Source-Channel Coding over Gaussian Sensor Networks with Active Sensing

Emrah Akyol, Urbashi Mitra
University of Southern California
Ming Hsieh Department of Electrical Engineering
University Park, Los Angeles, CA 90089, USA.
email: {eakyol, ubli}@usc.edu

***Abstract*—A limited energy budget is a major obstacle to the practical, wide deployment of sensor networks and hence necessitates the judicious optimization of available resources. In this paper, joint optimization of sensing and communication resources to minimize total energy spent within a sensor network is considered. A particular sensor network model with one Gaussian source observed by many sensors, subject to additive independent Gaussian observation noise, is examined. Sensors communicate with the receiver over an additive Gaussian multiple access channel. The aim of the receiver is to reconstruct the underlying source with minimum mean squared error. The fundamental tradeoff between communication and sensing over this sensor network model is characterized. Under symmetric conditions, for a single sensor, power is shared equally between communication and sensing. As the number of sensors increases, the sensing error dominates the overall error expression, hence sensing takes almost all power. The optimal power scheduling among sensors in the asymmetric case is determined, and it is shown that the power allocation schedule admits a simple decentralized implementation. Numerical results show that joint optimization of communication and sensing power yields significant power savings compared to the conventional approach of optimization of only communication power allocation.**

***Index Terms*—joint sensing and transmission, sensor networks, power allocation, underwater communications**

I. Introduction

In this paper, we study optimal power allocation strategies within a sensor network where each sensor can adjust its sensing and transmission power on the fly, during the sensing and transmission stage. Such highly adaptive sensor networks are particularly useful for certain types of sensor networks such as underwater sensor networks. In such networks, the cost of communication and sensing can be comparable, with both being expensive, see e.g. [1], [2] and the references therein.

Although sensing is a measurable part of total power budget, sensing power optimization was only very recently considered, partly because fixed sensing strategies (given the network conditions) were assumed in most prior work, see eg. [3]–[12]. In this paper, we consider the fundamental problem of joint sensing and communication, via "active sensing". Our problem constitutes an instance of a class of problems known as active classification problems, where the goal is to efficiently estimate an unknown phenomenon of interest by exploiting different sensing modalities (*e.g.*, sensor type, number of samples, location), see eg. [13]–[17] and the references therein.

In this paper, we study the joint optimization of sensing and communication power allocation over a sensor network. We focus on a particular sensor network model which involves a single Gaussian source observed by many sensors, subject to additive independent Gaussian observation noise. Sensors communicate with the receiver over an additive Gaussian multiple access channel. The aim of the receiver is to reconstruct the underlying source with minimum mean squared error (MSE). We consider a particular sensing model where sensing power acts as effective power over a Gaussian sensing test channel. We limit the communication strategies to amplify-and forward, motivated by the fact that this zero-delay scheme is indeed optimal for the symmetric case, among all encoding schemes with arbitrary delay [18]. Optimal communication power allocation strategies for this setting was analyzed in several recent work [4], [11], [12], [19].

The rest of the paper is organized as follows. In Section II, we describe the problem setting. In Section III, we present our results pertaining to the symmetric sensor network setting. We study optimal power assignment for an asymmetric sensor network in Section IV. In Section V, we demonstrate the effectiveness of optimization by comparative results. Finally, in Section VI, we discuss future directions.

II. Problem Definition

Let $\mathbb{E}(\cdot)$, $\mathbb{R}$ and $\mathbb{R}^+$ denote the expectation operator, and the sets of real numbers and positive real numbers respectively. Gaussian distribution with mean μ and variance σ^2 is denoted as $\mathcal{N}(\mu, \sigma^2)$. Let $(x)^+$ denote $\max(0, x)$. In this paper, we focus on scalar, Gaussian sources and additive white Gaussian noise. Communication policies are limited to zero-delay, amplify-forward schemes.

A. Sensing Model

The single sensor model is depicted in Figure 1. The source $\{S(i)\}$ is a sequence of i.i.d. real valued Gaussian random variables with zero mean and variance σ_S^2.The sensor has a total power budget P to be allocated across sensing and communication tasks, denoted by P_S and P_T respectively ($P = P_S + P_T$). The greater the power allowed for sensing, the better the ability of the sensor to estimate the source. To facilitate the computation, we model this improvement

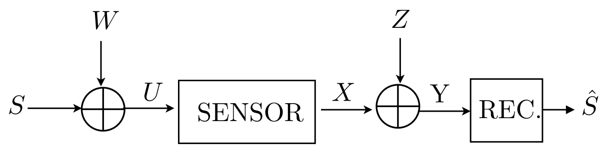

Fig. 1. Point-to-point (single sensor) setting

equivalent to the power for transmission of a source with i.i.d. sensing noise $W(i) \sim \mathcal{N}(0, \sigma_W^2)$, *i.e.,* the sensor receives

$$U(i) = \sqrt{\frac{P_S}{\sigma_S^2}} S(i) + W(i). \tag{1}$$

Note that the choice of sensing model determines the optimal power allocation. We focus on this Gaussian test channel approach in modeling the relationship between sensing power and the associated mean squared error. Clearly, the sensor has no control over the source signal, however our model captures sensing capability clearly.

B. Sensor Network

The sensor network of interest is illustrated in Figure 2. Suppose that there are M sensors each making observations on a common unknown random, memoryless scalar source S. The underlying source $\{S(i)\}$ is a sequence of i.i.d. real valued Gaussian random variables with zero mean and variance σ_S^2. Sensor $m \in [1\!:\!M]$ observes a sequence $\{U_m(i)\}$ defined as

$$U_m(i) = \sqrt{\frac{P_{S_m}(i)}{\sigma_S^2}} S(i) + W_m(i), \tag{2}$$

where $\{W_m(i)\}$ is a sequence of i.i.d. Gaussian random variables with zero mean and variance $\sigma_{W_m}^2$, independent of $\{S(i)\}$ and each other for all m. For notational convenience in the asymmetric setting (Section IV), we express the sensing channel with two parameters $\beta_m \in \mathbb{R}^+$ and $h_m \in \mathbb{R}^+$, where β_m is a given system parameter (sensing scaling coefficient) and h_m is the sensing control parameter, *i.e.,*

$$U_m(i) = \beta_m h_m S(i) + W_m(i), \tag{3}$$

Sensor $m \in [1\!:\!M]$ applies a linear function to $U_m(i)$ so as to generate sequence of channel inputs

$$X_m(i) = \sqrt{\frac{P_{T_m}}{\sigma_{U_m}^2}} U_m(i), \tag{4}$$

where $\sigma_{U_m}^2$ is the variance of $U_m(i)$. Similar to the sensing case, we represent this with two coefficients for each sensor: $\alpha_m \in \mathbb{R}^+$ representing the given channel parameter and $g_m \in \mathbb{R}^+$ is the communication parameter, *i.e.,*

$$X_m(i) = g_m U_m(i), \tag{5}$$

and the channel output is then given as

$$Y(i) = Z(i) + \sum_{m=1}^{M} \alpha_m X_m(i), \tag{6}$$

$$= Z(i) + \sum_{m=1}^{M} \left(\alpha_m g_m h_m \beta_m S(i) + \alpha_m g_m W_m(i)\right), \tag{7}$$

where $\{Z(i)\}$ is a sequence of i.i.d. Gaussian random variables of zero mean and variance σ_Z^2, independent of $\{S(i)\}$ and $\{W_m(i)\}$. The receiver applies a linear estimator to the received sequence $\{Y(i)\}$ to minimize the cost, which is measured as the MSE between the underlying source S and the estimate at the receiver $\hat{S}$:

$$J(\boldsymbol{g}, \boldsymbol{h}) = \mathbb{E}\left\{\left[S(i) - \hat{S}(i)\right]^2\right\}. \tag{8}$$

where $\boldsymbol{g} = [g_1, \dots g_M]$ and $\boldsymbol{h} = [h_1, \dots, h_M]$. Throughout this paper, we drop the time index i for notational convenience since sources and channels are memoryless, and transmission mappings are zero-delay. Recall that β_m is the sensing coefficient and α_m is the channel gain. The sensors have at their disposal the ability to adapt h_m (sensing control) and g_m transmit power control to minimize $J(\boldsymbol{g}, \boldsymbol{h})$ subject to the total power constraint.

III. Symmetric Setting

In this section, we analyze the effect of the number of sensors on the optimal power allocation between sensing and communication. To facilitate the analysis, we assume that $\alpha_m = \beta_m = 1$ and $\sigma_{W_m}^2 = \sigma_W^2$, $\forall m$ and each sensor has identical power limit P. The following theorem presents the optimal power allocation between the sensing power P_S and P_T, where $P_S + P_T = P$.

Theorem 1. *The optimal sensing power, P_S^* is:*

$$P_S^* = \begin{cases} P/2 & \text{if } \sigma_Z^2 = M\sigma_W^2 \\ \frac{MP\sigma_W^2 + \sigma_W^2\sigma_Z^2 - \sqrt{\sigma_W^2\sigma_Z^2(P+\sigma_W^2)(MP+\sigma_Z^2)}}{M\sigma_W^2 - \sigma_Z^2} & \text{o.w.} \end{cases} . \tag{9}$$

The proof directly follows from standard estimation theoretic principles and it is omitted here. In the following, we present two results which directly follow from Theorem 1. For simplicity, in the following we assume that $\sigma_W^2 = \sigma_Z^2$.

Corollary 1. *For a single sensor, i.e., the point to point setting, $P_S^* = P_T^* = \frac{P}{2}$.*

Corollary 2. *P_S^* is monotonically increasing function of M. As $M \to \infty$, $P_S \to P$ and hence $P_T \to 0$.*

Remark 1. *Although it might seem surprising that asymptotically, sensing takes all allocated power, this observation has an intuitive explanation. As $M \to \infty$, the communication channel approaches a noiseless channel due to the fact that we have $Z + \sum_{m=1}^{\infty}\left(\frac{P_S}{\sigma_S^2} S + W_m\right)$ at the decoder, and SNR of this channel is unbounded, provided that P_S is constant.*

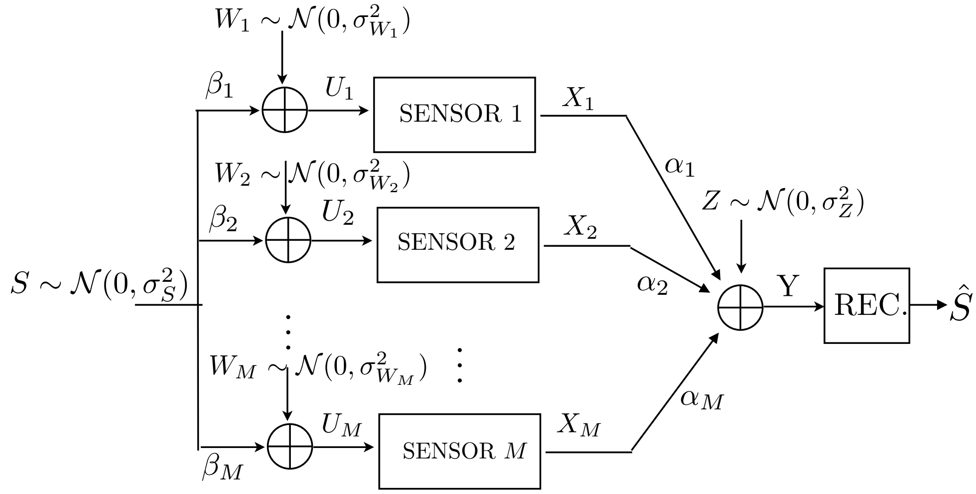

Fig. 2. Gaussian sensor network

That is we have infinite diversity on the observation of the common source. While the effective source variance increases with M^2 the noise variance increases only with M. Hence, the communication error vanishes asymptotically, irrespective of the communication power, and all power is allocated to sensing.

IV. Asymmetric Setting

We next consider the more practical, "asymmetric" setting of the sensor network, where the sensors minimize (8) subject to a total power constraint, by adapting g_m and h_m. In this section, without loss of generality we assume that $\sigma_S^2 = \sigma_{W_m}^2 = \sigma_Z^2 = 1, \forall m$. The following theorem presents our main result that pertains to this setting:

Theorem 2. *The optimal sensing and coding coefficients for sensor m are:*

$$g_m = \frac{1}{\beta_m}\sqrt{\left(\frac{\lambda_2 \alpha_m \beta_m}{2\sqrt{1+\lambda_1\alpha_m^2}} - 1\right)^+}, \tag{10}$$

and

$$h_m = g_m\sqrt{1+\lambda_1\alpha_m^2}. \tag{11}$$

Proof. Note that

$$\mathbb{E}\{SY\} = \sum_{m=1}^{M} h_m\beta_m g_m\alpha_m, \tag{12}$$

and

$$\mathbb{E}\{Y^2\} = 1 + \left(\sum_{m=1}^{M} h_m\beta_m g_m\alpha_m\right)^2 + \sum_{m=1}^{M}\alpha_m^2 g_m^2. \tag{13}$$

The distortion is (observing that $\sigma_S^2 = 1$):

$$D = 1 - \frac{\mathbb{E}\{SY\}}{\mathbb{E}\{Y^2\}}, \tag{14}$$

$$= 1 - \frac{\left(\sum_{m=1}^{M} h_m\beta_m g_m\alpha_m\right)^2}{1 + \left(\sum_{m=1}^{M} h_m\beta_m g_m\alpha_m\right)^2 + \sum_{m=1}^{M}\alpha_m^2 g_m^2}, \tag{15}$$

$$= \frac{1 + \sum_{m=1}^{M}\alpha_m^2 g_m^2}{1 + \sum_{m=1}^{M}\alpha_m^2 g_m^2 + \left(\sum_{m=1}^{M} h_m\beta_m g_m\alpha_m\right)^2}. \tag{16}$$

Note that this problem is not convex in g_m, h_m. By changing the variables, we can convert this problem into a convex form which is solvable in closed form. First, instead of minimizing the distortion with a power constraint, we can equivalently minimize the power with a distortion constraint. This a well known method in convex optimization and it is known that there is no duality gap since the distortion is a convex function of power [20]. The convexity of the distortion with respect to total power can be intuitively understood from the fact that sensors can use time-sharing among different powers to achieve the convex hull of total power-distortion curve. The modified problem is to minimize:

$$\sum_{m=1}^{M}(1+\beta_m^2 h_m^2)g_m^2 + h_m^2, \tag{17}$$

subject to

$$\frac{1 + \sum_{m=1}^{M}\alpha_m^2 g_m^2}{1 + \sum_{m=1}^{M}\alpha_m^2 g_m^2 + \left(\sum_{m=1}^{M} h_m\beta_m g_m\alpha_m\right)^2} \le D. \tag{18}$$

Note that

$$\frac{1}{D} = 1 + \frac{(\sum_{m=1}^{M} h_m \beta_m g_m \alpha_m)^2}{1 + \sum_{m=1}^{M} \alpha_m^2 g_m^2} \tag{19}$$

Next, we introduce a slack variable

$$r = \sum_{m=1}^{M} \alpha_m \beta_m g_m h_m. \tag{20}$$

The optimization problem is to minimize

$$\sum_{m=1}^{M} (1 + \beta_m^2 h_m^2) g_m^2 + h_m^2, \tag{21}$$

subject to

$$1 + \sum_{m=1}^{M} \alpha_m^2 g_m^2 \leq (D^{-1} - 1)^{-1} r^2, \tag{22}$$

and (20). This problem is convex in the variables g_m, h_m and r. Hence, we construct the Lagrangian cost as

$$J = \sum_{m=1}^{M} \left(1 + \beta_m^2 h_m^2\right) g_m^2 + h_m^2 + \lambda_1 \left(1 + \sum_{m=1}^{M} \alpha_m^2 g_m^2 - \frac{r^2}{(D^{-1} - 1)}\right)$$
$$+ \lambda_2 \left(r - \sum_{m=1}^{M} \alpha_m \beta_m g_m h_m\right), \tag{23}$$

where $\lambda_1 \in \mathbb{R}^+$ and $\lambda_2 \in \mathbb{R}$. Next, we apply the KKT optimality conditions:

$$\frac{\partial J}{\partial h_m} = 2 h_m g_m^2 \beta_m^2 + 2 h_m - \lambda_2 \alpha_m \beta_m g_m = 0, \tag{24}$$

$$\frac{\partial J}{\partial g_m} = 2 g_m (1 + \beta_m^2 h_m^2) + 2 \lambda_1 g_m \alpha_m^2 - \lambda_2 \alpha_m \beta_m h_m = 0, \tag{25}$$

$$\frac{\partial J}{\partial r} = -2 \lambda_1 (D^{-1} - 1)^{-1} r + \lambda_2 = 0, \tag{26}$$

and we have (20) and

$$1 + \sum_{m=1}^{M} \alpha_m^2 g_m^2 = \left(D^{-1} - 1\right)^{-1} r^2. \tag{27}$$

From (24), we obtain h_m in terms of the unknown g_m as

$$h_m = \frac{\lambda_2 \alpha_m \beta_m g_m}{2(1 + \beta_m^2 g_m^2)}, \tag{28}$$

and similarly, from (25), we have

$$g_m = \frac{\lambda_2 \alpha_m \beta_m h_m}{2\left(1 + \beta_m^2 h_m^2 + \lambda_1 \alpha_m^2\right)}. \tag{29}$$

Plugging (29) in (28), we obtain (24) and (25).

$$h_m = g_m \sqrt{1 + \lambda_1 \alpha_m^2}, \tag{30}$$

and

$$g_m^2 = \frac{1}{\beta_m^2} \left(\frac{\lambda_2 \alpha_m \beta_m}{2\sqrt{1 + \lambda_1 \alpha_m^2}} - 1\right)^+. \tag{31}$$

□

Remark 2. *The optimal coefficients can be computed for each sensor in a distributed manner. The central agent can compute the optimal values of λ_1 and λ_2 and then broadcast this information to all sensors. Next, each sensor can compute its own sensing and communication power allocation based on local parameters α_m and β_m and the broadcasted global parameters λ_1 and λ_2.*

Remark 3. *The optimal coefficients for two special cases are of interest: a) the communication variables g_m are fixed and the MSE is minimized only over the sensing variables h_m; b) the sensing coefficients h_m are fixed and the communication power allocation is optimized through g_m. Solutions to both of these problems follow simply from the proof of Theorem 2. An important distinction for joint optimization compared to such fixed optimization strategies is that joint optimization allows for shutting off the bad sensors, i.e., the power allocation has an intuitive waterfilling solution. However, fixed optimization does not yield such a result, sensing and/or communication power is allocated to each sensor, irrespective of the sensing or communication channel qualities, see eg., [4]. The loss due to not optimizing jointly will be seen in the Numerical Results.*

V. Numerical Results

In this section, we present numerical results demonstrating the effectiveness of joint optimization of sensing and communication power. We simulate the asymmetric sensor network with 10 sensors, α_m and β_m are generated from a uniform distribution over $[0, 5]$. To obtain statistically meaningful results, we average the performances over 100 trials. In Fig. 3, we plot the distortion in MSE versus total power in dB. We also simulated the performance of optimization of the communication powers only (denoted as "Only Comm. Optimized"), with a fixed randomly chosen sensing power: we set $h_m = 1, \forall m$ optimize g_m (see also [4]). As a naive competitor (denoted as "Naive"), which can change sensing and communication power, without optimization, we simulate the performance of $g_m = h_m = \gamma P, \forall m$, where gamma is adjusted so that the total power constraint is achieved.

Two observations can be made regarding the numerical results. Firstly, joint optimization of sensing and communication yields significant power savings for the same MSE, compared to a naive scheme that changes sensing and communication power in a simple, non-optimized manner and a scheme that only optimizes communication power. More importantly, in the high SNR regime, communication only optimization converges to a distortion significantly larger than that of the joint optimization. In that setting, even a naive optimization strategy performs superior to fixed sensing, communication only optimization method. This is simply due to the fact that communication only optimization cannot decrease its sensing error with increased total power.

VI. Discussion

In this paper, we studied optimal power allocation between two fundamental tasks of a sensor network: sensing and communication. We showed that in the symmetric setting, allocated

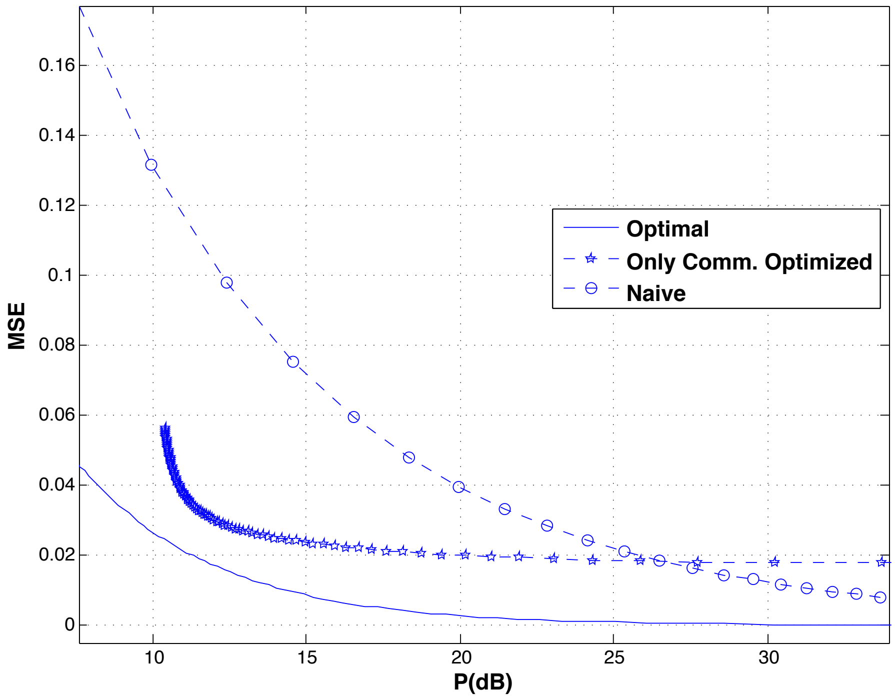

Fig. 3. Comparative results

sensing power increases with the number of sensors. In the asymmetric setting, we derived the optimal power allocation among sensors that minimizes the achieved MSE under a total power constraint. We also showed that optimal sensing and communication power can be calculated in a distributed manner by using local information. Numerical results show that optimizing sensing and transmit power jointly improves significantly over conventional strategies. As a future work, we will extend our analysis to other network topologies, including orthogonal MAC, and to multidimensional settings.

VII. Acknowledgements

This research was funded in part by one or all of these grants: ONR N00014-09-1-0700, CCF-1410009 , CCF-0917343, CCF-1117896, CNS-1213128, AFOSR FA9550-12-1-0215, and DOT CA-26-7084-00.

References

[1] G. Hollinger, S. Choudhary, P. Qarabaqi, C. Murphy, G Mitra, U.and Sukhatme, M. Stojanovic, and F. Singh, H.and Hover, "Underwater data collection using robotic sensor networks," *IEEE Journal on Selected Areas in Communications*, vol. 30, no. 5, pp. 899–911, 2012.

[2] G. Hollinger, C. Choudhuri, U. Mitra, and G Sukhatme, "Distortion metrics for robotic sensor networks," in *Proc. Int. Conf. Robotics and Automation*, 2013.

[3] J. Xiao, A. Ribeiro, Z. Luo, and G. Giannakis, "Distributed compression-estimation using wireless sensor networks," *IEEE Signal Processing Magazine*, vol. 23, no. 4, pp. 27–41, 2006.

[4] J. Xiao, S. Cui, Z. Luo, and A. Goldsmith, "Linear coherent decentralized estimation," *IEEE Transactions on Signal Processing*, vol. 56, no. 2, pp. 757–770, 2008.

[5] G. Thatte and U. Mitra, "Sensor selection and power allocation for distributed estimation in sensor networks: Beyond the star topology," *IEEE Transactions on Signal Processing*, vol. 56, no. 7, pp. 2649–2661, 2008.

[6] W.U. Bajwa, J.D. Haupt, A.M. Sayeed, and R.D. Nowak, "Joint source-channel communication for distributed estimation in sensor networks," *IEEE Transactions on Information Theory*, vol. 53, no. 10, pp. 3629–3653, Oct 2007.

[7] P. Dragotti, M. Gastpar, and M. Vetterli, "Sensing reality and communicating bits: A dangerous liaison," *IEEE Signal Processing Magazine*, vol. 23, no. LCAV-ARTICLE-2007-002, pp. 70–83, 2006.

[8] M. Gastpar and M. Vetterli, "Power, spatio-temporal bandwidth, and distortion in large sensor networks," *IEEE Journal on Selected Areas in Communications*, vol. 23, no. 4, pp. 745–754, 2005.

[9] S. Draper and G. Wornell, "Side information aware coding strategies for sensor networks," *IEEE Journal on Selected Areas in Communications*, vol. 22, no. 6, pp. 966–976, 2004.

[10] S. Cui, J. Xiao, A. Goldsmith, Z. Luo, and V. Poor, "Estimation diversity and energy efficiency in distributed sensing," *IEEE Transactions on Signal Processing*, vol. 55, no. 9, pp. 4683–4695, 2007.

[11] A. Leong and S. Dey, "On scaling laws of diversity schemes in decentralized estimation," *IEEE Transactions on Information Theory*, vol. 57, no. 7, pp. 4740–4759, 2011.

[12] F. Jiang, J. Chen, and L. Swindlehurst, "Parameter tracking via optimal distributed beamforming in an analog sensor network," in *Signals, Systems and Computers (ASILOMAR), 2012 Conference Record of the Forty Sixth Asilomar Conference on*. IEEE, 2012, pp. 1397–1401.

[13] D.-S. Zois, M. Levorato, and U. Mitra, "Energy–Efficient, Heterogeneous Sensor Selection for Physical Activity Detection in Wireless Body Area Networks," *IEEE Transactions on Signal Processing*, vol. 61, no. 7, pp. 1581–1594, April 2013.

[14] V. Krishnamurthy and D. V. Djonin, "Structured Threshold Policies for Dynamic Sensor Scheduling: A Partially Observed Markov Decision Process Approach," *IEEE Transactions on Signal Processing*, vol. 55, no. 10, pp. 4938–4957, 2007.

[15] G. K. Atia, V. V. Veeravalli, and J. A. Fuemmeler, "Sensor Scheduling for Energy-Efficient Target Tracking in Sensor Networks," *IEEE Trans. Signal Process.*, vol. 59, no. 10, pp. 4923–4937, Oct. 2011.

[16] S. Nitinawarat, G. K. Atia, and V. V. Veeravalli, "Controlled Sensing for Multihypothesis Testing," *IEEE Trans. on Automatic Control*, vol. 58, no. 10, pp. 2451–2464, October 2013.

[17] M. Naghshvar and T. Javidi, "Active Sequential Hypothesis Testing," *The Annals of Statistics*, vol. 41, no. 6, pp. 2703–2738, Dec. 2013.

[18] M. Gastpar, "Uncoded transmission is exactly optimal for a simple

Gaussian sensor network," *IEEE Transactions on Information Theory.*, vol. 54, no. 11, pp. 5247–5251, 2008.

[19] H. Behroozi, F. Alajaji, and T. Linder, "On the optimal performance in asymmetric Gaussian wireless sensor networks with fading," *IEEE Trans. on Signal Processing*, vol. 58, no. 4, pp. 2436–2441, 2010.

[20] S. Boyd and L. Vandenberghe, *Convex optimization*, Cambridge university press, 2004.