# A Probabilistic Analysis of Link Duration in Vehicular Ad Hoc Networks

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***Abstract*—The past decade has witnessed a phenomenal market penetration of wireless communications and a steady increase in the number of mobile users. Unlike wired networks where communication links are inherently stable, in wireless networks the lifetime of a link is a random variable whose probability distribution depends on mobility, transmission range and various impairments of radio communications. Because of the very dynamic nature of VANET and of the short transmission range mandated by the Federal Communications Commission (FCC), individual communication links come into existence and vanish unpredictably, making the task of establishing and maintaining routing paths between fast-moving vehicles a very challenging task. The main contribution of this work is to investigate the probability distribution of the lifetime of individual links in VANET under the combined assumptions of a realistic radio transmission model and of a realistic probability distribution model of inter-vehicle headway distance. Our analytical results were validated and confirmed by extensive simulation.**

***Index Terms*—Wireless communications, vehicular ad hoc networks, path-loss model, headway distance, probability distribution, log-normal distribution.**

## I. Introduction

In the past decade Vehicular Ad-hoc NETworks (VANET), a noteworthy variant of Mobile Ad-hoc Networks (MANET) specialized to Vehicle-to-Vehicle and Vehicle-to-Infrastructure wireless communications, have been proposed to make driving a safer and more enjoyable experience. In turn, the emergence of VANET as a credible partner and interlocutor has accelerated the convergence of Intelligent Transportation Systems (ITS) and wireless communications. This confluece of technologies is poised to revolutionize the way we think driving by creating a safe and secure environment that will eventually pervade our highways and city streets.

Recently, the U.S. Federal Communications Commission (FCC) has set aside a block of 75MHz of spectrum in the 5.850 GHz to 5.925 GHz band specifically dedicated to applications intended to enhance the safety and efficiency of our highway systems. Vehicular communications are governed by the stipulations of Dedicated Short Range Communications (DSRC) that restrict inter-vehicle radio communications to about 300 meters [1], [2]. As a result, individual communication links are short-lived and multi-hop routing paths built on top of such links are highly vulnerable to disconnection.

Most of the early applications of VANET were motivated by traffic safety. However, it was recently noticed that the DSRC spectrum set aside by the FCC, by far exceeds the needs of traffic-related safety applications. This observation has motivated the emergence of a host of other applications that can take advantage of the allocated spectrum. Not surprisingly, we see more and more third-party providers offering non-safety-related applications ranging from peer-to-peer applications, to location-specific services, to multimedia content delivery and various flavors of mobile entertainment.

The *headway distance* between two *co-directional* vehicles is defined as the time (or, equivalently, distance) between two vehicles passing the same point and traveling in the same direction. The headway distance plays a fundamental role in understanding traffic flow and in ensuring travel safety and related issues. While there were several attempts at suggesting a safe headway distance on roadways and streets, the task of legislating what the best headway distance should be is a complex problem fraught with technical, societal and political issues. This explains why the task of determining the probability distribution of headway distance is still an open question that has received a great deal of attention in the literature [3], [4], [5], [6].

Our main contribution is a probabilistic analysis of link duration in VANET, based on a realistic distribution of headway distance as well as on a realistic channel model. Since, as already mentioned, there is no consensus in the literature on the exact distribution of the headway distance, we begin by an empirical validation of the log-normal headway distribution assumption proposed by Greenberg [3]. Our empirical validation serves the dual purpose of anchoring our analytical work in a realistic headway distance model and also to calibrate our subsequent simulation environment. Next, assuming a log-normal distribution of headway distance and a realistic channel propagation model, we derive the probability distribution as well as the expected duration of links in VANET. We assume that while they have not reached the speed limit, or have not stopped, vehicles have constant, non-empty, acceleration. Our analytical results were confirmed by extensive simulation.

## II. Related Work

In spite of the fundamental importance of predicting the duration of communication links in VANET, to the best of our knowledge, up to this point there have been only empirical studies or else papers reporting results built on top of simplistic radio propagation models.

To make this paper as self-contained as possible, we now provide a succinct survey of papers that have addressed similar issues. Panichpapiboon *et al.* [7] studied links and routing

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paths on the basis of signal strength. Kiese *et al.* [8] adapted received power levels and improved antenna gains to find better links. Yang *et al.* [9] and Kesting *et al.* [10] proposed using statistical and real-time density data to select wireless links in VANET. Bai *et al.* [11] proposed a model for path duration distribution in MANET. Based on experiments with Dynamic Source Routing (DSR), they proposed an approximate probability density function (exponential distribution) for the path duration. However, the assumption of path duration was not validated in [11].

Gruber and Hui [12] assumed link durations to be independent, exponentially distributed random variables; with this assumption, they derived the probability distributions of path duration which, not surprisingly, is exponential as well.[1] However, the details of the underlying mobility model supporting such an assumption were not discussed in [12].

Pascoe *et al.* [13] derived the time duration of an $n$-node path. However, their mobility model only includes velocity without considering acceleration. Nekovee [14] proposed a model to determine the probability of a link in VANET under the assumptions that (1) the headway distance is constant, (2) the radio propagation model only accounts for slow fading and ignores path loss due to distance, and (3) vehicle mobility patterns are ignored. Nekovee *et al.* [15] assumed that car velocities are normally distributed. From this assumption, the throughput is modeled by various formulas. The path-loss is also formalized as an exponential function of velocity. This formula is also the basis of Nekovee's work [14].

Su *et al.* [16] proposed an analytical model for the probability density function (pdf) of link lifetime. Their model is based on several assumptions, namely that nodes are equally spaced, and that speed is normally distributed. Building on these assumptions, Sun *et al.* [16] computed the probability of link lifetime. However, their first assumption is not reasonable since, as widely known, inter-vehicle distance is a random variable and certainly not a constant.

## III. A Closer Look at the Headway Distance

As already mentioned, the headway distance between consecutive cars on a roadway plays a fundamental role in understanding traffic flow and in ensuring travel safety and related issues. Not surprisingly, many headway distance models have been developed since the 1960s. As pointed out by Cowan [5], typical representatives of such distribution models include the exponential distribution, the normal distribution, the gamma distribution and the log-normal distribution. For instance, the log-normal distribution was proposed to model headways under car-following situations [3]. A major assumption for the log-normal headway model is that a vehicle maintains a safe distance while following its leading vehicle closely at variable speeds. This assumption makes sense and is apparent in real traffic data [17], [18], [19]. For example, Krbálek and Šeba [19] studied the statistics of public transportation in and around Mexico-City. Chen and Li employed a Markov model to study the headway distance and confirmed that the headway distance is log-normally distributed [20]. Chowdhury *et al.* [17] proposed a different distribution of headways, i.e. a function of speed limit. The headways between two successive particles is defined based on the number of empty boxes between them. Panwai *et al.* [4] studied headway distance in microscopic mobility simulators as a car following model. Some mixed distribution models are proposed on the assumption that a road consists of two components, tracking/following and free components. For example, Cowan [5] proposed a mixed distribution consisting of a constant distribution (tracking/following component) and an exponential distribution (free component). Griffiths and Hunt [18] proposed a mixed model called Double Displaced Negative Exponential Distribution.

Given the large variety of opinions in the literature concerning the probability distribution of the headway distance, we decided to begin our investigations by validating these models in relation to their suitability as basis for analytical studies of link distribution in VANET. Towards this goal, we have carried out experiments using the open source simulator written by Treiber [21]. Specifically, we have recorded and plotted the headway distance. It is important to note that Treiber [21] *does not* assume a specific model for the headway distance and, as a consequence, our empirical measurement were not biased towards any distribution. Having plotted the resulting headway distance, we then plotted, on the same graph, the various candidate probability distributions just mentioned. A detailed discussion can be found in the Appendix. As discussed below, and as illustrated in Figure 1, we found that the best fit between a classic distribution function and the simulation results is provided by the *log-normal distribution*. It is interesting to mention that our conclusion agrees with a similar an experiment conducted independently by Puan [6]. Given the good fit between our simulation results shown in Figure 2, and Paun's data collected using video cameras to record traffic movement at four sites [6], we have adopted the log-normal distribution of headway distance as the basis for our analytical derivations.

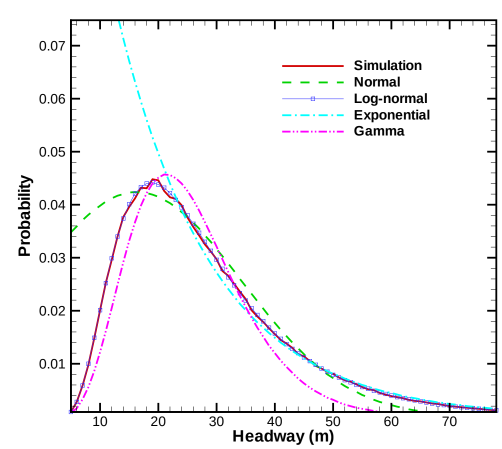



Fig. 1. *The pdf of headway distance versus the normal, log-normal, exponential and gamma distribution. The log-normal distribution best matches our simulations.*

[1]Recall that the minimum of several independent exponential random variables is also exponential.

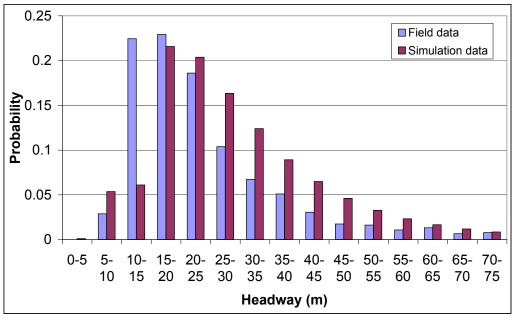



Fig. 2. *Contrasting our headway simulation vs. Paun's field data.*

## IV. The Probability Distribution of a Link

### A. Path-Loss

The *path-loss model* [22] is a radio propagation model that predicts the signal attenuation (in dB) at a distance $X$ from the transmitter. Visser *et al.* [23] used a patch antenna and studied the path-loss of a DSRC link. The path-loss relevant to inter-vehicle communication can be modeled by *two-ray model* which takes the reflection signal from the road itself into consideration. This suggests defining the path-loss in dB as a random variable $L(X)$ defined by writing:

$$L(X) = 40\log X - (10\log G_t + 10\log G_r + 20\log h_t + 20\log h_r) \quad (1)$$

where[2] $G_t$ and $G_r$ are the antenna gains of the transmitter and the receiver, respectively; $h_t$ and $h_r$ are, respectively, the heights of the transmitting and receiving antennas [24].

### B. On the Link Distance

We are interested in the link distance $X$, the distance between a source vehicle and a destination vehicle (i.e., between a sender and a receiver). Write $X = \sum_{i=1}^{m} X_i$ where the $X_i$s are independent log-normal random variables with a common distribution, specifically $X_i \in \log\mathrm{N}(\mu_i, \sigma_i)$. As illustrated in Figure 3, $X$ represents the convolution of $m$ independent headway distances. As it turns out [25], [26], $X$ is approximately log-normal; the commonly-used Fenton-Wilkinson approximation [27] of $X$ is obtained by setting

$$\sigma_X^2 = \log\left[\frac{\sum e^{2\mu_i+\sigma_i^2}(e^{\sigma_i^2}+1)}{(\sum e^{\mu_i+\sigma_i^2/2})^2}+1\right]$$

$$\mu_X = \log(\sum e^{\sigma_i^2}) - \frac{\sigma_Z^2}{2}.$$

To simplify the notation, we write $\sigma = \sigma_X$ and $\mu = \mu_X$ and

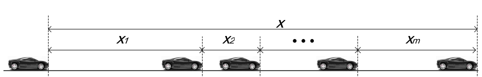



Fig. 3. *Illustrating the convolution* $X = X_1 + X_2 + \cdots + X_m$.

we use the notation $X \in \log\mathrm{N}(\mu, \sigma)$.

[2]Here, and in the remainder of this paper, we use log to represent $\log_{10}$ and ln to represent the natural logarithm $\log_e$.

### C. The Probability Distribution of Path-Loss

A quick look at (1) reveals that the path-loss $L(X)$ is the convolution of a random variable: $Y = 40\log X$ and a constant value $-(10\log G_t + 10\log G_r + 20\log h_t + 20\log h_r)$.

*Lemma 1:* Assuming that $X \in \log\mathrm{N}(\mu, \sigma)$, the random variable $L(X) = 40\log X - (10\log G_t + 10\log G_r + 20\log h_t + 20\log h_r)$ is normally distributed.

The proof of the lemma is routine and is, therefore, omitted; we note that since $L(X)$ is normally distributed, its probability density function (pdf) reads $l(z) \in \frac{1}{40}N\left(\frac{\mu}{40}+b, \sigma\right)$ where $b = -(10\log G_t + 10\log G_r + 20\log h_t + 20\log h_r)$. To simplify the notation, we write $L(X) = Z \in aN(\mu_z, \sigma_z^2)$, where $a = \frac{1}{40}$, $\mu_z = a\mu + b$ and $\sigma_z^2 = \sigma^2$.

### D. The Probability Distribution of the Existence of a Link

The existence of a communication link between a source vehicle and a destination vehicle depends on the path-loss at the receiver's side. For a link between these vehicles, the path-loss between them needs to be smaller than a given threshold $PL_{thr}$. Thus, the probability distribution $F(z)$ of the existence of a link between two vehicles separated by a distance of $z$:

$$\begin{aligned} F(z) &= P\{L(X) \le z\} \\ &= \int_{-\infty}^{z} \frac{a}{\sigma_z\sqrt{2\pi}} \exp\left(-\frac{(t-\mu_z)^2}{2\sigma_z^2}\right) dt \\ &= \frac{C_1 a}{2}\left[1 + erf\left(\frac{z-\mu_z}{\sigma_z\sqrt{2}}\right)\right] \end{aligned} \quad (2)$$

where $C_1$ is a normalization coefficient. Since $\lim_{z\to\infty} F(z) = 1$, it follows that $C_1 = \frac{1}{a}$ and, thus, we write

$$F(z) = \frac{1}{2} + \frac{1}{2} erf\left(\frac{z-\mu_z}{\sigma_z\sqrt{2}}\right). \quad (3)$$

## V. The Link Duration Model

Referring to Figure 4, assume that at time $t_0 = 0$ a link is established i between co-directional vehicles $i$ and $j$ with $j$ ahead of $i$. Let the random variable $X$ denote the distance separating the two vehicles at link setup time. Mindful of the 300m DSRC transmission range constraint, we have

$$0 \le X < 300. \quad (4)$$

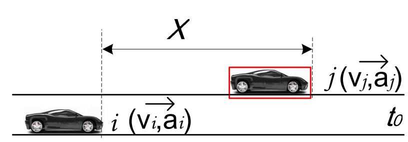



Fig. 4. *Illustrating our basic scenario.*

Recall that $X$ is the convolution of $m$ independent headway distances with a common log-normal distribution and that $X$ is approximately log-normal with parameters $\mu$ and $\sigma$. We assume that the speed limit on the roadway is $v_m$ and that no vehicle will travel faster than $v_m$. For $t \ge 0$, we define $a(t)$, the acceleration of the vehicle at time $t$ as follows:

- if $a(0) = 0$, then $a(t) = 0$ for all $t \ge 0$;

- if $a(0) > 0$, then

$$a(t) = \begin{cases} a(0) & \text{for } t \leq \frac{v_m - v(0)}{a(0)} \\ 0 & \text{otherwise;} \end{cases} \quad (5)$$

- if $a(0) < 0$, then

$$a(t) = \begin{cases} a(0) & \text{for } t \leq \frac{-v(0)}{a(0)} \\ 0 & \text{otherwise.} \end{cases} \quad (6)$$

In other words, (5) and (6) indicate that as long as the vehicle has not reached the speed limit $v_m$ or has not stopped (in case $a(0) < 0$), its acceleration remains $a(0)$. However, once the vehicle reaches the speed limit (or has stopped), its acceleration becomes 0.

Given a generic vehicle with initial speed $v(0)$, the instantaneous speed $v(t)$ at time $t$ is defined as

$$v(t) = v(0) + \int_0^t a(u)du, \quad (7)$$

where for all $u \in [0,t]$, $a(u)$ is the instantaneous acceleration at time $u$ defined above.

Now, (5) and (6) and (7), combined imply that

- if $a(0) = 0$, then $v(t) = v(0)$ for all $t \geq 0$;
- if $a(0) > 0$, then

$$v(t) = \begin{cases} v(0) + a(0)t & \text{for } t \leq \frac{v_m - v(0)}{a(0)} \\ v_m & \text{otherwise;} \end{cases} \quad (8)$$

- if $a(0) < 0$, then

$$v(t) = \begin{cases} v(0) + a(0)t & \text{for } t \leq \frac{-v(0)}{a(0)} \\ 0 & \text{otherwise.} \end{cases} \quad (9)$$

Similarly, with $v(x)$ defined above, the distance that our generic vehicle travels in the time interval $[0,t]$ is defined as

$$S(t) = \int_0^t v(x)dx. \quad (10)$$

We now return to our vehicles $i$ and $j$. To simplify the notation, we write $v_i = v_i(0),\ a_i = a_i(0)$ and $v_j = v_j(0),\ a_j = a_j(0)$. The instantaneous speeds and accelerations $v_i(t)$ and $a_i(t)$, respectively, $v_j(t)$ and $a_j(t)$ are obtained by suitably instantiating (5), (6), (8), (9). Now, (10) guarantees that the distances traversed in the time interval $[0,t]$ by vehicles $i$ and $j$ are, respectively,

$$S_i(t) = \int_0^t v_i(x)dx$$

and

$$S_j(t) = \int_0^t v_j(x)dx.$$

Assuming that at connection setup (i.e., at time 0) the distance between the two vehicles was $x$, it follows that the distance between $i$ and $j$ at time $t$ can be written as

$$S_j(t) - S_i(t) + X. \quad (11)$$

It is important to note that (11) defines a *signed* distance: indeed, if at time $t$, $S_j(t) - S_i(t) + X > 0$, then vehicle $j$ is ahead of $i$; otherwise, vehicle $i$ is ahead of $j$.

We find it convenient to define the indicator function $I(i,j)$ intended to capture information about which of the two vehicles is ahead when the communication link between them breaks

$$I(i,j) = \begin{cases} 1 & \text{if } S_j(t) - S_i(t) + X > 0 \\ -1 & \text{otherwise} \end{cases}$$

Given that DSRC links break at 300 meters, it follows that when the link breaks the following relation holds:

$$S_j(t) - S_i(t) + X = 300 \cdot I(i,j). \quad (12)$$

We refer the reader to Figure 5 where the various possible combinations of $v_i, v_j, a_i, a_j$ are illustrated. The boxes next to the speed axis indicate a possible combination of $v_i$ and $v_j$. The planar regions $A'$,$A''$ and $A'''$ are several sections of the area formed by the speed lines. As shown in Figure 5, we distinguish special time instances $t_\alpha, t_\beta, t_\gamma$ where

- $t_\alpha$ is the time when two vehicles have same speed,
- $t_\beta$ is the time when iexactly one vehicle has stopped,
- $t_\gamma$ is the time when both vehicles have stopped.

Due to the existence of a speed limit, we have $t_\varepsilon$ and $t_\zeta$ as special time moments where $t_\varepsilon$ is the time when one vehicle reaches the speed limit $v_m$, $t_\zeta$ is the time when both vehicles reach the speed limit. The reason for discussing these special time instances is that they affect in a crucial way the link duration. We now define a number of time instances that will be used in our analysis:

- provided that $\frac{v_j - v_i}{a_j - a_i} > 0$, we write

$$t_\alpha = \frac{v_j - v_i}{a_j - a_i};$$

- define $t_\beta$ as follows

$$t_\beta = \begin{cases} \frac{-v_i}{a_i} & \text{if } \frac{-v_i}{a_i} > 0 \text{ and } \frac{-v_j}{a_j} < 0 \\ \frac{-v_j}{a_j} & \text{if } frac{-v_j}{a_j} > 0 \text{ and } \frac{-v_i}{a_i} < 0 \\ \min\{\frac{-v_i}{a_i}, \frac{-v_j}{a_j}\} & \text{if } \frac{-v_i}{a_i} > 0 \text{ and } \frac{-v_j}{a_j} > 0 \\ \text{undefined} & \text{otherwise.} \end{cases}$$

- similarly, define $t_\gamma$ as follows

$$t_\gamma = \begin{cases} \max\{\frac{-v_i}{a_i}, \frac{-v_j}{a_j}\} & \text{if } \frac{-v_i}{a_i} > 0 \text{ and } \frac{-v_j}{a_j} > 0 \\ \text{undefined} & \text{otherwise.} \end{cases}$$

- define $t_\varepsilon$ as follows

$$t_\varepsilon = \begin{cases} \frac{v_m - v_i}{a_i} & \text{if } \frac{-v_i}{a_i} > 0 \text{ and } \frac{v_m - v_j}{a_j} < 0 \\ \frac{v_m - v_j}{a_j} & \text{if } \frac{-v_j}{a_j} > 0 \text{ and } \frac{v_m - v_i}{a_i} < 0 \\ \min\{\frac{v_m - v_i}{a_i}, \frac{v_m - v_j}{a_j}\} & \text{if } \frac{v_m - v_i}{a_i} > 0 \text{ and } \frac{v_m - v_j}{a_j} > 0 \\ \text{undefined} & \text{otherwise} \end{cases}$$

- define $t_\zeta$ as follows

$$t_\zeta = \begin{cases} \max\{\frac{v_m - v_i}{a_i}, \frac{v_m - v_j}{a_j}\} & \text{if } \frac{v_m - v_i}{a_i} > 0 \text{ and } \frac{v_m - v_j}{a_j} > 0 \\ \text{undefined} & \text{otherwise} \end{cases}$$

It is important to note that $\{t_\alpha \leq t_\beta \leq t_\gamma\}$ and $\{t_\varepsilon, t_\zeta\}$ only depend on the speeds and acceleration of the two vehicles at connection setup time and on the value of the speed limit $v_m$.

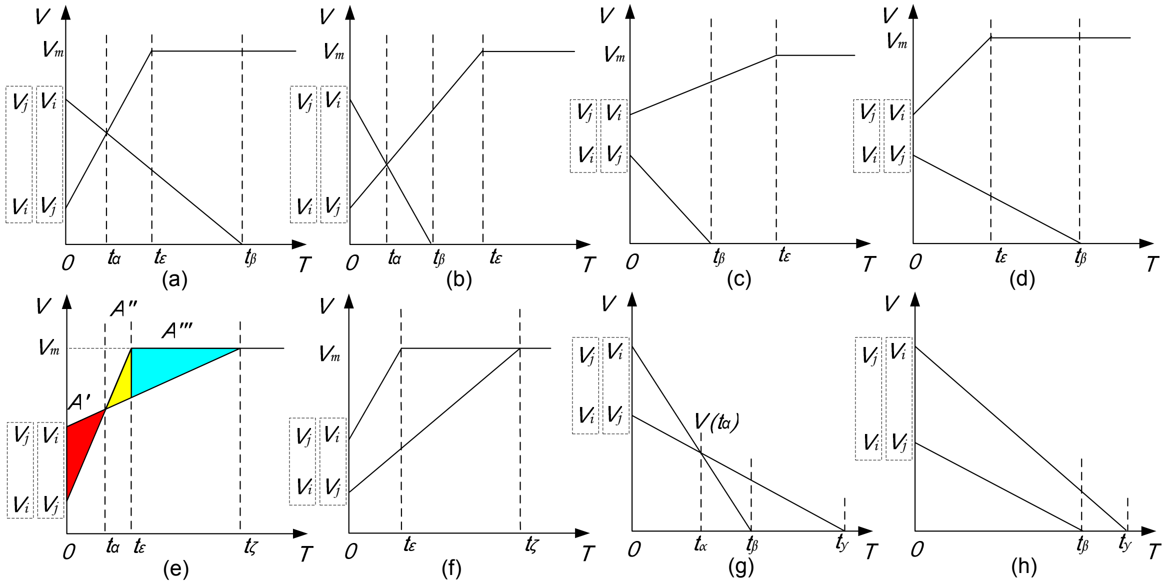



Fig. 5. *Illustrating possible combinations of $v_i, v_j, a_i, a_j$.*

### A. Deriving the Link Duration

Because of obvious similarities, we only discuss the scenario illustrated in Figure 5(e) as an example of solving (12). In this scenario, both vehicle move in the same direction and have positive accelerations. Sooner or later they will reach the speed limit and will, thereafter, cruise at the maximum speed.

*1) $0 \le t \le t_\alpha$:* The time region where the link breaks is $A'$ in Figure 5(e). Since $i, j$ are in $A'$, when the link breaks, vehicle $j$ must be ahead of $i$ and, thus, $I(i,j) = 1$. Recalling that in DSRC the link breaks when the distance between $i$ and $j$ is 300 meters, we can write (12) as

$$S_j(t) - S_i(t) + X = 300. \tag{13}$$

On the other hand, by (10), $S_j(t) - S_i(t) = \frac{1}{2}a_r t^2 + v_r t$. where $a_r = a_j - a_i$, $v_r = v_j - v_i$. All that remains, is to substitute $S_j(t) - S_i(t)$ in (13) and to solve for $t$. Since $t < t_\alpha$, we obtain

$$t = \frac{-v_r + \sqrt{v_r^2 + 2a_r(300 - X)}}{a_r}.$$

*2) $t_\alpha < t \le t_\varepsilon$:* In this case, the time region where the link breaks is $A''$ in Figure 5(e). Since $i, j$ are in $A''$, when the link breaks vehicle $i$ must be ahead of vehicle $j$; thus $I(i,j) = -1$ and by (12) we can write

$$S_j(t) - S_i(t) + X = -300.$$

We know that $S_j(t) - S_i(t) = \frac{1}{2}a_r t^2 + v_r t$. By substituting the value of $S_j(t) - S_i(t)$ in (V-A2) and by solving for $t$ we obtain

$$t = \frac{-v_r - \sqrt{v_r^2 - 2a_r(300 + X)}}{a_r}.$$

*3) $t_\varepsilon < t \le t_\zeta$:* In this case, the time region where the link breaks is denoted by $A'''$ in Figure 5(e). Since $i, j$ are in $A'''$, when the link breaks, vehicle $i$ must be ahead of vehicle $j$ and so $I(i,j) = -1$. Thus, we can write

$$S_j(t) - S_i(t) + X = -300.$$

Observe that by (10), $S_j(t) = \frac{1}{2}a_j t^2 + v_j t$ and $S_i(t) = v_m t - \frac{v_m - v_i}{2} t_\varepsilon$. After substituting the value of $S_j(t) - S_i(t)$ in (V-A3) and after solving for $t$ we obtain

$$t = \frac{-(v_j - v_m) - \sqrt{(v_j - v_m)^2 - 2a_j(300 + x + \frac{v_m - v_i}{2} t_\varepsilon)}}{a_j}.$$

*4) $t_\zeta < t$:* In this case, the link will not break because the two vehicles move at the same speed (alternatively, the link breaks at $+\infty$).

The scenario captured in Figure 5(e) shows that vehicle $i$ catches up with vehicle $j$, passes $j$ and, finally, breaks the link with vehicle $j$. Experience tells us that this is a very frequent occurrence in highway traffic.

## VI. The Distribution Function of Link Duration

The duration of a link is the lifetime of an established communication link between two vehicles. The main goal of this section is to derive analytical expressions for the probability distribution and density function of the duration of a link.

With the preamble of the previous section out of the way, we are now ready to state and prove the following important result.

*Lemma 2:* Assuming $X \in \log\mathrm{N}(\mu, \sigma)$, the random variable $T = \sqrt{aX + b} + c$ is log-normally distributed, where $a, b, c \in \mathbb{R}$, $a, b, c \neq 0$ and $aX + b \ge 0$.

*Proof:* Let $G_T$ be the probability distribution function of $T$. For every positive $t$, we write

$$G_T(t) = \Pr[\{T \le t\}]. \tag{14}$$

Since $T$ is obviously continuous, (14) allows us to write

$$\begin{aligned} G_T(t) &= \Pr[\{T \le t\}] \\ &= \Pr[\{\sqrt{aX+b}+c \le t\}] \\ &= \Pr[\{aX \le (t-c)^2 - b\}] \\ &= \begin{cases} F_X\left(\frac{(t-c)^2-b}{a}\right) & \text{for } a>0 \\ 1-F_X\left(\frac{(t-c)^2-b}{a}\right) & \text{for } a<0 \end{cases} \end{aligned}$$

where $F_X$ is the probability distribution function of $X$. When $a > 0$, it is clear that $T$ is log-normally distributed.

Next, we propose to show that $T$ is also log-normally distributed when $a < 0$. For this purpose, letting $z = \frac{(t-c)^2-b}{a}$ and using (3), we write

$$\begin{aligned} 1-F_X\left(\frac{(t-c)^2-b}{a}\right) &= \frac{1}{2}-\frac{1}{2}erf\left(\frac{\ln z-\mu(X)}{\sigma(X)\sqrt{2}}\right) \\ &= F_Y\left(\frac{a}{(t-c)^2-b}\right) \end{aligned}$$

where $Y$ is a log-normal random variable with parameters $-\mu(X)$ and $\sigma(X)$, $z = \frac{(t-c)^2-b}{a}$. Note that we use the fact $-erf(x) = erf(-x)$. Thus, in all cases, $T$ obeys a log-normal distribution, completing the proof. ■

*Lemma 3:* Assuming that $X$ is log-normal with parameters $\mu$ and $\sigma$, the random variable $T = aX + b$ is log-normally distributed, where $a,b,c \in \mathbb{R}$ and $a,b,c \neq 0$.

*Proof:* This lemma can be easily proved using the arguments employed in the proof of Lemma 2. ■

*Lemma 4:* Suppose that the communication link between two vehicles $i$ and $j$ breaks at time $t$. The link duration time is either a linear function of $X$ or a square root function of $X$.

*Proof:* Recall that when the link breaks, $t$ satisfies (12). By the definition of $S_i(t)$, we know that $S_i(t) = \int_0^t v_i(x)dx$ is a linear function of $t$ when the speed $v_i$ is constant, i.e., $v_i(t) = v_m$. Let $S_i(t) = at + b$. Similarly, $S_j(t)$ is a linear function of $t$ when $v_j$ is constant, i.e. $v_j(t) = v_m$. Let $S_j(t) = ct + d$. When both $S_i(t)$ and $S_j(t)$ are linear function of $t$, substituting the corresponding values of $S_j(t)$ and $S_i(t)$ in (12), we obtain

$$(a-c)t + b - d + X = 300I(i,j)$$
$$t = 300\frac{I(i,j) - X - b + d}{a-c}.$$

Clearly, the link duration $t$ is a linear function when both $v_i(t)$ and $v_j(t)$ are constant. If any of $v_i(t)$ and $v_j(t)$ are not a constant, the distance function will be a quadratic polynomial. Without loss of generality, we let $v_i(t) = v_i(0) + a_i t$, by definition, the distance function

$$\begin{aligned} S_i(t) &= \int_0^t v_i(0) + a_i x dx \\ &= v_i(0)t + \frac{1}{2}a_i t^2. \end{aligned}$$

Therefore, $S_j(t) - S_i(t)$ will be a quadratic polynomial. Suppose $S_j(t) - S_i(t) = at^2 + bt + c$ and $a \neq 0$. Substitute $S_j(t) - S_i(t)$ in (12), to get the quadratic equation

$$at^2 + bt + c + X - 300I(i,j) = 0.$$

Clearly, $t$ exists because the link breaks at time $t$. Therefore, the solution must be a square root function of $X$. This completes the proof. ■

*Theorem 1:* The duration $T$ of the link between vehicles $i$ and $j$ is log-normally distributed.

*Proof:* By Lemma 4, the link duration can be expressed as either $aX + b$ or $\sqrt{aX+b}+c$. By Lemma 2, the expression $\sqrt{aX+b}+c$ is log-normally distributed. By Lemma 3, $aX + b$ is log-normally distributed. Thus, in all cases, the duration time of the link has a log-normal distribution. This completes the proof of the theorem. ■

Let $\Phi$ be a set of all real combination of $v_i, v_j, a_i, a_j$ on roads and $\phi$ be the size of $\Phi$. Let $P_k$ be the probability of the case $k \in \Phi$ and $T_k$ be the link duration time of case $k$. By the law of total expectation, we can obtain the overall expected duration of a link $E[link]$,

$$E[link] = \sum_{k=1}^{\phi} P_k T_k. \tag{15}$$

Theoretically, $E[link]$ can be computed by (15). However, to the best of our knowledge, there are no analytical results or field-test data on $P_k$ and $T_k$ in literature. We will leave the computation of the expected link duration for future investigations.

## VII. Simulation Results

The main goal of this section is to discuss the details of the simulation model that we used to validate the theoretical results obtained in the previous sections.

### A. Experimental Setup

The defining characteristic of VANET is their dynamic topology and the often unpredictable mobility of individual vehicles. State of the art VANET simulators involve two components: a mobility simulator and a wireless network simulator. These two components may be tightly integrated (two components in one simulator) or else loosely integrated (two components in two separate simulators connected by trace files). In our simulator, we use loosely integrated entities. Our mobility simulations were performed on the basis of the open source code of IDM [21]. We recorded the trace files of vehicle mobility and then imported them into NS-2.30. In NS-2, each node represents one vehicles in the mobility simulations, moving based on the represented vehicle movement history in the trace file. Nodes in NS-2 are also entities of wireless communication nodes integrating network protocol stacks. The general parameters we used in simulations are detailed in Table I. The total number of vehicles $n$ is varying because in our mobility simulation, vehicles may enter and exit.

We distinguish the following cases in our simulation. In case I, both vehicle $i$ and $j$ have positive speeds and accelerations. Case I covers the scenarios 5(e) and (f). In case II, both vehicle $i$ and $j$ have positive speeds but they have opposite accelerations (for example, $i$ has a positive acceleration and $j$ has a negative acceleration). Case II covers scenarios 5(a)-(d).

To begin, we were interested to discover the impact of velocity on link duration. To this end, we investigated the

TABLE I
*The environment configuration*

| Name | Value |
|---|---|
| Total number of vehicles (n) | 1000-2000 |
| Application | CBR |
| Network protocol | IEEE 802.11 |
| Network connections | $n/3$ |
| Simulation map | Highway |
| Road length | 10 miles |
| Traffic density | 1500 vehicles/hour |

relationship between speed and link duration in the two cases discussed above. The purpose of this simulation was to confirm the analytical expression of link duration, given the speed and acceleration of vehicles $i$ and $j$, as discussed in Section V. The parameters used are shown in the Tables II and III. The parameters $\mu$ and $\sigma$ are from log-normal distribution of headway distance. Given $\sigma = 0.55$, we assume that the mean value of headway distance is 30 meters, i.e. $e^{\mu+\sigma^2/2} = 30$ from which $\mu$ can be determined. The values of the parameters of path loss model $\sigma$ are taken from [28]. $C_1$ and $C_2$ are normalizing coefficient in (2). The assumed speed limit $v_m$ was 33 m/s. We varied the speed (both $v_i$ and $v_j$) in cases I and II

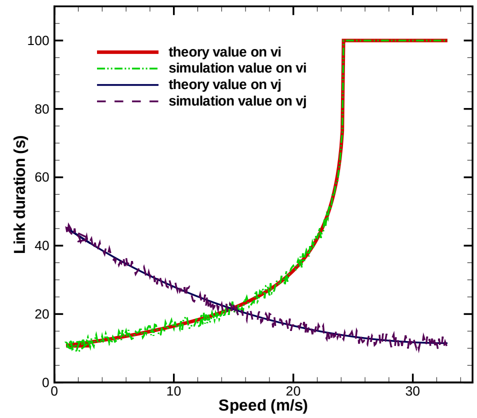
TABLE II
*Parameters used in simulation*

| $\mu$ | $\sigma$ | $G_t$ | $G_r$ | $h_t$ | $h_r$ | $X$ |
|---|---|---|---|---|---|---|
| 2.95 | 0.55 | 1 | 1 | 0.5m | 0.5m | 100m |

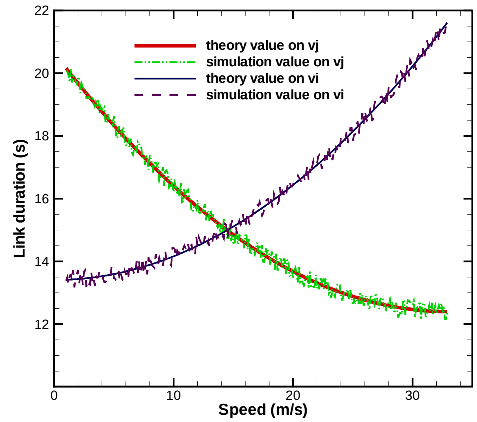
TABLE III
*Mobility parameters used in the speed simulation*

| case | $a_i\ m/s^2$ | $a_j\ m/s^2$ | $v_{i0}\ m/s$ | $v_{j0}\ m/s$ |
|---|---|---|---|---|
| I | 0.1 | 1 | 15 | 15 |
| II | -1 | 1 | 20 | 10 |

and recorded the link duration. Figure 6 illustrated the results we obtained. We assume that the link duration 100 seconds represent infinite link duration. Figure 6(a) shows that larger speed of vehicle $i$ will cause longer link duration and larger speed of vehicle $j$ will cause shorter link duration because both $i$ and $j$ are accelerating in case I. When $v_i$ increases, it increases the likelihood of reaching $v_m$. Therefore both $i$ and $i$ reach the speed limit $v_m$ before the link breaks. Tus, in this case, the link tends to be stable. As we expected, the simulation results match well the theoretically-predicted values, as shown in Figure 6(b). The higher speed of vehicle $j$ will cause a shorter link duration, and the higher speed of vehicle $i$ will cause a longer link duration. If vehicle $j$ has higher speed, the relative speed between $j$ and $i$, i.e. $v_j - v_i$ will be higher. Therefore, the link duration will be shorter.
The previous simulation shows that link duration is affected by speed. Since acceleration will affect the link duration as well, we simulated the relationship between the link duration and acceleration in cases I and II. The purpose of this set




(a) *Speed impact in case I.*




(b) *Speed impact in case II.*

Fig. 6. *Speed has significant impact on link duration.*

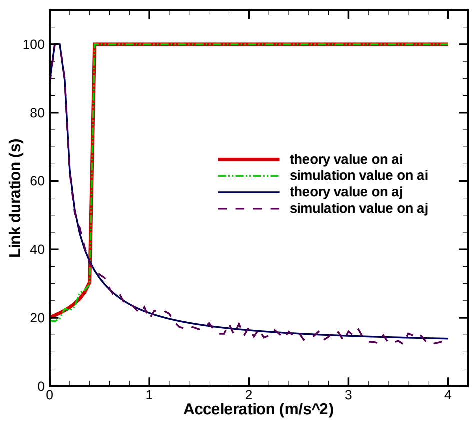
of simulation was to validate the discussion of Section V. We only modified the mobility parameters comparing with the previous simulations. The change of the mobility parameters are shown in Table IV. Figure 7 was generated by varying

TABLE IV
*Mobility parameters used in the acceleration simulation*

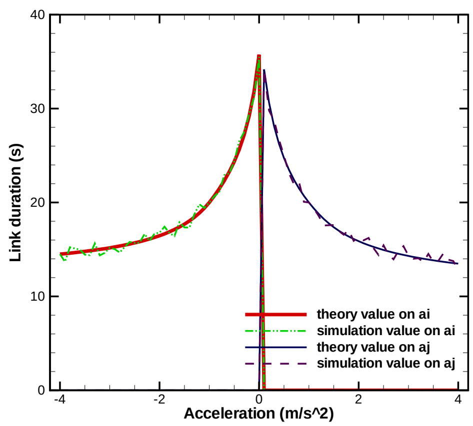
| case | $a_i\ m/s^2$ | $a_j\ m/s^2$ | $v_{i0}\ m/s$ | $v_{j0}\ m/s$ |
|---|---|---|---|---|
| I | 0.2 | 1 | 30 | 1 |
| II | -1 | 1 | 20 | 10 |

acceleration and by recording the link duration. Figure 7(a) shows that, as expected, the simulation results match the analytical predictions. The increment of $a_j$ will cause the decrement of link duration, and the increment of $a_i$ will cause the increment of link duration in Case I. The increment of $a_i$ will cause the speed $v_i$ to quickly reach the speed limit. Therefore, both vehicles are moving at the same speed and the link duration tends to be stable. Figure 7(b) shows that the increment of $a_i$ will cause the increment of link duration and the increment of $a_j$ will cause the decrement of link duration in Case II. Figure 7(b) shows a symmetric situation. This is because that the decrement of $a_i$ and the increment of

$a_j$ can explained as contributions to the increment of relative acceleration which will cause the same effect: the decrement of link duration.




(a) *Acceleration impact in case I.*




(b) *Acceleration impact in case II.*

Fig. 7. *Acceleration has significant impact on link duration.*

We were also interested in the pdf of link duration. Since our analytical derivation shows that the link duration has a log-normal distribution, the purpose of this simulation was to validate the pdf of link duration which is covered in Section IV. For each cases I and II, we created 2000 links among vehicles and recorded the durations of each link. As expected, the pdf of link duration for each case has the shape of the log-normal distribution, shown in figure 8. Case I has a flat curve but case II has a relative sharp curve. This is because case II tends to form a centralized relative speed and acceleration. Therefore the link duration values tend to congregate in a certain range.

## VIII. Concluding Remarks and Directions for Future Work

In this work we have studied link duration in VANET where the individual nodes in the network are vehicles on roadways and city streets and the mobility pattern is described in terms of the instantaneous inter-car distance. Because of the very dynamic nature of VANET, and of the rather modest transmission range mandated by the FCC, individual links come into existence and vanish unpredictably, making the task of establishing and maintaining routing paths between fast-moving vehicles a very challenging task. In order to help mitigate the problem, we have undertaken a probabilistic analysis of the lifetime of links in VANET under the assumption of a realistic radio transmission model and of the fact that instantaneous inter-car distance follows a log-normal distribution.

In addition to obtaining a probability distribution model for the duration of links, we have also obtained the expected duration of links. This result can be extrapolated to multi-hop paths consisting of a number of individual links. Our analytical results were validated and confirmed by extensive simulations.

The next obvious task is to extend our analytical derivations to selecting and maintaining *stable* routing paths in VANET. The basic idea is to predict the expected duration of links and to select the link with the longest expected duration. For those links which are about to break, local repair can be undertaken by selecting other links with longer link duration. In the worst case, another backup routing path can be set up before the current routing path breaks. As a future work, the constant acceleration in mobility model can be extended. For example, acceleration can be a function of time. We are interested to integrate the idea of link selection with existing protocols to increase the stability of routing paths; similarly, we will pursue the implementation of stable link selection in intelligent transportation system addressed by Wang [29].

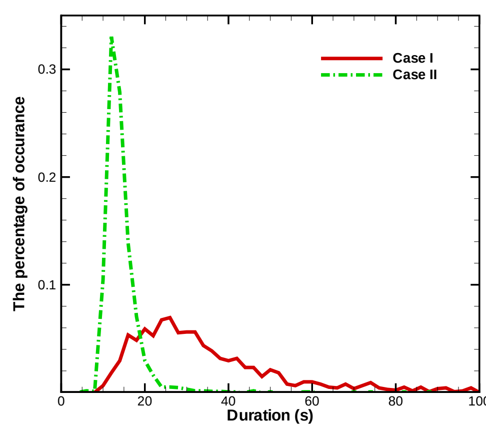



Fig. 8. *The pdf of link duration.*

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# APPENDIX

*Fitting the pdf of headway distance:* To fit the simulation result and fit the pdf curve by log-normal distribution, we define a *general* form of log-normal distribution.

$$f(x,\mu,\sigma,a,b,c) = \frac{b}{(cx-a)\sigma\sqrt{2\pi}} e^{-\frac{(ln(cx-a)-\mu)^2}{2\sigma^2}}$$

where $x > 0$. Since we have five unknown parameters, we establish five equations to solve them. The mean headway distance can be written $\frac{b}{c^2}e^{\mu+\sigma^2/2} + \frac{ab}{c^2}$.

The parameters we applied to simulations are the following, the traffic density: 2800 vehicle/hour, maximum speed: 60 km/h; Minimum speed: 20 km/h. The road is a straight line with two lanes. Total 48257 headway distance readings are collected. Table V shows part of the simulation results. We have the mean value of headway distance and randomly select other four (x, Pr) pairs, i.e. (x=8.0, Pr=0.0100), (x=19.0, Pr=0.0442), (x=33.0, Pr=0.0241), (x=47.0,Pr=0.0098). Therefore, five equations are created. With the aid of computer, we can numerically solve the solution of $\mu = 2.95, \sigma = 0.55, a = 2.93, b = 1, c = 0.80$. Therefore, the log-normal distribution of headway distance is

$$f(x) = \frac{1}{(0.80x-2.93)0.55\sqrt{2\pi}} e^{-\frac{(ln(0.80x-2.93)-2.95)^2}{2*0.55^2}}$$

Similarly, a general form of normal distribution is defined as

$$N(x,\mu,\sigma,a,b,c) = \frac{b}{\sigma\sqrt{2\pi}} e^{-\frac{(cx-a-\mu)^2}{2\sigma^2}}$$

We obtained $\sigma = 9.51, \mu = 0.085, a = 8.55, b = 1.01, c = 0.53$. A general form of exponential distribution is defined as $Expo(x,\lambda,a,b,c) = b\lambda e^{-\lambda(cx-a)}$. We obtained $\lambda = 2.01, a = 1.05, b = 0.01, c = 0.03$. A general form of gamma distribution is defined as

$$\Gamma(x,\theta,k,a,b,c) = b(cx-a)^{k-1}\frac{e^{-(cx-a)/\theta}}{\theta^k\Gamma(k)}.$$

We obtained $\theta = 4.01, k = 6.085, a = 1.05, b = 1.01, c = 1.03$. The comparison of these fitting curves is shown in Figure 1. It is clear that log-normal distribution fits the simulation results better than other distributions.

TABLE V
*Simulation headway distance values (x) and frequencies (Pr)*

| H | Pr | H | Pr | H | Pr |
|---|---|---|---|---|---|
| 5.0 | 0.0010 | 6.0 | 0.0028 | 7.0 | 0.0059 |
| 11.0 | 0.0253 | 12.0 | 0.0296 | 13.0 | 0.0340 |
| 17.0 | 0.0432 | 18.0 | 0.0431 | 19.0 | 0.0442 |
| 23.0 | 0.0411 | 24.0 | 0.0399 | 25.0 | 0.0376 |
| 29.0 | 0.0312 | 30.0 | 0.0297 | 31.0 | 0.0275 |
| 35.0 | 0.0222 | 36.0 | 0.0201 | 37.0 | 0.0189 |
| 41.0 | 0.0145 | 42.0 | 0.0138 | 43.0 | 0.0130 |
| 47.0 | 0.0098 | 48.0 | 0.0091 | 49.0 | 0.0085 |
| 53.0 | 0.0066 | 54.0 | 0.0061 | 55.0 | 0.0057 |
| 59.0 | 0.0043 | 60.0 | 0.0041 | 61.0 | 0.0038 |