Full-Diversity Group Space-Time-Frequency (GSTF) codes from Cyclic codes

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***Abstract*— It is known that multi-antenna transmissions over frequency-selective channels can provide a diversity gain that is product of the number of transmit antennas, the receive antennas and the length of the channel impulse response. Liu, Xin and Giannakis have studied multi-antenna orthogonal frequency division multiplexing (OFDM) through frequency-selective Rayleigh-fading channels and have introduced the concept of Space-Time-Frequency (STF) coding to enable maximum diversity and high coding gains.**

It is known that under some conditions, an n-length cyclic code C over $F_{q^m},\ (n|q^m-1,\ and\ m\ \leq\ n)$ can have full-rank i.e $Rank_q(\mathbf{C})=m$. Designs for Space-Time codes suitable for both quasi-static fading channels and block-fading channels have been derived from n length cyclic codes over F_{q^m}. In this paper, we present a simplified design of STF codes using designs derived from cyclic codes to obtain Group Space-Time-Frequency (GSTF) codes for frequency selective Rayleigh fading channels. These codes achieve maximum diversity gain.

I. INTRODUCTION

Liu, Xin and Giannakis [1] have studied multi-antenna orthogonal frequency-division multiplexing (OFDM) transmissions through frequency-selective Rayleigh fading channels i.e joint Space-Time-frequency (STF) coding over space, time and frequency. Subchannel grouping [7],[8] has been used to convert the STF system into group STF (GSTF) subsystems which preserves maximum diversity gains and simplifies the code construction. In [4] and [5], designs for full-rank and full-diversity STBCs for quasi-static and block-fading channels respectively were derived using conventional cyclic codes over a finite field. In this paper we obtain codes suitable for GSTF systems. These codes achieve full-diversity and can be easily configured to an arbitrary number of transmit antennas.

A. Preliminary Concepts [1]

Consider a multi-antenna wireless communication system with N_t transmit antennas and N_r receive antennas, where OFDM using N_c subcarriers is employed per antenna transmission as shown in Fig.1.

The fading channel between the μ^{th} transmit antenna and ν^{th} receive antenna is assumed to be frequency selective but time flat and described by the base band equivalent impulse response vector $\mathbf{h}_{\mu\nu} \triangleq [h_{\mu\nu}(0),\ h_{\mu\nu}(1),\ldots,h_{\mu\nu}(L)]^T \in \mathbb{C}^{(L+1)\times 1}$ where L denotes the length of the channel impulse response.

Let $x_n^{\mu}(p)$ be the data symbol transmitted on the p^{th} subcarrier (frequency bin) from the μ^{th} transmit antenna during the n^{th} OFDM symbol interval. The symbols $\{x_n^{\mu}(p),\ \mu=1,2,\ldots,N_t,\ p=0,1,\ldots,N_c-1\}$ are transmitted in parallel on N_c subcarriers by N_t transmit antennas. The three variables $\mu,\ n,\ p$ respectively index the antenna (space), time and frequency dimensions associated with the transmission of $x_n^{\mu}(p)$. Thus, each $x_n^{\mu}(p)$ can be viewed as a point in a three dimensional (3-D) space-time-frequency (STF) parallelepiped. We denote the projection of the STF codeword on the p^{th} subcarrier as $\mathbf{X}(p)$ and $\mathbf{X} = [\mathbf{X}(0)\ \mathbf{X}(1)\ \ldots\mathbf{X}(N_c-1)]$.

At the receiver, each antenna receives a noisy superposition of the multi-antenna transmissions through the fading channels. After FFT processing, the received data sample $y_n^{\nu}(p)$ at the ν^{th} receive antenna can be expressed as,

$$y_n^{\nu}(p) = \sum_{\mu=1}^{N_t} H_{\mu\nu}(p)x_n^{\mu}(p) + w_n^{\nu}(p), \tag{1}$$

$\nu = 1,2,\ldots,N_r,\ p = 0,1,\ldots,N_c-1$ where $H_{\mu\nu}(p)$ is the subchannel gain from the μ^{th} transmit antenna to the ν^{th} receive antenna evaluated on the p^{th} subcarrier

$$H_{\mu\nu}(p) \triangleq \sum_{l=0}^{L} h_{\mu\nu}(l)e^{-j(\frac{2\pi}{N_c})lp}. \tag{2}$$

and the additive noise $w_n^{\nu}(p)$ is circularly symmetric, zero mean, complex Gaussian with variance N_0 that is assumed to be statistically independent with respect to n, ν and p. Equation (1) represents a general model for multi-antenna OFDM systems.

B. Subchannel Grouping and Design Criteria

The idea of subchannel grouping was introduced in [7],[8] to reduce design complexity. In [1], the concept of subchannel grouping has been used to simplify the STF system. The steps are:

1. Choose the number of subcarriers equal to an integral multiple of the channel impulse response length. i.e $N_c = N_g(L+1)$, where N_g is a positive integer denoting the number of groups.
2. Split the $N_t \times N_c N_x$ STF codeword $\mathbf{X}$ into N_g group STF (GSTF) codewords $\mathbf{X}_g,\ g = 0,1,\ldots,N_g-1$.

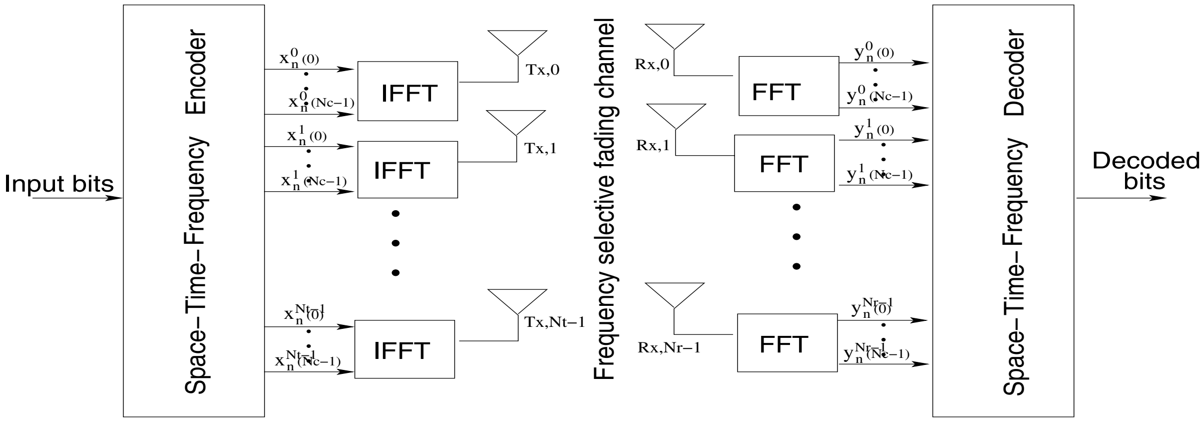

Fig. 1. MIMO-OFDM system model

$$\mathbf{X}_g = [\mathbf{X}_g(0), \mathbf{X}_g(1), \ldots, \mathbf{X}_g(L)] \in \mathbb{C}^{N_t \times N_x(L+1)}. \quad (3)$$

where $\mathbf{X}_g(l) \triangleq \mathbf{X}(N_g l + g)$. Thus the STF system is divided into N_g GSTF (sub)systems, which can be described through the input-output relationships,

$$\mathbf{Y}_g(l) = \mathbf{H}_g(l)\mathbf{X}_g(l) + \mathbf{W}_g(l) \quad (4)$$

$l = 0, 1, \ldots, L,\ g = 0, 1, \ldots, N_g - 1$. where,
$\mathbf{Y}_g(l) \triangleq \mathbf{Y}(N_g l + g), \mathbf{H}_g \triangleq \mathbf{H}(N_g l + g)$,
$\mathbf{W}_g(l) \triangleq \mathbf{W}(N_g l + g)$.
Hence each GSTF system is a simplified STF system with a much smaller size in the frequency dimension, as compared with the original STF system. Liu, Xin and Giannakis [1] consider STF coding within each GSTF system and also show that doing this does not result in any reduction in the diversity advantage and the design complexity is considerably reduced. The design criteria for GSTF coding are:
1. (Sum of Ranks criterion): Design $\mathcal{A}_{x_g}$ (the set comprising of GSTF codewords) such that $\forall\ \mathbf{X}_g \neq \mathbf{X}'_g \in \mathcal{A}_{x_g}$, the matrices $\Lambda_e(l) = [\mathbf{X}_g(l) - \mathbf{X}'_g(l)][\mathbf{X}_g(l) - \mathbf{X}'_g(l)]^H\ \forall\ \ l\ \in\ [0, l]$ have full-rank.
2. (Product-of-determinants criterion): For the set of matrices satisfying the rank criterion, design $\mathcal{A}_{x_g}$ such that, $\forall \mathbf{X}_g \neq \mathbf{X}'_g \in \mathcal{A}_{x_g}$, the minimum of

$$\prod_{l=0}^{L} det[\Lambda_e(l)]$$

is maximized.

In this paper, we present simple techniques to obtain GSTF codes from the full-rank designs derived from conventional cyclic codes over finite fields [4], [5] and study their performance. Our design process can be extended easily to obtain codes for systems with more number of transmit antennas though we illustrate our design for 2 transmit antennas.

The rest of the paper is organized as follows. In Section II, we review the theorems connected with the rank-characterization for cyclic codes and the procedure used to derive designs suitable for STBCs from them. In Section III, we present a number of GSTF codes derived from designs based on cyclic codes and study their performance.

II. Designs from Cyclic codes

We summarize the results relevant to the design of Space Time Block codes in Theorems 2.1 and 2.2 the proofs of which are available in [2], [3].

Theorem 2.1: Let $\mathbf{C}$ be the cyclic code of length $n|q^m - 1$, $(m \leq n)$ over F_{q^m} characterized by the transform component $A_{jq^s} \in A_{[j]}, |[j]| = e_j,\ e_j|m$ being free and all other transform components are constrained to zero. Then $rank_q(\mathbf{C}) = e_j$.
In practice, we choose the free transform domain component $A_{jq^s} \in A_{[j]}$ where $e_j = |[j]| = m$ from a full size q-cyclotomic coset. Then from Theorem 2.1 it follows that the rank of the resulting code is m. This code has q^m codewords.

Theorem 2.2: Let $\mathbf{C}$ be a cyclic code of length $n|q^m - 1$ over F_{q^m} whose free transform domain components are A_{jq^r} and $A_{jq^{r+s}}$. (the indices of the free transform domain components belong to the same q- cyclotomic coset and s denotes the separation between them. $(1 \leq s \leq e_j - 1)$, $(0 \leq r \leq e_j - 2)$. Let all other transform components be constrained to zero. Then, $rank_q(\mathbf{C}) = (e_j - gcd(s, e_j))$.
From Theorem 2.2 it follows that if we try to increase the number of codewords by considering codes with two free transform components from the same q-cyclotomic coset and constraining all other transform components to zero, we can no longer obtain full-rank cyclic codes. Hence, in our search for full-rank STBCs for block-fading channels, we shall confine our study to one dimensional cyclic codes of length n over $F_{q^m}, (n|q^m - 1)$.

We shall use Theorem 2.1 to derive derive designs for STBCs from n-length cyclic codes over F_{q^m} from which GSTF codes can be derived.

Definition 1: A rate-k/n, $n \times l$ linear design over a field $F \subset \mathbb{C}$ (the complex field) is an $n \times l$ matrix with all its entries F-linear combinations of k complex variables and their conjugates which are allowed to take values from the field F.

Let $(n,q)=1$ and $n|q^m-1$, where q is either 2 or a prime of the form $4k+1$. We define $I_n \triangleq \{0,1,2,\ldots,n-1\}$. Let $[j]$ be a q-cyclotomic coset of I_n of size m. By restricting $A_j,\ j\in[j]$, to F_{q^m} and constraining all other transform components to zero, we have an n-length cyclic code over F_{q^m} whose codewords are of the form,

$$[\ A_j,\quad \beta^{-j}A_j,\quad \beta^{-2j}A_j,\quad \ldots,\quad \beta^{-(n-1)j}A_j\ ]$$

where β is a primitive n-th root of unity in F_{q^m} and $A_j\in F_{q^m}$. Viewing A_j as an m-length column vector over F_q, the codewords can be viewed as $m\times n$ matrices over F_q given by,

$$\begin{bmatrix} a_{0,0} & a_{0,1} & a_{0,2} & \ldots, & a_{0,n-1} \\ a_{1,0} & a_{1,1} & a_{1,2} & \ldots, & a_{1,n-1} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{m-1,0} & a_{m-1,1} & a_{m-1,2} & \ldots, & a_{m-1,n-1} \end{bmatrix} \quad (5)$$

where $\beta^{-kj}A_j=\sum_{i=0}^{m-1}a_{i,k}\alpha^i$, $a_{i,k}\in F_q$ and α is a primitive element of F_{q^m}. Notice that (5) is a design over F_q. Also, note that this is in general, possible for any linear code, however, we have information about the rank, only in the case of cyclic codes.

Example 1: Let the number of transmit antennas $N_t=3$. We take $n=7$. The 2-cyclotomic coset of 1 modulo 3 is $\{1,2,4\}$. With A_1 taking all of F_8 and other transform components constrained to zero, we obtain a one dimensional cyclic code $\mathbf{C}$ such that $Rank_2(\mathbf{C})=3$. Let α be a cube root of unity in F_8. With $F_8=F_2[x]/(x^3+x+1)$, the codewords of $\mathbf{C}$ are of the form given in (6) where $a_0,a_1,a_2\ \in F_2$.

This is an example of a rate $\frac{3}{7}$ design. To obtain STBCs from the above designs, we have to map the elements of F_q into the complex field such that the full-rank property of the finite field design is preserved. We call a signal set to be matched to F_q, if there exists a map from F_q to the signal set which is an isometry for the F_q-rank to the complex field rank. There are two such maps, the Hammons and El Gamal map [9] (suitable for codes over extension fields of F_2) and the map proposed by Lusina Gabidulin and Bossert [10] (suitable for codes over F_{q^m} where q is a prime of the form $4k+1$). An n-length cyclic code over F_{2^m} will give rise to an $m\times n$ STBC with 2^m codewords for m transmit antennas. Hence, the code rate in bits per channel use is $\frac{1}{n}\log_2(2^m)=m/n$. Now, assuming that we want full-rank STBCs, we have the condition $m\le n$. Therefore, for the case of cyclic codes over F_{2^m}, the data rate is always upper bounded by 1 bit per channel use. Hence, we will not consider the Hammons and El Gamal map here. To achieve higher code rates, we have to derive STBCs from non binary cyclic codes by making use of the map proposed by Lusina, Gabidulin and Bossert [10].

A. Map proposed by Lusina Gabidulin and Bossert [10]

Let q be a prime of the form $q=4k+1$. A Gaussian integer w is a complex number defined as $w=a+ib,\ a,\ b\ \in\mathbf{Z},\ i=\sqrt{-1}$ and it is known that every prime number q of the form $q\ \equiv 1\ mod\ 4$ can be written as $q=(u+iv)\times(u-iv)=u^2-v^2$. The number $\Pi=u+iv$ is known as Gaussian prime number where $u,\ v\ \in\mathbf{Z}$. Let $\Pi'=u-iv$. Then calculation modulo Π is defined as, $\zeta=w\ modulo\ \Pi=w-\left[\frac{w\Pi'}{\Pi\Pi'}\right]\Pi$ where [.] performs the operation of rounding to the nearest Gaussian integer. The Gaussian integers modulo Π form a field, $G_\pi=\{\zeta_0=0,\ \zeta_1=1,\ \zeta_2,\ldots,\zeta_{q-1}\}$ and the map $\xi: F_q\Rightarrow G_\Pi$ given by $\zeta_i=i\ mod\ \Pi=i-\left[\frac{i\Pi'}{\Pi\Pi'}\right]\Pi,\ i=0,\ 1,\ 2,\ldots,,\ p-1$ is an isomorphism [10]. Therefore when we map codewords from a linear cyclic code over $F_{q^m},\ q=5,13,17,\ldots$, which are $m\times n$ matrices over F_q to $m\times n$ matrices over the complex Gaussian field, the full-rank property of the code over F_{q^m} is preserved. An example of the map between F_5 and the corresponding complex Gaussian field is:

$0\ \mapsto\ 0,\ 1\ \mapsto\ 1,\ 2\ \mapsto\ 0+i1,\ 3\ \mapsto\ 0-i1,\ 4\ \mapsto\ -1.$

III. Designs and Simulation Results

We illustrate the process of obtaining GSTF codes from designs derived from cyclic codes through several examples.

Example 2: We will first study the GSTF code derived from $\mathcal{C}_1$ which is obtained from the $(4,1)$ code $\mathbf{C}_1$ over F_{5^2} that is derived from a $(6,1)$ cyclic code over F_{5^2} by dropping the last two columns of every codeword. The original cyclic code is characterized by parameters $n=6,\ q=5,\ m=2$. The 5-cyclotomic coset of 1 mod 6 is $\{1,5\}$. With A_1 taking all of $F_{25}=F[x]/(x^2+x+1)$ and all other transform components constrained to zero, we have a rank-2 cyclic code over F_5 with 25 codewords which can be expressed as 2×6 matrices over F_5. We make use of the map proposed by Lusina *et al.* to derive a full-rank 2×6 STBC. Let us denote this STBC by $\mathcal{C}$. The codewords of $\mathcal{C}$ are of the form given in (7) where $a_0,a_1\in F_5,\ \xi: F_5\mapsto G_{1+2i}$. We can reduce the length of this STBC by dropping the last two columns. The corresponding STBC is denoted by $\mathcal{C}_1$. The codewords of $\mathcal{C}_1$ are of the form given in (8), where $a_0,a_1\in F_5,\ \xi: F_5\mapsto G_{1+2i}$. We assume that the channel impulse response length $L+1=2$. The parameters chosen for this GSTF code derived from $\mathcal{C}_1$ are, $N_t=2,\ N_r=1$, number of groups $N_g=32$, OFDM symbol interval $N_x=2$, number of subcarriers $N_c=N_g\times(L+1)=64$. We pick up a set of 32 codewords from the code $\mathcal{C}_1$ randomly. Let these codewords be indexed by parameter $k,\ 0\le k\le 31$. Since $N_x=2$, each GSTF symbol has length 2. We split each codeword into two parts in the following manner. Let $\mathbf{c}_1(k)$ denote the k^{th} codeword of $\mathcal{C}_1$. We represent $\mathbf{c}_1(k)$ as,

$$\mathbf{c}_1(k)=\begin{bmatrix} x_{11}(k) & x_{12}(k) & x_{13}(k) & x_{14}(k) \\ x_{21}(k) & x_{22}(k) & x_{23}(k) & x_{24}(k)\end{bmatrix}.$$

Codeword $\mathbf{c}_1(k)$ is split into two GSTF symbols $\mathbf{X}(k)$ and $\mathbf{X}(k+32)$ in the following manner,

$$\mathbf{X}(k)=\begin{bmatrix} x_{11}(k) & x_{12}(k) \\ x_{21}(k) & x_{22}(k)\end{bmatrix},\ \mathbf{X}(k+32)=\begin{bmatrix} x_{13}(k) & x_{14}(k) \\ x_{23}(k) & x_{24}(k)\end{bmatrix}$$

Symbol $\mathbf{X}(k)$ is conveyed on subcarrier k and symbol $\mathbf{X}(k+32)$ is conveyed on subcarrier $k+32$. (over two antennas and two time slots). Thus each of the 64 GSTF symbols obtained from these 32 codewords is modulated on the corresponding subcarrier. Simulation proceeds by repeatedly selecting 32

$$\begin{bmatrix} a_0 & a_0+a_1+a_2 & a_0+a_1+a_2 & a_1+a_2 & a_0+a_2 & a_1 & a_2 \\ a_1 & a_2 & a_0 & a_0+a_1 & a_0+a_1+a_2 & a_1+a_2 & a_0+a_2 \\ a_2 & a_0 & a_0+a_1 & a_0+a_1+a_2 & a_1+a_2 & a_0+a_2 & a_1 \end{bmatrix} \quad (6)$$

$$\begin{bmatrix} \xi(a_0) & \xi(4a_0+a_1) & \xi(3a_0+a_1) & \xi(4a_0) & \xi(a_0+4a_1) & \xi(2a_0+4a_1) \\ \xi(a_1) & \xi(2a_0+2a_1) & \xi(2a_0+a_1) & \xi(4a_1) & \xi(3a_0+3a_1) & \xi(3a_0+4a_1) \end{bmatrix} \quad (7)$$

$$\begin{bmatrix} \xi(a_0) & \xi(4a_0+a_1) & \xi(3a_0+a_1) & \xi(4a_0) \\ \xi(a_1) & \xi(2a_0+2a_1) & \xi(2a_0+a_1) & \xi(4a_1) \end{bmatrix} \quad (8)$$

codewords at random, obtaining 64 GSTF symbols from them and transmitting them on 64 carriers. ML decoding has been performed and the probability of GSTF symbol error has been plotted as a function of signal to noise ratio (SNR). In Fig. 2, the performance of this code has been compared with the performance of STF block code and the STF trellis code proposed by Liu, Xin and Giannakis [1]. The rate of the GSTF code (in bits/sec/Hz) is defined as,

$$rate = \frac{log_2(|\mathbf{C}|)}{(L+1)\times N_x}\ bits/sec/Hz. \quad (9)$$

where $|\mathbf{C}|$ denotes the number of codewords in the code. Hence, the $(4,1)$ code over F_{5^2} is characterized by rate $\frac{log_2(25)}{2\times 2} = 1.16\ bits/sec/Hz$. The STF Block code and the STF trellis code proposed by Liu, Xin and Giannakis are characterized by rate $2\ bits/sec/Hz$. From Fig. 2 we observe that our code gives a performance advantage of about approximately 1 dB over the STF block code but has a lower rate

Example 3: In a similar manner, the $(4,1)$ code over F_{13^2} $\mathbf{C}_2$ is obtained by dropping the last three columns of the $(7,1)$ cyclic code over F_{13^2}. Let the STBC derived from this by using the Lusina map be denoted by $\mathcal{C}_2$. The codewords of $\mathcal{C}_2$ are of the form,

$$\begin{bmatrix} \xi(a_0) & \xi(3a_0+5a_1) & \xi(4a_0+11a_1) & \xi(7a_0+a_1) \\ \xi(a_1) & \xi(9a_0+3a_1) & \xi(a_0+3a_1) & \xi(6a_0+a_1) \end{bmatrix} \quad (10)$$

where $a_0,\ a_1 \in F_{13},\ \xi : F_{13} \mapsto G_{2+3i}$.
Assuming that the channel impulse response length $L+1 = 2$, the GSTF code derived from $\mathcal{C}_2$ is characterized by the parameters, $N_t = 2,\ N_r = 1,\ N_c = 64,\ N_g = \frac{N_c}{L+1} = 32,\ N_x = 2$. To perform simulation, we choose 32 codewords from $\mathcal{C}_2$ randomly and assign GSTF symbols to codewords in a manner identical to that done for the GSTF system in Example 2. From (9), we conclude that the rate of this GSTF code is $\frac{log_2(169)}{2\times 2} = 1.85\ bits/sec/Hz$. Simulation results pertaining to the performance of this code are plotted in Fig. 2. We observe that the performance of this code in terms of error rate is inferior to the STF block code proposed by Liu, Xin and Giannakis but superior to the STF trellis code proposed by the same authors. The diversity order of the GSTF codes is by definition, $N_tN_r(L+1)$. In this case the diversity for both the codes is equal to $2\times 1\times 2\ = 4$.

Example 4: Let us now consider three different GSTF codes of diversity order $N_t \times N_r \times (L+1) = 2\times 1\times 3 = 6$ derived respectively from length 6 cyclic code over

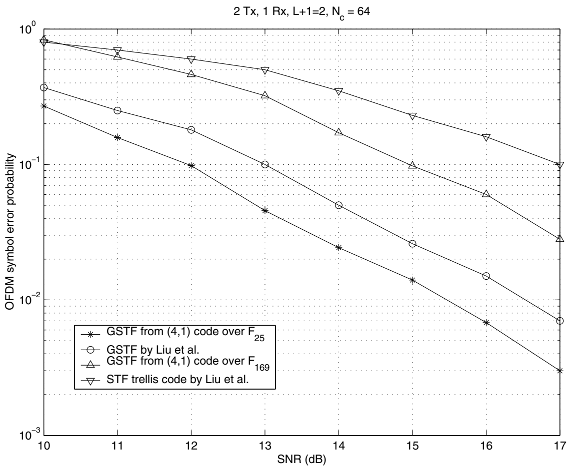

Fig. 2. Performance comparison of GSTF code derived from reduced length $(4,1)$ code over F_{5^2} , and the GSTF code derived from reduced length $(4,1)$ code over F_{13^2} with the STF Block code and the STF trellis code proposed by Liu, Xin and Giannakis.

F_{5^2}, $\mathbf{C}_3$, the length 6 code over F_{13^2}, $\mathbf{C}_4$ and the length 6 cyclic code over F_{17^2}, $\mathbf{C}_5$. (We will assume in the following two examples that the channel impulse response length $L+1 = 3$ and $N_c = 63$). The length 6 code over F_{13}^2, $\mathbf{C}_4$ is derived from the cyclic code of length 7 over F_{13^2} by dropping the last column of every codeword. Let us denote by $\mathcal{C}_3,\ \mathcal{C}_4,\ \mathcal{C}_5$ respectively the STBCs derived from $\mathbf{C}_3,\ \mathbf{C}_4,\ \mathbf{C}_5$. The codewords of $\mathcal{C}_3$ are of the form given in (11), where $a_0,\ a_1 \in F_5,\ \xi : F_5 \mapsto G_{1+2i}$. Let us first consider the GSTF derived from $\mathcal{C}_3$. This is characterized by, $N_t = 2,\ N_r = 1,\ N_g = \frac{N_c}{(L+1)} = \frac{63}{3} = 21,\ N_x = 2$. Therefore each GSTF symbol is a (2×2) matrix over F_5. Each GSTF codeword consists of three GSTF symbols.

We first pick up a set of 21 codewords from $\mathcal{C}_3$ randomly. Let the codewords of $\mathcal{C}_3$ be indexed by parameter $k,\ 0 \le k \le 20$. (Codeword k of $\mathcal{C}_3$ is represented by $\mathbf{c}_3(k)$). Let $\mathbf{c}_3(k)$ denote the k^{th} codeword of $\mathcal{C}_3$. We represent $\mathbf{c}_3(k)$ as,

$$\mathbf{c}_3(k) = \begin{bmatrix} x_{11}(k) & x_{12}(k) & x_{13}(k) & x_{14}(k) & x_{15}(k) & x_{16}(k) \\ x_{21}(k) & x_{22}(k) & x_{23}(k) & x_{24}(k) & x_{25}(k) & x_{26}(k) \end{bmatrix}$$

Codeword $\mathbf{c}_3(k)$ is split into three GSTF symbols $\mathbf{X}(k)$, $\mathbf{X}(k+21), \mathbf{X}(k+42)$ in the following manner,

$$\mathbf{X}(k) = \begin{bmatrix} x_{11}(k) & x_{12}(k) \\ x_{21}(k) & x_{22}(k) \end{bmatrix}$$

$$\begin{bmatrix} \xi(a_0) & \xi(4a_0+a_1) & \xi(3a_0+a_1) & \xi(4a_0) & \xi(a_0+4a_1) & \xi(2a_0+4a_1) \\ \xi(a_1) & \xi(2a_0+2a_1) & \xi(2a_0+a_1) & \xi(4a_1) & \xi(3a_0+3a_1) & \xi(3a_0+4a_1) \end{bmatrix} \quad (11)$$

$$\begin{bmatrix} \xi(a_0) & \xi(3a_0+5a_1) & \xi(4a_0+11a_1) & \xi(7a_0+a_1) & \xi(a_0+12a_1) & \xi(3a_0+2a_1) \\ \xi(a_1) & \xi(9a_0+7a_1) & \xi(a_0+3a_1) & \xi(6a_0+a_1) & \xi(7a_0+7a_1) & \xi(12a_0+4a_1) \end{bmatrix} \quad (12)$$

$$\begin{bmatrix} \xi(a_0) & \xi(15a_0+12a_1) & \xi(14a_0+8a_1) & \xi(16a_0+5a_1) & \xi(2a_0+a_1) & \xi(3a_0+2a_1) \\ \xi(a_1) & \xi(12a_0+7a_1) & \xi(12_0+7a_1) & \xi(3a_1) & \xi(5a_0+a_1) & \xi(5a_0+4a_1) \end{bmatrix} \quad (13)$$

$$\mathbf{X}(k+21) = \begin{bmatrix} x_{13}(k) & x_{14}(k) \\ x_{23}(k) & x_{24}(k) \end{bmatrix}$$
$$\mathbf{X}(k+42) = \begin{bmatrix} x_{15}(k) & x_{16}(k) \\ x_{25}(k) & x_{26}(k) \end{bmatrix}$$

Symbol $\mathbf{X}(k)$ is conveyed on subcarrier k, symbol $\mathbf{X}(k+21)$ is conveyed on subcarrier $(k+21)$ and symbol $\mathbf{X}(k+42)$ is conveyed on subcarrier $(k+42)$. Simulation proceeds by repeatedly selecting 21 codewords of $\mathcal{C}_3$ at random, obtaining 63 GSTF symbols from them using the outlined procedure and transmitting these symbols on 63 carrier frequencies. for $k = 0, 1, 2, \ldots, 20$, $\mathbf{W}$ is a 3×2 matrix representing the noise terms. The entries of $\mathbf{W}$ are independent complex Gaussian with zero mean and unit variance. ML decoding has been performed and the probability of GSTF codeword error has been plotted as a function of signal to noise ratio (SNR) in Fig. 3. This GSTF code has a rate equal to $\frac{log_2(25)}{3\times 2} = 0.774\ bits/sec/Hz$.

Example 5: Let us now consider the GSTF code derived from $\mathcal{C}_4$. The codewords of $\mathcal{C}_4$ are of the form given in (12) where $a_0, a_1 \in F_{13}$, $\xi : F_{13} \mapsto G_{2+3i}$. The corresponding GSTF code is characterized by parameters $N_t = 2$, $N_r = 1$, $N_g = 21$. We have carried out simulations for the GSTF code derived from this design in a similar manner as in Example 4 and have plotted the results in Fig. 3. This GSTF code has a rate of $\frac{log_2(169)}{3\times 2} = 1.233\ bits/sec/Hz$.

Example 6: Let us now consider the GSTF code derived from $\mathcal{C}_5$. This code has 289 codewords of the form given in (13), where $a_0, a_1 \in F_{17}$. $\xi : F_{17} \mapsto G_{4+i}$. The corresponding GSTF code is characterized by parameters $N_t = 2$, $N_r = 1$, $N_g = 21$, We have carried out simulations for the GSTF code derived from this design in a similar manner as in Example 4 and have plotted the results in Fig. 3. This GSTF code has a rate of $1.3626\ bits/sec/Hz$.

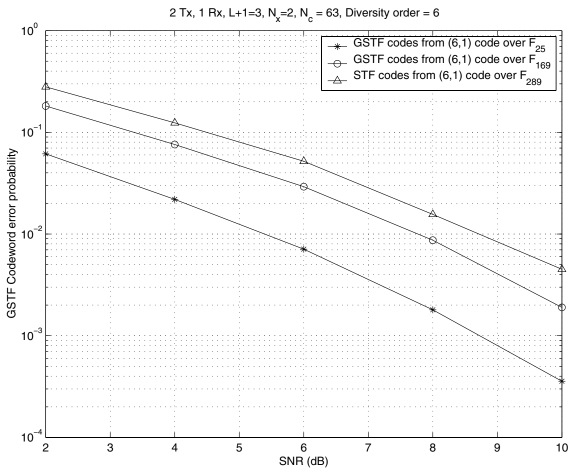

Fig. 3. Performance comparison of GSTF code derived from $(6, 1)$ cyclic code over F_{5^2} with GSTF code derived from $(6, 1)$ code over F_{13^2} and GSTF code derived from $(6, 1)$ cyclic code over F_{17^2}.

ACKNOWLEDGMENTS

This work was partly funded by the DRDO-IISc Program on Advanced Research in Mathematical Engineering and also by the Council of Scientific and Industrial Research (CSIR), India, through research grant (22(03065)/04/EMR-II) to B.S.Rajan.

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