

Distributed Energy Efficient Spectrum Access in Cognitive Radio Wireless Ad Hoc Networks

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Abstract—In this paper, energy efficient spectrum access is considered for a wireless cognitive radio ad hoc network, where each node is equipped with cognitive radio, has limited energy, and the network is an OFDMA system operating on time slots. In each slot, the users with *new* traffic demand will sense the spectrum and locate the available subcarrier set. Given the data rate requirement and maximal power limit, a constrained optimization problem is formulated for each individual user to minimize the energy consumption per bit over all selected subcarriers, while avoid introducing harmful interference to the existing users. Because of the multi-dimensional and non-convex nature of the problem, a fully distributed subcarrier selection and power allocation algorithm is proposed by combining an unconstrained optimization method with a constrained partitioning procedure. Due to the non-cooperative behavior among new users, they will execute distributed power control to manage the co-channel interference when needed. Simulation results demonstrate that the proposed scheme performs tightly to the global optimal solution. In addition, the comparison between the proposed *energy efficient* allocation scheme and the well established *rate* or *power* efficient allocation algorithms is carried out to demonstrate the advantage of the proposed scheme in terms of network lifetime.

Index Terms—Resource allocation, cognitive radio, ad hoc network, OFDMA.

I. INTRODUCTION

ALTHOUGH the U.S. government frequency allocation data [1] shows that there is fierce competition for the use of spectra, especially in the bands from 0 to 3 GHz, it is pointed out in several recent measurement reports that the assigned spectrum are highly under-utilized [2], [3]. The discrepancy between spectrum allocation and spectrum use suggests that “spectrum access is a more significant problem than physical scarcity of spectrum, in large part due to legacy command-and-control regulation that limits the ability of potential spectrum users to obtain such access” [2]. In order to achieve much better spectrum utilization and viable frequency planning, Cognitive Radios (CRs) are under development to dynamically capture the unoccupied spectrum [4], [5]. Furthermore, in many challenging situations, the spectrum condition

and usage information may not be available a priori, such as in battlefield applications [38]. It is up to the CR users to sense the spectrum and obtain the spectrum occupancy. Many challenges arise with such dynamic and hierarchical means of accessing the spectrum, especially for the dynamic resource allocation of CR users by adapting their transmission and reception parameters to the varying spectrum condition while adhering to power constraints and diverse quality of service (QoS) requirements (see, for example, [12]–[15], [18]).

In this paper, an energy constrained wireless CR ad hoc network is considered, where each node is equipped with CR and has limited battery energy. One of the critical performance measures of such networks is the network lifetime. Moreover, due to the infrastructureless nature of ad hoc networks, distributed resource management scheme is desired to coordinate and maintain communications between each transmitting receiving pair. In this context, the present paper provides a framework of distributed energy efficient spectrum access and resource allocation in wireless CR ad hoc networks that employ orthogonal frequency division multiple access (OFDMA) [8], [9], [34] at the physical layer. OFDMA is well suited for CR because it is agile in selecting and allocating subcarriers dynamically and it facilitates decoding at the receiving end of each subcarrier [26]. In addition, multi-carrier sensing can be exploited to reduce sensing time [6].

The CR OFDMA network operates on time slots. An existing user transmits a pilot signal periodically on occupied subcarriers [27]. By detecting the presence of such a pilot signal, emerging CR users can determine the available subcarrier set in a target spectral range, and then select subcarriers and transmission parameters (based on the proposed algorithm) without introducing harmful interference to the existing users [5], [7]. In this work, the primary users (existing users) are users with on-going communications, while the secondary users refer to CR users with new traffic demand. After the emerging CR users start their data transmissions, they become existing users [38].

Each emerging CR user will select its subcarriers and determine its transmission parameters individually by solving an optimization problem. The optimization objective is to minimize its energy consumption per bit¹ while satisfying its QoS requirements and power limits. Compared with the power minimization with respect to target data rate constraints [12] or throughput maximization under power upper bound [13], this objective function, which measures the total energy consumed for reliable information bits transmitted, is particularly suitable

Manuscript received October 3, 2008; revised February 9, 2009 and June 1, 2009; accepted June 29, 2009. The associate editor coordinating the review of this paper and approving it for publication was W. Lou.

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An earlier version of the manuscript was published in IEEE WCNC 2008. This research work is supported in part by the U.S. Army Research Office under Cooperative Agreement No. W911NF-04-2-0054. Contents are solely the responsibility of the authors and do not necessarily represent the official views of the Army Research Office or the U. S. Government.

Digital Object Identifier 10.1109/TWC.2009.081323

¹which is defined as the ratio of the total transmission and reception power consumption over available subcarrier set to its achieved throughput

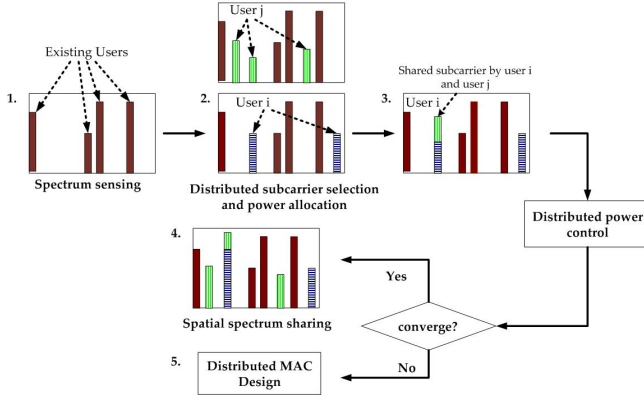


Fig. 1. Block diagram of the proposed distributed resource allocation algorithm.

for energy constrained networks where the network lifetime is a critical metric. The multi-dimensional and non-convex nature of the optimization problem in multi-carrier systems makes it more challenging than the throughput maximization/power minimization problems or the energy efficiency problem in a single carrier system [21]. Hence, we propose a two-step algorithm by first decoupling it into an unconstrained optimization problem, and a constrained partitioning method is applied thereafter to obtain the constrained optimal solution by partitioning the solution space according to power and QoS constraints.

Although the emerging CR users will not cause harmful interference to the existing users, they may choose the same subcarriers in the same time slot independently, and thus co-channel interference may be introduced. In this work, we allow multiple new users to share the same subcarriers as long as their respective Signal-to-Interference-and-Noise-Ratio (SINR) is acceptable. This may be achieved by distributed power control [17], which converges very fast. The flow chart of the proposed distributed energy efficient spectrum access and resource allocation scheme is highlighted in Fig. 1, where step 2 corresponds to the constrained optimization performed by each emerging user individually. More detailed illustrations of the flow chart are given in section IV.

The remainder of this paper is organized as follows. In section II, the system model and the problem formulation are given. A fully distributed channel selection and power allocation scheme for single user case is proposed in section III. In section IV, a distributed power control algorithm is suggested to manage potential co-channel interference caused by concurrent *new* users. Section V contains simulation results and discussions. Related works are discussed in section VI. Section VII gives concluding remarks.

II. SYSTEM MODEL

We consider an energy constrained CR OFDMA network of N communicating pairs. Both transmitter i and receiver j is indexed by $\mathcal{N} := \{1, 2, \dots, N\}$. If $j = i$, receiver j is said to be the intended receiver of transmitter i . The transmission system is assumed to be a time-slotted OFDMA system with fixed time slot duration T_S . Slot synchronization is assumed to be achieved through a beaconing mechanism. Before each time

slot, a guard interval is inserted to achieve synchronization, perform spectrum detection as well as resource allocation (based on the proposed scheme). Inter-carrier interference (ICI) caused by frequency offset of the side lobes pertaining to transmitter i is not considered in this work (which can be mitigated by windowing the OFDM signal in the time domain or adaptively deactivating adjacent subcarriers [19]).

A frequency selective Rayleigh fading channel is assumed at the physical layer, and the entire spectrum is appropriately divided into M subcarriers to guarantee each subcarrier experiencing flat Rayleigh fading [10]. We label the subcarrier set available to the transmitter receiver pair i after spectrum detection by $\mathcal{L}_i \subset \{1, 2, \dots, M\}$. Let $\mathbf{G} := \{G_{i,j}^k, i, j \in \mathcal{N}, k \in \mathcal{L}_i\}$ denote the subcarrier fading coefficient matrix, where $G_{i,j}^k$ stands for the sub-channel coefficient gain from transmitter i to receiver j over subcarrier k . $G_{i,j}^k = |H_{i,j}^k(f)|^2$, where $|H_{i,j}^k(f)|$ is the transfer function [33]. It is assumed that \mathbf{G} adheres to a block fading channel model which remains invariant over blocks (coherence time slots) of size T_S and uncorrelated across successive blocks. The noise is assumed to be additive white Gaussian noise (AWGN), with variance $\sigma_{i,k}^2$ over subcarrier k of receiver i . We define $\mathbf{P} := \{p_i^k, p_i^k \geq 0, i \in \mathcal{N}, k \in \mathcal{L}_i\}$ as the transmission power allocation matrix for all users in \mathcal{N} over the entire available subcarrier set $\bigcup_{i \in \mathcal{N}} \mathcal{L}_i$, where p_i^k is the power allocated over subcarrier k for transmitter i . For each transmitter i , the *power vector* can be formed as

$$\mathbf{p}_i := [p_i^1, p_i^2, \dots, p_i^M]^T \quad (1)$$

If the k^{th} subcarrier is not available for transmitter i , $p_i^k = 0$. Each node is not only energy limited but also has peak power constraint, i.e., $\sum_{k \in \mathcal{L}_i} p_i^k \leq p_i^{\max}$. The set of all feasible power vector of transmitter i is denoted by \mathcal{P}_i

$$\mathcal{P}_i := \left\{ \mathbf{p}_i \in \prod_{k \in \mathcal{L}_i} [0, p_i^{\max}], \sum_{k \in \mathcal{L}_i} p_i^k \leq p_i^{\max} \right\} \quad (2)$$

The signal to interference plus noise ratio (SINR) of receiver i over subcarrier k (γ_i^k) can be expressed as

$$\begin{aligned} \gamma_i^k(p_i^k) &= \alpha_i^k(p_j^k) \cdot p_i^k \\ \alpha_i^k(p_j^k) &= \frac{G_{i,i}^k}{\sum_{j \neq i, j \in \mathcal{N}} G_{j,i}^k \cdot p_j^k + \sigma_{i,k}^2} \end{aligned} \quad (3)$$

where α_i^k is defined as the channel state information (CSI) which treats all interference as background noise. α_i^k can be measured at the receiver side and is assumed to be known by the corresponding transmitter through a reciprocal common control channel.

When all users divide the spectrum in the same fashion without coordination, it is referred to as a *Parallel Gaussian Interference Channel* [20] which leads to a tractable inner bound to the capacity region of the interference model. The achievable maximum data rate for each user (Shannon's capacity formula) is

$$\frac{c_i(\mathbf{p}_i)}{B_i} = \sum_{k \in \mathcal{L}_i} \frac{c_i^k(p_i^k)}{B_i} = \sum_{\substack{k \in \mathcal{L}_i, \\ p_i^k \in \mathcal{P}_i}} \log_2 \left(1 + \alpha_i^k(p_j^k) \cdot p_i^k \right) \quad (4)$$

where B_i^k is the equally divided subcarrier bandwidth for transmitter i . Without loss of generality, B_i^k is assumed to be unity in this work. The noise is assumed to be independent of the symbols and has variance σ^2 for all receivers over entire available subcarrier set. Furthermore, all communicating transmitter and receiver pairs are assumed to have diverse QoS requirements specified by $\sum_{k \in \mathcal{L}_i} c_i^k \geq r_i^{tar}$, where r_i^{tar} is the target data rate of transmitter i .

In an energy constrained network (such as a wireless sensor network), reception power is not negligible since it is generally comparable to the transmission power [21]–[23]. In this work, we denote the receiving power as p_i^r which is treated as a constant value for all receivers².

Aiming at achieving high energy efficiency, the energy consumption per information bit for transmitter receiver pair i in each time slot is

$$e_i(\mathbf{p}_i, c_i) := \frac{\sum_{k \in \mathcal{L}_i} p_i^k + p_i^r}{\sum_{k \in \mathcal{L}_i} c_i^k} \quad (5)$$

Let $\mathcal{S}_i(\mathbf{p}_i, c_i)$ denote the set of all power and rate allocations satisfying QoS requirements and power limit constraints for transmitter i , and it is given by

$$\mathcal{S}_i(\mathbf{p}_i, c_i) = \{\mathbf{p}_i, c_i : \mathbf{p}_i \in \mathcal{P}_i, c_i \geq r_i^{tar}, i \in \mathcal{N}\} \quad (6)$$

Given the above system assumptions and the objective defined in (5), we end up with the following constrained optimization problem.

$$\begin{aligned} & \min_{\mathbf{p}_i^k, c_i^k \in \mathcal{S}_i} e_i(\mathbf{p}_i, c_i) \\ & s.t. \quad c_i(\mathbf{p}_i) \geq r_i^{tar}, \forall i \in \mathcal{N} \\ & \quad \sum_{k \in \mathcal{L}_i} p_i^k \leq p_i^{max}, \forall i \in \mathcal{N} \end{aligned} \quad (7)$$

III. OPTIMAL SUBCARRIER SELECTION AND POWER ALLOCATION

The problem (7) is a combinatorial optimization problem and the objective function is not convex/concave. Constrained optimization techniques, such as multi-dimensional interior-point method [32], can be applied here but with considerable computational complexity. Hence, we propose a two-stage algorithm to decouple the original problem into an unconstrained problem in order to reduce the search space. After the optimal solution for the unconstrained problem is obtained in stage 1, the power and data rate constraints will be examined in search of the final optimal solution. It should be noted that the solution of the unconstrained problem provides the optimal operating point which can be taken as the benchmark for the system design.

²In this work, we consider an energy constrained CR ad-hoc wireless network where the throughput requirement is usually not as high as the throughput demanding networks such that the baseband symbol rate is not very high. Thus this baseband power consumption is quite small compared with the power consumption in the RF circuitry. Hence, we neglect the energy consumption of baseband signal processing blocks to simplify the model, and the receiving power equals to the power consumption in the RF circuitry and can be treated as a constant [21]

A. Unconstrained Energy Efficient Allocation

We define the unconstrained energy per bit function as

$$f(\hat{\mathbf{p}}_i, \boldsymbol{\alpha}_i) := \frac{\sum_{k \in \mathcal{L}_i} \hat{p}_i^k + p_i^r}{\sum_{k \in \mathcal{L}_i} \log_2(1 + \alpha_i^k \cdot \hat{p}_i^k)} \quad (8)$$

where $\hat{\cdot}$ is used to represent the variables in the unconstrained optimization domain and $\boldsymbol{\alpha}_i = [\alpha_i^1, \alpha_i^2, \dots, \alpha_i^k]$. It is assumed $f(\hat{\mathbf{p}}_i, \boldsymbol{\alpha}_i)$ is a continuous function in \mathbb{R}_M^+ . We define the unconstrained optimal energy per bit for transmitter i of (8) as $\hat{\zeta}_i^* = \min f(\hat{\mathbf{p}}_i, \boldsymbol{\alpha}_i)$.

1) Energy Efficient Waterfilling:

Theorem 1: Given the channel state information $\boldsymbol{\alpha}_i$ and noise power, power allocation $\hat{\mathbf{p}}_i^* = [\hat{p}_i^{1*}, \hat{p}_i^{2*}, \dots, \hat{p}_i^{k*}, k \in \mathcal{L}_i]$ is defined as the *unconstrained optimal power allocation* by satisfying

$$f(\hat{\mathbf{p}}_i^*, \boldsymbol{\alpha}_i) \leq f(\hat{\mathbf{p}}_i, \boldsymbol{\alpha}_i), \forall \hat{\mathbf{p}}_i \subset \mathbb{R}^{M+} \quad (9)$$

Then the *unconstrained optimal power allocation* can be obtained by solving the following equations:

$$\begin{aligned} \hat{p}_i^{k*} &= \max \left\{ \log_2 e \cdot \hat{\zeta}_i^* - \frac{1}{\alpha_i^k}, 0 \right\} \\ \hat{\zeta}_i^* &= \frac{\sum_{k \in \mathcal{L}_i} \hat{p}_i^{k*} + p_i^r}{\sum_{k \in \mathcal{L}_i} \log_2(1 + \alpha_i^k \cdot \hat{p}_i^{k*})} \end{aligned} \quad (10)$$

Proof: Differentiating $f(\hat{\mathbf{p}}_i, \boldsymbol{\alpha}_i)$ with respect to \hat{p}_i^k (which stands for the power allocated for transmitter i on subcarrier k), we obtain the equations (10). The details of the derivation are given in **Appendix A**. ■

The value of $\hat{\zeta}_i^*$ can be obtained by using a numerical method which will in turn determine $\hat{\mathbf{p}}_i^*$. It is observed that $\hat{\mathbf{p}}_i^*$ has similar type of *rate-adaptive / margin-adaptive* waterfilling results, and we name it *energy-efficient* waterfilling. Whereas, the fundamental difference among them lies in the positions of their respective optimal points. The *rate-adaptive* waterfilling maximizes the achievable data rate under power upper bound, and *margin-adaptive* waterfilling minimizes the total transmission power subject to a fixed rate constraint [16], both of which achieve their optimality at the boundary of the constraints. On the contrary, the proposed *energy-efficient* waterfilling selects the most energy-efficient operating point (in other words, it selects the optimal data rate that minimizes the energy consumption per information bit) while adhering to the QoS requirements and power limits. In this case, optimality is usually obtained in the constraint interval rather than on the boundary. In fact, the *rate-adaptive* and *margin-adaptive* waterfilling can be considered as special cases of the *energy-efficient* waterfilling solved in this paper. If we set $\sum_{k \in \mathcal{L}_i} p_i^k = p_{con} \leq p_i^{max}$ or $\sum_{k \in \mathcal{L}_i} c_i^k(p_i^k) = r_i^{tar}$, the *energy-efficient* allocation problem is reduced to the well explored *rate-adaptive* or *margin-adaptive* waterfilling problem.

2) *Feasibility Region:* The existence of the solution for the unconstrained optimization ($\min f(\hat{\mathbf{p}}_i, \boldsymbol{\alpha}_i)$) depends on the subcarrier condition α_i^k if we assume other system parameters

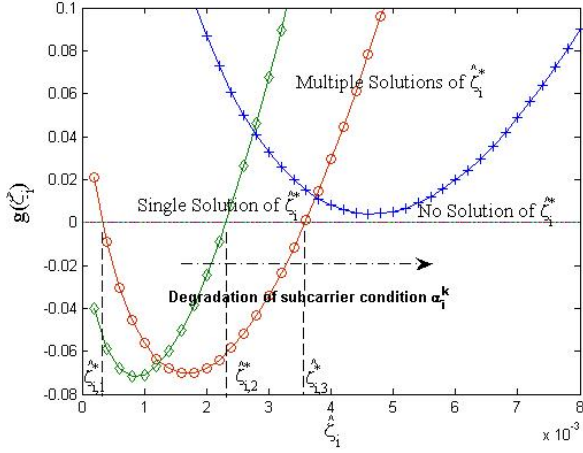


Fig. 2. Feasible solution vs. subcarrier condition.

(e.g. bandwidth, maximal power, etc.) are fixed. From (10), if we take \hat{p}_i into the expression of $\hat{\zeta}_i^*$, we can get

$$\hat{\zeta}_i = \frac{\Gamma(\hat{p}_i^*) \cdot \log_2^e \hat{\zeta}_i - \sum_{k \in \mathcal{L}_i} \frac{1}{\alpha_i^k} \cdot I(\hat{p}_i^{k*}) + p_i^T}{\Gamma(\hat{p}_i^*) \cdot \log_2(\log_2^e \hat{\zeta}_i) + \sum_{k \in \mathcal{L}_i} \log_2(\alpha_i^k) \cdot I(\hat{p}_i^{k*})}$$

$$I(\hat{p}_i^{k*}) = \begin{cases} 1, & \hat{p}_i^{k*} > 0 \\ 0, & \hat{p}_i^{k*} \leq 0 \end{cases} \quad (11)$$

where $\Gamma(X)$ is defined as the cardinality of nonzero elements in vector X . The optimal solution $\hat{\zeta}_i$ can be determined by solving equation (11), and the existence of the optimal solution is influenced by the subcarrier condition α_i^k . This is illustrated in Fig.2. A unique optimal solution ($\hat{\zeta}_{i,2}^*$) is obtained when the subcarrier condition is good; while no feasible solution exists when the subcarrier condition is bad. Multiple solutions may be obtained when the subcarrier condition is in the middle range. In this case, only the larger solution ($\hat{\zeta}_{i,3}^*$) is the feasible solution, and this can be verified by checking the corresponding power allocation, i.e., all the allocated power should be non-negative.

The feasibility condition of the unconstrained optimization problem is given in the following theorem.

Theorem 2: Denote the maximal optimal solution of $\hat{\zeta}_i^*$ as $\hat{\zeta}_i^{max}$ and the channel gain of the *best* subcarrier as $\alpha_i^T, \alpha_i^T = \max\{\alpha_i^k, \forall k \in \mathcal{L}_i\}$. The feasibility condition for the existence of the optimal solution of the *energy efficient* waterfilling (10) is given by $\alpha_i^T \geq \frac{\ln 2}{\hat{\zeta}_i^{max}}$.

Proof: 1) Necessity: From the optimal solution of *energy efficient* waterfilling (10), it is observed the amount of allocated power is determined by the subcarrier condition α_i^k , specifically, more power should be allocated on better subcarrier. Thus, if the optimal solution exists, at least the power allocated on the *best* subcarrier should be non-negative, i.e., $\hat{p}_i^T = \log_2^e \hat{\zeta}_i^{max} - \frac{1}{\alpha_i^T} \geq 0 \implies \alpha_i^T \geq \frac{\ln 2}{\hat{\zeta}_i^{max}}$.

2) Sufficiency: We prove this part by contradiction. If $\alpha_i^T \geq \frac{\ln 2}{\hat{\zeta}_i^{max}}$ and still no optimal solution exists, which implies that the power allocated on the entire subcarrier set is negative, i.e., $\hat{p}_i^{k*} < 0, \forall k \in \mathcal{L}_i$, then $\hat{\zeta}_i^{max} - \frac{\ln 2}{\alpha_i^T} < 0 \implies \alpha_i^T < \frac{\ln 2}{\hat{\zeta}_i^{max}}$,

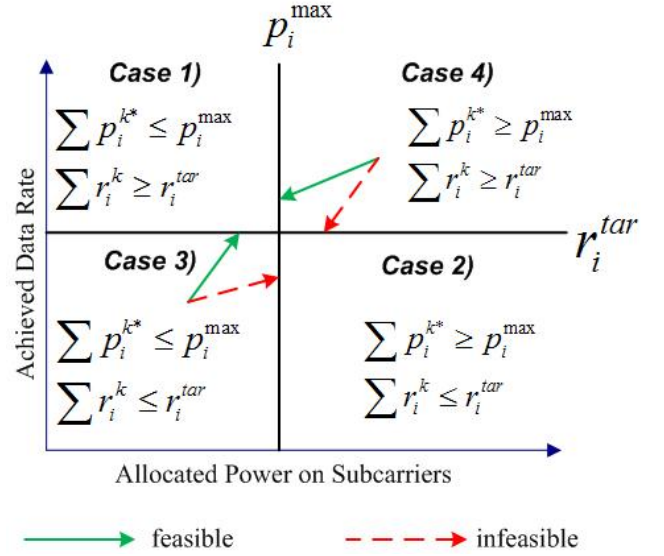


Fig. 3. Partition of the solution space of the constrained optimization problem.

which contradicts the condition $\alpha_i^T \geq \frac{\ln 2}{\hat{\zeta}_i^{max}}$. This completes the proof. ■

Theorem 2 suggests that it is sufficient to check the best available subcarrier in order to determine the feasibility of the unconstrained optimization problem.

B. Constrained Energy-Efficient Allocation Algorithm

Given the unconstrained optimal solution $\hat{p}_i^*, i \in \mathcal{N}$, the previous section offers the optimal operating point with best energy efficiency of each individual user. However, some users may not satisfy their respective data rate and/or power constraints when operating at this point. In this section, we partition the solution space of the constrained optimization problem (7) into four sub-spaces based on the power and data rate constraints, as highlighted in Fig.3.

$$1) \sum_{k \in \mathcal{L}_i} \hat{p}_i^{k*} \leq p_i^{max} \text{ and } \sum_{k \in \mathcal{L}_i} c_i^k(\hat{p}_i^{k*}) \geq r_i^{tar}.$$

In this case, the unconstrained optimal solution \hat{p}_i^* of (10) satisfies the sum-power and rate requirement constraints. Apparently \hat{p}_i^* is the optimal solution of the original problem (7).

$$2) \sum_{k \in \mathcal{L}_i} \hat{p}_i^{k*} \geq p_i^{max} \text{ and } \sum_{k \in \mathcal{L}_i} c_i^k(\hat{p}_i^{k*}) < r_i^{tar} \text{ or } \sum_{k \in \mathcal{L}_i} \hat{p}_i^{k*} > p_i^{max} \text{ and } \sum_{k \in \mathcal{L}_i} c_i^k(\hat{p}_i^{k*}) \leq r_i^{tar}.$$

In this case, the allocated power has already exceeded the sum-power constraint, but the rate requirement is still not met, even under the optimal subcarrier selection and power allocation. Therefore, there is no feasible solution for the original problem (7).

$$3) \sum_{k \in \mathcal{L}_i} \hat{p}_i^{k*} < p_i^{max} \text{ and } \sum_{k \in \mathcal{L}_i} c_i^k(\hat{p}_i^{k*}) < r_i^{tar}.$$

If both the power allocated on all subcarriers does not reach the maximal power bound and the data rate requirement is not met, the power should be increased to achieve data rate requirement under the maximal

power bound. Based on (7) and (10), we can modify the original problem as

$$\begin{aligned} & \min_{\hat{p}_i^k + \Delta p_i^k \in \mathcal{S}_i} \frac{\sum_{k \in \mathcal{K}_i} (\hat{p}_i^{k*} + \Delta p_i^k) + p_i^r}{\sum_{k \in \mathcal{K}_i} \log_2 (1 + \alpha_i^k \cdot (\hat{p}_i^{k*} + \Delta p_i^k))} \\ & \text{s.t. } \sum_{k \in \mathcal{K}_i} c_i^k (\hat{p}_i^{k*} + \Delta p_i^k) \geq r_i^{tar}, \forall i \in \mathcal{N} \\ & \sum_{k \in \mathcal{K}_i} (\hat{p}_i^{k*} + \Delta p_i^k) \leq p_i^{max}, \forall i \in \mathcal{N} \end{aligned} \quad (12)$$

where \mathcal{K}_i is defined as the selected subcarrier set through the optimal *energy efficient* waterfilling solution, $\mathcal{K}_i \subset \mathcal{L}_i$. If we increase the power on any one of the subcarriers, such as the k^{th} subcarrier, the corresponding constrained energy consumption per bit can be expressed as

$$\begin{aligned} \zeta_i^k &= \frac{\Delta p_i^k + \sum_{k \in \mathcal{K}_i} \hat{p}_i^{k*} + p_i^r}{\sum_{k \in \mathcal{K}_i} \log_2 (1 + \alpha_i^k \cdot \hat{p}_i^{k*}) + \Delta c_i^k} \\ \Delta c_i^k &= \log_2 \left(\frac{1 + \alpha_i^k \cdot (\hat{p}_i^{k*} + \Delta p_i^k)}{1 + \alpha_i^k \cdot \hat{p}_i^{k*}} \right) \end{aligned} \quad (13)$$

From (10), Δc_i^k can be simplified to $\Delta c_i^k = \log_2 \left(1 + \frac{\Delta p_i^k}{\log_2^e \cdot \hat{\zeta}_i^*} \right)$. It is observed that given the increased power Δp_i^k on subcarrier k , the increased data rate does not rely on its subcarrier condition α_i^k , since $\log_2^e \cdot \hat{\zeta}_i^*$ is a constant value for the entire selected subcarrier set. In other words, for any two subcarrier $k, l \in \mathcal{K}_i$ of transmitter $i \in \mathcal{N}$, if $\Delta p_i^k = \Delta p_i^l$, then $\Delta c_i^k = \Delta c_i^l$. And the constrained energy consumption per bit ζ_i^k and ζ_i^l will not vary due to different chosen subcarriers. If we presume, in order to reach the data rate requirement r_i^{tar} , the additional required power Δp_i over the selected subcarrier set is known and denoted as $\Delta p_i = \sum_{k \in \mathcal{K}_i} \Delta p_i^k$. Then, problem (12) is equivalent to

$$\begin{aligned} & \min_{\hat{p}_i^k + \Delta p_i^k \in \mathcal{S}_i} \frac{\Delta p_i + \sum_{k \in \mathcal{K}_i} \hat{p}_i^{k*} + p_i^r}{\sum_{k \in \mathcal{K}_i} \log_2 (1 + \alpha_i^k \cdot \hat{p}_i^{k*}) + \sum_{k \in \mathcal{K}_i} \Delta c_i^k} \\ & \text{s.t. } \sum_{k \in \mathcal{K}_i} c_i^k (\hat{p}_i^{k*} + \Delta p_i^k) \geq r_i^{tar}, \forall i \in \mathcal{N} \\ & \sum_{k \in \mathcal{K}_i} (\hat{p}_i^{k*} + \Delta p_i^k) \leq p_i^{max}, \forall i \in \mathcal{N} \end{aligned} \quad (14)$$

If we assume Δp_i has been pre-determined, in order to minimize energy consumption per bit ζ_i , $\sum_{k \in \mathcal{K}_i} c(\hat{p}_i) + \sum_{k \in \mathcal{K}_i} \Delta c_i^k$ need to be maximized. In other words, maximizing $\sum_{k \in \mathcal{K}_i} \Delta c_i^k$ will result in a classical *rate-*

adaptive waterfilling problem.

$$\begin{aligned} & \max \sum_{k \in \mathcal{K}_i} \log_2 \left(1 + \frac{\Delta p_i^k}{\log_2^e \cdot \hat{\zeta}_i^*} \right) \\ & \text{s.t. } \sum_{k \in \mathcal{K}_i} (\hat{p}_i^{k*} + \Delta p_i^k) \leq p_i^{max}, \forall i \in \mathcal{N} \end{aligned} \quad (15)$$

Because $\log_2^e \cdot \hat{\zeta}_i^*$ is a constant value for the entire selected subcarrier set \mathcal{K}_i , the solution of the above water filling problem implies that the optimal solution Δp_i^k for (15) should be the same for all chosen subcarriers. In other words, given the total required additional power Δp_i , the power should be equally allocated on all subcarriers, $\Delta p_i^k = \frac{\Delta p_i}{\Gamma(\mathcal{L}_i)}$. Thus, problem (12) can be rewritten as

$$\begin{aligned} & \min \frac{\Delta p_i + \sum_{k \in \mathcal{K}_i} \hat{p}_i^{k*} + p_i^r}{\sum_{k \in \mathcal{K}_i} \log_2 (1 + \alpha_i^k \cdot \hat{p}_i^{k*}) + \sum_{k \in \mathcal{K}_i} \Delta c_i^k} \\ & \text{s.t. } \sum_{k \in \mathcal{K}_i} \hat{p}_i^{k*} + \Delta p_i \leq p_i^{max}, \forall i \in \mathcal{N} \end{aligned} \quad (16)$$

where $\sum \Delta c_i^k = \Gamma(\mathcal{K}_i) \cdot \log_2 \left(1 + \frac{\Delta p_i}{\Gamma(\mathcal{K}_i) \log_2^e \cdot \hat{\zeta}_i^*} \right)$. Given the unconstrained optimal solution \hat{p}_i^* from stage 1, (16) can be considered as an objective function in terms of variable Δp_i bounded by $p_i^{max} - \sum_{k \in \mathcal{K}_i} \hat{p}_i^{k*}$.

Lemma 1: The constrained energy consumption per bit of problem (16) which is denoted as ζ_i is always worse than the unconstrained optimal energy efficiency $\hat{\zeta}_i^*$ with respect to the power increase Δp_i^k , i.e. $\zeta_i \geq \hat{\zeta}_i^*$, $\forall \Delta p_i \in \mathbb{R}^+$.

The proof of Lemma 1 is given in [39]. Due to the optimality of the unconstrained solution \hat{p}_i , the minimal deviation from $\hat{\zeta}_i$ will result in the optimal energy efficiency. Thus, the optimal power increase to satisfy the target data rate will be the minimal required additional power as illustrated in Fig.4. Therefore, the optimal required additional power (min Δp_i) to satisfy the data rate requirement r_i^{tar} can be calculated as min $\Delta p_i = \sum_{k \in \mathcal{K}_i} \Delta p_i^{k*}$.

The minimal required additional power $\Delta p_i^{min} = \min \Delta p_i$ can be derived by

$$\log_2 \left(1 + \frac{\Delta p_i^{min}}{\Gamma(\mathcal{K}_i) \cdot \log_2^e \cdot \hat{\zeta}_i^*} \right) = \frac{r_i^{tar} - \sum_{k \in \mathcal{K}_i} c_i^k (\hat{p}_i^{k*})}{\Gamma(\mathcal{K}_i)} \quad (17)$$

From (17), the optimal power increase on k th subcarrier Δp_i^{k*} is given by

$$\frac{\Delta p_i^{k*}}{\log_2^e \cdot \hat{\zeta}_i^*} = \exp \left(\frac{r_i^{tar} - \sum_{k \in \mathcal{K}_i} c_i^k (\hat{p}_i^{k*})}{\log_2^e \cdot \Gamma(\mathcal{K}_i)} \right) - 1 \quad (18)$$

If Δp_i^{min} exceeds the remaining power, i.e., $\sum_{k \in \mathcal{K}_i} \hat{p}_i^{k*} + \Delta p_i^{min} \geq p_i^{max}$, there is no feasible solution for (7). If $\sum_{k \in \mathcal{K}_i} \hat{p}_i^{k*} + \Delta p_i^{min} \leq p_i^{max}$, the optimal solution for the original problem (7) is

$$p_i^{k*} = \hat{p}_i^{k*} + \Delta p_i^{k*} \quad (19)$$

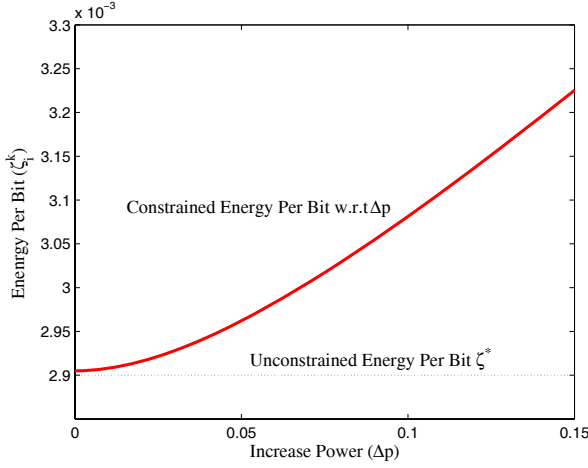


Fig. 4. Illustration of constrained ζ_i^k with respect to Δp_i .

$$4) \sum_{k \in \mathcal{L}_i} \hat{p}_i^{k*} > p_i^{max} \text{ and } \sum_{k \in \mathcal{L}_i} c_i^k(\hat{p}_i^{k*}) > r_i^{tar}.$$

In this case, the data rate requirement is satisfied but the allocated power exceeds the limit. In order to obtain a feasible solution, the allocated power should be decreased. The derivation of the optimal solution follows similar procedures as given in case 3) and is available in [39].

The inter-relationship and evolution of the four cases partitioned by the power and data rate constraints are highlighted in Fig.3. Excellent and terrible subcarrier conditions will lead to case 1) (feasible) and case 2) (infeasible), respectively. When the subcarrier conditions are “good”, the solid lines from case 3) and case 4) lead the problem into the feasible region case 1) of the constrained optimization problem when it reaches the maximal power and target data rate bounds, respectively. Whereas, the dashed lines suggest that the problem enters the infeasible region case 2) when the current subcarrier condition cannot accommodate the target data rate under the maximal power limits.

IV. DISTRIBUTED POWER CONTROL

In the previous section, each emerging new user obtains its optimal subcarrier selection and power allocation individually without considering other new users. Although no interference will be introduced to the existing users, due to the non-cooperative behavior of each user, multiple new users may choose the same subcarriers and co-channel interference will be introduced among themselves. In order to maintain user's QoS, we propose an iterative and distributed algorithm for reaching an equilibrium point among multiple transmitter and receiver pairs based on the distributed power control scheme [17]. The distributed power control algorithm is given by

$$p_i^k(t+1) = \min \left\{ \frac{\gamma_i^{k*}}{\gamma_i^k(t)} p_i^k(t), p_i^{max} \right\} \quad (20)$$

where γ_i^{k*} is the individual target SINR of the i^{th} transmitter receiver pair over each subcarrier k , which is determined by the constrained optimal solution \mathbf{p}^* , $\gamma_i^{k*} = \exp(\ln 2 \cdot c(p_i^{k*})) - 1$.

In the power control stage, each node only needs to know its own received SINR (γ_i^k) at its designated receiver to update its transmission power. This is available by feedback from the receiving node through a control channel. As a result, the proposed scheme is fully distributed. Convergence properties of this type of algorithms were studied by Yates [17]. An interference function $I(P)$ is standard if it satisfies three conditions: positivity, monotonicity and scalability. It is proved by Yates [17] that the standard iterative algorithm $P(t+1) = I(P(t))$ will converge to a unique equilibrium that corresponds to the minimum use of power. The distributed power control scheme (20) is a special case of the standard iterative algorithm.

In summary, the proposed energy efficient spectrum access and resource allocation scheme includes the following steps, as highlighted before in Fig. 1.

Distributed Energy Efficient Spectrum Access and Resource Allocation

1) Initialization

- Each transmitter receiver pair obtains their respective available subcarrier set \mathcal{L}_i through spectrum detection.

2) Individual Energy Efficient Resource Allocation

- Each transmitter receiver pair derives its respective unconstrained optimal solution from equation (10).
- Based on the power limit and data rate constraint, each transmitter receiver pair adjusts its power allocation according to the constrained optimal solution given in Section III. B.
- Each transmitter receiver pair also calculates its corresponding optimal target SINR γ_i^{k*} based on the constrained optimal solution.

3) Multiuser Distributed Power Control

- Through a control channel, each transmitter acquires the measured SINR $\gamma_i^k(t)$ from the designated receiver.
- If $\gamma_i^k(t) \neq \gamma_i^{k*}$, the transmission power will be updated according to (20).
- If $|\gamma_i^k(t) - \gamma_i^{k*}| \leq \epsilon$, $\forall i$, where ϵ is an arbitrary small positive number, the power control algorithm converges to a unique equilibrium point. Otherwise, it is infeasible to accommodate all the new users in the current time slot.

The detailed flow chart of the entire procedures of the proposed distributed spectrum access and resource allocation is given in Fig.5. During the power control stage, if the target SINR γ_i^{k*} cannot be maintained when transmitter i hits its power bound p_i^{max} , the network is unable to accommodate all the new users. In this case, a multi-access control (MAC) scheme is required to guarantee the fairness among the users. This will be one of our future efforts.

V. SIMULATION RESULT

In this section, we evaluate the performance and convergence of the proposed distributed energy efficient channel selection and power allocation algorithm. The proposed algorithm is firstly investigated for each individual user to validate the theoretical results. The impact of system parameter

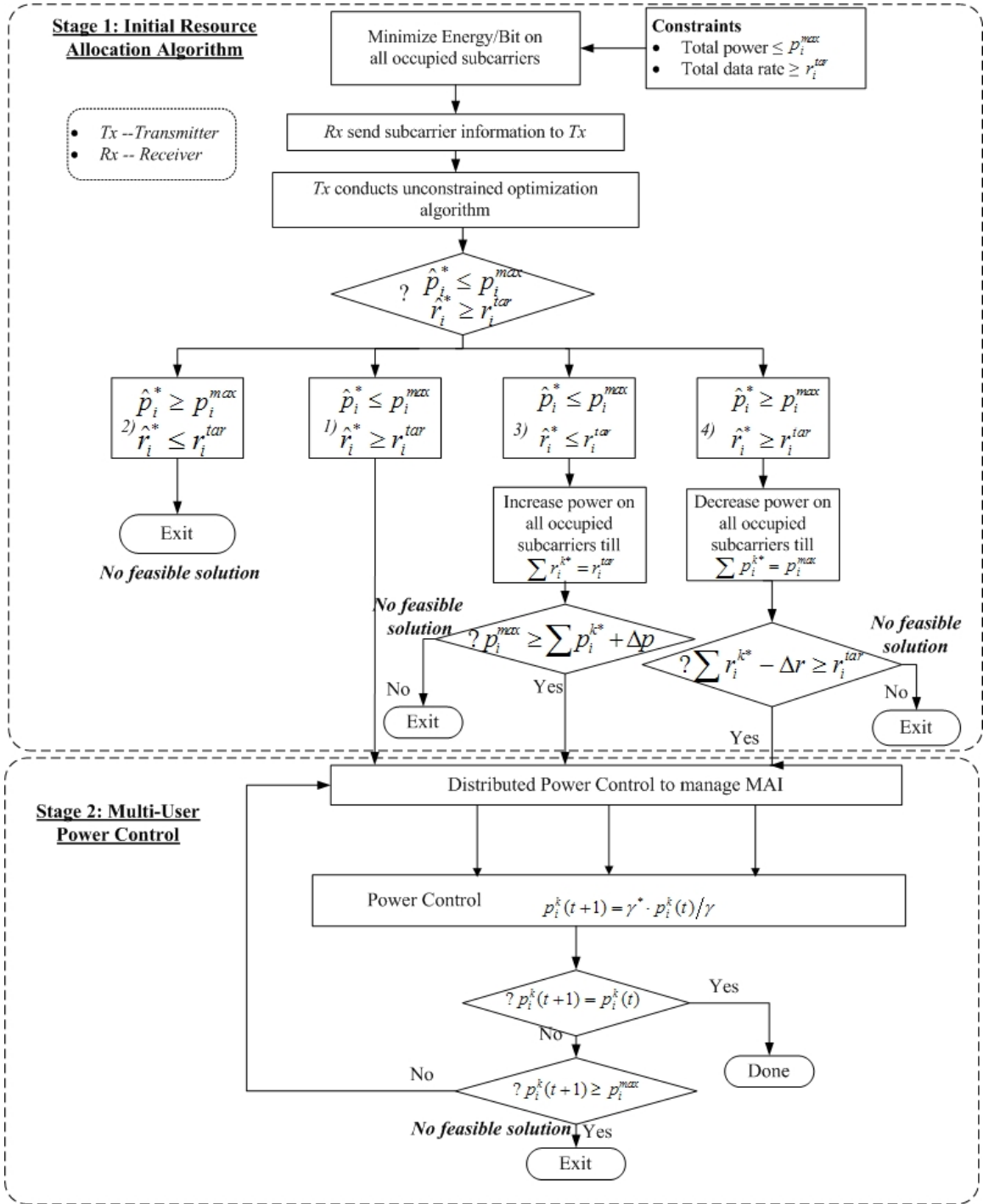


Fig. 5. The block diagram of the proposed distributed energy efficient spectrum access scheme.

settings on energy efficiency is also analyzed. Furthermore, the convergence of the distributed power control scheme for multiple new users sharing the same subcarriers is studied. In addition, we demonstrate that the proposed *energy-efficient* waterfilling algorithm always outperforms the well-established *rate-adaptive* and *margin-adaptive* waterfilling algorithms in terms of network lifetime. Finally, we compare the proposed distributed allocation algorithm with the global optimal solution for benchmarking.

A. Simulation Setup

In the simulation, we consider a wireless ad hoc network with cognitive radio capability. Specifically, the parameters of mica2/micaz Berkeley sensor motes [23] are used here. The sensor motes operate on 2 AA batteries and the output of each battery is about 1.5 volts, 25000 mAh. The channel gains are assumed to be sampled from a Rayleigh distribution with mean equals to $0.4d^{-3}$, where d is the distance from the transmitter to the receiver. The power bound for the transmission power is 150 mW. The entire spectrum is equally divided into subcarriers with bandwidth 100 kHz. The duration

TABLE I
UNITS OF SYSTEM PARAMETERS

Symbols	Description	Value
p_i^r	receiving power	$48 \times 10^{-3} W$
p_i^{max}	maximal power limit	$150 \times 10^{-3} W$
B	Bandwidth of each subcarrier	100KHz
T_S	Duration of time slot	10ms
σ^2	Power of thermal noise	10^{-8}

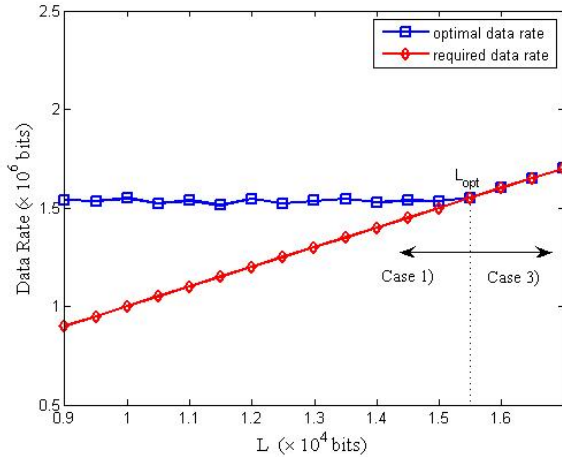


Fig. 6. Impact of L to optimal data rate.

of each time slot T_S is assumed to be $10ms$ in which L bits need to be transmitted. Thus, the target data rate is assumed to be $r_i^{tar} = L/T_S$. The thermal noise power is assumed to be the same over all subcarriers and equals to $10^{-8}W$. The system parameters are summarized in Table I and they are set such that the target data rate is feasible.

B. Performance of Individual Resource Allocation Algorithm

For each individual user, we first investigate the impact of the target data rate on energy efficiency. We consider a transmitter receiver pair with available subcarrier set $\Gamma(L_i) = 18$, the required data rate $r_i^{tar} = L/T_S$ ranges from 9×10^5 bps to 1.7×10^6 bps. In Fig.6, the *squared* line represents the optimal data rate allocation with the increase of r_i^{tar} , while the *diamond* line represents the required data rate r_i^{tar} . It can be observed from Fig. 6 that the optimal rate and power allocation remains approximately³ unchanged given the channel conditions of the available subcarriers as long as $r_i^{tar} < r_i^{opt} = 1.55 \times 10^6$. After the two lines converge at $L_{opt} = 15500$ bits, the optimal data rate coincides with r_i^{tar} , i.e., the required rate can only be obtained at the cost of lower energy efficiency. It is noticeable that L_{opt} is an important system design parameter, and its optimal value can be pre-calculated given the channel conditions.

Fig. 7 illustrates the effect of L (thus the target data rate r_i^{tar} for fixed T_S) on energy efficiency. We define $E_i = \zeta_i^* \times L$ as the energy consumption per time slot which is jointly determined by ζ_i^* and L . It is observed that in case 1) with the increase of L , E_i increases linearly with respect to L and the energy consumption per bit remains approximately unchanged.

³due to numerical round-off errors

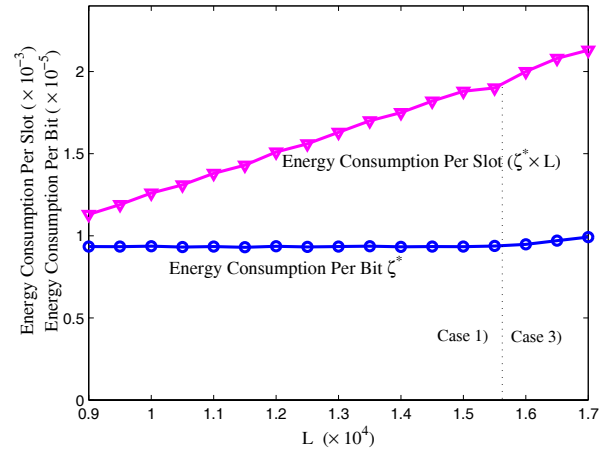


Fig. 7. Impact of L to energy efficiency.

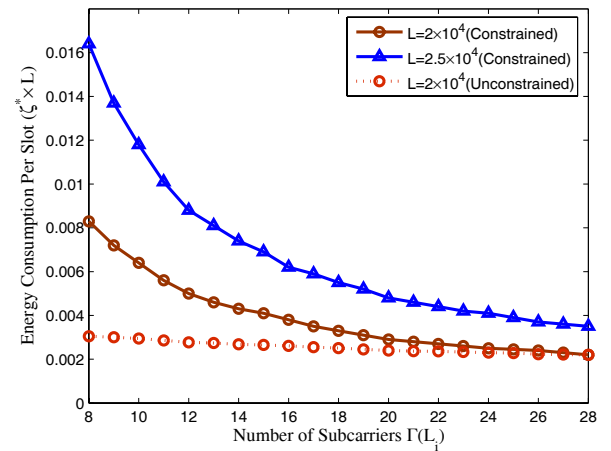


Fig. 8. Impact of number of available subcarriers to energy efficiency.

When the system enters case 3) due to the increase of r_i^{tar} , ζ_i^* degrades which suggests that the required data rate r_i^{tar} is satisfied with the expense of energy efficiency.

The impact of the number of available subcarriers on energy efficiency is plotted in Fig. 8. It is shown that the increase of the number of available subcarriers ($\Gamma(L_i)$) improves energy efficiency by providing more available bandwidth. In fact, the total optimal allocated power to satisfy a fixed target data rate is reduced with the increase of $\Gamma(L_i)$. It can be seen in Fig. 8 that the dashed circle line (which represents the unconstrained optimal energy consumption) converges with the constrained energy consumption (solid circle line) when the number of available subcarriers reaches 28. It implies that when the available subcarriers are less than 28, the unconstrained optimal solution corresponds to case 3) in Section III-B. The system will enter case 1) when $\Gamma(L_i) \geq 28$.

The performance of the proposed *energy-efficient* waterfilling with respect to network lifetime (which is a critical metric for energy constrained CR ad hoc networks) is investigated. Assuming uniform traffic patterns and persistent traffic flow across the network, we define the network lifetime as $T_l = E_{max}/(L \times \zeta_i^*)$, where E_{max} is the maximal energy source

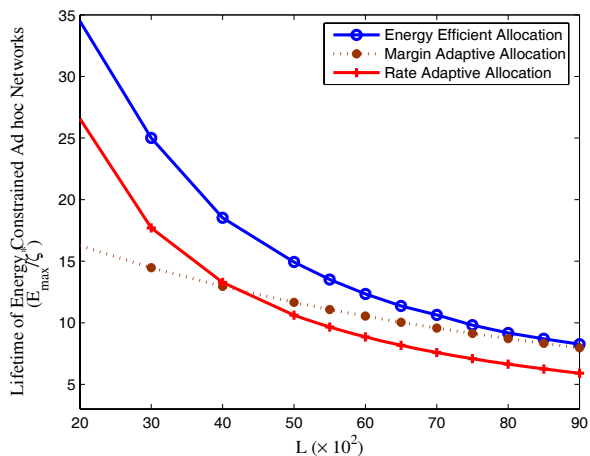


Fig. 9. Performance comparison among different allocation schemes.

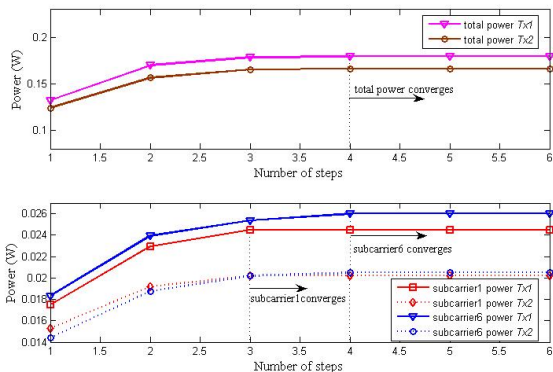


Fig. 10. Convergence of distributed power control.

of each transmitter. Compared with *rate-adaptive* and *margin-adaptive* waterfilling (for transmitting the same amount of information bits in the network), it is observed in Fig.9 that the proposed scheme outperforms the other two allocation schemes in terms of network lifetime. As the optimal allocated rate approaches the target data rate, *energy-efficient* waterfilling will converge with *margin-adaptive* waterfilling as expected. However, since the target data rate in a typical energy constrained ad hoc network is usually low, it is expected that the proposed scheme will improve network lifetime in most applications.

C. Performance Evaluation for Multiuser Allocation Scheme

After each new user obtains its optimal subcarrier selection and power allocation independently, distributed power control (20) may be triggered to manage the co-channel interference if multiple new users happen to choose the same subcarriers. The convergence of allocated power is shown in Fig. 10 (including the total required power and the power allocated on two randomly chosen subcarriers of two randomly chosen Tx-Rx pairs). It is observed that the convergence occurs in 3-4 steps.

In this part of the simulation (Fig. 11), the performance of the proposed distributed scheme is compared with the

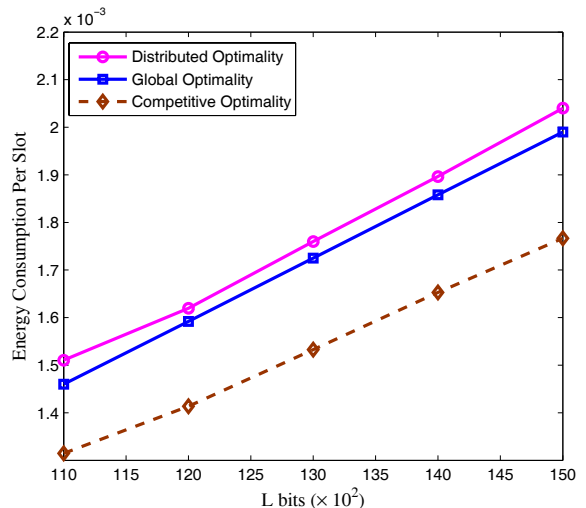


Fig. 11. Performance comparison between distributed scheme and global optimality.

centralized optimal solution, where it is assumed that a central controller collects all the $M \times N^2$ channel gain information from all the N new users, and calculates the global optimal solution by considering all the co-channel interference. The case for 8 users and each user with 16 available subcarriers is investigated here [25]. It is observed that the proposed distributed scheme (the upper two lines) performs closely to the centralized optimal solution (the middle line). In addition, the competitive optimal solution is also shown in Fig. 11, where each user calculates its own solution *without* considering co-channel interference (thus optimistic).

VI. RELATED WORK

The multi-user resource allocation problem based on multi-carrier modulation such as Orthogonal Frequency Division Multiplexing (OFDM), where subcarrier band, data rate and power are adaptively allocated to each user, has been widely addressed for cellular systems [35], [36]. In multi-carrier direct-sequence CDMA (DS-SS) cellular system, a non-cooperative power control game for resource allocation with respect to maximizing the energy efficiency is proposed in [24] which leads to the *best subcarrier selection* scheme by assuming the realized *SINR* on each subcarrier is the same. It is assumed in these works that the spectral utilization information is known a priori with the aid of base stations, which is not realistic in scenarios where an infrastructure is not available. Furthermore, it is worth noting that the optimal solution of energy efficient resource allocation is *not* best subcarrier selection for multiple transmitting receiving pairs in ad hoc networks [25].

In [11], the resource allocation problem is explored for OFDMA-based wireless ad hoc network by directly adopting distributed power control scheme for the power and bits allocation on all subcarriers to improve power efficiency. A greedy algorithm is proposed for best subcarrier selection in CR networks employing multicarrier CDMA [38], and distributed power control is performed thereafter to resolve co-channel interference. An Asynchronous Distributed Pricing

(ADP) scheme is proposed in [37], where the users need to exchange information indicating the interference caused by each user to others. In the context of CR enabled wireless sensor network (WSN) [12], a two-step algorithm is proposed to tackle the allocation problem: channel assignment with objective of minimizing transmission power and channel contention to reserve the subcarrier set for transmission by intended transmitters, while the interference spectrum mask is assumed to be known a priori. The authors of [13] address the opportunistic spectrum access (OSA) problem in WSN, in which a distributed channel allocation problem is modeled by a partially observable Markov decision process framework (POMDP) while assuming the transition probability of each channel is known. In [29], the CR spectrum sharing problem is formulated in multi-hop networks with objective to minimize the *space-bandwidth product* (SBP). However, the transmission power allocated on each subcarrier is assumed to be the same which may lead to significant performance loss. The effect of power control is analyzed in a subsequent paper [30]. Dynamic Frequency Hopping Community (DFHC) is proposed in [31] for the spectrum sharing in CR based IEEE 802.22 wireless regional area networks (WRANs) to ensure QoS satisfaction and reliable protection to licensed users.

A. Contributions of this paper

In summary, the contributions of this work include:

- 1) A new constrained optimization problem is formulated and solved that minimizing energy per bit across users subject to QoS and power constraints in a multi-user OFDMA network. This problem is significantly more difficult comparing to that in a single carrier system [21]. Furthermore, the problem considered in this paper is also significantly more difficult than the power minimization with respect to target data rate constraints or throughput maximization under power upper bound, because of the multi-dimensional and non-convex nature of the problem. The proposed performance criterion (minimizing energy per bit) is critical for energy constrained networks. It is a better choice than minimizing total power or maximizing throughput when energy efficiency is the major concern.
- 2) A novel concept, “*energy-efficient* waterfilling”, is given in this paper that is fundamentally different from the *rate-adaptive* waterfilling or *margin-adaptive* waterfilling⁴. In this case the optimal point is located in the constraint interval rather than on the boundary. In fact, the *rate-adaptive* and *margin-adaptive* waterfilling can be considered as special cases of the *energy-efficient* waterfilling solved in this paper.
- 3) The results obtained in this paper provide a valuable insight that the optimal solution of energy efficient resource allocation is *not* best subcarrier selection for multiple transmitting receiving pairs in an OFDMA network [25].

⁴The optimal allocation strategy with objective to minimize power or maximize throughput is named *margin-adaptive* and *rate-adaptive* waterfilling over frequency channels [28], respectively.

- 4) The proposed distributed subcarrier selection and power allocation scheme provides an *efficient* and *practical* solution for dynamic spectrum access in CR wireless ad hoc networks employing OFDMA. By combining the optimal resource allocation of individual users and distributed power control, the proposed method guarantees fast convergence speed, computational efficiency and implementation simplicity. Motivated by iterative waterfilling (IWF) algorithm in [16], another distributed solution may be obtained by solving the multi-user distributed channel and power allocation problem iteratively. However, it may take many steps for the iterative algorithm to converge if it converges at all and the delay may be too large to be tolerable. The cost of the additional computation complexity is high. On the contrary, the proposed optimal resource allocation of individual users is easy to obtain and distributed power control algorithm has well-known fast convergence speed. Furthermore, it is shown in this paper that the proposed distributed algorithm performs closely to the global optimal point in the simulations.

VII. CONCLUSION

In this paper, a framework of distributed energy efficient resource allocation is proposed for energy constrained OFDMA-based cognitive radio wireless ad hoc networks. A multi-dimensional constrained optimization problem is formulated by minimizing the energy consumption per bit over the entire available subcarrier set for each individual user while satisfying its QoS constraints and power limit. A two-step solution is proposed by first decoupling it into an unconstrained problem, and a constrained partitioning procedure is applied thereafter to obtain the constrained optimal solution by branching the solution space according to power and rate constraints. Co-channel interference may be introduced by concurrent *new* users and the distributed power control scheme may be triggered to manage the interference and reach the equilibrium point in the multiuser environment.

The proposed spectrum sharing plus resource allocation scheme provide a practical distributed solution for a CR wireless ad hoc network with low computational complexity. It is important to point out that the proposed algorithm for CR networks can be easily modified and applied to multi-channel multi-radio (MC-MR) networks which can be considered as a special case of the CR based wireless networks [29].

In this work, it is assumed that the subcarrier detection is perfect. The effects of detection errors will be investigated in our future work.

VIII. APPENDIX A

The unstrained optimization problem (8) is

$$f(\hat{\mathbf{p}}_i, \boldsymbol{\alpha}_i) := \frac{\sum_{k \in \mathcal{L}_i} \hat{p}_i^k + p_i^r}{\sum_{k \in \mathcal{L}_i} \log_2(1 + \alpha_i^k \cdot \hat{p}_i^k)} \quad (21)$$

The first order derivative of (21) with respect to \hat{p}_i^k can be derived as

$$\frac{\partial f(\hat{\mathbf{p}}_i, \boldsymbol{\alpha}_i)}{\partial \hat{p}_i^k} = \frac{1}{\log_2 e} \cdot \left(\frac{\partial \Phi(\hat{\mathbf{p}}_i, \boldsymbol{\alpha}_i)}{\partial \hat{p}_i^k} \right)$$

$$\Phi(\hat{\mathbf{p}}_i, \boldsymbol{\alpha}_i) = \frac{\hat{p}_i^k + \sum_{l \in \mathcal{L}_i, l \neq k} \hat{p}_i^l}{\ln(1 + \alpha_i^k \hat{p}_i^k + p_i^r) + \sum_{\substack{l \in \mathcal{L}_i \\ l \neq k}} \ln(1 + \alpha_i \hat{p}_i^l)} \quad (22)$$

If $k \neq l$, $c_i(\hat{p}_i^l)$ is taken as constant with respect to \hat{p}_i^k since the mutual interference between subcarriers is not considered in this work. Therefore, (22) can be expressed as

$$\frac{\partial \Phi(\hat{\mathbf{p}}_i, \boldsymbol{\alpha}_i)}{\partial \hat{p}_i^k} = \frac{\sum_{k \in \mathcal{L}_i} \frac{c_i(\hat{p}_i^k)}{\ln 2} - \left(\sum_{k \in \mathcal{L}_i} \hat{p}_i^k + p_i^r \right) \left(\frac{\alpha_i^k}{1 + \alpha_i^k \hat{p}_i^k} \right)}{\left[\sum_{k \in \mathcal{L}_i} \ln(1 + \alpha_i^k \cdot \hat{p}_i^k) \right]^2} \quad (23)$$

We assume the data rate $\sum_{k \in \mathcal{L}_i} c_i(\hat{p}_i^k) \geq 0$ in this work, thus for $\frac{\partial f(\hat{\mathbf{p}}_i, \boldsymbol{\alpha}_i)}{\partial \hat{p}_i^k} = 0$, (23) can be reduce to

$$\frac{\alpha_i^k}{1 + \alpha_i^k \cdot \hat{p}_i^k} = \frac{\sum_{k \in \mathcal{L}_i} \ln(1 + \alpha_i^k \cdot \hat{p}_i^k + p_i^r)}{\sum_{k \in \mathcal{L}_i} \hat{p}_i^k} \quad (24)$$

From (24), we can derive the unconstrained optimal power allocated for transmitter i over subcarrier k as

$$\hat{p}_i^k = \frac{\sum_{k \in \mathcal{L}_i} \hat{p}_i^k + p_i^r}{\sum_{k \in \mathcal{L}_i} \ln(1 + \alpha_i^k \cdot \hat{p}_i^k)} - \frac{1}{\alpha_i^k} \quad (25)$$

From the definition of unconstrained energy consumption per bit $\hat{\zeta}_i$, the first term of (25) is in the similar type of $\hat{\zeta}_i$. If we assume the optimal solution of (A1) does exist (the subcarrier condition resides in the feasible region), there must be a corresponding optimal value of energy per time slot $\hat{\zeta}_i^*$ with respect to \hat{p}_i . Then (25) can be expressed in terms of $\hat{\zeta}_i^*$ as

$$\hat{p}_i^{k*} = \log_2 e \cdot \hat{\zeta}_i^* - \frac{1}{\alpha_i^k} \quad (26)$$

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