can provide some evidence/access to different possible motivations.

My hypothesis ... is that people will still be solving problems once this project is "complete" – just a different kind of problems. Or, rather, a new order of problems. So that the new frontier isn't the Congo or the West, but in human minds, "cyberspace", maybe also in cities, etc.

Keeping in mind that what we're doing here is not *so* dissimilar to what went on with logarithm tables (so it would be good to tell that story).

However, that's not all, since even though building a better logarithm table is interesting, it's probably not enough to hold my attention for a decade. There's a hyperreal dictionary to build!

(And I've written some more general thoughts about non-mathematical implications this in the FTG work.)

Note: a figure could go showing what the curve of participation in PlanetMath might look like in the next decade.

Autonomy, Mastery, and Purpose Among other thinkers of (self-)motivation and regulation, Daniel Pink has had some things to say about these topics. What's interesting is that a specific *reward* – or indeed, anything specific – narrows the focus. A lot. So much so that sometimes it can *consume* the focus – and, ironically, kill motivation.

If we widen back out, we see:

- The urge to direct my life
- The desire to get better at something that matters
- The yearning to do something that serves a purpose bigger than just "myself"

Maybe the key is *attention* and how background noise can pull it away from a given foreground (or vice versa). Without the "reward" to distract you, your mind can search a broader horizon.

This might be a good place to bring up another popular writer, NNT, who writes about a sort of "80/20" rule for investing. Put 80% of your capital into "safe investments", and use 20% to expose yourself to possibly hugely profitable but low probability outcomes. On average you might lose frequently, but the big wins will be worth it.

(Note: In a certain sense, "mastery", "autonomy", and "purpose" provide good candidate types of answers to look for in the PAR.)

Demotivators In addition to poor motivation (clearly), *Lack of accountability* and *negative interdependence* are two notable issues. Also *technical problems*.

The scenarios are likely to be different in formal and informal learning, since in formal learning, there's always recourse to the motivator of *the grade*.

Making recommendations I haven't much about this (yet), but I'm aware of the classic Pólya heuristic, various Minskyan heuristics, and even some *modern* heuristics. And it's also my belief that *a heuristic is a recommendation* – or a potential recommendation, which suggests a course of action which you may or may not accept.

More generally, these systems are about implementing new kinds of "senses". Consider the functional form: "Do you know another example like this?" Clearly this suggests doing a search.

I ought to be able to sum up the *kinds* of recommendations that we could give, and meaningfully assess. There significant prior work in this area by Joseph Konstanz (sp?).

Back in the realm of popular authors, there's also the current notion of "nudge" (which I've written some criticisms about).

What's not like this? Drupal.org – for one.

> I'm sorry, but although you can create issues in Drupal Core and marke the category as 'support request', we don't really handle support requests in the Drupal Core issue queue as a regular practice [...]

My point being: building the links between questions and actual *documentation* isn't a 100% *typical* activity. There's a sort of conservative attitude towards *dealing with questions* that I tend to find a bit frightening.

But, that said usually these attitudes are only implicit ones. For instance, IRC logs are frequently *saved*, but not explicitly *processed* – and it wouldn't be entirely fair to focus the critique only on overt cases, when there are so many implicit ones like this to consider!

(Compare my anti-pattern about "navel gazing"... and maybe China Mièlville's post on neo-novels...)

Almost Wikipedia This is a great talk (and paper?) by Benjamin Mako Hill. In it, he says, to be Wikipedia, you should firstly be something people are familiar with, and secondly, make it easy for people to contribute.

So, PlanetMath as a "problem workbook" makes a good amount of sense.

The Free Technology Guild THe main idea is that of the *multi-sided market*, which is similar to the ideas I wrote about in the Crowdsourcing Education paper. (So, in particular, *multi-sided* and multi-*segmented* markets are similar.)

The question: how can things be more efficient rather than everyone beating their heads against the same wall? An idea (from Aaron Krowne): *hyperbarter*.

For my current thesis work, this is an inspiration – and it points to a more general context where some of this work could apply. (E.g. process IRC logs with NNexus?)

The intention Build an environment that can adequately support people who are learning mathematics, and research how this works.

The ideas in W. P. Thurston's paper provide a key starting point, and we have now looked at some other good starting points as well. The *methods* can actually be fairly simple – but with room to grow as our understanding improves.

☛ Methodology

Open online learning environments provide researchers with access to all of the same data that participants use to communicate with one another[145]. Researchers thereby have access to a natural data set similar to that provided by "talk-aloud protocols", which can provide detailed evidence of learning and development. This kind of data – and the informal, ad hoc interactions that generate it – matches well with some of the ideas and techniques of microgenetic analysis. The microgenetic approach studies learning as a process, rather than the outcome of a process[93]. The approach examines moment-by-moment changes observed in a short period of time, often for a high number of separate observations, but not necessarily subject to the same staged treatment patterns found in longitudinal studies. Observations tend to be analyzed intensively, both qualitatively and quantitatively. Microgenetic approaches have been used to take into account the social process of development, in which individuals learn concurrently in a distributed fashion[56]. This method seems particularly well suited to our informal peer-based learning context, where a pre-test/post-test method for assessing learning quality would generally be inappropriate or infeasible. Statistical analysis of this sort of data presents some unique challenges[34]. A higher-level challenge appears when we try to take what we've learned and apply it to shape practice. We give an indication of how we plan to address this challenge in the following section.

Put together, these various methods should help support various aspects of learning (recall Zaretskii's perspectives on the Zone of Proximal Development).

	form	**substance**
content	Language: relative to a static word list (the defined terms) or graph (the encyclopedia); relative to time (i.e., which words follow one another in sequence); and indeed even language creation over time, relative to a given community (recalling that a peer-produced encyclopedia is *not* a fixed entity)	Behavior (such as it is modeled within our system)
expression	Silence (i.e. the passage of time)	Growth and change of the system itself (considered as a sort of grammar of behavior)

Table 16: An Hjemslevian division of observables

But how do we detect "learning"? Table 16 provides one possible big picture overview of some likely places to look, dividing the space into observed *language*, nonlinguistic *behavior*, *the passage of time*, and *system change*.

Examples of activities in these four categories would include *using a given technical term (e.g. 'group')*, *submitting a solution to a problem*, *taking a two month hiatus*, and *submitting a feature request to the ticket tracker*.

Table 17 applies the division from Section .1 to PlanetMath's *entity relation diagram* ("ER diagram", see Figure 1) to expand on substance/content quadrant, i.e. the non-linguistic behaviours that we are able to model. In particular, this table suggests a range of different vectors that could be pulled out of the database: *new links between articles*, *new questions*, *new corrections*, *new problems*, and *joining a group*, for example. Figure 5 (from page 73) may suggest various interpretations for these actions, e.g. changing the topology of the encyclopedia might help future newcomers orient themselves to a given topic.

Context	Feedback	Quality	Structure	Heuristic
$A \leftarrow A$ $A \stackrel{\ell}{\leftarrow} A$	$X \leftarrow T$ $X \leftarrow Q$	$A \leftarrow C$	$A \leftarrow P \leftarrow S \leftarrow R$ $L \leftarrow A, P$ $M \leftarrow A$	$G \hookleftarrow U$ $S \hookleftarrow H$ $Q \to C, W, P$
A article ℓ link	X object T post Q question	C corr.	P problem S solution R review L collection M classific.	G group U user W request H heuristic

Table 17: A tentative decomposition of PlanetMath's activities

The purpose of the section is to design a rubric for assessment of informal learning in undergraduate level mathematics. In our main study, we will mostly ignore the substance/expression quadrant from Table 16 (at least as it applies to the Planetary system), but we will take this up briefly later on. The other three quadrants will all be used, though our initial experiments require very simple interpretations. Some of what cannot be studied quantitatively (yet) *may still* be amenable to a qualitative approach. In this section we will discuss both more-ideal and more-practical approaches, which we consider in three layers (which cut across the dimensions proposed in Table 16).

The first layer would keep track of what happens in the learning environment at the *micro-developmental* level, looking at what is essentially individual data. Here we would:

- *Take note of the range of user activities*: such as solving a problem, asking a question, or giving a hint.
- *Take note of vocabulary growth*: Examine the specialized terms that people use in texts they submit to the site. (We can assume that when this vocabulary is used in a solution that has been marked correct, then the vocabulary has also been used correctly.)
- *Take note of silence*: The passage of time is presumably one of the key aspects of learning!

The second layer assesses aspects of the development of *collective knowledge*, inspired by Chan, Lee and van Aalst's[33] adaptation of principles designed by Scardamalia[134]. The things to look for here include:

- *Working at the cutting edge*: participants work to advance individual and collective knowledge, e.g. this could be modeled by overall vocabulary growth, or by attempting problems that no one has attempted before.
- *Collaborative effort*: participants share understanding in order to advance collective knowledge, e.g. by asking and answering questions, coauthoring, etc.
- *Identifying high points in the discourse*: participants might flag up contributions that help them understand a problem better in such a way that this recommendation advances collective knowledge of the problem domain.

The third layer would focus explicitly on *mathematical problem solving*. This includes the traditional "grammar of proof" (e.g. proof by induction, reductio ad absurdum, and so forth). However, importantly, it also includes *heuristics*. In one of his memos for the One Laptop per Child (OLPC) project, Marvin Minsky wrote[108]:

> Children [...] learn words for various objects and processes – such as addition, multiplication, fraction, quotient, divisor, rectangle, parallelogram, and cylinder, equation, variable, function, and graph. But they learn only a few such terms per year – which means that in the realm of mathematics, our children are mentally starved, by having to live in a "linguistic desert." It is hard to think about something until one learns enough terms to express the important ideas in that area.

His concern, however, is, not merely with increasing the rate of vocabulary acquisition, but with learning ways to think and problem solve. He quotes Allen Newell[112]:

> The essential point of efficient learning is that, after you have solved a problem, it is not enough just to remember the answer: you need to remember the strategies that you used to discover that answer.

If a problem seems familiar, try *reasoning by analogy*. If you solved a similar one in the past, and can adapt to the differences, you may be able to re-use that solution.

If the problem still seems too hard, *divide it into several parts*. Every difference you recognize may suggest a separate subproblem to solve.

If it seems unfamiliar, *change how you're describing it*. Find a different description that highlights more relevant information.

If you get too many ideas, then *focus on a more specific example* – but if you don't get enough ideas, make the description more general.

If a problem is too complex, *make a simpler version* of it. Solving a simpler instance may suggest how to solve the original problem.

Asking "*what makes a problem hard?*", may suggest another approach – or a better way to spend your time.

When your ideas seem inadequate, remember someone more expert at this, and *imagine what the expert would do*.

Whenever you find yourself totally stuck, *stop what you're doing* now and let the rest of your mind find alternatives.

The best way to solve a problem is to already *know how to solve it* – if you can manage to retrieve that knowledge.

If none of these methods work, you can always *ask another person for help*.

Table 18: Problem-solving heuristics suggested by Minsky

Minsky then proposes several example heuristics (ways of thinking about problems) that might be taught, or detected when they are employed (Table 18). He also suggests some meta-level heuristics:

- *Select appropriate representations*: Building an understanding of the problem or goal at levels ranging from deciding its domain, finding suitable parameters or making an intuitive sketch.
- *Find appropriate analogies*: Some of the most useful results in mathematics combine ideas from disparate sub-fields (like geometry and algebra); further, some say that mathematics is the science of patterns, so knowing how to look for useful patterns is a vital skill.
- *Deploy negative expertise*: Knowing what has failed to work in the past can be useful.
- *Construct more realistic self-models*: As one gains experience, one can understand better how one thinks.

These heuristics are of course not specific to mathematical problem solving per se, but the mathematical domain is indeed rather keenly focused on problem solving. In some sense, we could say that all "informal" mathematical speech represents use of heuristic reasoning. Many of these could be identified from *textual features* drawn from the informal parts of mathematical speech ("by analogy" or even "it easily follows"), or else via explicit discourse markers, following the example of the "Dangerous Bend" sign employed by Bourbaki – one possible use for the icons supplied in Table 18.

Quantitative analysis

1. In our quantitative study, which will focus on legacy data, we will take *vocabulary growth* as the main indicator of learning.

2. Considering vocabulary growth over time to be the *outcome* of interest, we will consider various different *treatments*, which take place over time, and model their effect on vocabulary acquisition.

3. Treatments will be formed from vectors pulled out of Table 17. Some of these vectors have an interpretation in terms of *collective knowledge building* (e.g. corrections), others blur the lines (e.g. the effect of posting in the forum versus the encyclopedia).

Design and implementation work

1. Many new interactions will be built into the new system, fleshing out Table 17 (e.g. problems, solutions, questions, answers, and groups are all new kinds of data, which will have new interactions associated with them).

2. We will also enable heuristic-based tagging following Table 18. Subsequent studies could then follow the model described in the "quantitative" section, using heuristic labels as either outcome or treatment. Future scholars who are prepared to do more detailed textual and/or hypertextual analysis can expand on this approach.

3. We will come back to the substance/expression quadrant of Table 16, and look at how the current set of "future" issues in Planetary's issue tracker expand what we saw in Figure 5.

Qualitative analysis

1. Where possible, we will seek to "validate" results of both the quantitative analysis and design work in interviews and focus groups. The three layers we've discussed in this chapter will in inform both questions and analysis of user feedback.

2. A future-looking aspect of this is "technology acceptance"[154]; a backwards-looking aspect is, where possible, to understanding whether learning as we've modelled it corresponds to learning in the user's experience.

3. Analysis of the *methodology itself* will naturally form part of the discussion and conclusion of this work.

☛ Implementation

In the transition from reference resource to learning environment, the following major changes to the platform were required.

(I) General improvements to basic functionality (a modern look and feel based on the contemporary content management Drupal, using LaTeXML for mathematics rendering* – including extensive work on feature cloning and data porting)

(II) Problem solving support (problems, solutions, reviews, collections, questions)

*HTTP://DLMF.NIST.GOV/LaTeXML/

The blood-dimmed tide is loosed, and ev

(**III**) Data collection and management (personalized tracking and analytics, SPARQL integration via Virtuoso and PyRDFa)

(**IV**) Other new features (real-time collaborative editing and assistance for authors, Javascript-based interaction for readers, mathematics-aware search)

Note that from a research standpoint, the new platform is the main intervention (cf. Chapter 5.1). As is the nature of open source collaborative work, the new platform builds on and synthesizes a tremendous amount of already-existing material.[85] (That said, the only code we've taken from the legacy codebase is the NNexus autolinking module, which has been modernized into object oriented Perl.[*])

The Planetary project also actively incorporates work from quite a few contributors.[†] My role has been exclusively to drive the PlanetMath rebuild, which is the first major public demo of the Planetary platform. (Another flavor of Planetary, called Panta Rhei, is already in use as a LaTeX – indeed sTeX[81;82] – enhanced course management tool.[47]) Thus item **I**, above, has been a collaborative effort in which my role has mostly been recreating old interactions and porting old data. Items **II** and **III** have been mostly novel work by me, as these features are needed to develop the main ideas in this thesis. Finally, item **IV** integrates work from other contributors, in which I've mostly provided testing support, though I have taken a serious role in outlining *future* enhancements.[‡]

Item **I**, though laborious, is not worth describing in detail here (relevant information is presented in Appendix .1). Thus, after a short overview of the system as a whole, this chapter will focus on items items **II** and **III**. Aside from some brief notes in the following overview, details on item **IV** can be found in various papers and working notes.[84;71;83;123]

Overview of the Planetary System

Figure 2: The current PlanetMath webpage under Noösphere 1.5, and the new "beta" version.

Indeed, it is best known for its mathematics *encyclopedia* which contains over 9000 entries, and defines around 16000 concepts (see Chapter 0.1). Another key feature of PlanetMath is that every entry is *discussable* via its own attached, threaded, forum. PlanetMath has a variety of other features (like mathematics rendering, a term autolinker, and a workflow and authority model suitable to distributed encyclopedia authoring), most of which were developed in a custom system based on Perl and XSLT (called "Noösphere"), which was written up in Aaron Krowne's 2003 Master's thesis[89]. While this feature set has provided a (mostly) stable and functional basis for a popular community mathematics website for over a decade, the custom nature of the software made extensions and adaptations relatively scarce.

PlanetMath's knowledge rich, systematically constructed, peer reviewed encyclopedia will supplement problem sets with background material that undergraduate students will not find in online mathematics learning environments like OpenStudy[§], Khan Academy[¶], MathOverflow[‖], or the

[*] https://github.com/dginev/nnexus

[†] http://trac.mathweb.org/planetary/wiki/people

[‡] https://github.com/cdavid/drupal_planetary/issues?milestone=1&page=1&state=open

[§] http://openstudy.com

[¶] http://www.khanacademy.org

[‖] http://mathoverflow.net

Math Forum's Virtual Math Teams*, though we draw ideas and inspiration from each of these projects.

Full support for MathML lays the foundation for many other future services. For example, our Executable Paper demo integrated JOBAD, a Javascript tool for interacting with mathematical documents while reading.

After our first round of prototyping[85], Drupal 7 emerged as a good candidate solution for both of these issues. It is a popular system, with a wide variety of contributed modules – and it also supports a healthy marketplace for professional services. So far, the Planetary team has 14 contributors (most of them computer science students at Jacobs University, Bremen), with the current second author focusing on developing Drupal support for features and workflow similar to those found on PlanetMath.

Indeed, if we are going to do anything about the "$500 million pricetag" for building a math-capable AI†, we either have to bring about greater efficiencies, or spread the cost out over a relatively large number of people. Without making promises about just when this will be accomplished, we assert that we have, with PlanetMath and now Planetary, taken some vital steps in this direction.

We have found that there are some modules that can be installed and used directly, with minimal configuration (e.g. *privatemsg*, for the exchange of private messages between users) – others needed to be custom-built (e.g. support for *corrections*, essentially a custom form of bug report used to maintain accuracy and quality in PlanetMath's encyclopedia). Some others, like the *userpoints* module can be installed and used with minor tweaks. All in all, we depend on around 25 existing contributed modules, and have written a comparable number of custom modules. For a few legacy features, we took things in a new direction.

In place of, or alongside, the legacy autolinking service, we have a new interactive ("semi-automated") autolinker, which should provide greater precision for links – and, again, open the door to a range of new interactive services during the document-authoring/editing process[71]. In addition, this feature is made possible by building on top of a real-time collaborative editor Etherpad, so we will get real-time collaboration on mathematics documents "for free".

Our aim will be to develop some *semantically aware* activity tracking and "heads up" information for people using this system (see Figure 3).

Figure 3: Important information about articles (like outstanding corrections) in a "heads up" style display, shown here for a recent alpha version.

Features of the problem solving environment

Some of the core strengths of our system will be:

- A low floor (easy to participate just by asking a question: keywords will be automatically linked to their definitions) and a high ceiling (the possibility to explore advanced topics and help others);

*HTTP://VMT.MATHFORUM.ORG/VMT/

†HTTP://THECONVERSATION.EDU.AU/IF-I-HAD-A-BLANK-CHEQUE-ID-TURN-IBMS-WATSON-INTO-A-MATHS-GENIUS-1213

- Simple models of learning (vocabulary acquisition, situational/relational data like "working at the cutting edge" or "helping others", and use of identified problem-solving heuristics), which will help us keep track of students progress, making the system suitable for peer-supported self-study[40];
- Teachers will be able to use the site to run their own courses (cf. earlier classroom experiments by David Smith[140] and Robert Milson[107]);
- Solutions will be available to logged in users only;
- Activity tracking can be used to identify abuses, and, more importantly, data mined to make recommendations and generate new heuristics for learning and problem solving.

After some initial prototyping work we decided to develop the system as a collection of plugins and modules for the popular open source content management system, Drupal. The current version of the system supports core "Web 2.0" features (like editing and comments, which are also present in the legacy version of PlanetMath's software), and thanks to an OMDoc-based backend, provides a basis for subsequent semantic extensions, such as term disambiguation and linking within formulas.

We're aware that there are many existing repositories of problems (in textbooks, lecture notes, and other sources), and we hope to encourage authors to contribute their problems into our repository by providing long-term storage, dissemination, and maintenance, as well as improvements like cross-linking the problems with expository material in the encyclopedia.

We plan to add support for problems written in a simple automatic marking facility, (e.g. STACK*, WeBWork[137]), from which automatic assessment (for multiple choice style problems) can be obtained. This will be useful for creating some "standardised testing" to accompany the nonlinear, distributed, self-directed learning processes.

Finally, we also also plan to add support for tangibly interactive problems-cum-learning objects, e.g. written in HTML5 (compare the Flash interactives in the National Schools Observatory† and the beautiful examples on WorryDreams‡).

Metadata management

TODO: Say more here.

For access to the encyclopedia by Mathematics Subject Classification (MSC), we used a new Linked Open Data (SKOS) implementation of the classification system[92]. This was motivating partly because it allowed us to develop and demo an integration between Drupal, LaTeXML, and the Virtuoso triple store, which, again, will be useful in a range of future applications (e.g. we will be able to generate RDFa that can then be used to maintain backlinks, for example from an image to all of the articles that include that image).

Learner profiles and basic activity logging, keeping track of **(something like the following... need to be a bit more clear about what we can and cannot detect, why, why it matters, what it would take, etc. – as we will discuss in the methodology section, what we can't detect directly, we can incorpaorate into interviews, focus groups, participant observation techniques)**

*HTTP://WWW.STACK.BHAM.AC.UK

†HTTP://WWW.SCHOOLSOBSERVATORY.ORG.UK

‡HTTP://WORRYDREAMS.COM

Summary/Evaluation

We have described the Planetary system, and discussed its relevance to PlanetMath's continued project of building "a central repository for mathematical knowledge on the web, with a pedagogical slant." We expect the phase of work we will complete this summer to fully renovate and modernize PlanetMath. But once we have readied and deployed an extensible – and re-deployable – core, in a sense, our main work will just be beginning.

For example, we recall the meaning of "planet" from the blogosphere, i.e. planet-as-aggregator. Thinking in this way, PlanetMath might best fulfill its promise not *just* with a great new platform, but by successfully integrating content from other math on the web projects. This sort of aggregation service has yet to be realized, but forms a highly interesting direction for future work.

The realized system is anticipated to help cut time and other costs for both learners and teachers, by being a source of problems and solved examples, cross-referenced with prerequisite readings, all of which can be remixed in purpose-made study guides. We expect that our approach to knowledge reuse and peer-to-peer learning will be applicable in related technical fields.

It seems to us that moving the kinds of resources usually found in textbooks into an online, peer-to-peer, context changes the meaning and dynamic of their use. For example, while it is entirely reasonable to suggest that students must solve problems with a high degree of independence in order to learn deeply, in the context of "PlanetMath Redux", it is not clear that we should require the same style of independent learning from all users. Some may prefer to browse textbook solutions and move on to applications; others may be interested in looking for simpler derivations, or expanding existing solutions with deeper formality or nice graphics. We feel that all of these contributions are interesting and useful, and we feel we can look forward another exciting decade of "math for the people, by the people" on PlanetMath.

The new PlanetMath will render LaTeX blazingly fast, have better links, and a range of new features that address long-standing user concerns and also some nice surprises.

But deeper pros and cons of working with this system include:

1. Although our early prototype of the Planetary system was convincing enough to be named a finalist in Elsevier's Executable Papers Challenge[85], but it was not until this year that an end to the PlanetMath rebuild appeared to be within sight. In other words, things have been slow! (Software development is said to always take 3 times as much time as developers *think* it will take, so it's not clear if this is a unique feature of Drupal; however, the Drupal learning curve is known to be a steep one.)

2. It is relatively easy to make new content types in Drupal, and we are introducing "problem" and "solution" node types, and allowing people to attach them to encyclopedia articles, and discuss problems with attached "questions" and solutions with attached "reviews".

3. It is within reach to provide "related problems" using the MSC classification, but further analysis of theory dependencies (or approximations to the same) should allow us to give links to "simpler related problems", automating a key Pólya heuristic. Even without sophisticated tools, our hope is that a new generation of *students* will feel more encouraged to participate when problems and solutions become "first class" objects in the system.

4. Our hypothesis is that the introduction of problems and solutions will provide a vital quality check, and enhancement. In short: encyclopedia articles that do not have attached exercises (or applications) should not necessarily be presumed to be useful. At the same time, exercises that do not have attached solutions may be too hard, i.e., the relevant subjects in the encyclopedia may not be sufficiently developed.

☛ Studies

Preliminary Interviews

Dear PlanetMath Contributor:

You've been invited to participate in this survey because we want your feedback on a new platform for PlanetMath that is currently under construction. We also want to know more about who you are! People often ask me: who are PlanetMath's contributors, and I always say, well, no one's ever done a survey to find out! So this is that survey.

The survey is being sent to the top 30 users on PlanetMath.org, along with an invitation to take a look at http://alpha.planetmath.org and http://beta.planetmath.org (two slightly different demo versions of the new PlanetMath software that I've been working on, together with a research group at Jacobs University, in Bremen, Germany). This is very much work-in-progress so please don't be surprised that many things do not work (you don't need to keep track of a detailed list!). A few key things to note are that rendering is very fast:

HTTP://BETA.PLANETMATH.ORG/NODE/42297

And we have a new syntax-highlighting, collaborative, editing tool:

HTTP://BETA.PLANETMATH.ORG/NODE/42297/EDIT

Other features may be in any degree of brokenness, but your existing PlanetMath usernames and passwords should let you play around with what's there! And the dev team and I are hoping to get everything fully finished in the first half of the coming year.

I'm planning to use the results of this survey (and the new platform) in my Ph. D., which is called "Peer Supported Problem Solving and Mathematical Knowledge". And we will also use the results to make the "new PlanetMath" as useful as possible to you. The survey will remain open until the first week of January. I'm guessing it will take between 10 and 20 minutes to fill in.

Thanks in advance for your participation, and please feel free to contact me with any comments or questions about the survey or the new software.

Joe Corneli, PlanetMath Board Member, Postgraduate Research Student (The Open University, UK)

The ceremony of innocence is drowned

In addition to basic demographic data, the questions got at the mathematical background and goals, previous experience with PlanetMath, and impressions of the new site. The useful feedback is summed up Figure 4.

Study 1: Quantitative analysis of legacy data on learning

Needs a lot more work...

A reasonable "target paper" for this section is Barabási's recent paper on "The origin of bursts and heavy tails in human dynamics".[10] ❖

Introduction

This section provides a quantitative modeling and analysis of learning effects. We're interested in:

- The effect of *self-motivation*, which effectively we will try to "rule out", since we have effectively no control over this.*
- The effect of system features and interactions, like getting corrections. This is the main question we look at: can we measure non-endogenous (to the learner) factors of learning?
- A "geometric component" which says that if you've learned a given concept, idea, or approach, you're most likely to learn something nearby or related. We may not have a chance to get into detail on this one, but we could give a very rough approximation by saying that terms are most likely to be learned according to their PageRank.

The study will utilize a new approach to the study of time series data.[166] Our first aim is to extend this with a Dirichlet prior that takes into account "different kinds of people", roughly, people who bring tons of intrinsic motivation to stay involved, people with a moderate amount, and people who are just passing through.

Other analytic techniques

There are lots of these techniques (described e.g. in that last chapter of the Data Mining book that Siemmens pointed out)... Social connectedness, measured by *betweenness centrality*. Various *corpus techniques*. SNA is similar to this, but concerns a text that we admit to be changing. In general, the better we can understand texts, the more we can identify working *concepts* and *constructions* in writing; etc.

Temporality Matters Cited in the "It's about time" bibliography. This paper introduces "lag-sequential analysis", using "quantitative content analysis" as a foil (otherwise known as "counting and coding"). That method throws out temporal data. LsA is actually defined by Bakeman and Gottman 1997, and by Wampold 1992. It applies to *process categories* (which is interesting, but which isn't "real" time series data).

And, of course, temporal data is qualitatively interesting. I'm reminded of those plots that Dan made of drop-out rates in the P2PU context. Take a look also at the "convening a group" citations that talk about how people only really start working about half-way through a project!

*Coaching and positive thinking notwithstanding.

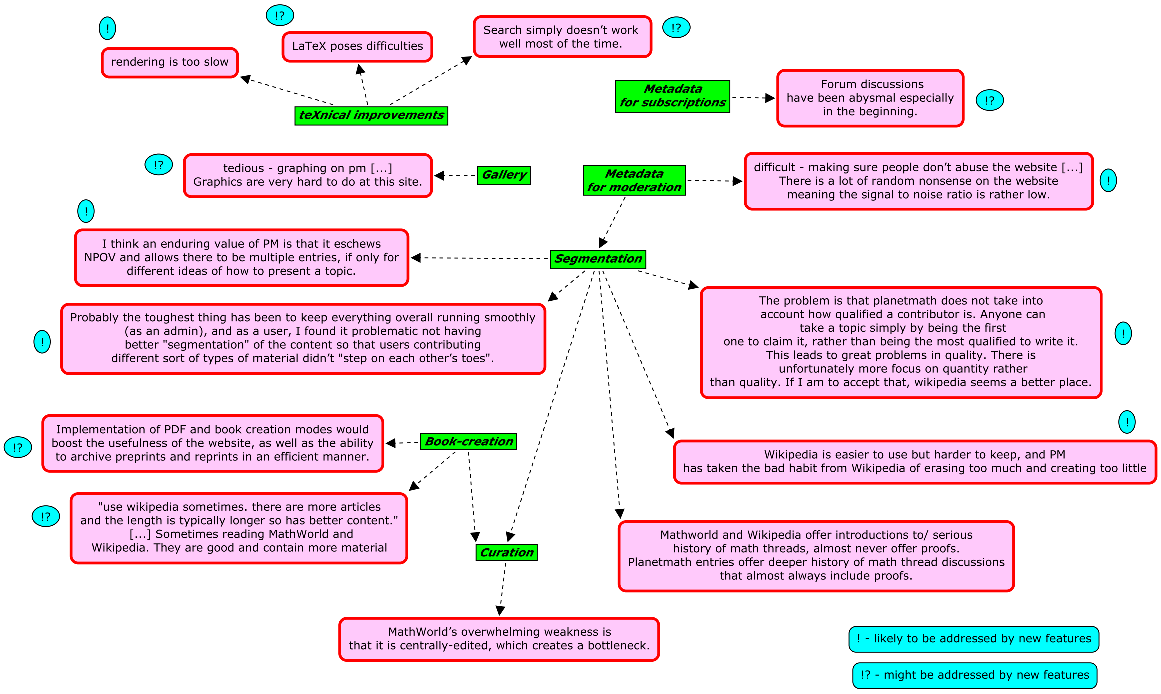

Figure 4: Initial answers to survey questions

In the analysis realm, others used “hidden markov modeling”[142;69]... however the specific relevance to learning *outcomes* may be vague. (A bit hard to tell.)

It *is* interesting to wonder how much NNexus could be used for tagging (and other things). There are some interesting clues in the Functional Category System of Poole and Holmes[121]. (For specific applications to problem solving, see work by Kapur, Voiklis, and Kinzer.[75])

Some more sophisticated techniques for disambiguation have been developed in the mean time.[127]

And all kinds of interesting stats gewgaws.

Further thoughts on the geometric component

This sort of analysis would be especially useful for making recommendations. There are 15000 terms to choose from, and we’d of course want to make suggestions that make sense.

We could, conceivably, flip the matrix, so that instead of looking at terms-per-user, we could look at users-per-term.

Study 2: Qualitative analysis of the new approach

Also needs a lot more work...

A reasonable “target paper” for this section is Venkatesh and Bala’s “Technology Acceptance Model 3 and a Research Agenda on interventions”.[154] ❖

Introduction

We should really start with some focus groups to deploy the Technology Acceptance model, since if we’re ever going to get data on how the new software impacts learning, we need people to actually *use* the new software. And they won’t do that if they don’t accept it.

The other interesting qualitative thing to do is speak with people about what we’re getting out of a quantitative analysis (similar to Study One) of new data on learning. Can people confirm that they were in fact learning when the data says they were, or explain just how they were affected by a given learning-conducive event?

☛ Discussion

The best lack all conviction, whi

FAQ #1: “What if people use it cheat?” If the result of the above “innovations” is that mathematics problems become easier solve (including by looking up the answer), this may indeed have dramatic consequences.

FAQ #2: ... We witness an increasing number of open online groups populated by self-organized and self-managed individuals who actively generate content in the attempt to foster learning communities open to all who wish to participate. These groups attempt to deploy the educational affordances of Web 2.0 tools and technologies for information sharing, communication, content creation and collaboration.[78] These affordances can enable participatory processes that can support multiple modes of learning in new open spaces, and blur the boundaries between production and consumption[29;5]. Calls for learning approaches (especially those linked to socio-cultural theory) that are able to better exploit the educational affordances of Web 2.0 have been proposed. However, claims about the educational and affective dimensions of online learning, e.g. that “these affordances stimulate the development of a participatory culture in which there is genuine engagement and communication, and in which members feel socially connected with one another"[104] (p. 667) invite further grounding in empirical research. Similarly, there seems to be a dearth of empirical studies addressing the learning-specific impact of Web 2.0. In this paper, we will begin to tackle these broad issues in one concrete case. We describe an analytical framework for detecting when and how learning is taking place among participants in the open, peer-produced, mathematics website, PlanetMath.org. For this to work, we require learning-specific metrics: for example, "participation" is not in and of itself a particularly salient measure of learning. We follow the popular view that "mathematics is a language". In this view, learning mathematics can be modelled as a process of learning a specialized vocabulary, along with the technical grammar of proof. We will flesh out this simple understanding in several ways. We agree with David Smith when he writes, "learning mathematics is, first of all, learning, and only secondarily about mathematics."[140] We will draw on microgenetic analysis, a collection of methods and techniques that have been used to study learning "as it happens" in a wide range of fields. Although we have not yet been able to validate this framework in practice, it has informed the design of the platform we are building, as we will describe below. The paper is organized as follows: we describe PlanetMath and the ongoing project to turn it into a learning environment. We then discuss some theoretical perspectives that are relevant to our project and integrate these into our analytical framework. We conclude with some discussion of the implications of this endeavour for mathematical education.

FAQ #3: ... Learning scientists should be interested in some of the results here, but the real relevance of the work would be felt by practitioners (or, even more to the point: users and potential users). Here we can cite Anderson, Reder, and Simon's call for a sort of "Federal Drug Administration" for learning.[6] Note that the CMU systems themselves haven't always fared perfectly with ❖ downstream studies... but it all depends on what you're trying to measure.

So for this reason it seems best to target papers like Thurston's 1990 paper[152] and Quinn's ❖ more recent one[126], both published in the Notices of the AMS.

A reasonable claim to make would be that the dynamic aspects of this tool offer "extra features" that relatively fixed resources like, say, the Princeton Companion to Mathematics could never offer, while PlanetMath could (in theory) contain all the same words and symbols in the PCM.

The "extra features" may be so far reaching that it would be best to speak of them in terms of *diversity*. Diverse different kinds of users, diverse different uses of the material, and so forth. Here we could turn to Scott Page's book "The Difference: How the Power of Diversity Creates Better Groups, Firms, Schools, and Societies" for some intuitions about the sorts of benefits we would expect.[117]

But are users and potential users really as *different* as they seemed, say, when we did that initial SECI-style analysis? Would a Rowe-style analysis of the different roles help?

And more broadly: what are some of the shortcomings of this work, both technically, and socially? What has changed or shifted relative to our "epistemic object"? Why does what we've learned matter?

And: is the Zone of Proximal Development really a good way to think about these results? Consider the list of things that Sloterdijk describes as *nurturing* from the first volume of Spheres, and maybe the other dimensions that are relevant to humans from the later volumes.

FAQ #4... Note that PlanetMath, unlike, say, Wikipedia, is not *constrained* to be either "just a wiki" or "just an encyclopedia" – so, interactive problem sets and/or peer tutoring are welcome in PlanetMath, though they might not fit so cleanly within the existing Wikimedia family. Rather, the encyclopedic approach envisioned here connects interactions to a carefully curated and *systematic* knowledge base – in contrast with, for example, the StackExchange sites, at least in their current implementation.

In any event, the perspective developed above should brings up some big questions: in brief, what happens to mathematics teaching when students have access to a universal solutions manual for their mathematics course work? We may be able to *measure* whether lecture/homework/test is as effective for learning, as, say, participating in applied research projects.

Still, the hazard here would be to imagine that this can all happen overnight. It has taken PlanetMath 10 years to *define* 16000 terms, how much time will it take to provide a good *exposition* of those terms (always assuming that we do find users who want to participate in this process)?

Furthermore, as we have learned in the last few years, programming Drupal is not equally easy for everyone, documentation is not always clear (or available), and development work is generally a slow process (even with skilled programmers onboard). If the potentially revolutionizing changes (sketched above for mathematics education, but relevant also to research) of math on the web are to be realized, most aspects of this project will have to scale up a lot – and hopefully coding will be less of a bottleneck.

☛ Future Work

Are full of passionate intensity

Here we will talk about things that were opened up by this work (for which there wasn't time to do yet!)

- A possible classroom based comparison study
- How we might make recommendations based on further textual and hypertextual analysis, to suggest simpler or harder problems, related expository writings, and relevant reasoning steps. This could draw on the Social Network Analysis approaches that have been used on Wikipedia.[73] ❖
- The possibility to extend PlanetMath with an Open Mathematical Marketplace
- Broadening of this idea into a Free Technology Guild, along with an open letter to researchers that describes how this sort of work could impact research practice
- A brief note on "massively-multiauthor.net" which discusses some technical changes we would want to make
- Peeragogy.EDU, a project that would take up the thread in academia, construed as an education enterprise and not just a research enterprise
- and finally, a summary of the eMath 3.0 proposal, which after this far ranging visionary journey, returns with a very technical focus to mathematics, and possible next steps for the PlanetMath project or other related projects in mathematical communication

A proposed classroom study

Throughout this section, a reasonable "target paper" is Benjamin Bloom's often-cited "2-sigma problem: the search for methods of group instruction as effective as one-to-one tutoring".[25] This ❖ paper is mentioned in the popular press articles about mathematical tutoring systems, which have apparently become popular enough to be mentioned in the popular press quite frequently.

Here, it may go without saying, we would take a very different approach. Rather than looking at *automated* tutoring, we would look at *peer-supported learning*. Indeed, the idea that this sort of learning is might be better than traditional classroom learning isn't a surprise. (But this isn't the claim of the current thesis: first, the system may not be better yet, and second, our aim is to provide some method for studying whether it is better or not.)

For reasons of ethics, supposing the system really is better, we should use version of the "waiting-list control" method. This approach has been used, for example, to study the effects of meditation training.[101]

An open maths marketplace

This project undertakes to build a market for online tutoring contracts in mathematics. Our strategy is to build these services around an existing free/open mathematical knowledge repository, the encyclopaedia that has been developed over the past decade at PlanetMath.org. The key idea in this proposal is that transcripts of paid online tutoring sessions will be integrated into an increasingly comprehensive set of freely available open educational resources (OERs).

Thus, people will have a concrete financial motive to benefit from their investment of time and expertise, but these investments will be made in such a way so as to pay dividends to the whole learning community. The real challenge of this project is thus a subtle mix of software engineering, social engineering, and economics. We may expand these points in terms of the specific objectives outlined in Paragraph 6 of the call:

This project will provide online tutoring and good practices around coordinating both paid tutors and volunteer contributors, as well as strategies for managing a knowledge base of mathematical and pedagogical content. We will be providing services not just to students, but also to tutors, for example, by helping tutors build their reputation with badges that certify tutoring quality.

The typical aim of a market is to provide a place for exchange between individuals. The market itself does not benefit, except financially. Our context is different: here, the aim is to capture the "positive externalities" of educational transactions. In particular, we are developing not just a business model, but also a strategy for extracting value from learning interactions in order to provide better quality services and freely available learning resources. The knowledge base we will build will provide further opportunities for downstream users to develop added-value services. In particular, we expect service providers using our site to up-skill so as to be able to offer increasingly complex mathematical services in our open maths marketplace.

We will be working with cutting-edge tools that are being developed entirely as open source. Key ingredients for the asynchronous and archival aspects of the project include the popular content management system Drupal, and the up-and- coming Linked Open Data paradigm. We are mixing these general-purpose technologies with special-purpose tools for working with mathematical content, ranging from LaTeXML for building interactive mathematical documents, to Geogebra for interactive geometrical figures. For real-time tutorial sessions, we will use the open source collaborative editor, Etherpad, along with other collaboration tools. Text-based interactions will be particularly easy to integrate into the larger knowledge base, but audio and video material can also provide added value by being tagged according to subject matter and the heuristic or pedagogical strategies employed.

Tutors will be motivated to tap into a growing demand for high-quality mathematics instruction, delivered globally, at a flexible pace. Our approach exploits the power of Web 2.0 to add value to student- and tutor-contributed materials. We will build an increasingly complete and well-integrated knowledge resource, using valuable material that is otherwise wasted: frequently asked questions and common responses will be documented in easy-to-follow pedagogical guides. Not only do students need access to qualified, accomplished, teachers and tutors: teachers, and other professionals also, increasingly need to be able to learn new skills while on the job. We expect the market to offer services tailored not only for students in formal education, but also to adult learners who do not have time to go back to school.

We do not yet know what balance of financial motivation and open sharing works best, nor can we say what is the best balance of individual study and tutorial support. Presumably there is no one-size-fits-all answer. What we are confident about is that an online learning environment that facilitates a wide variety of mathematical interactions will help us to better understand what works best under which conditions. We will be in a good position with this project to both apply what we learn to improve our own practices, and, because of our commitment to openness, to serve as a key reference point for others who are interested in improving mathematics education.

We will build a maths-enhanced version of the open source real-time editor, Etherpad, which will function as a shared interactive blackboard in tutorial sessions. Etherpad will be integrated with other existing open source real-time communication technologies, including both mainstream tools (voice and video chat), and mathematics-specific tools (Geogebra for geometrical figures, SAGE for computer algebra). Transcripts of tutorial sessions, and advice on problem sets, will be shared publicly under a non-restrictive Creative Commons license. Doing tutoring in the open will have the following benefits: first, members of the PlanetMath community can go over previous tutoring transcripts to extract and highlight frequently asked questions, and prepare corresponding pedagogical guides and other expository enhancements to PlanetMath's encyclopaedia. Second, openness provides a degree of security to both the consumer and the service provider, in the form of quality assurance, dispute arbitration, and reputation building. We will work with JISC to disseminate best practices and related research results.

Leading this effort will put us close contact with knowledge workers across the UK and around the world who are able to provide increasingly complex mathematical services. In the process we will have the opportunity to explore new approaches to lifelong learning that we anticipate shall be useful in other educational contexts.

A Free Technology Guild

In response to Topic 22, "Research in the Dynamic Support of Future Business Environments", this proposal considers building an marketplace for services related to free/open source software and other free/open technologies. The purpose of this marketplace is to support various kinds of exchanges, both for-fee and voluntary, such as mentoring, training, project coaching, and freelance consulting. Participants will do useful projects for one another or for external parties, and on-the-job, learn from one another, and get better at what they do. On the technical side, problems to solve include representing requests and certify transactions spanning a wide range of different modes of engagement, and vastly different project domains. On the business and economics side, this project will enable us to explore the interface between "commons", "markets", and "firms" – and potentially to invent new forms of online collaboration appropriate to the age of universal connectivity and 24/7 productivity. The project will develop in the context of the Free Technology Academy (FTA), which has successfully been running Masters-level courses in Free Software with a set of academic partners across Europe, but which is seeking a business model that will make it financially sustainable. We foresee the Free Technology Guild as a way to help both the Free Technology Academy and many other small-scale free software service providers achieve their financial, professional development, and networking goals.

Description of Proposed Research

Our proposed project blends lifelong learning opportunities with work opportunities, reusing infrastructure common to the two where possible. In the domain of free software* (also referred to as "open source software"† or "libre software"), learning and productivity tend to go hand in hand[42;136]. Free software communities include beginner and advanced users who deploy the software tools in various applied settings, and who also study, modify and redistribute program source code. Our aim in this project is twofold: first, to make a system that is useful for dispersed communities of learners as they form peer groups, become apprentices in real work situations, and receive guidance when needed. Second, we want to enable professionals and consumers of professional services to contract for high-quality work done with free software tools. Thus, our platform will provide a range of economic opportunities for skilled technologists, and learning opportunities for beginners, via a suitable "matchmaking" service or marketplace.

The idea of a "guild" seems to fit – at least as a loose approximation. While membership should be open, and while anyone can work on free software whether they join our organization or not, the system is "guild-like" when it comes to reputation building. While the system is open to anyone to work, teach, or learn, people can only get "rated up" if they do a good job. Building a good reputation within the system would confer benefits like repeat customers or the legitimate

*Free Software Definition, Free Software Foundation, HTTP://WWW.GNU.ORG/PHILOSOPHY/FREE-SW.HTML
†Open Source Definition, Open Source Initiative, HTTP://WWW.OPENSOURCE.ORG/OSD.HTML

ability to charge a premium for services. So, with the inherent scarcity of reputation in mind, we refer to the project as the Free Technology Guild (acronym: "FTG").

In the following sections, we sketch our project plan in detail.

Functional design The FTG is a relatively thin layer that sits above (potentially) all other free/open communities, and provides them with a lightweight common set of services and information exchange. This "thin" information infrastructure can be developed in an iterative fashion. The initial phases are simple. To get started, users would need to supply metadata like the following:

- interests (e.g. "Drupal")
- skill level (e.g. "beginner")
- budget or fee (e.g. "I could come up with about $500.")
- brief description of proposed project (e.g. "I need a few days of skilled help with such-and-such a platform, with the time split between 60% development work, 40% coaching.")

When a new user comes to the FTG's homepage, they would see a "word cloud" of interests (e.g. "Drupal", "python", etc.). They can click on one to specify their profile in that area, giving their skill level, whether they want to teach, learn, or do development work, whether they want to pay or receive money, etc. Each user's profile will thus also be viewable as word cloud showing their interests, which can be scaled by things like "contribution level". This is where the modeling tasks becomes more complex: we would like to be able to keep track of users contributions not only on our site, but across the internet, pulling in metadata from GitHub, StackExchange, Wikipedia, and so on. These facilities will be developed over the course of the project, using the techniques of the Open Web.

Theoretical design A relevant article is "The Open Code Market"[30] from 2003, in which Carrasco-Muñoz envisioned "end-users [of free software] becom[ing] customers by attaching a monetary value to the creation of the software they commission." It is possible that the strong division between "users" and "developers" in this essay is what accounts for the fact that we do not have a viable (or visible) open code market around today. The problem with this earlier approach is that it overlooks the learning-specific needs of both users and creators of free software (many of whom are *both* users and creators). Frequently in this ecosystem, the relevant transaction is not commissioning software, but building a skill. Establishing expertise as a technologist is a complex affair: understanding and clearly representing the pathways by which this works is an important component of this research project. As we better understand how people build expertise, we will also get better price-signalling, creating a more useful market.

Learning design Having recognized that modeling learning needs is important for the project, let's take one person as a case study: Sacha Chua, noted collaboration evangelist and hacker. On her website, she has lists of things she can help *you* learn[*], and things you can help *her* learn.[†] This is a great strategy for building a peer learning network. The idea of a "learning/teaching profile" could be further formalized and exploited in the FTG, so, for example, a "course" might emerge when we see that all of the requirements are satisfied: that is, when we have enough people who are interested in learning a given skill or topic, and enough people with the ability and

[*] HTTP://SACHACHUA.COM/BLOG/2009/12/WHAT-CAN-I-HELP-YOU-LEARN-LOOKING-FOR-MENTEES/
[†] HTTP://SACHACHUA.COM/LEARNING.HTML

interest to teach them. But how do we certifying learning this sort of context? A simple approach would be to contract for "grading services", so, for example, answers to textbook-style exercises could be marked by a Guild service provider for a fee (related issues are discussed at length in Joseph Corneli's thesis work[*]). Contributions to software projects to some degree have their own "certification" built in (does the code compile? was it accepted by project maintainers?). Of course, transactions in the FTG will have to do not just with "software topics" but many "social" topics (such as frequently appear on Sacha's lists). Learning about topics like "how to delegate" is a paradigmatic example of the type of learning we want to support.

Business model Supporting non-monetary exchanges (you don't have to pay or charge money to participate) recovers the standard no-cost model of free software mailing lists, encyclopedia projects, and software collaborations via distributed version control, taking such exchanges as the free basis of a "freemium" business model for FTG. The FTG could be "bootstrapped" using a prototype version of its own platform, with hackers charging fees to improve this platform itself (e.g. Gun.IO or any other "code bounty" management service could be taken as a "prototype" in this sense). Subsequently, FTA (or its members) could presumably become a "service provider" (providers) within FTG - providing all the services that the FTA is currently known for - building curricula, securing formal accreditation for learning experiences, finding quality teachers, etc. There is also the possibility of instituting a system-wide "tax" on monetary transactions (which could go to ongoing maintenance and development of the system itself), or a membership fee structure (similar to the Free Software Foundation). Various crowdfunding models (e.g. Goteo[†]) could be deployed, e.g. courses or other trainings would run only when sufficient funds had been deposited.

Certain aspects of this project have been explored previously by Advogato[‡], Gun.IO[§], and Peer-2-Peer University (P2PU).[¶] FOSSFactory[‖], focuses on "free/open source software production, including design, funding and development" – without the learning and knowledge building features we emphasize in this proposal. As our project evolves, increasingly sophisticated "analytics" can be applied to gain added value from user profiles, cf. HTTP://WWW.SOLARESEARCH.ORG. Some early inspiration for the project came from ComputerMinds' "mindshare" consultancy/training programme.[**]

[*] HTTP://METAMESO.ORG/~JOE/THESIS-OUTLINE.HTML
[†] HTTP://WWW.GOTEO.ORG/
[‡] HTTP://WWW.ADVOGATO.ORG
[§] HTTP://GUN.IO
[¶] HTTP://P2PU.ORG
[‖] HTTP://WWW.FOSSFACTORY.ORG/
[**] HTTP://WWW.COMPUTERMINDS.CO.UK/DRUPAL-FREELANCE-WEB-DEVELOPERS

☛ Conclusion

The main contribution in this thesis is a way to measure the effectiveness of a class of generalized learning strategies. At present, the common-place advice to "solve problems" is like a good home remedy. But what kind of problem solving is most conducive to learning? What can help problem solving work better? Are there any "helps" that are actually not so helpful? This thesis will deliver a method for answering questions like the above, using statistical techniques, and the basic components of a "lab" for testing claims about mathematics learning.

☛ Research Statement

Research Question

How can the ability to deliver learning interactions online in a peer-produced context be used to deliver improved support for informal study of mathematics at the undergraduate level?

Approach

Everyone who has studied mathematics is familiar with thick, expensive, mathematics text books. When paired with lectures and homework, textbooks provided an adequate method for delivering mathematics instruction in the 20th Century. But now that we have the internet, can we do better?

Online peer-production communities have successfully built operating systems and reference works. I am taking one of the later, the mathematics encyclopedia at PlanetMath.org, and using it as the core knowledge resource in an online learning environment that will host interactions around mathematical problem solving. The first step for me in my Ph. D. project has been to collaborate on building a tool – namely, a new version of PlanetMath's online platform – ready to accommodate the enhancements that will be needed to support problem solving interactions. The next step has been to devise a methodology that is suitable for understanding and supporting learning in this new space. While it would be possible to evaluate the system as a whole by using standard pre- and post-test techniques, I'm more interested in what happens in the middle. Thus, the next phase of my research will be focused on text and hypertext/interaction analysis that aims to measure, support, and advise on learning. For example, a typical user question I would like to be able to answer automatically is: "Help me find an easier related problem". Starting in January, I plan to evaluate user experiences with the system in small-scale pilots (classroom studies) and, hopefully, "live on the internet", on the "new" PlanetMath.org.

Main things I have learned about doing research

> "We solve the *whole* crime. We find the *whole* person. Phone today for the *whole* solution to your problem."[2]

When I started my Ph. D., everyone kept telling me to focus. I don't think I knew how to do that when I started. I was used to a very "holistic" approach, in which everything from anthropology to 'zines were somehow relevant. Working remotely with a disciplined and productive group of collaborators, and meeting weekly with my supervisors, I've seen how important it is to keep progressing on a concrete body of work. It's not just about being willing to put in long hours, but engineering a workflow that feels productive (and satisfying) on a daily basis. In the end, my revised approach is probably no less holistic, just more disciplined.

Major hurdles

As I explored the problem more deeply, it became clear to me that there was enough work in the first phase to keep a person busy for most of the available time, particularly me, since I'm a novice with the software systems we're using. I decided it would be useful to apply for a grant to bring in a full-time developer for a few months. In my first attempt at this, I failed to get the grant out the door, due to difficulties in the team-building phase, and annoying details in the call's small print. I believe I will have everything in order to bid for a smaller amout of money in October, 2011. I've also tried to restructure my work into "good enough" and "more ideal" packages, so that I have a reasonable degree of security about completing, with or without the grant.

☛ One more effort

> Praxis, a noble activity, is always one of use, as distinct from poesis which designates fabrication. Only the former, which plays and acts, but does not produce, is noble.[13] (p. 101)

This appendix documents a thought process, rather than a result. The process helped formulate Table 17, which a fairly central artifact in this thesis; it also helped organize the literature review. The mode is "inspired" rather than scientific. To briefly summarize:

- Thurston says that mathematics is the study of patterns. Patterns form a generalized 'kinship diagram', as in, X is related to Y in some manner Z. Thus, while I might *jump in the river to save two brothers or eight cousins*, I'm willing to stake a much higher mathematical career risk on an effort to reengineer the mathematical "game". It's a pity that this is only described in my notes as having to do with connections between articles ($A \leftarrow A$). Still, if we're looking for *patterns in knowledge*, *patterns in the growth of knowledge*, or *relationships at the conceptual level*, then $A \leftarrow A$ is roughly what it comes down to.
- The "social process" is actually a lot like a big (m)Other which contributes to 'nurturing' the 'child', for example with *corrections* $A \leftarrow C$, but also more generally by keeping the learner/trainee within the Zone of Proximal Development (typically ZPD). (However note the irony, and cf. Žižek's "For they know not what they do"... "just because you understand

what you're doing doesn't mean you're doing the right thing." [...] "the surprising characteristic [...] is its insensitivity to reflection." (LRB)... maybe relevant? The injunction to enjoy, to "make learning fun", etc., seems pretty thick these days... But, in particular, this sort of injuction seems strongest in a direct reciprocity mode, where face-to-face power runs most effectively.) What's maybe less studied is how the Zone works with adults, e.g. that paper about "Teaching Smart People how to Learn" seems like a very useful/usable example. I wonder how that would work together with the Zaretskii stuff. (And the PAR is a related/relevant practice; 1.1.)

- Clearly this is related to *stigmergy* on which the main authority these days is probably Peter Pirolli, but which was actually introduced by Pierre-Paul Grassé. You can also see some interesting writings about stigmergy in the context of *knowledge based reasoning* (Greer et al.), and even *stigmergic epistemology* (which might formalize Stahl's stuff?). Mainly, in the PlanetMath context, being able to leave a comment ($X \leftarrow T$) or ask a question ($X \leftarrow Q$) is a good example. Ideally this would make the resource "self-healing" or "self-correcting". Of course, actual corrections and other aspects of interaction can play a similar role, but the important distinction is that those are more one-to-one, not *one-to-many* signals.

- "Architecture" sounds spatial – and there's a nice quote from Nick "Sheep" Dalton that says that "space is the machine". Thus the ability to weave structures within a given "loom" is interesting, i.e. $A \leftarrow P \leftarrow S \leftarrow R$ for example, or even just $M \leftarrow A$. We can experience this sort of architecture as a machine for weaving space! And of course, the idea that this space is explored/woven/created through the individual learning process – seems pretty keen.

- It's worth keeping in mind that a website's groups are *themselves* little "online communities", so it would be nice to support the range of different nice communities features for these smaller sub-communities. As for the claim that $G \hookleftarrow U \approx S \hookleftarrow H$ or that $Q \rightarrow C,R,P$ are also examples of ways of thinking based on heuristics that are associated with particular group memberships... i.e. that "joining a group" and "solving a problem" are similar... *well, why not?* – i.e. people could indeed be put into groups (level sets) based on which problems they have solved. And isn't this actually how the school/university works, as an associative and stratification mechanism? This is certainly what's being employed in the $Q \rightarrow C,R,P$ transformation, or any other similar triage process.

The remainder of the appendix will expand on these ideas, together with further musing about analogues, parallels, and other connections. I would specifically like to acknowledge discussion with Andrea Kohlhase, Timothy Teravainen, Raymond Puzio, and Howard Rheingold as relevant to the development. I confess to be inspired in this essay by Bernhard Reimann's *On the Hypotheses which lie at the Bases of Geometry* (republished by Michael Spivak[143] (pp. 135-153) and available online*).

(1) the theory of paragogy outlined in Chapter 1.1;

(2) the media theory of Vilém Flusser[55] (pp. 91–98);

(3) the basics of a light-weight metamathematical network theory, from discussions with Raymond Puzio†;

(4) Martin Nowak's theory of evolutionary games.[115]

*HTTP://WWW.MATHS.TCD.IE/PUB/HISTMATH/PEOPLE/RIEMANN/GEOM/
†HTTP://METAMESO.ORG/~JOE/MATH/METAMATHEMATICS.PDF

(5) Gregory Bateson's criteria of mind.[12] (p. 92 et seq.)

This might seem like an odd mix of topics but in addition to having about the right number of elements (5 or so each), every one of the frameworks I mentioned also lends itself to a graphical or pictoral interpretation. I was struck by the idea that they might all rely on the same sorts of pictures. Aligning these various frameworks might help us see how *communication* relates to *collaboration*, and how a given *symbolic expression* encodes a given *pattern of thought*.

In the mathematics context, for some activities we rely on humans (creativity), and for others, we rely on symbols (communication) or even on machines (collaboration platforms, proof checking). Translating between those regimes isn't easy.

But if we start to see that the same kinds of diagrams describe each of these different layers, then maybe that will make translation easier. This note – by necessity a "poetic" one – will give us some things to look for in subsequent sections. The sort of thing we can get out of this work is depicted in Figure 5, which shows how various problem solving *heuristics* (to be discussed in Section 18) and *patterns of peer learning* might be modelled within Nowak's framework.

It convenient if we could additionally map Pease and Martin's four-term framework of *concepts*, *examples*, *conjectures*, and *proofs* onto this framework-of-frameworks.[118]

Thus, a condensed synthesis, is as follows:

1. Taxonomy is one way to say 'what is it' – but one doesn't need to rely only on basis-derived taxonomy, one can also speak of a freer "affinity" ('what is it like').

2. What do you get (or, more generally and dryly, what does one obtain) from doing it? This is a somewhat exchange-oriented question in my view. (What do you give to get.) The mode of thinking here is *inductive*. The motivating feature is *purpose*.

3. Here, it is maybe interesting to consider mirror neurons as one of our fundamental symmetries. If you've answered the previous question (that is: the draw, the inherent why), you can follow up with "implications", to address the not-inherent "so what, who cares?". I don't want to suggest that this is purely social: the point is that geometric thinking is itself a matter of taking up different "perspectives". The mode of thinking here *abductive*, rooted in fantasy. The motivating feature is *autonomy*. E

4. I'd suggest that the preference in computation is always strictly local (reduce everything to bitwise operations) over global. The same thinking works in reverse when thinking about "architectures". The mode of thinking is *deductive*. The motivating feature is *mastery*.

5. Presumably synthesis is a part of any new system or new heuristic. Even though it might seem ephemeral, I think it's actually implied by any tangible outcome. (Consider that the word "focus" means fire, as in the communal firepit, something to gather around.)

Network - Program - Context - Kin Selection - Conceptualization

$$\{A \xrightarrow{B} A'\}$$

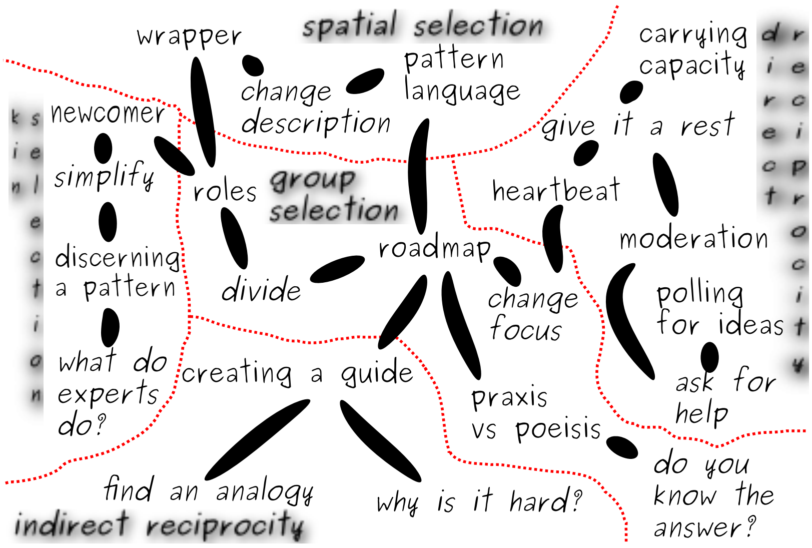

Figure 5: A tentative decomposition of heuristics and patterns for peer learning

This is a set of relationships between objects. We can also say, in a more active way, that "if we subject A to the process B, we obtain A'." Networks can contain both specified and unspecified terms; unspecified terms can be thought of as "terminals" (details below).

Narrative version: The picture shows us what's akin to what, what's related to what: in other words, it shows a *pattern*. This pattern may be thought of as a *context*. Thus, in our context, the sun will rise tomorrow, green is green, I can get myself a cup of tea more or less whenever I like, etc. – and I can also act on the context in various ways. Humans think a lot about how they fit into bigger patterns. (And, in particular, if I could "see" DNA, then from a strictly evolutionary standpoint, I might do well to "jump in the river to save 2 brothers or 8 cousins.")

Mathematical example: The PlanetMath encyclopedia is a network.

Theater - Map - Engagement - Direct Reciprocity - Induction

$$\{A \xrightarrow{B \mapsto X} A'\}$$

This tells us that all B's in A will be substituted with something, but we don't yet know with what.

Narrative version: Let's think of A as a story and B as some of the active elements of the story. The audience will interpret the story in some way, but at the outset, we don't know what their associations are ("gun $\mapsto$ flower" would be a surprise; etc.). In a scenario like this, the "listener" acts in a direct relationship with the speaker. In the end, they may react by throwing tomatoes – or money – or by jumping on stage themselves. The format is "I give you this (A), and you give me that (A with some changes in the B part)." Note that if we repeat this game, then we can see, on average, how things get mapped.

Mathematical example: "Let f be a real-valued function on the complex plane." (We don't yet know what the values are, and may never know – although in this case we know more about the function than we would if we said it was complex-valued.)

Observe from this example that "substitutions" like $B \mapsto X$ can take place on a purely atomic/lexical/syntactic level, or, as we did here, on a higher semantic level.

Pyramid/Tree - Operation - Quality - Indirect Reciprocity - Abduction

$$\{A \xrightarrow{X \mapsto B} A'\}$$

We are invited to specify just which elements of A will have B's substituted in for them.

Narrative version: Perhaps A is a job description, B is an element that will have to be used to fulfil the job. (Later, someone can check whether the job was done well.) We do not need to say how the job should be done, and, indeed, in general, the actual implementation can be as complicated or refined as you wish, requiring its own micro-specializations and sub-contracting.* In any case, the general format: "You use that B on my A somehow, and others will observe your good work and wise choices." Thus, however "instrumental" this relationship may be, it can also have an important "informational" aspect to it.

Mathematical example: "Let f be a function into the real numbers with at least one zero." Here, we don't know what value(s) gets mapped to zero, but we know that there is at least one (unlike in the previous example).

Amphitheater - Binder - Structure - Spatial Selection - Deduction

$$\{X \xrightarrow{A \mapsto B} X'\}$$

This tells us the specific outcomes that await a subject (all A's will have B's substituted).

Narrative version: Here, A is a story element, B is a specified association. The deal: "I will give you something as yet unspecified, and you turn all the A's into B's, OK?" What's interesting is that this is in some sense "geographical": if you're not exposed to the machine, you're not exposed to the effect. Also note that if we can sample various effects, we can understand the a broader pattern of their distribution by reasoning backwards.

Mathematical example: "Let S be the sum of k^2 for all k in K." Here, we don't know what K is, but we know that whatever it is, its k's will be mapped to k^2's (and the results will be summed).

Circle - Evaluation - Heuristic - Group Selection - Creativity This describes a class of "meta-operations" that "resolve" networks.

Narrative version: Given a network that includes various maps, binders, and operations, we will decide (as a group) what to do with it. Different groups will have different ways of thinking about things. Clearly, we want to be part of a group that will make decisions that are beneficial to our own interest. Typically it is only after attaining membership that one has the opportunity to critique the group's way of making decisions (but this isn't always the case: it's possible to critique an editorial body without being part of it, for example)..

*I think that in Flusser's language, the more we specify how the job is supposed to be done, the more "pyramid-like" and less "tree-like" the regime is.

Mathematical example: Set theory; Lisp-1 REPL; or what have you.

We are left with numerous questions. What sorts of diagrams describe "how people learn mathematics"? "How people communicate a proof"? "How I decide to use one proof strategy or another?"

These sorts of questions invite us to think about "assemblages" – sometimes what matters in the assemblage are people, sometimes symbols, sometimes machines. Often, all three are hooked up together. Do the diagrams that "work" on one level resemble the diagrams that work on other levels? I think the answer is "yes" - but as this is only a preliminary note, I'm not entirely sure!

I can however sum up my thinking. *Grammar* is inherently *diagrammatic*, and mathematical grammar all the more so. Mathematics (at its best) represents how we *think* about things, and our thinking is based on low-level *cooperation* at the level of nerves and neurons. Our brains and bodies are also structured in such a way that our thinking itself is inherently *social* – and in particular, we often think in *language* (as well as pictures or other sensory data).

What would confirm this, deny it, or better yet, make it useful? I've done some sketching with words above, but I'd love to see some more diagrams of mathematical sentences and of mathematical behavior, to see if I can spot the same sorts of patterns appearing on small and large scales.

This may remind you of something.

> ...connects any point to any other point, and its traits are not necessarily linked to traits of the same nature; it brings into play very different regimes of signs, and even nonsign states... It is composed not of units but of dimensions, or rather directions in motion. It has neither beginning nor end, but always a milieu from which it grows and which it overspills... The rhizome proceeds by variation, expansion, conquest, capture, offshoots. ...the rhizome pertains to a map that ...is always detachable, connectable, reversible, modifiable, and has multiple entryways and exits.[48] (p. 203)

But I have another quote for you...

> In the *Kafka: Pour une Littérature Mineure* of Deleuze-Guattari, the transcendental Law such as it is found in The Castle is opposed to the immanence of desire in the adjacent offices. How can we fail to see that the Law of the Castle has its "rhizomes" in the corridors and the offices – the "bar" or the break constituted by the law has simply been geared down ad inifinitum in cellular and molecular succession. Desire is therefor only the molecular version of the Law. And what a strange coincidence to find schemas of desire and schemas of control everywhere. It is a spiral of power, of desire, and of the molecule which is now bringing us openly toward the final peripeteia of absolute control. Beware of the molecular![14] (p. 35)

That is to say, being "rhizomatic" and "decentered" isn't an excuse. From what? From anything. In other words, if we accept Baudrillard's idea that the Revolution has already happened (by in large), then where does that put us? What's left?

I think we should take these concerns very seriously. Do we need to agitate and make plans and roadmaps, if we're already "living the dream", as it were? (And how much is that the case?)

In other words, what should we make of the "theory" of paragogy? Does it express a good idea? Does it express an important (non-obvious and also useful) idea? F

To echo Socrates of the *Phaedrus*: I would say that even if we were implementing an effective and efficient roadmap, if we were not learning anything about these fundamental questions in the process, I think we would not be doing justice to the subject.

I think we *have* learned something so far, which is to be skeptical of revolutionary projects and grand claims. We know, thanks to Baudrillard, that the very mention of the word "power" should make us skeptical, and we know from Žižek that there's nothing "natural" about emancipation. We've gotten good at not listening when people talk about "the new paradigm of value-based relations replacing institutional power-based relations" or similar.

☛ Getting Your Hands Dirty with Drupal

Introduction

This report aims to capture the specific Drupal knowledge that was needed (and gained) during the process of re-building the PlanetMath website in Drupal 7. It adds considerable detail to the "work in progress" paper I previously presented at the 2012 Conference on Intelligent Computer Mathematics.[37]

The report is not a replacement for a good technical book[129] – which is itself no replacement for the slow learning process that happens via support requests and bug reports submitted through DRUPAL.ORG and IRC. One can also learn a lot in person, at Drupal meetups. Mostly, however, one learns by doing – and so I'm aiming to capture what I've learned that way, here. The key themes become sections, below.

The basics of working with Drupal

Backups, drush ard all,default –destination=../Backups/... : scp that to disk from time to time.

Load the latest code onto Github.

find . -mtime -10 or N instead of 10. Copy files over.

Merging everything else into D7.12

Clicking through the main links, what happens?

Which version of latex_field is current?

Schema: add the metadata field? Drupal 7.12 was needed.

Check out Drupal's "cron" setup.

Updated to D7.12, uninstalled, truncate cache, delete from field_config, reinstall... map latex_field to article type.

files, filter, drutexml, latex: We need a way to rebuild all of the XHTML and metadata upon re-importing.

Workflow: problem, add solution.

Rewrite, engine update, how to apply patches?

Get URL aliasing (e.g. for images and other uploads) sorted out...

We will want a new dump from PM. Do I know how to make these, or where to find them? Yeah, I can find them. However I'm not sure if what we have there gives me all the info we need for a Git conversion. (It does, but we still haven't hooked Git up into the systeme.)

Finishing up the alpha This was the kind of tasklist that confronted me:

- Complete the different functionality around problem solving
- Check into github those things which need to be there
- Import content, using a recent database dump
- Check that all the needed interactions are there

- Look at the list and refine the list of things that happen
- Collections (browsing the list of all problems for example is too much).

Feature overview

Topics Questions, requests, profile, private message, tags/sections, scoring, notices, "ads", orphanage, formatting, speed, backlinks. Fix groups, editing UI. Integrate everything.

Motivation "A student's use of the resource helps her learn and improves the resource in the process". (Look at the USE CASE I wrote up, consider extensions, like uploading a problem or cross-referencing a problem to find similar problems.) The idea of improving course materials via crowdsourcing can be used by other parties at various levels. Enhancing content and platforms to enable enhanced usage tracking... visualizing use... providing dashboards to manage analytics data. (Here I wonder if some of the Khan stuff can be used directly?)

Note that this grant wasn't funded, with the stated reason: it looks more like a continuation of an existing project than a "new" project. Maybe it was also just vague, or concerned with building something that's not modular enough. But it is, nevertheless, an interesting sort of approach.

Business model?

Comparison with the legacy system It was a relatively quick change to Noosphere 1.5 to add problems and solutions. However, it's not as flexible about manipulations thereof.

Specific dependencies notify_user : create a "system message" node? Notify when:

(watch) an article is edited, a forum is updated (correction) correction accepted/rejected (article) a solution is checked, a request is filled, orphaning is pending

_watcher_user_set_watching_mode

The installation profile

This is likely to be slightly annoying with lots of point and click, but a good chance to try and compare and debug my 'profile' too.

Features need to be loaded later. Where is info about the installed modules stored in Drupal's database?

Note also: selectively disabling some of the modules should help make the profile work better for dev work, i.e. faster.

The profile needs testing.

Blocks: done Config for pathautho: done

Figure out what the problem is with OG configuration in the profile.

What not to do

Patching core: what does it take to get a given patch into D7?

Anatomy of a module

Something simple: setting up fields Do we know which collaborations are "public" and which are "private"?

planetmath_edit_article might be a good example Canonical name shouldn't be made editable to the user. There shouldn't be spaces at the beginning or end of a title.

We needed to make some things disappear from the editing UI.

Commit: code for dealing with preambles (check)

Interaction block might be another nice one Do we need things like "unattach problem from this article"? How about "unattach correction"? The workflows are slightly different, that's for sure. Corrections need an "accept" logic, whereas "reviews" do not. Note, this is similar to the question about reviewing articles on their way into the encyclopedia (see above).

SPARQL and metadata

Why this is interesting Things link like: $A \to \{C,P\}SR$.

Add certification field to reviews: done

Show connections appropriately Get list of problems without solutions

Move the links at the bottom of the page into an "interact" box at the right.

"Revisions" to problems need to show up

Why Drupal seemed like a good idea Add back: sparql and sparql_registry

PyRDFa module should clean up after itself!

Maybe we should ship the code with ARC2 lib installed? Actually that doesn't seem to work.

A virtuoso API. Maybe emphasize adding RDFa more?

```
\msc{47A32}
\author{jac}
\title{Happy Theorem}
```

Get metadata like this into the XHTML, extract it, put it into the store (all done: still need to check the deletion case however!).

We needed to install and populate the triple store with classification data. We didn't find the famous sparql_views module to be particularly useful for our needs, maybe we just didn't understand it?

With new articles: hook_node_insert (populate them with a suitable preamble) when updating: hook_node_presave.

What happens if the user has deleted some metadata fields or put in illegal values? (Well, we don't let the user edit the TeX directly, we just take what they put into the form... but nevertheless, we might need to do some checking for `}`, for example, since that could screw up the tex processor).

Later: svn commit myarticle.tex -m "Bla" (and extract metadata as part of the post-commit).

We should also be updating the triple store information when the user *transfers ownership*.

Rather than storing the metadata field "within" the user-editable preamble field, we just decided to go ahead and create an extra textarea as part of the latex_field.

If we want to store triples like "16puzzle is related to 17 puzzle" then we will have some work to do.

Triple or other way to find things:

MSC Categories→Articles→Problems

Problems could also connect to other problems (hasAssociatedProblem).

We could do some kind of "fuzzy annotations" but that might be expensive.

We'd like to be able to do: "Give me problems about games that have solutions". (This might be a place for SPARQL views?)

Deyan: design an RDFa binding for something like `\addproblem{SquarePuzzleProblem}`.

We need a way to get the sparql store location set. (Actually we can just stick this in msc_browser).

Hook so that RDFa extractor runs on "save" event. Replace the old data with new data. Tricky, because we need to get it at the right point in time.

Extra bonus: nice displays of query results.

(Note: caching the MSC information would provide a nice speed up...)

`\pmproposedsoln{SquarePuzzleSoln1}`

Add a solution. `\pmsolves{SquarePuzzle}`.

Press save.

Instead of using a triple-based solution, we ended up just maintaining a table of article nids and associated problem nids.

Problem 1: use a flag and node_load to do the metadata processing?

Why Drupal may actually have been a good idea Views! We need a feed of problems without solutions, solutions without reviews, etc., and these are extremely easy to make.

Why virtuoso was also a good idea What links here? (Ask Virtuoso.)

Tracking user behavior

This is basically an example of the "metadata" topic discussed in the previous section – though the term "paradata" has been advanced to describe data about use (as opposed to data about content), and I tend to prefer this term.

In particular: if you're not logged in, you don't get any personal metadata!

Considering using Virtuoso or Drupal for activity data...

Export current config for userpoints. Userpoints_nc?

We need a "scoresheet" extracted from the user score information.

Collections (or "playlists") "Add to playlist" link (like on Youtube).

Suitable link to (my) playlists

Re-order items in a playlist, get the metadata/paradata display working: collections can show tasks, and "my progress".

Users and workflow

Description logics. Users. Can we say "What a user knows about"? Can we say who is an "expert"?

Is there a "review" step on the way to publishing something in the encyclopedia?

Cylinder sets... can show us if two people are working on "similar topics" at a given point in time. Then we might want to put them together to form a little ad hoc study group.

Groups and permissions

Every article is a group? Not any more.

Coauthor groups

Groups and permissions - Note: for the most part, groups have been our method for handling permissions (describe this)

They are: site-wide, user-specific, generated partly on the fly.

Apply to become a co-author?

Designate an existing team as a buddy list?

Workflow related to teams: Can Lucas have a list of algebra articles by Joe in his team (that he can't edit, until he invites Joe, who can then transfer some subset of the articles, etc)?

(Is my workplan article publicly editable? Maybe I need to share it with specific co-authors. Note that there is still that "group loophole" to deal with.)

Information about buddy lists had to be recorded in a couple of places (the user object, and the group system)...

Joining a group: Request, confirm, add? Request, Add?

Different places where we may want to have different levels of permissions, questioning, triple handshakes, etc

Button/block to transfer articles: done

Text on nodes should say "owner" not "submitted by" or whatever.

The "groups audience" should be shown on the article.'

Need: group permissions to be squared away

Groups policy: 1. Coauthor group of article A – all coauthors of A can edit A. 2. Team – all members of a team can edit all articles shared by the team 3. Buddy list of U – all members can edit all items created by U

Groups: who's in them? Search across them? Search within them? Associate with content.

Group stuff: information needs to show up on user pages, group pages, articles, the group list...

Adding content should be easy...

"Forums intersect group" - to show all postings from people in e.g. a course

How will people coordinate work?

Can we catch up with what people (e.g. in a group) are doing by using a shared activity stream?

Migration

Migration-related TODO items: migrate requests, thinking of them as a type of question

Comments: Node, OP, reply to etc. all had to be sorted out

Errors mid-migration: where do these come from?

Does Correction import work?

There should be a fairly simple way to overwrite the Drupal-generated score with the old score. And maybe some way to replay the "activities" that went into making that score (though that seems a bit doubtful).

Conclusion

Personal stuff: Time, presence, money, emotion, care, boundaries...

Feeling like it's a chore vs feeling like it's a creative process

"fractal knowledge"

Me: a "researcher" (what does this mean in my case?)

☛ Endnotes

A "Marx's struggle in this realm will always have a passionate value, and our goal is to confirm the right, not merely for youth, but for every individual, to realize themselves according to their free desires in autonomous creation and consumption. The focus of such a development could right away be UNESCO, from the moment when the SI takes command of it; new types of popular university, broken away from the passive consumption of the old culture; lastly, utopian educational centers which through the relation of leisure to certain arrangements of social spaces, they must come to be more completely free of the dominant daily life, and at the same time functioning as bridgeheads for an invasion of this daily life, instead of pretending to be separated from it." (OPEN CREATION AND ITS ENEMIES, Asger Jorn)

B The points made in "The Innovator's Dilemma" are interesting to consider here.[35]

C With a more technical bent, this is an area of current interest in economics research,[160] which is in fact quite relevant to our work here.[54]

D Although it is a genuine backronysm, it is interesting to compare Vygotsky's ZPD with the Zone of Strugatsky and Strugatsky from *Roadside Picnic*.[149]

E If for argument's sake the previous item was the Id, ascertaining an "end" in the form of primitive hedonic pleasure, then this one's the Super-Ego, situating a "cause" in global phenomena.

F And is really a *theory of production*? If it manages that, what else do we need to know?

> Seduction is that which is everywhere and always opposed to production; seduction withdraws something from the visible order and so runs counter to production, whose project is to set everything up in clear view, whether it be an object, a number or a concept.[14]

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