ELSEVIER Journal of Econometrics 88 (1999) 227–250

JOURNAL OF Econometrics

Conduct parameters and the measurement of market power

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Received 1 November 1993; received in revised form 1 January 1998

Abstract

This paper examines a simple version of the conduct parameter method widely used in empirical industrial organization and argues that the conduct parameter fails to measure market power accurately. It is shown analytically and with simulations that in a dynamic oligopoly model this mismeasurement can be quite severe.

JEL classification: C1; L0

Keywords: Conduct parameters; Conjectural variations; Market power

1. Introduction

Empirical industrial organization economists have long been concerned with measuring the degree of competition in markets and understanding its underlying determinants. The 'competitiveness' of a market – where it falls in the mysterious realm between perfect competition and monopoly – determines the extent to which prices and costs diverge, with important ramifications for consumer welfare, firm profits, and the efficiency of the market. The price–cost margin is the natural measure of a market's competitiveness; however, while prices are often readily observable, marginal cost is rarely so easily measured. The shortcomings of accounting cost data for economic analysis are well-known; among the most salient issues are the somewhat arbitrary rules for the

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PII: S0304-4076(98)00028-1

treatment of phenomena like depreciation and for the designation of costs as marginal and fixed.

In response to the important challenge of measuring price–cost margins in the absence of good cost data, a 'New Empirical Industrial Organization', surveyed in Bresnahan (1989), has offered a myriad of techniques with which to address this problem. Some studies, including Bresnahan (1981a), Suslow (1986), Baker and Bresnahan (1988), and the more recent discrete choice literature, carefully estimate the demand function faced by the firm to determine the extent of its price-setting power. Others attempt to estimate the parameters of the firm's supply relation instead, especially in industries where products are likely to be close substitutes and analysis of demand is likely to yield little insight.

A significant portion of the literature focusing on supply relations relies on an econometric approach that I term the conduct parameter method (CPM), which employs an empirical model based on the theory of 'conjectural variations' to estimate a 'conduct parameter'. This parameter is purported to measure the competitiveness of a market in a very general way, yielding an elasticity-adjusted price–cost margin and simultaneously nesting the perfectly competitive, monopoly, and classical Cournot models. The conduct parameter method's simplicity, relatively undemanding data requirements, and easily interpreted measure of market power have made it extremely popular among empirical IO economists. Recent papers that invoke some version of this methodology include Graddy (1995), Ellison (1994), Berg and Kim (1994), Seldon et al. (1993), Rubinovitz (1993), Brander and Zhang (1993), Claycombe and Mahan (1993), Suzuki et al. (1993), Stalhammer (1991), Brander and Zhang (1990), Conrad (1989), Gelfand and Spiller (1987), Spiller and Favaro (1984), Roberts (1984), Porter (1983), Applebaum (1982), Gollop and Roberts (1979), and Iwata (1974).

Though conjectural variations models have fallen out of favor with theorists, and despite the fact that the CPM is explicitly derived from a conjectural variations model, the empirical literature has continued to employ the CPM, often interpreting its results with an 'as–if' interpretation. The as–if interpretation of the conduct parameter is based on the observation that, for given demand and cost conditions, one can compute the conjecture that would yield the observed price–cost margins if firms were playing a conjectural variations equilibrium, even if observed behavior is in fact generated by some other oligopoly game. Bresnahan (1989), p. 1029) summarizes this point of view:

> The crucial distinction here is between (i) what firms believe will happen if they deviate from the tacitly collusive arrangements and (ii) what firms do as a result of those expectations. In the 'conjectural variations' language for how supply relations are specified, it is clearly (ii) that is estimated. Thus, the estimated parameters tell us about price- and quantity-setting behavior; if the estimated 'conjectures' are constant over time, and if breakdowns in the

collusive arrangements are infrequent, we can safely interpret the parameters as measuring the average collusiveness of conduct.

A typical empirical paper in this literature takes an agnostic stance toward the behavioral model governing imperfect competition in the market in question and simply interprets the empirical results as indicating that the result of that behavior is as competitive 'as–if' the firms were in fact playing a conjectural variations game with the estimated conjectural variations parameter.

I argue in this paper that such inferences are invalid. Without stipulating the true nature of the behavior underlying the observed equilibrium, no inference about the extent of market power can be made from analysis of the observed variables. The CPM can be thought of as having two distinct steps in its inference of market power. First, it estimates the slope of the supply relation to measure 'equilibrium variation' – how equilibrium behavior responds to perturbation of demand conditions. Second, this equilibrium variation is implicitly mapped into the inferred 'equilibrium value' of the elasticity-adjusted price–cost margin. The first step is fraught with difficulties; in particular, one risks confusing increases in marginal cost with increases in the price–cost margin. However, the literature has been quite careful to get this step of this process right, identifying at least three methods by which increasing costs and increasing margins can be disentangled: by assuming constant marginal cost (Iwata, 1974), by including shifters of demand elasticity (Bresnahan (1982) and Lau (1982)), and by permitting supply shocks or multiple pricing regimes (Porter, 1983). In contrast, the argument of this paper concerns not the first step, but the second. The model analyzed here will be simplified to render trivial the estimation of the slope of the supply relation, in order to show that the second step – relying on the conjectural variations model to provide the mapping from equilibrium variation to equilibrium values – is fundamentally flawed.

To demonstrate the potential severity of this mismeasurement in a theoretically coherent alternative model, I analyze the application of the CPM to data generated by tacit collusion supported by repeated interaction. I show that CPM estimates of market power can be seriously misleading. In fact, the conduct parameter need not even be positively correlated with the true measure of the elasticity-adjusted price–cost margin, so that some markets are deemed more competitive than a Cournot equilibrium even though the price–cost margin approximates the fully collusive joint-profit maximizing price–cost margin.

Section 2 discusses the conjectural variations model and elasticity-adjusted price–cost margin in more detail. It then formalizes the argument that the CPM can measure only 'equilibrium variation', which cannot be mapped into the 'equilibrium values' of interest in the absence of a theoretical model of competitive interaction. Section 3 demonstrates the severity of the mismeasurement when equilibrium behavior in fact results from tacit collusion supported by repeated interaction. Section 4 concludes by discussing empirical support for the analysis of this paper.

2. Conduct parameters and conjectural variations equilibria

While a conduct parameter may be imbedded in a firm's first order conditions in several ways, Bresnahan's (1989) survey emphasizes the approach in which the conduct parameter θ_i is estimated as a part of the following supply relation:

$$P = c_i'(q_i) - \theta_i P'(Q) q_i. \tag{1}$$

Here, $c_i'(\cdot)$ denotes firm i's marginal cost, $P(\cdot)$ is the inverse industry demand function, q_i is firm i's quantity, and $Q = \sum_i q_i$. Such a supply relation is typically derived from a conjectural variations model, in which firms formulate conjectures about their rivals' reactions to their output decisions. Bowley (1924) first introduced conjectures to the classical Bertrand and Cournot models by permitting firms to hold non-zero conjectures about their rivals' responses to changes in their strategies. If each firm i anticipates that its rivals' aggregate output is some function $R_i(q_i)$ and if $R_i'(q_i) = r_i$, firm i's first-order condition is $P = c_i'(q_i) - (1 + r_i)P'(Q)q_i$, which is equivalent to (1) when $\theta_i = 1 + r_i$.

This modification to the classical model allows simple price- and quantity-setting games to generate a wide range of outcomes; varying r_i generates the entire range of outcomes from the perfectly competitive to the monopolistic or joint-profit-maximizing outcome. Further, this model nests the three standard models – perfect competition, monopoly, and classical Cournot competition – in a single supply relation. In the conjectural variations model, a conjecture of $r_i = -1$ ($\theta_i = 0$) corresponds to the competitive model, as output reductions by one firm are completely offset by output expansions by other firms, leaving the price unchanged. A conjecture of $r_i = 0$ ($\theta_i = 1$) yields the classical Cournot model, in which rivals' quantities are taken as given. Finally, a conjecture of $r_i = N - 1$ ($\theta_i = N$) corresponds to a model of joint profit maximization, where firm i's output changes are matched by all other firms (and N represents the number of firms).

Bresnahan (1981b) argued that by restricting attention to consistent conjectures, one could reduce the theoretical ambiguity of these models to arrive at a unique 'consistent conjectural equilibrium' in some circumstances. A long theoretical debate ensued, but the work of Daughety (1985) and Lindh (1992) eventually showed that in the absence of peculiar informational assumptions, the no-response Cournot conjectures are actually the only truly consistent equilibrium conjectures. These findings reduced the theoretical viability of and interest in conjectural variations models; nonetheless, the CPM literature has continued to employ this model.

If the econometrician observes prices and costs and can consistently estimate demand parameters, then construction of this measure of market power – the 'as–if' conjectural variations parameter – is simple. Rearranging Eq. (1) and substituting for prices, costs, quantities, and the demand parameters yields the following full-information measure of the conduct parameter, referred to throughout this paper as the as–if conjectural variations parameter $\tilde{\theta}_i$:

$$\tilde{\theta}_i = \frac{P - c_i'}{-P'q_i} = \frac{P - c_i'}{P} N\varepsilon, \tag{2}$$

where ε is the elasticity of aggregate demand.[1] It is now apparent that this parameter can be interpreted as an elasticity-adjusted Lerner index; it therefore provides a measure of the price–cost margin that is normalized by both the price level (like all Lerner indices) and the demand elasticity (to distinguish markets that have high margins because demand is inelastic from markets that have high margins because they are less competitive or perhaps collusive). The criticism of this paper does not center on the usefulness of this measure or its interpretation, but rather on the inability of the CPM to estimate this parameter accurately.

Since the goal of this paper is to strip away all complicating factors to show that even when the model and the data are extremely well-behaved, the CPM may not accurately measure market power, I assume that marginal costs are constant, that demand is linear, and that a symmetric equilibrium is observed. No further assumptions on the nature of the equilibrium are made in this section; the results presented here demonstrate a failure of the conduct parameter method that is not specifically related to any particular oligopoly model. In subsequent sections, a model of efficient supergame collusion will be analyzed in order to draw more specific conclusions about the severity of the potential mismeasurement of market power demonstrated in this section.

I assume a linear inverse demand relationship

$$P(q_t; x_t) = a_0 + a_1 x_t + a_2 Q_t + e_t, \tag{3}$$

[1] In a slightly different, but analogous framework, aggregate industry data are used to estimate the supply relation. If $\theta_i = \theta$ for all i, then summing Eq. (1) over i and dividing both sides by N yields $P = \bar{c}_i' + \theta^a P'(Q)Q$, where $\theta^a = \theta/N$. This transformation changes only the interpretation of the conduct parameter. Since the aggregate conduct parameter θ^a is equal to θ/N, $\theta^a = 1$ corresponds to $\theta = N$ and indicates monopoly market power. Similarly, $\theta^a = 1/N$ corresponds to $\theta = 1$ and indicates one-shot Cournot equilibrium behavior. In this case, the interpretation of the conduct parameter as an elasticity-adjusted Lerner index is even clearer: $\theta^a = [(P - \bar{c}_i')/P]\varepsilon$. The choice between these frameworks amounts to a normalization of the conduct parameter, and the analysis of this paper applies equally to both approaches.

where a_0, a_1, and a_2 are constants known to the firms but not to the econometrician, x_t is a vector of demand shifters observable to all parties in period $t, Q_t = Nq_t$, and e_t is an unobservable i.i.d. mean zero random error term. The true marginal cost function for all firms i is

$$c_i'(q_i) = c_0 + c_1 w_t, \tag{4}$$

where c_0 and c_1 are cost parameters known to the firms but not to the econometrician, and w_t is a vector of cost shifters observable to all parties in period t. A typical CPM study would in this case employ two-stage least squares to estimate the two-equation simultaneous system consisting of the demand relation

$$P_t = \alpha_0 + \alpha_1 x_t + \alpha_2 Q_t + \varepsilon_t \tag{5}$$

and the supply relation

$$P_t = \beta_0 + \beta_1 w_t + \beta_2 q_{it} + \xi_{it}, \tag{6}$$

which correspond to Eqs. (1) and (4) in Bresnahan's (1989) survey.

The criticism of this paper is not a small-sample criticism, but pertains to the asymptotic parameter estimates, which are denoted by 'hats' throughout. The asymptotic estimate of the conduct parameter θ_i is given by $\hat{\theta}_i = -\hat{\beta}_2/\hat{\alpha}_2$ since, comparing equations (6) and (1), the coefficient on quantity in the supply relation must be scaled by the demand derivative $-P'$. From (5), the estimate of $-P'$ is $-\hat{\alpha}_2$. Since two-stage least squares allows consistent estimation of the demand parameters ($\hat{\alpha}_2 = a_2$), it remains only to derive the value of $\hat{\beta}_2$ to determine whether the CPM accurately measures market power.

If the first-stage regression of q on x and w yields estimates $\hat{q}_{it}$, the 2SLS estimate of β_2 is given by

$$\begin{aligned}\hat{\beta}_2 &= (\hat{q}_i' M_w \hat{q}_i)^{-1}(\hat{q}_i' M_w P)\\ &= (\hat{q}_i' M_w \hat{q}_i)^{-1}(\hat{q}_i' M_w x a_1 + \hat{q}_i' M_w Q a_2 + \hat{q}_i M_w e a_2)\\ &= (\hat{q}_i' M_w \hat{q}_i)^{-1}(\hat{q}_i' M_w x a_1 + \hat{q}_i M_w e a_2) + N a_2,\end{aligned} \tag{7}$$

where $M_w = (I - w(w'w)^{-1}w')$. The demand equation substitutes for P in moving from the first expression to the second, and the assumption of symmetric equilibrium is used in deriving the third. It follows that the asymptotic estimate of β_2 is

$$\hat{\beta}_2 = \frac{a_1}{\gamma} + N a_2, \tag{8}$$

where $\gamma = \text{plim }(x'M_w x)^{-1}(x'M_w q)$ is the asymptotic linear projection coefficient of q on x from the first-stage regression. The asymptotic estimate of the conduct parameter θ_i is then

$$\hat{\theta}_i = \frac{\hat{\beta}_2}{-\hat{\alpha}_2} = -\frac{a_1}{a_2\gamma} - N. \tag{9}$$

This analysis demonstrates that the estimated conduct parameter $\hat{\theta}$ is a function of only the demand parameters and the parameter γ, the responsiveness of equilibrium quantity to the demand shifter x. Thus, the estimated conduct parameter $\hat{\theta}$ is fully determined by 'equilibrium variation', the extent to which equilibrium quantities respond to perturbations of demand.

It is immediately clear from Eq. (1) that the conduct parameter measures something having to do with the slope of the supply relation. Since outside of the competitive model supply relations are not simple curves with well-defined slopes, it is not immediately clear what this means. For simplicity, assume that the firm's optimal quantity rule $q_t^*(\cdot)$ is linear in x_t so that $\gamma = \mathrm{d}q_t^*/\mathrm{d}x_t$. This is satisfied, for example, by the classical Cournot supply relation. In this case, it is easily verified that the estimated conduct parameter measures the 'slope' of the price–cost margin with respect to demand-driven fluctuations in quantity. Formally,

$$\hat{\theta} = \frac{1}{-P'}\frac{\mathrm{d}(P-c')}{\mathrm{d}x}\bigg/\frac{\mathrm{d}q^*}{\mathrm{d}x}. \tag{10}$$

The 'as–if' conjectural variations parameter $\tilde{\theta}$, on the other hand, measures 'equilibrium values', the *level* of the price–cost margin, not its responsiveness to fluctuations of output. Comparison of Eqs. (2) and (10) proves the first proposition.

Proposition 1. For any underlying supply process generating q^*, the estimated conduct parameter accurately measures market power ($\hat{\theta} = \tilde{\theta}$) if and only if

$$\frac{P-c'}{x}\bigg/\frac{q^*}{x} = \frac{\mathrm{d}(P-c')}{\mathrm{d}x}\bigg/\frac{\mathrm{d}q^*}{\mathrm{d}x}. \tag{11}$$

Fig. 1 depicts the competitive and joint profit-maximizing supply relations (S^{PC} and S^{M}, respectively) traced out by demand curves D_1 and D_2. It is easy to

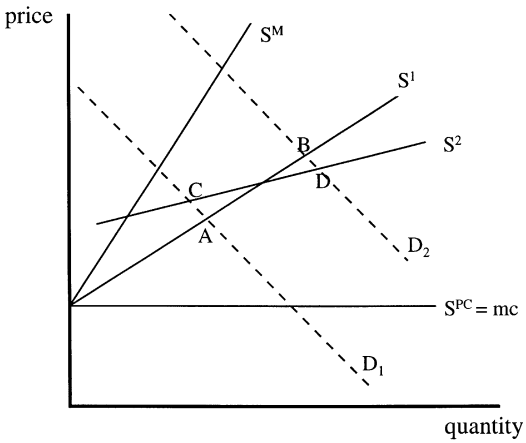

Fig. 1. Supply relationships under different models of conduct.

show that the supply relation for conjectural variations models is always a ray through the marginal cost intercept; one possible conjectural variations supply relation is depicted as S^1. Higher values of θ rotate the conjectural variations supply relation toward S^M. Some other (non-conjectural variations) model might generate supply relation S^2, which should be considered approximately as competitive as the model generating S^1 since prices at points A and B are on average as high as prices at C and D. However, the estimated conduct parameters for these two industries might diverge significantly, since supply curves S^1 and S^2 have different slopes. Thus, implementation of the CPM is problematic even when the system (5)–(6) is correctly specified, which is the case when the firm's supply rule q_t^* is linear in (w_t, x_t). The problem is that $\beta_0 + \beta_1 w$ need not be the marginal cost c' (even if marginal cost is independent of q and x), and hence β_2 need not be equal to $(P - c')/q$. Therefore, a consistent estimate of $\beta_2/(-\alpha_2)$, as delivered by 2SLS, will not provide a consistent estimate of the as–if conjectural variations parameter $\tilde{\theta}$ defined in Eq. (2), except of course when firms do behave according to the conjectural variations model (1).

In the conjectural variations model, the average relationship of price–cost margin to quantity (the LHS of Eq. (11), or the slope of the ray from the marginal cost intercept through the observed data) is identical to its marginal relationship to demand-driven variation in quantity (the RHS of Eq. (11), or the slope of the line defined by the observed data). Equilibrium values of the price-cost margin can therefore be inferred from observed equilibrium variation. Outside of the conjectural variations model, this need not hold and inferences based on this relationship may be invalid. Put another way, the CPM is valid only if the true process underlying the observed equilibrium generates behavior that is identical *on the margin*, and not just *on average*, to a conjectural variations game. This seems unlikely to hold across a wide range of models, and the validity of the inference of market power from measurements of the conduct parameter is therefore suspect. The next section demonstrates just how misleading this rule can be when firm behavior in fact results from tacit collusion through repeated interaction.

3. The conduct parameter method and efficient supergame collusion

Proposition 1 suggests that *sometimes* the CPM will accurately measure market power; however, it is not immediately clear whether theoretically coherent models of imperfect competition generate supply relations that differ from those implied by conjectural variations models in important ways. For this reason, in this section I explicitly evaluate, using both simulation and analytical methods, the conduct parameter's mismeasurement of market power in the leading theoretical model of imperfectly competitive behavior – a model of repeated interaction.[2]

Specifically, I focus on an N-firm symmetric oligopoly game in which the firms play an efficient supergame equilibrium in which deviations are punished by reversion to one-shot Cournot equilibrium strategies forever. A supergame equilibrium is *efficient* if it prescribes quantities that maximize joint profits subject to the firms' incentive constraints, given the realization of the random exogenous observables x_t and w_t. Let $\pi(q;\ x_t,\ w_t) = q(P(Nq;\ x_t) - c_0 - c_1 w_t)$ denote the profits when quantities q are chosen in state x_t, w_t. Best-response profits (which will determine optimal deviation profits as a function of rivals' equilibrium strategies) are given by $\pi^b(q;\ x_t,\ w_t) = \max_{q_0} q_0(P((N-1)q + q_0;\ x_t) - c_0 - c_1 w_t)$. Punishment payoffs in period k are given by the one-shot Cournot equilibrium payoffs, which are denoted $\pi^c(x_k,\ w_k)$.[3]

In an efficient supergame equilibrium the equilibrium quantity is defined by

$$
\begin{aligned}
q^*(x_t, w_t, \delta) = \operatorname*{argmax}_q\ & \pi(q;\ x_t,\ w_t) \\
\text{subject to } & \pi^b(q;\ x_t,\ w_t) \\
& + \sum_{i=1}^{\infty} \delta^i \mathrm{E}_t \pi^c(x_{t+i},\ w_{t+i}) \leqslant \pi(q;\ x_t,\ w_t) \\
& + \sum_{i=1}^{\infty} \delta^i \mathrm{E}_t \pi(q^*(x_{t+i},\ w_{t+i},\ \delta);\ x_{t+i},\ w_{t+i}),
\end{aligned}
\tag{12}
$$

[2] The perfect information repeated quantity-choice game analyzed here is closely related to the models studied by Rotemberg and Saloner (1986) in the i.i.d. case and by Kandori (1991) in the case of serially correlated demand.

[3] Earlier versions of this paper allowed more general punishments. It is straightforward to extend the analysis that follows to finite-period Nash reversion and other related punishment schemes.

where E_t denotes expectations conditional on the information available to the firms in period t, and δ is the firms' common discount factor. Note that I have imposed stationarity of the equilibrium by writing continuation payoffs as a function of $q^*(x_{t+i}, w_{t+i}, \delta)$, which is consistent with the assumption that firms solve Eq. (12) in each period with respect to a time-invariant punishment scheme.

It is easy to show that the maximand attains its unconstrained maximum at

$$q^m(x_t, w_t) = \frac{a_0 + a_1 x_t - c_0 - c_1 w_t}{-2Na_2}. \tag{13}$$

If the incentive constraint does not bind then $q^*(x_t, w_t, \delta) = q^m(x_t, w_t)$. If the incentive constraint does bind then $q^*(x_t, w_t, \delta)$ is defined implicitly by the solution to the constraint in Eq. (12) when written as an equality. For the case of linear demand, $\pi^b(q; x_t, w_t)$ has a simple closed-form expression. Substituting for π and π^b in the incentive constraint and solving for the optimal quantity when the incentive constraint binds yields

$$\tilde{q}(x_t, w_t, \delta) = \frac{a_0 + a_1 x_t - c_0 - c_1 w_t}{-(N+1)a_2} - \frac{2\sqrt{-a_2 L(x_t)}}{-(N+1)a_2}, \tag{14}$$

where

$$L(x_t) = \sum_{i=1}^{\infty} \delta^i \mathrm{E}_t[\pi(q^*(x_{t+i}, w_{t+i}, \delta); x_{t+i}, w_{t+i}) - \pi_i^c(x_{t+i}, w_{t+i})]. \tag{15}$$

$L(x_t)$ represents the total expected discounted loss of profits incurred by entry into the punishment phase in period t. Combining Eqs. (13) and (14), the efficient supergame equilibrium quantity rule is

$$q^*(x_t, w_t, \delta) = \begin{cases} \tilde{q}(x_t, w_t, \delta) & \text{if } \tilde{q}(x_t, w_t, \delta) \geqslant q^m(x_t, w_t), \\ q^m(x_t, w_t) & \text{otherwise.} \end{cases} \tag{16}$$

3.1. Simulation results

Before presenting analytical results, I illustrate the severity of the CPM's mismeasurement of market power by simulating the application of the CPM to data generated by a symmetric duopoly playing an efficient supergame equilibrium. The industry faces linear inverse demand (3) and constant marginal cost

(4). Firms know the demand and cost parameters and observe x_t and w_t in each period t. The demand and cost shifters x_t, w_t are discrete random variables with K states.[4] The x_t's follow a Markov process with transition matrix Λ that is known to the firms. Restricting attention to Markov processes ensures that no variables are omitted from the supply relation (6), since expectations of future demand states are fully determined by the current state. I restrict attention to the class of transition matrices with $\Lambda_{ii} = \lambda$ and $\Lambda_{ij} = (1 - \lambda)/(K - 1)$ for all $i \neq j$. Thus, the parameter λ represents varying degrees of permanence of the demand shocks, with $\lambda = 1/K$ representing the i.i.d. case. If $\lambda > 1/K$, the demand states are persistent in the sense that the observation of $x_t = \hat{x}$ makes the observation of $x_{t+1} = \hat{x}$ more likely. Note that as λ tends to unity the demand shocks become completely permanent. The process for the cost shocks is kept simple since their primary role is to permit estimation of the demand parameters. The w_t's are i.i.d. with a uniform distribution over the K states.

The firms' optimal quantities are calculated through an iterative numerical computation that begins by giving $L(x_t)$ some large value for all x_t. Then, the optimal quantities $q^*(\cdot)$ that are sustainable in each demand state for this $L(x_t)$ are computed in accordance with the theoretically derived supply relation (16). Given these optimal quantities, the vector $L(x_t)$, which represents the difference in profits between continuing to play q^* and shifting to the one-shot Cournot punishment payoffs, that is consistent with such a quantity rule is computed from Eq. (15), calculating expectations according to the known Markov transition matrix Λ. This process is repeated with the new value of $L(x_t)$ and is iterated until convergence. Finally, a sample of x_t, w_t is taken and the conduct parameter method as described in Section 2 is applied to the resulting data.

The results for the case in which $\lambda \geqslant 1/K$ (and demand states are therefore somewhat persistent) are presented in Fig. 2. This graph plots the estimated conduct parameter $\hat{\theta}$ corresponding to several different values of λ against the discount factor δ. The as–if conjectural variations parameter $\tilde{\theta}$, calculated according to (2), is also shown. Results suggested by the simulation include: (a) *for high discount factors, the CPM accurately measures market power regardless of the demand process*; (b) *at low discount factors, the CPM fails to detect any market power if demand is i.i.d.*; (c) *when demand is fully persistent the CPM accurately measures market power*; (d) *as demand becomes more persistent, the CPM becomes a more accurate measure of market power.*

The simulations demonstrate that specific conclusions about conduct drawn from estimates of $\hat{\theta}$ may not be valid; specifically, comparisons of the degree of market power across industries or across time within an industry may be

[4] In the simulations reported in Figs. 2 and 3, $K = 10$. In addition, the K discrete states are evenly dispersed, i.e., the difference between the ith and $(i + 1)$th highest demand states is the same for all i.

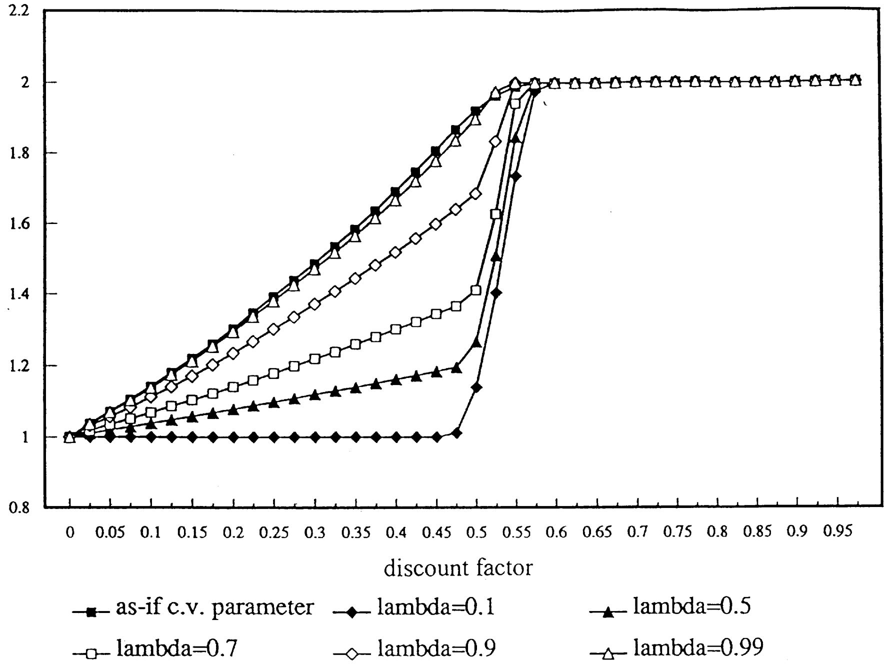

Fig. 2. Conduct parameters with positive serial correlation in demand.

misleading. For example, in the simulations $\hat{\theta} = 1.2$ is consistent with $\tilde{\theta} = 1.3$ in an industry where $\lambda = 0.9$ and with $\tilde{\theta} = 1.9$ in an industry where $\lambda = 0.5$. Here, one might infer, based on estimates of the conduct parameter, that the level of competition in the two industries is comparable when in fact there is a dramatic difference in the level of collusiveness between the two industries as measured by the as–if conjectural variations parameter $\tilde{\theta}$. Similarly, an industry in which the degree of persistence of the demand shocks undergoes a dramatic change may appear to the econometrician to have become markedly more or less competitive over time when in fact the degree of collusiveness, as measured by the as–if conjectural variations parameter $\tilde{\theta}$, has not changed.

Precisely why low values of δ and λ cause a divergence between $\hat{\theta}$ and $\tilde{\theta}$ will become clear in the analysis of the next subsection. Roughly, increases in demand simply scale up any one-shot model ($\delta = 0$) since current demand fully determines equilibrium price. As a result, the marginal and average relationships between demand and price are the same. Similarly, when δ is high or $\lambda = 1$, current demand simply scales up the model since current demand again fully determines equilibrium price (either because the monopoly price obtains or because expected future demand is equivalent to current demand). For intermediate values of δ and low values of λ, however, increases in demand are only partially exploited by the firms through an increase in price. While the increase in *current* demand raises the optimal price, it also raises the temptation to cheat, without correspondingly raising the cost of *future* punishment when $\lambda < 1$. This limits the firms' ability to capture the additional surplus generated by the increase in demand, and drives a wedge between the average and marginal relationships of price and demand.

Simulation results for demand state transition matrices exhibiting negative correlation are presented in Fig. 3. The line labeled $\lambda = 0.01$ corresponds to a transition matrix Λ as previously defined with $\lambda = 0.01$. The line labeled 'secondary diagonal 0.55' corresponds to a transition matrix with entries on the secondary diagonal equal to 0.55 and all other entries equal to $(1 - 0.55)/(K - 1)$. Both of these transition matrices exhibit negative correlation in the sense that higher values of x_t shift weight to the left in the distribution of x_{t+1}. Inspection of Fig. 3 suggests a fifth result: (e) *when demand states are negatively correlated, higher values of the conduct parameter correspond to lower values of the as–if parameter.*

When demand states are negatively correlated, the CPM may indicate that an industry quite near to achieving joint-profit-maximization is more competitive

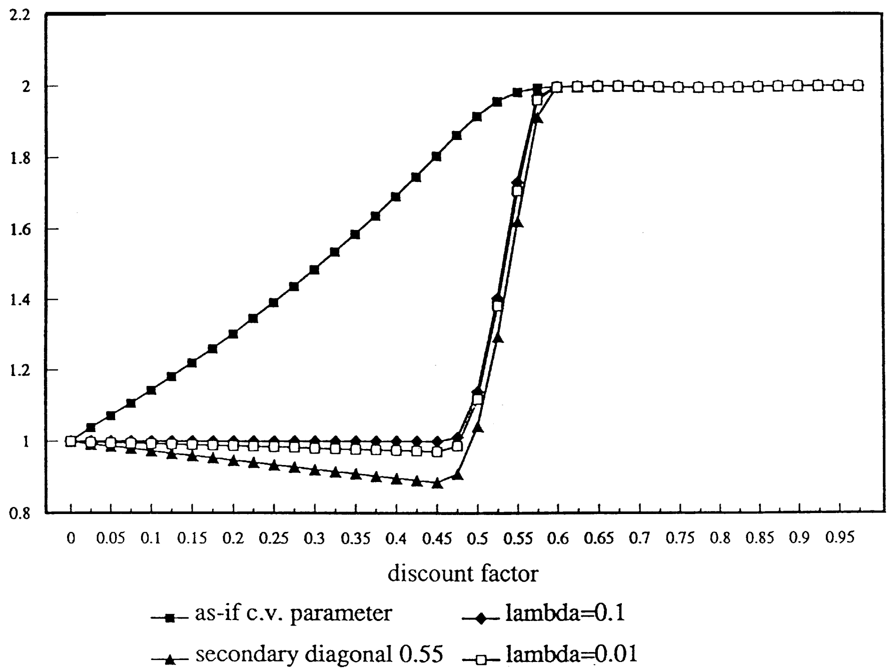

Fig. 3. Conduct parmeters with negative serial correlation in demand.

than if the firms were playing a one-shot Cournot equilibrium. This case is of particular interest because the switching process that negative correlation produces in a Markov process (higher demand in this period signals lower demand in the next period, but higher demand in the following period, and so on) is somewhat similar to a process of seasonal variation. These results suggest that if the exogenous demand shifters used to identify the supply relation are seasonal dummies, the conduct parameter method may fail to detect collusive behavior or, worse yet, yield an estimate of the conduct parameter $\hat{\theta}$ that is negatively correlated with the as–if parameter $\tilde{\theta}$ and which is less than one despite collusive behavior raising prices and profits (perhaps significantly) above the one-shot Cournot level.

3.2. Analytical results

This subsection continues the analysis of efficient supergame equilibria under a range of discount factors to clarify the cause of the mismeasurment illustrated in Section 3.1. In particular, I identify the source of the divergence between average and marginal responsiveness of margins to demand in this model of repeated interaction.

As the discount factor increases, the discounted value of future profit losses, $L(x_t)$, increases. This in turn reduces the optimal quantity and allows higher profits to be sustained, which further increases $L(x_t)$. For $\delta < 1$ large enough, the joint profit maximizing outcome is sustainable in all states; i.e., $q^*(x_t, w_t, \delta) = q^m(x_t, w_t)$ for all x_t, w_t. Let $\bar{\delta} = \inf\{\delta \mid q^*(x_t, w_t, \delta) = q^m(x_t, w_t)\ \forall x_t, w_t\}$ represent the lowest discount factor for which this is the case. The following proposition demonstrates that when the joint profit maximizing quantity is sustained in equilibrium in all demand states the conduct parameter correctly measures the conjectural variations parameter $\tilde{\theta}$. Note that the proof of this result is extremely general and does not rely on the particularities of the model of efficient supergame collusion or the demand process. Simply put, if the joint profit maximizing quantity is sustained in every period it does not matter how it is sustained; the model generating that quantity choice is irrelevant and all models of collusion that predict full joint profit maximization are indistinguishable. This confirms observation (a) suggested by the simulations.

Proposition 2. If the joint profit maximizing quantity is sustained in equilibrium in all demand states, then the estimated conduct parameter $\hat{\theta}$ correctly measures the conjectural variations parameter $\tilde{\theta}$. Formally, for all $\delta \in (\bar{\delta}, 1]$, $\hat{\theta} = \tilde{\theta} = N$.

Proof. For $\delta \in (\bar{\delta}, 1]$,

$$q^*(x_t, w_t, \delta) = q^m(x_t, w_t) = \frac{a_0 + a_1 x_t - c_0 - c_1 w_t}{-2Na_2} \quad \forall x_t, w_t.$$

It is straightforward to verify that the condition of Proposition 1 is satisfied. Further, $\gamma = \mathrm{d}q_t^*/\mathrm{d}x_t = a_1/(-2Na_2)$, which implies from Eq. (9) that $\hat{\theta} = N$. □

Now, consider the case in which the discount factor is too low to sustain the joint profit maximizing quantity in equilibrium. As δ decreases towards 0, $L(x_t)$ approaches 0 for all x_t and $q^*(x_t, w_t, \delta)$ approaches the Cournot quantity

$$q^c(x_t, w_t) = \frac{a_0 + a_1 x_t - c_0 - c_1 w_t}{-(N+1)a_2} > q^m(x_t, w_t)$$

in all states. For $\delta > 0$ small enough, the joint profit maximizing quantity is sustained in no state; i.e., $q^*(x_t, w_t, \delta) > q^m(x_t, w_t)\ \forall x_t, w_t$. Let $\underline{\delta} = \sup\{\delta \mid q^*(x_t, w_t, \delta) > q^m(x_t, w_t)\ \forall x_t, w_t\}$ represent the highest discount factor for which this is the case. Clearly, $\underline{\delta} < \bar{\delta}$. The following proposition demonstrates that when demand states are i.i.d., the conduct parameter fails to detect all collusion that falls short of full joint profit maximization. This confirms observation (b) suggested by the simulations.

Proposition 3. If the joint-profit maximizing quantity is sustained in equilibrium in no demand state and demand states are intertemporally independent, then the estimated conduct parameter fails to detect any collusive behavior. Formally, for $\delta \in [0, \underline{\delta})$, $\hat{\theta} = 1$.

Proof. For $\delta \in [0, \underline{\delta})$,

$$q^*(x_t, w_t, \delta) = \frac{a_0 + a_1 x_t - c_0 - c_1 w_t}{-(N+1)a_2} - \frac{2\sqrt{-a_2 L(x_t)}}{-(N+1)a_2}.$$

When demand shocks are i.i.d., observations of x_t do not influence expectations of future demand, and $L(x_t)$ is constant with respect to x_t. This implies $\gamma = \mathrm{d}q^*/\mathrm{d}x_t = a_1/(-(N+1)a_2)$. From Eq. (9), this implies $\hat{\theta} = 1$. □

As δ increases on the interval $[0, \underline{\delta})$, q^* decreases in each demand state as higher discounted values of punishments allow higher current profit levels to be sustained. Since lower-quantity and higher-profit equilibria are associated with higher values of the as–if conjectural variations parameter, $\tilde{\theta}$ increases with the discount factor. However, Proposition 3 demonstrates that the estimated conduct parameter $\hat{\theta}$ is constant over discount factors in $[0, \underline{\delta})$. Over this range, the conduct parameter method yields results that are consistent with one-shot

Cournot equilibrium behavior even though equilibrium prices and profits are increasing. In this model, the conduct parameter method fails to detect all collusive behavior that falls short of joint profit maximization. Note that this rather strong result is independent of the punishment scheme employed by the firms.

Given the analysis of Section 2, the intuition for this somewhat surprising result is quite simple. Since the econometrician has incomplete information on the cost parameters of the firms and must simultaneously estimate the cost parameters and the conduct parameter, the marginal response of the equilibrium quantity to demand shocks identifies the slope of the supply relation and therefore the value of the conduct parameter. Eq. (14) shows that for $\delta \in [0, \underline{\delta})$, the equilibrium quantity under efficient supergame collusion is simply the Cournot quantity less a function of the punishment incurred by deviation. With intertemporally independent shocks, the present value of future punishments is invariant to the demand state. Therefore, while the *equilibrium value* of quantities and profits may differ between the collusive equilibrium and the one-shot Cournot equilibrium, the *equilibrium variation* – the marginal response of quantities to demand shocks – is the same. As the discount factor increases, the conjecture consistent with the equilibrium behavior increases, but the estimated conduct parameter does not change. Propositions 2 and 3 give results for $\delta \in [0, \underline{\delta}) \bigcup (\bar{\delta}, 1]$. It is easy to show that as the supports of the cost and demand shifters shrink, $\underline{\delta}$ and $\bar{\delta}$ converge.

From Eq. (9) it is clear that the estimated conduct parameter $\hat{\theta}$ is determined by the parameter γ, which reflects the covariance of q_t and x_t. However, it is extremely difficult to draw specific conclusions about the relationship of q_t and x_t when the demand shocks are serially correlated since $L(x_t)$ varies with x_t in a complicated and nonlinear way; only if q_t^* is linear in x_t is $\gamma = \mathrm{d}q_t^*/\mathrm{d}x_t$. However, there is a special case in which the efficient supergame equilibrium quantity supply rule is, in fact, linear in the demand state and one can calculate $\gamma = \mathrm{d}q_t^*/\mathrm{d}x_t$ and therefore $\hat{\theta}$. In the limiting case of complete persistence of the demand state, $\hat{\theta}$ can be calculated and, perhaps surprisingly, the estimated conduct parameter accurately measures the conjectural variations parameter; that is, $\hat{\theta} = \tilde{\theta}$. Note that the permanence condition of the following proposition is satisfied by the limiting cases of two simple processes: an AR(1) $x_t = \rho x_{t-1} + \eta_t$ with $\rho \to 1$ and $\sigma_\eta^2 \to 0$, and a discrete Markov process with the transition matrix going to an identity matrix. Let σ_w^2 be the variance of the cost shocks w_t. I assume that the conditional p.d.f. of x_{t+i} at time t depends only on x_t and is given by $f_{t+i}^{x_t}$. $F_{t+i}^{x_t}$ denotes the corresponding c.d.f. This assumption ensures that $L(x_t)$ in fact captures the expected profit losses associated with deviation, since the current demand state is sufficient for the entire history of demand states in calculating the expectations over the future demand states. The following proposition confirms observation (c) suggested by the simulations.

Proposition 4. If demand shocks are completely persistent, then the conduct parameter accurately measures the as–if conjectural variations parameter $\tilde{\theta}$. Formally, let

$$f_{t+i}^{x_t} \xrightarrow{\mathrm{d}} \begin{cases} 1, & \text{if } x_{t+i} = x_t, \\ 0, & \text{otherwise.} \end{cases}$$

Then, in the limiting case as $\sigma_w^2 \to 0$, $\hat{\theta} \to \tilde{\theta}$.

Proof. In this limiting case $\mathrm{E}_t\tilde{q}_{t+i} = \mathrm{E}_t\tilde{q}_{t+j}$ for all $i, j \geqslant 1$ and this $\mathrm{E}_t\tilde{q}_{t+i}$ can be solved for by rewriting Eq. (14) for $\mathrm{E}_t\tilde{q}_{t+i}$ and taking expectations. The resulting supply relation forms one equation in one unknown and $\mathrm{E}_t\tilde{q}_{t+i}$ can be calculated explicitly. Solving for $\mathrm{E}_t\tilde{q}_{t+i}$, substituting back into the supply relation Eq. (14), and rearranging yields

$$\tilde{q}_t(x_t, w_t, \delta) = \left[\frac{(N+1)^2 - \delta(N-1)(N+3)}{(N+1)^2 - \delta(N-1)^2}\right]\frac{(a_0 + a_1x_t - c_0)}{-(N+1)a_2} - \frac{c_1w_t}{-(N+1)a_2}. \tag{17}$$

Ignoring the w_t term in Eq. (17) since $\sigma_w^2 \to 0$, it is easy to verify that this supply relation satisfies the condition of Proposition 1. An essentially identical proof demonstrates that the condition on the vanishing variance of w_t could be replaced with an assumption of the complete permanence of w_t. In that case, $\tilde{q}(x_t, w_t, \delta)$ is proportional to $a_0 + a_1x_t - c_0 - c_1w_t$ and $\hat{\theta} = \tilde{\theta}$ without approximation. □

The efficient supergame equilibrium supply relation (14) gives the quantity that balances current gains to deviation against discounted expected future profit losses that would result from deviation. The demand state x_t may therefore have two distinct effects on the equilibrium quantity. First, a higher x_t increases the short-term gains to deviation. If expected future profit losses do not change, then q_t must increase in x_t in order to offset this increased incentive to deviate. This is precisely the reason the Cournot equilibrium quantity is increasing in the demand state. If demand states are i.i.d. then this effect on q_t through the changing gains to deviation is the only effect present. If the x_t's are not intertemporally independent, then the value of x_t may provide new information about the probability distributions of future demand states and may therefore change the expected discounted future profit losses incurred by deviation. This provides a second means by which the demand state may influence the efficient supergame equilibrium quantity. For example, if the observation of a higher x_t indicates an increased probability of observing high future demand states, and if the gains to collusive behavior are larger in higher demand states, then the expected future losses resulting from punishment for a current deviation will increase with the observation of a higher x_t. This increase in future punishments allows higher profits (lower quantities) to be sustained in the current period. This second effect of x_t on q_t through changing expected future punishments leads to a lower quantity when the demand state is higher and will therefore at least partially offset the first effect. This makes q_t less responsive to x_t, which increases the estimated conduct parameter. The following proposition, which is proved in the appendix, demonstrates that positive intertemporal correlation is in fact a sufficient condition for the estimated conduct parameter to exceed one.

Proposition 5. If observations of higher x_t shift weight to the right in the distributions of all x_{t+i}, then $\hat{\theta} > 1$ for all $\delta \in (0, \underline{\delta})$.

When demand is i.i.d., only the first of these effects – the increase in the incentive to deviate – is present. Since the first effect concerns current conditions and is therefore independent of the discount factor, the marginal response of quantity to the demand state is independent of the discount factor. For this reason, the conduct parameter method fails to distinguish Cournot behavior from efficient supergame collusion for $\delta \in (0, \bar{\delta})$. Clearly, when shocks are not i.i.d., x_t and $L(x_t)$ are correlated and efficient supergame collusion may be distinguishable from Cournot behavior. Furthermore, since the second effect deals with the change in expected future profit losses, and since expected future losses depend on the discount factor, it seems reasonable that the second effect becomes increasingly important as δ increases. For this reason, $\hat{\theta}$ may be systematically correlated with the discount factor and therefore with $\tilde{\theta}$; that is, not only may the estimated conduct parameter exceed one, its magnitude may vary with the discount factor (see the simulations).

Alternatively, if a higher value of x_t indicates that lower values of the future demand state x_{t+i} are more likely (demand states are negatively correlated in some sense), then the observation of a higher demand state reduces the expected value of future punishments and leads to lower equilibrium profits (higher quantities) in the current period. In this case the second effect makes q_t even more increasing in x_t, which tends to lower the estimated conduct parameter. This is consistent with the simulation results that indicate that $\hat{\theta} < 1$ when demand shocks are negatively correlated.

4. Conclusion

This paper has demonstrated that the conduct parameter estimated in many empirical studies of market power cannot, in general, be interpreted as an as–if conjectural variations parameter indexing intermediate levels of collusive behavior if the underlying behavior is not the result of a conjectural variations equilibrium. In particular, if observed equilibrium behavior results from efficient supergame collusion, the estimated conduct parameter underestimates the degree of market power if demand shocks are not fully permanent, and may fail to detect any market power whatsoever when demand shocks are completely transitory, even if average price–cost margins are near the monopoly level.

Even those skeptical of an interpretation of the conduct parameter based on a conjectural variations equilibrium argue that the conduct parameter method is useful as a means of testing hypotheses about well-specified behavioral extremes. Part of the appeal of the CPM is that it nests the competitive ($\hat{\theta} = 0$), Cournot ($\hat{\theta} = 1$) and joint-profit maximizing ($\hat{\theta} = N$) models. While it is possible to do hypothesis testing of these extreme cases, these tests may lack power. Proposition 3 shows that $\hat{\theta} = 1$ may be consistent with any level of market power when demand is i.i.d., so that the econometrician would fail to reject the Cournot model over a large range of collusive equilibria. Thus, a failure to reject one of these nested static models does not necessarily provide much information about observed behavior, and the conduct parameter method may prove ineffective as a means of testing these well-specified theoretical models.

The arguments presented in this paper are grounded in a particular version of the conduct parameter method, but the spirit of the argument is quite general. The estimated conduct parameter measures how equilibrium output varies with shifts in the exogenous variables; however, different oligopoly models that produce the same degree of market power *on average* may generate behavior that, *on the margin*, varies with the exogenous variables in very different ways. For this reason, it is in general impossible to infer the equilibrium values of the market power measures of interest from the observed equilibrium variation that the estimated conduct parameter captures. Further, in a model of supergame collusion this mismeasurement can be quite severe; not even the positive correlation of the conduct parameter with the as–if conjectural variations parameter can be maintained across all models.

A small empirical literature has recently begun to test the CPM directly, by comparing estimates of the conduct parameter obtained by the standard conduct parameter methodology with more reliable direct measures of the price–cost margin in industries where good cost data are available. Wolfram (1997) tests the accuracy of the CPM in the British electricity spot market; Genesove and Mullin (1997) pursue the same issue in a historical study of the U.S. sugar industry. In both studies, the authors obtain CPM estimates smaller than the direct estimates of the as–if conjectural variations parameter. While the authors of both studies emphasize that the difference between the estimates is not large, in both cases one can reject the hypothesis that the two parameters are equal. These findings are broadly consistent with the analysis of this paper, lending additional credibility to the theoretical criticism of the conduct parameter method set forth here.

Acknowledgements

This paper is based on Chapter 1 of my 1994 Princeton University PhD dissertation. I thank Mike Boozer, Penny Goldberg, Martin Lettau, Robin Lumsdaine, Jim Powell, Catherine Wolfram, and especially Doug Bernheim for many helpful discussions. Seminar participants at Princeton, Harvard, Stanford, Chicago, Yale, Brown, Northwestern, and UCLA contributed useful comments on an earlier version of this work. Financial support from a National Science Foundation Graduate Research Fellowship and an Alfred P. Sloan Foundation Dissertation Fellowship is gratefully acknowledged.

Appendix A. Proof of Proposition 5

$L(x_t)$ can be rewritten as

$$L(x_t) = \sum_{i=1}^{\infty} \delta^i \int_X l_i(x_{t+i}) \, \mathrm{d}F_{t+i}^{x_t},$$

where

$$l_i(x_{t+i}) = \int_W [\pi(q^*(x_{t+i}, w_{t+i}, \delta); x_{t+i}, w_{t+i}) - \pi_i^c(x_{t+i}, w_{t+i})] \, \mathrm{d}G(w_{t+i}),$$

and $G(\cdot)$ is the c.d.f. of the cost shifter w. Here, $l_i(x_{t+i})$ represents the expected profit loss associated with being in the punishment phase in period $t+i$ if x_{t+i} is realized. Again, $L(x_t)$ represents the total discounted expected future profit loss associated with entering the punishment phase following a deviation in period t. Throughout this appendix, I assume that the $F_{t+i}^{x_t}$'s vary sufficiently smoothly in x_t that, for any bounded and continuously differentiable function $h(x)$, the derivative $[\mathrm{d}E_t h(x_{t+i})]/\mathrm{d}x_t$ exists. Specifically, I require that $\mathrm{d}f_{t+i}^{x_t}/\mathrm{d}x_t$ is bounded and continuously differentiable for all i.

I show that under plausible conditions, $\hat{\theta} > 1$ if demand states are positively intertemporally correlated in the sense that a higher value of the demand state in one period shifts weight to the right in the density functions of the future demand states. One technical definition is required before defining this condition.

Definition. Let F and G be c.d.f.'s on $X = [\underline{x}, \bar{x}]$. F (strictly) first-order stochastically dominates ((S)FOSD) G if

$$\int_{\underline{x}}^{x_0} \mathrm{d}F \leqslant \int_{\underline{x}}^{x_0} \mathrm{d}G \quad \forall x_0 \in X$$

(and this inequality holds strictly for some $x_0 \in X$).

Definition. The demand state x is positively intertemporally correlated (PIC) if

$$x_t > x'_t \;\Rightarrow\; F^{x_t}_{t+i} \text{ FOSD } F^{x'_t}_{t+i} \quad \forall i \geqslant 1$$

and this expression holds with SFOSD for some $i \geqslant 1$.
It is easy to verify that an AR(1) process, $x_{t+1} = \rho x_t + \eta_{t+1}$, where the η_t's are i.i.d., satisfies the above assumptions on differentiability and, for $\rho > 0$, the definition of PIC. In addition, this condition is satisfied by the Markov processes used in the simulations when $\lambda > 1/K$. For such a transition matrix, $F^{x_t}_{t+1}$ SFOSD $F^{x'_t}_{t+1}$ if and only if $x_t > x'_t$. It can be shown through an inductive argument that the demand states generated by this transition matrix satisfy PIC if $\lambda > 1/K$.

The difficulty in analyzing this model is in determining the value of γ. When demand states are not i.i.d., the equilibrium quantity is not, in general, linear in x_t and $\gamma = \mathrm{Cov}(x_t, q^*_t)/\mathrm{Var}(x_t) \neq \mathrm{d}q^*_t/\mathrm{d}x_t$. This complicates the analysis of $\hat{\theta}$; however, the following lemma demonstrates that if the derivative $\mathrm{d}q^*_t/\mathrm{d}x_t$ is bounded then γ is bounded as well, which will permit some conclusions about the estimated conduct parameter $\hat{\theta}$.[5]

Lemma 1. Let $q(x)$ be continuously differentiable on X and let the c.d.f. of x be some distribution F such that the variance of x exists. Then if $q'(x) > c$,

$$\frac{\mathrm{Cov}(x,\ q(x))}{\mathrm{Var}(x)} > c.$$

[5] This analysis is related to the Angrist et al. (1995) interpretation of 2SLS estimates of coefficients in simultaneous equations models as weighted average derivatives of endogenous response functions.

Proof. Substituting for $q(x)$ by the mean value expansion $q(x) = q(\bar{x}) + q'(\tilde{x})(x - \bar{x})$,

$$\mathrm{Cov}(x, q(x)) = \int_X x[q(\bar{x}) + q'(\tilde{x})(x - \bar{x})]\,\mathrm{d}F - \bar{x}\int_X [q(\bar{x}) + q'(\tilde{x})(x - \bar{x})]\,\mathrm{d}F$$

$$= \int_X q'(\tilde{x})(x - \bar{x})^2\,\mathrm{d}F$$

$$> \int_X c(x - \bar{x})^2\,\mathrm{d}F = c\,\mathrm{Var}(x). \qquad \square$$

The following lemma shows that if cooperation is more profitable in higher demand states ($l_i'(x_{t+i}) > 0$) and if the demand states are positively intertemporally correlated, then the expected excess profits in any future period $t + i$ are increasing in the current demand state x_t.

Lemma 2. Assume $l_i'(x_{t+i}) > 0$ and that the demand state x is PIC. Then

$$\frac{\mathrm{d}E_t l_i(x_{t+i})}{\mathrm{d}x_t} \geqslant 0$$

and this inequality holds strictly for some i.

Proof. Given that this derivative exists, it is sufficient to show that

$$x_t > x_t' \;\Rightarrow\; \int_X l_i(x_{t+i})\,\mathrm{d}F_{t+i}^{x_t} - \int_X l_i(x_{t+i})\,\mathrm{d}F_{t+i}^{x_t'} \geqslant 0.$$

After integration by parts, this expression becomes

$$\int_X l_i'(x_{t+i})(F_{t+i}^{x_t'} - F_{t+i}^{x_t})\,\mathrm{d}x.$$

By assumption, $l_i'(x_{t+i}) > 0$. By PIC, $F_{t+i}^{x_t}$ SFOSD $F_{t+i}^{x_t'}$, which implies $F_{t+i}^{x_t'} - F_{t+i}^{x_t} \geqslant 0$. Together, these prove the result for the weak inequality. By the definition of PIC, the above inequalities hold strictly for some i such that the conditions of PIC are satisfied for that i with SFOSD. □

Lemma 3. Assume $l_i'(x_{t+i}) > 0$ *for all i and that the demand state x is PIC. Then* $\hat{\theta} > 1$.

Proof. To show that $\hat{\theta} > 1$, it suffices to show that $\gamma < a_1/(-(N+1)a_2)$. Using the results of Lemma 2, it is easy to show that $\mathrm{d}L(x_t)/\mathrm{d}x_t > 0$. Differentiation of (16) shows that this implies $\mathrm{d}q_t^*/\mathrm{d}x_t < a_1/(-(N+1)a_2)$. By Lemma 1, this is sufficient for $\gamma < a_1/(-(N+1)a_2)$. □

It is straightforward to show that $l_i'(x_{t+i}) > 0$ for Nash reversion punishments. Thus, Lemmas 2 and 3 together prove Proposition 5.

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