ELSEVIER

Ultrasonics 39 (2001) 27–32

Ultrasonics

www.elsevier.nl/locate/ultras

Acoustic dispersion and attenuation measurement using both transmitted and reflected pulses

Ping He [*], Jun Zheng

Department of Biomedical, Industrial and Human Factors Engineering, 207 Russ Engineering Center, Wright State University, Dayton, OH 45435, USA

Received 1 December 1999; received in revised form 1 May 2000

Abstract

Traditional broadband transmission method for measuring acoustic dispersion and attenuation requires the measurement of the thickness of the specimen, the transmission coefficient at the water–specimen interface, and the Fourier spectra of two transmitted pulses. A new method has recently been developed that can determine both the thickness and dispersion of the specimen by utilizing the phase spectra of two additional pulses reflected back from the front and back surfaces of the specimen. In this paper, the method is further extended to the measurement of attenuation. If the density of the specimen is known, the frequency-dependent transmission coefficient can be determined based on the measured phase velocity, and only the amplitude spectra of the two transmitted pulses are used to determine the attenuation. If the density of the specimen is unknown, the attenuation can be determined from the amplitude spectra of all the four pulses. In both cases, the thickness estimated from the phase spectra of the four pulses is utilized. Experimental results from two specimens are presented to demonstrate the application of the new method. © 2001 Elsevier Science B.V. All rights reserved.

PACS: 43.35
Keywords: Attenuation; Dispersion; Phase velocity; Ultrasound

1. Introduction

Accurate measurement of acoustic attenuation and dispersion (phase velocity as a function of frequency) has important applications in theoretical acoustics, nondestructive testing, and ultrasound tissue characterization [1–5]. The attenuation and dispersion properties of a layer of immersible material can be determined using a broadband, through transmission technique [1,5,6]. In a typical setup, two broadband transducers, one for transmitting and one for receiving the ultrasound pulses, are placed in a water tank and aligned properly. Two pulses are recorded: one without and one with the specimen inserted between the two transducers. The amplitude spectra of the two pulses are then used to determine the attenuation coefficient, and the phase spectra of the two pulses are used to determine the phase velocity of the specimen, both as functions of frequency within the frequency range of the measurement system. In addition to recording the two pulses, one also needs to know the transmission coefficient at the water–specimen interface in order to determine the attenuation, and the thickness of the specimen in order to determine both the attenuation and dispersion. Consequently, the overall accuracy of attenuation and dispersion measurements may be limited by the uncertainties in determining the transmission coefficient and the thickness of the specimen under the test.

We have recently developed a new method for measuring dispersion without the need for measuring the specimen's thickness [7]. The instruments and setup used in the new method are the same as that used in the traditional method. In addition to recording two transmitted pulses, the new method requires recording two reflected pulses, one from the front surface and one from the back surface of the specimen. The phase velocity and the thickness of the specimen can both be determined from the phase spectra of the four recorded pulses. In this paper, the method is further extended to the measurement of attenuation. First of all, the thickness determined from the phase spectra of the four pulses can

---

[*] Corresponding author. Tel.: +1-937-775-5069; fax: +1-937-775-7364.
E-mail address: phe@cs.wright.edu (P. He).

PII: S0041-624X(00)00037-8

be directly used in the attenuation measurement. Secondly, if the density of the specimen is known, the phase velocity determined from the phase spectra of the four pulses can be used to determine the frequency-dependent transmission coefficient at the water–specimen interface. Finally, if the density of the specimen is unknown, it is shown that by using the amplitude spectra of all the four pulses, the need for measuring the transmission coefficient can be eliminated. Experimental results from two polyethylene samples are presented to demonstrate the application of the new method.

2. Theory

Fig. 1 shows the signal paths in the immersion experiment for the attenuation and dispersion measurements. $P_0(t)$ is the initial pulse launched by the transducer labeled as T_1. $P_1(t)$ and $P_2(t)$ represent the pulses reflected back from the front and back surfaces of the specimen, respectively. $P_s(t)$ and $P_w(t)$ represent the transmitted pulses with and without the specimen inserted, respectively. If we use $A_i(f)e^{-j\theta_i(f)}$ to represent the Fourier transform of a pulse $P_i(t)$, the Fourier transforms of the above four pulses can be found as

$$U_1(f) = A_1(f)e^{-j\theta_1(f)} = U_0(f)e^{-2(\alpha_w+j\beta_w)D_1}r, \tag{1}$$

$$\begin{aligned} U_2(f) &= A_2(f)e^{-j\theta_2(f)} \\ &= U_0(f)e^{-2(\alpha_w+j\beta_w)D_1}e^{-2(\alpha+j\beta)L}(-r)T, \end{aligned} \tag{2}$$

$$U_w(f) = A_w(f)e^{-j\theta_w(f)} = U_0(f)e^{-(\alpha_w+j\beta_w)D} \tag{3}$$

and

$$\begin{aligned} U_s(f) &= A_s(f)e^{-j\theta_s(f)} \\ &= U_0(f)e^{-(\alpha_w+j\beta_w)(D-L)}e^{-(\alpha+j\beta)L}T, \end{aligned} \tag{4}$$

where $U_0(f)$, $U_1(f)$, $U_2(f)$, $U_w(f)$, and $U_s(f)$ are the Fourier transforms of $P_0(t)$, $P_1(t)$, $P_2(t)$, $P_w(t)$, $P_s(t)$, respectively. α_w and α are the attenuation coefficients, and β_w and β are the propagation constants, of water and the specimen, respectively. L is the thickness of the specimen. D_1 and D are two distances as shown in Fig. 1.

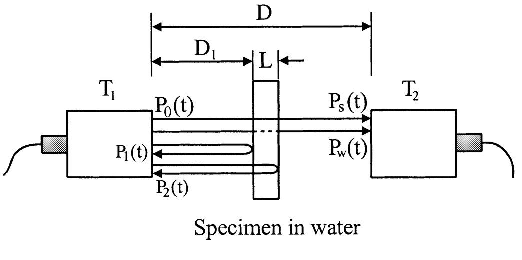

Fig. 1. Signal paths in the immersion experiment for measuring attenuation, dispersion and thickness using a broadband-pulse technique.

r is the reflection coefficient at the water–specimen interface, and T the overall transmission coefficient when the pulse passes through the interface twice.

2.1. Determination of the phase velocity and thickness of the specimen

The details of the derivation of the method for determining the phase velocity as well as the specimen's thickness have been reported in a previous paper [7]. The important results are summarized here for the sake of completeness. Using the relation $\beta = 2\pi f/V_p$, the phase velocity V_p can be determined separately from the phase spectra of the two reflected pulses using Eqs. (1) and (2), and from the phase spectra of the two transmitted pulses using Eqs. (3) and (4)

$$\frac{1}{V_p(f)} = \frac{\hat{\theta}_2(f) - \theta_1(f)}{4\pi f L} \tag{5}$$

and

$$\frac{1}{V_p(f)} = \frac{\theta_s(f) - \theta_w(f)}{2\pi f L} + \frac{1}{c_w}, \tag{6}$$

where c_w is the speed of sound in water; $\hat{\theta}_2$ is the phase spectra of $-P_2(t)$ (to take account of the inherent extra 180° phase shift between P_2 and P_1, the recorded P_2 is always inverted first in all the data processing described in this paper). By canceling the thickness L from the above two expressions of V_p, we obtain

$$V_p(f) = c_w\left[1 + 2\frac{\theta_w(f) - \theta_s(f)}{\hat{\theta}_2(f) - \theta_1(f)}\right]. \tag{7}$$

Direct application of Eq. (7) requires phase unwrapping to obtain the absolute phases θ_w, θ_s, θ_1 and $\hat{\theta}_2$. To avoid the $2m\pi$ phase ambiguity in calculating the phase spectrum from the arctangent function [8,9], the center of the pulse is first shifted (circularly-rotated) to the beginning of the sampling window and the phase spectrum of the shifted pulse is then calculated. Using this procedure, the new equation for determining phase velocity becomes

$$V_p(f) = c_w\left[1 + 2\frac{\phi_w(f) - \phi_s(f) + 2\pi f(t_w - t_s)}{\phi_2(f) - \phi_1(f) + 2\pi f(t_2 - t_1)}\right], \tag{8}$$

where $\phi_w(f)$, $\phi_s(f)$, $\phi_2(f)$, $\phi_1(f)$ are the phase spectra of shifted $P_w(t)$, $P_s(t)$, $-P_2(t)$, $P_1(t)$, respectively, and t_w, t_s, t_2, t_1 are the respective total time shift which includes both the time shift within the sampling window and the trigger delay of the sampling window [7].

By canceling V_p in Eqs. (5) and (6), we obtain an estimate of the thickness L of the specimen

$$\hat{L} = \frac{c_w}{4\pi f}\left[\hat{\theta}_2 - \theta_1 + 2(\theta_w - \theta_s)\right]$$
$$= \frac{c_w}{4\pi f}[\phi_2 - \phi_1 + 2(\phi_w - \phi_s) + 2\pi f(t_2 - t_1 + 2t_w - 2t_s)]. \tag{9}$$

The mean of the calculated $\hat{L}$ within the frequency range of the measurement system is used as the estimated thickness L.

2.2. Determination of the attenuation coefficient

Traditionally, the attenuation coefficient of the specimen is obtained from the amplitude spectra of the two transmitted pulses using Eqs. (3) and (4)

$$\alpha = \alpha_w + \frac{\ln(T) + \ln(A_w) - \ln(A_s)}{L} \cong \frac{\ln(T)}{L} + \ln\left(\frac{A_w}{A_s}\right), \tag{10}$$

where α_w is the attenuation coefficient of water which is normally much smaller than α and therefore can be ignored.

If we use the mean of $\hat{L}$ as determined by Eq. (9) to replace L in Eq. (10), the need for measuring L is first eliminated. The term ln(T) can be dealt with in two ways. If the density of the specimen is known, T can then be calculated from the following equation:

$$T = 4(\rho V_p)(\rho_w c_w)/(\rho V_p + \rho_w c_w)^2, \tag{11}$$

where ρ and ρ_w are the density of the specimen and water, respectively; V_p is the phase velocity determined by Eq. (8). Since V_p is a function of frequency, T is frequency-dependent. If the density of the specimen is unknown, we may cancel the term $\ln(T)$ using an approach parallel to the one used to cancel L in the measurement of the phase velocity. In addition to the expression of α shown in Eq. (10), we can obtain a second expression of α by using the amplitude spectra in Eqs. (1) and (2)

$$\alpha = \frac{\ln(T)}{2L} + \frac{1}{2L}\ln\left(\frac{A_1}{A_2}\right). \tag{12}$$

By canceling the term $\ln(T)$ from Eqs. (10) and (12), we obtain

$$\alpha = \frac{1}{L}\left[\ln\left(\frac{A_1}{A_2}\right) - \ln\left(\frac{A_w}{A_s}\right)\right]. \tag{13}$$

Eqs. (8), (9) and (13) form the basis of a new method that fully utilizes the amplitude and phase spectra of the two transmitted and two reflected pulses to determine the attenuation and dispersion without a prior knowledge of the thickness and density of the specimen.

2.3. Correction for diffraction effects in attenuation measurement

When the transmitting and receiving transducers have a finite size, diffraction, or beam spreading causes additional changes in the phase and amplitude spectra of the received signal. A number of investigators have studied the effects of diffraction and provided formulas for diffraction correction [10–12]. Although the explicit expressions of these formulas are different from each other, the end results are very close when the value of ka is large enough, where $k = \omega/c$ is the wavenumber and a is the radius of the transducer. An expression for diffraction correction that is easy to use is provided by Rogers and Van Buren [11]

$$D_L = 1 - e^{-j(2\pi/s)}[J_0(2\pi/s) + jJ_1(2\pi/s)]. \tag{14}$$

According to Eq. (14), the effects of diffraction are solely determined by the parameter $s = zc/(fa^2)$ where z is the distance. Referring to Fig. 1, the values of this parameter (called the Fresnel parameter) for each of the four pulses, $P_1(t)$, $P_2(t)$, $P_w(t)$, and $P_s(t)$ are

$$s_1 = \frac{2D_1 c_w}{fa^2}, \qquad s_2 = \frac{2D_1 c_w}{fa^2} + \frac{2LV_p(f)}{fa^2},$$
$$s_w = \frac{Dc_w}{fa^2}, \qquad s_s = \frac{(D-L)c_w}{fa^2} + \frac{LV_p(f)}{fa^2}. \tag{15}$$

If the transducer is focused, both the distance z and radius a need to be normalized by a factor $\gamma = F/|F - z|$ where F is the focal distance [13].

The effects of diffraction on the phase of the received signal have been found very small [4,14]. In addition, the effects of diffraction in the attenuation measurement based on the ratio of the amplitude spectra of the two transmitted pulses are also found negligible [8]. Consequently, diffraction correction is only applied to the amplitude spectra of the two reflected pulses.

3. Experiment and results

Two specimens, a high-density polyethylene sample and a low-density polyethylene sample are used in the experiment. The density and thickness of each sample are listed in Table 1. A Panametrics V382 (3.5 MHz, 13-mm aperture, 8.9-cm focal distance) is used as the transmitting/receiving transducer (T_1 in Fig. 1) and a Panametrics V384 (3.5 MHz, 6.35-mm aperture, non-focused) is used as the receiving transducer (T_2 in Fig. 1). The distance D_1 is 10 cm and the distance D is 18 cm. The specimen is carefully aligned parallel to the surface of transducer T_1 by maximizing the amplitude of the pulse reflected from the near surface of the specimen. The pulser/receiver used in the experiment is Panametrics 5052PR. The amplified pulse is A/D converted by a

Table 1
Density, thickness measured using a digital caliper and thickness estimated using Eq. (9) of the two samples used in this study

	Density (g/cm^3)	Measured thickness (mm)	Estimated thickness (mm)
High-density polyethylene	0.966	18.76	18.71
Low-density polyethylene	0.918	3.16	3.15

SONY/TEK 390AD programmable digitizer which has an adjustable digital delay for triggering the sampling window. Each sampling window contains 512 samples and the sampling frequency is 60 MHz. The samples are transferred to a PC and processed using a software package MATLAB (Math Works, MA). The water temperature is 21.2°C which gives $c_w = 1486$ m/s [15].

Fig. 2 shows a water-path pulse (P_w) recorded in the experiment (top), its amplitude spectrum (middle) and its phase spectrum without phase unwrapping (bottom). The largest positive peak of the pulse is located at sample number 204. In order to improve the resolution of the phase spectrum, the original pulse is first padded with 7680 zeros before performing FFT, a typical procedure used to reduce the phase uncertainty. Fig. 3 (top) shows the same pulse (P_w) after circularly-rotating to the left. The largest positive peak is now located at sample number 510. The phase spectrum of the shifted pulse (without zero padding) is shown at the bottom. Since this phase spectrum does not contain any discontinuities within the frequency range of interest (1–5 MHz), phase-unwrapping is not needed.

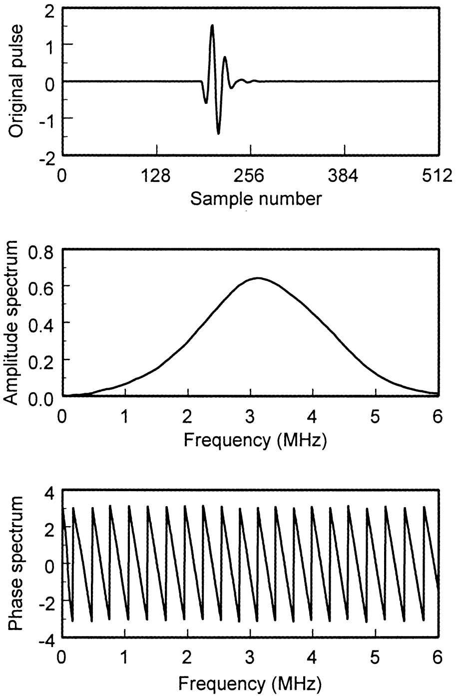

Fig. 2. Top: original water-path pulse (P_W) in a sampling window of 512 samples; middle: amplitude spectrum of the pulse; bottom: phase spectrum of the pulse (to improve the resolution in the frequency domain, the original pulse was padded with 7680 zeroes before performing FFT).

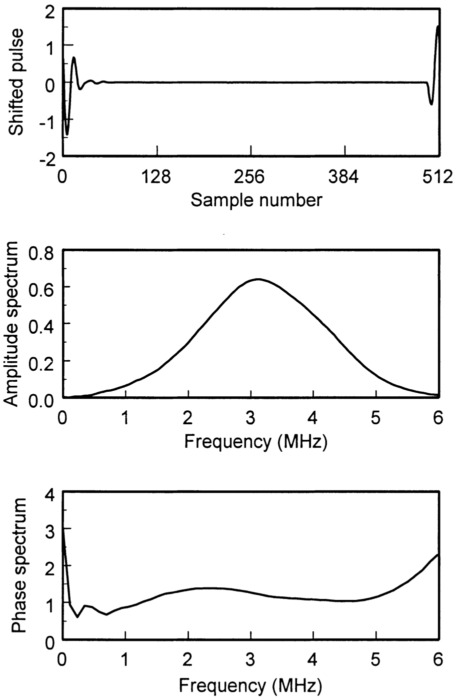

Fig. 3. Top: shifted water-path pulse (P_W); middle: amplitude spectrum of the shifted pulse; bottom: phase spectrum of the shifted pulse without zero padding.

Fig. 4 reports the experimental results from the high-density polyethylene sample. Fig. 4(a) plots the phase velocity of the specimen determined by Eq. (8). This phase velocity is used in Eq. (11) to determine the transmission coefficient T as well as in Eq. (15) to determine the Fresnel parameter s for diffraction correction in attenuation measurement. The mean of the thickness estimated using Eq. (9) is reported in Table 1, and is used in attenuation measurement (in place of L in Eqs. (13) and (15)). Fig. 4(b) compares the results of attenuation measurement using the two methods described in Section 2.2. The line with solid circles is obtained from the amplitude spectra of the two transmitted pulses using Eqs. (10) and (11), and the line with open squares is obtained from the amplitude spectra of all the four pulses using Eq. (13) after applying diffraction correction to A_1 and A_2 by dividing each amplitude spectrum by the respective value of D_L determined by Eqs. (14) and (15).

Fig. 5 reports the experimental results from the low-density polyethylene sample. Again, the mean of the thickness estimation is listed in Table 1.

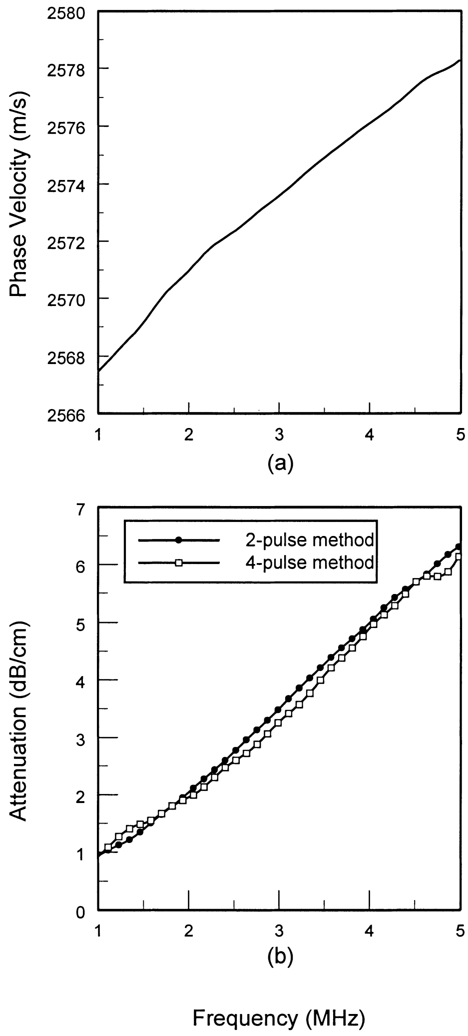

Fig. 4. Experimental results from the high-density polyethylene sample: (a) phase velocity and (b) attenuation determined by the two-pulse method and the four-pulse method.

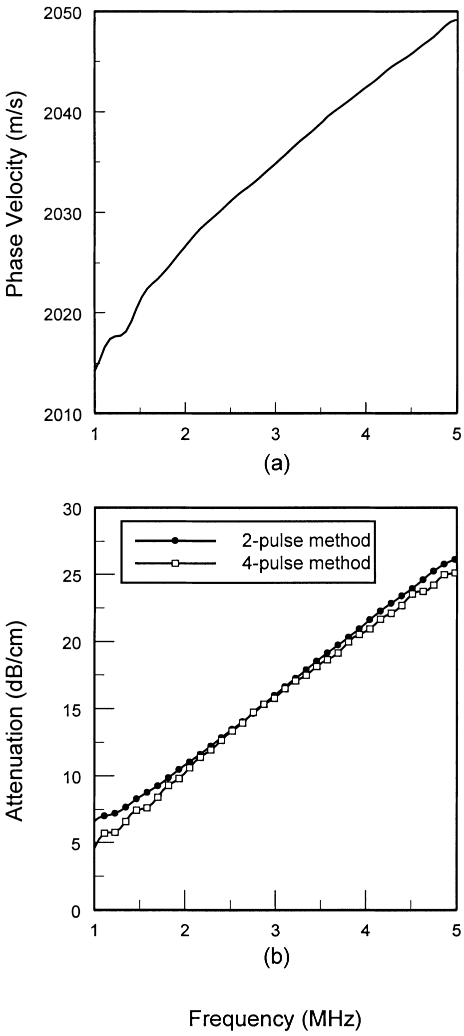

Fig. 5. Experimental results from the low-density polyethylene sample: (a) phase velocity and (b) attenuation determined by the two-pulse method and the four-pulse method.

4. Summary and discussion

As discussed in a previous paper [7], the accuracy and precision of the measurement of phase velocity and thickness of the specimen are mainly determined by the accuracy of the time measurement which is usually very high. The main difficulty in the measurement of phase velocity when a conventional procedure of phase unwrapping is used is the $2m\pi$ phase ambiguity. By using the new procedure to determine the absolute phase, as defined by Eq. (8), this phase ambiguity is avoided, as demonstrated by Fig. 3. As a result, the phase velocity and thickness of the specimen determined from the phase spectra of the four recorded pulses can be reliably used in determining the attenuation of the specimen.

Two methods for measuring attenuation are presented, both utilize the estimated thickness of the specimen. The first method uses only the amplitude spectra of the two transmitted pulses. The method requires a prior knowledge of the density of the specimen and uses the measured phase velocity to determine the frequency-dependent transmission coefficient at the water–specimen interfaces. In the second method, the transmission coefficient is canceled by using the amplitude spectra of all the four pulses to determine the attenuation. As a result, the need to know the specimen's density is eliminated. The attenuation coefficients of the two polyethylene samples measured by the two methods are both in good agreement with the results obtained by Wu [8]. On the other hand, it is evident from Figs. 4(b) and 5(b) that the two methods do not produce the exact results, indicating a relatively large uncertainty in the attenuation measurement. In general, the accuracy and precision of the attenuation measurement are more difficult to control than that of the velocity measurement. This is because the amplitude spectrum of the received pulse may be affected by many factors. In addition to the diffraction loss discussed in this paper, other factors include phase-cancellation at the receiving aperture [16], deviation from perfectly-normal incidence at both the front and back surfaces of the specimen, the flatness of the gain spectrum of the receiving amplifier [17], etc. Comparing with the transmitted pulse, the amplitude spectrum of the reflected pulse is more sensitive to most of the above factors. This is supported by the fact that for both samples, the attenuation curve obtained from the four pulses is less smooth than the curve obtained by the two transmitted pulses. Consequently, use of Eq. (13) to determine the attenuation (method 2) requires more careful experimental control, and is advantageous only if the density of the specimen is not available.

The method proposed in this paper may be particular useful for real-time measurement of dispersion/attenuation when the thickness and/or density of the specimen change with time. Rokhlin et al. [3] reported a measurement of frequency dependence of ultrasound velocity and attenuation during the curing reaction of epoxy resin. According to the authors, the sample underwent a certain degree of shrinkage during the curing reaction, resulting a change in both thickness and density. Using the method described in this paper, the thickness of the specimen can be measured in real-time from the recorded pulses and the accuracy of attenuation and dispersion measurement will not be deteriorated by the changes in thickness and density.

There are several limitations of this new method; all are related to the requirement of obtaining two reflected pulses. First of all, in order to obtain two non-overlapping echoes, the thickness of the specimen cannot be smaller than the axial resolution of the ultrasound system. Secondly, two surfaces of the specimen should be parallel to each other and should be kept strictly normal to the beam. Finally, if the acoustic impedance of the specimen is very close to that of the surrounding medium, or if the attenuation of the specimen is too large, the magnitude of the echo from the back surface may be reduced to the noise level. This last limitation may be the major obstacle in applying this method to certain biological tissues.

References

[1] C.C. Lee, M. Lahham, B.G. Martin, Experimental verification of the Kramers–Kronig relationship for acoustic waves, IEEE Trans. Ultrason. Ferroelec. Freq. Contr. 37 (1990) 286.

[2] D. Zellouf, Y. Jayet, N. Saint-Pierre, J. Tatibouët, J.C. Baboux, Ultrasonic spectroscopy in polymeric materials. Application of the Kramers–Kronig relations, J. Appl. Phys. 80 (1996) 2728.

[3] S.I. Rokhlin, D.K. Lewis, K.F. Graff, L. Adler, Real-time study of frequency dependence of attenuation and velocity of ultrasonic waves during the curing reaction of epoxy resin, J. Acoust. Soc. Am. 79 (1986) 1786.

[4] R.A. Kline, Measurement of attenuation and dispersion using an ultrasonic spectroscopy technique, J. Acoust. Soc. Am. 76 (1984) 498.

[5] A.C. Kak, K.A. Dines, Signal processing of broadband pulsed ultrasound: measurement of attenuation of soft biological tissues, IEEE Trans. Biomed. Engng. 25 (1978) 321.

[6] W. Sachse, Y.H. Pao, On the determination of phase and group velocities of dispersive waves in solids, J. Appl. Phys. 49 (1978) 4320.

[7] P. He, Measurement of acoustic dispersion using both transmitted and reflected pulses, J. Acoust. Soc. Am. 107 (2000) 801.

[8] J. Wu, Determination of velocity and attenuation of shear waves using ultrasonic spectroscopy, J. Acoust. Soc. Am. 99 (1996) 2871.

[9] J. Mobley, J.N. Marsh, C.S. Hall, M.S. Hughes, G.H. Brandenburger, J.G. Miller, Broadband measurements of phase velocity in Albunex® suspensions, J. Acoust. Soc. Am. 103 (1998) 2145.

[10] R. Bass, Diffraction effects in the ultrasonic field of a piston source, J. Acoust. Soc. Am. 30 (1958) 602.

[11] P.H. Rogers, A.L. Van Buren, An exact expression for the Lommel diffraction correction integral, J. Acoust. Soc. Am. 55 (1974) 724.

[12] W. Xu, J.J. Kaufman, Diffraction correction methods for insertion ultrasound attenuation estimation, IEEE Trans. Biomed. Engng. 40 (1993) 563.

[13] M. Fink, J.F. Cardosos, Diffraction effects in pulse echo measurements, IEEE Trans. Ultrason. Ferroelec. Freq. Contr. 31 (1984) 312.

[14] W. Sachse, Measurement of the elastic moduli of continuous-filament and eutectic composite materials, J. Compos. Mater. 8 (1974) 378.

[15] V.A. Del Grosso, C.W. Mader, Speed of sound in pure water, J. Acoust. Soc. Am. 52 (1972) 1442.

[16] L.J. Busse, J.G. Miller, Detection of spatially nonuniform ultrasonic radiation with phase sensitive (piezoelectric) and phase insensitive (acoustoelectric) receivers, J. Acoust. Soc. Am. 70 (1981) 1377.

[17] P. He, Experimental verification of models for determining dispersion from attenuation, IEEE Trans. Ultrason. Ferroelec. Freq. Contr. 46 (1999) 706.