Mixed Sensitivity H_2/H_∞ Control of a Flexible-Link Robotic Arm

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***Abstract*— Dynamics of multi-link manipulators with flexible links include complex high-order equations which make their control problem very challenging. This paper presents a new method for simultaneous motion and vibration control of a two-link flexible manipulator using H_2/H_∞ control. A multi-element finite element model of the manipulator is casted into the generalized plant model, and H_2 and H_∞ controllers are synthesised employing the LMI-based optimization algorithm of MATLAB. Finally, a mixed sensitivity H_2/H_∞ control is proposed based on an H_2 norm constrained H_∞ control design. It is shown that the method of control design can be used successfully for the control of the joint parameters as well as suppression of vibration at the tip (end-effector) of the manipulator.**

***Index Term*-- Flexible manipulators; motion and vibration control ; mixed H_2/H_∞ control ; finite element modelling**

I. INTRODUCTION

Flexible-link manipulators (FLMs) originally were developed to achieve advantages such as lightness, high payload capacity, accessibility to wider workspaces, and so on. These advantages are of particular importance in space robots [1]. In order to achieve the advantages, robotic arms can be designed with long and slender links; which in turn introduce flexibility to the system due to the elastic behavior such as bending and torsion of the links. Abundant research has been done to cope with modeling and control of the FLMs; and it is yet an open field.

Many methods have been proposed for modelling the dynamics of FLMs. Some famous examples include the assumed mode method (AMM), finite element method (FEM), perturbation methods, and Ritz expansion. Some researchers have reported studies on the dynamics of FLMs, without proposing a controller for the manipulator; while others propose methods for control, because most researchers are interested in models that can be used in control design [2]. Dwivedy and Eberhard [3] represented modeling of a two-link FLM using AMM with four modes. Supriyono and Tokhi [4] proposed a model of a single-link FLM based on biologically-inspired optimization technique of bacterial foraging algorithms.

The dynamics of a FLM has challenging complexities for control engineers. Many research were carried out attacking the complexities in the control of the FLMs. Chapnik, *et al.* [5] proposed an open-loop control system using frequency-domain techniques to compute a desired hub torque profile. Khorrami, *et al.* [6] studied a feedback linearization method for control of a two-link FLM. Bai, *et al.* [7] studied identification of friction in a two-link FLM. An inversion based controller that cancels the effect of the unstable zeros in a single-link FLM was presented in [8]. Cole and Wongratanaphisan [9], suggested an adaptive method for feed-forward control of a two-link FLM to achieve zero residual vibration in rest-to-rest motion. Shawky *et al.* [10], studied nonlinear control using end-point position feedback for a single-link FLM. Feliu *et al.* [11], studied passivity-based control of a single-link FLM considering robustness against payload variations and friction of the joints.

Perhaps the most important control problem in the FLMs is vibration suppression during or after a maneuver of the manipulator. In order to control the vibrations, we will need to have a standard norm that shows the severity of vibration. Depending on how such 'norm' is defined, the objectives and methods of control synthesis will vary. Two common tools to quantify vibration levels are the H_2 and H_∞ norms. Then, a controller that minimizes the H_2 (H_∞) norm of an output signal is known as H_2 (H_∞) controller. It is known that noise or random disturbances are more naturally expressed in H_2 or RMS terms. An LQR/LQG controller in fact attacks the H_2 performance aspects, and therefore is a good candidate for controlling the seemingly random vibrations of flexible systems. Konno and Uchiyama [12] studied LQR control of a two-link manipulator. Milford and Asokanthan [13] also used an LQG controller.

Due to uncertainties of the dynamic model, robust control of the FLMs has received particular attention in the literature. Along with various robust control methodologies, H_∞ control provides methods to deal with the stability and performance robustness. As some examples, Hisseine and Lohmann [14] considered sliding mode as well as nonlinear H_∞ control of a single-link FLM. Ming-Tzu and Yi-Wei [15], employed the H_∞ framework for achieving good performance of PID control for a single-link FLM.

In practical applications, sometimes standard H_∞ synthesis methods are not adequate to capture all design specifications. Therefore imposing an additional H_2 performance requirement to the H_∞ synthesis may include the advantages of an LQG design. Likewise, including an H_∞ performance requirement can improve LQG design [16]. Banavar [17] added an H_∞ controller to an LQG to improve stability of the LQG control of a single-link FLM. [18] designed a mixed-sensitivity H_∞ controller for a single-link FLM including gravity terms.

The Mixed-Sensitivity H_2/H_∞ Control has been used successfully for various control engineering applications. Safonov, *et al.* [19] used a mixed-sensitivity H_∞ control for robust control of a large scale space structure. Ohishi, *et al.* [20] proposed a force control technique for a manipulator realized by an acceleration controller and a force observer, both designed by the mixed-sensitivity H_∞ design method. Toker and Ozbay [21] presented a method for H_∞ optimal and suboptimal controllers for a class of infinite-dimensional distributed SISO systems. Chaudhuri, *et al.* [22] designed a decentralized H_∞ damping control based on the mixed-sensitivity formulation in the LMI framework for power system damping control problem. Khosrowjerdi, *et al.* [23] formulated the problem of simultaneous fault detection and control as a mixed H_2/H_∞ optimization problem and proposed a solution using Riccati equations. Sheng, *et al.* [24] used an H_∞-based mixed sensitivity analysis method for improving the dynamics and robustness of a flexible plate. Jingjun, *et al.* [25] simulated active vibration control of a cantilevered beam. Yi, *et al.* [26] applied the mixed-sensitivity method on a pneumatic actuator. Guo, *et al.* [27] used mixed-H_∞ norm sensitivity minimization for designing insensitive output feedback controllers for linear continuous-time systems.

In this work, the problem of H_2/H_∞ control design for a planar two-link manipulator with flexible links is investigated. For this aim, first two controllers, *i.e.* one H_2 and one H_∞ controller, are designed and compared in terms of speed (settling time) and level of vibration of the tip of the manipulator. Then, the final mixed H_2/H_∞ controller is achieved by solving the H_∞ optimization problem under H_2 constrains. The model used in this work is a multi-element FEM model of the manipulator. The dynamic equations of FEM models with multiple elements have big matrices which make the feedback control design more complex. The numerical method for H_2/H_∞ control design using linear matrix inequality LMI which is provided in MATLAB, was employed successfully for the control design.

The remainder of this paper is organized as follow. First, in section 2, the model of the two-link FLM is introduced. In section3, the methodology of the H_2/H_∞ LMI optimization is represented. Section 4 is devoted to H_2 control design which will be used as the first cut or reference to evaluate the performance of control system in terms of suppression of vibration. Then in section 5, an H_∞ controller is presented. In section 6 the mixed H_2/H_∞ controller is proposed to make a trade-off between the advantages of the H_2 and the H_∞ control. Finally, the conclusions will be summarized.

II. The Model of the Flexible Manipulator

In this paper, similar to [28], FEM is used for modeling the two-link FLM. For this reason each link is divided into ten Euler-Bernoulli beam elements. The FEM model of the manipulator is shown in Figure 1. Denoting the degrees of freedom of each node i by v_i , φ_i , which show the linear and angular displacements of the node; and showing the joint angles with θ_1, θ_2, the vector of generalized coordinates can be written as

$$\vec{q} = \{q_1, q_2, q_3, ..., q_n\}^T = \underbrace{\{\theta_1, v_1, \varphi_1, v_2, \varphi_2, ..., v_{10}, \varphi_{10}}_{coordinates\ of\ the\ upper\ arm} \Big| \underbrace{\theta_2, v_1, \varphi_1, v_2, \varphi_2, ..., v_{10}, \varphi_{10}\}^T}_{coordinates\ of\ the\ forearm} \quad (1)$$

The superscript ^{T} stands for transpose of the vector. The partitioning line in equation (1) separates the coordinates of the first and the second links.

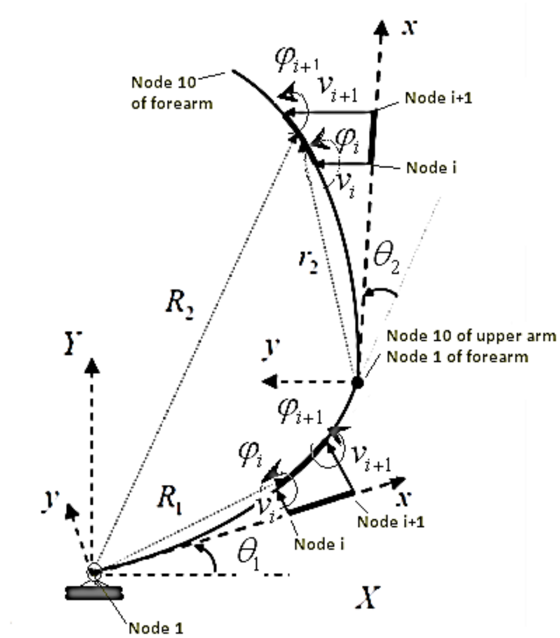

Fig. 1. Model of the flexible arm

In order to derive the dynamic equations, the potential and kinetic energy of the system is measured by summation of energies of all the Euler-Bernoulli elements. For this aim, the elemental values of the kinetic and strain energies of each element are measured using integration over Hermite shape functions. The kinetic energy will be then

$$T = \sum_{Link1} \frac{1}{2} \int_{element} \mu \dot{\vec{R}}_1 . \dot{\vec{R}}_1 dx + \sum_{Link2} \frac{1}{2} \int_{element} \mu \dot{\vec{R}}_2 . \dot{\vec{R}}_2 dx \quad (2)$$

where μ is the mass per unit length, and $\dot{\vec{R}}_1$, and $\dot{\vec{R}}_2$ are the time derivatives of the vector of position of a particle on the first and the second link, as shown in Figure 1. Next, supposing EI_1, and EI_2 are the flexural rigidity of the first and the second link, the potential energy of the system can be obtained as

$$U = \sum_{Link1} \frac{1}{2} \int_{element} EI_1 \left(\frac{\partial^2 v}{\partial x^2} \right) dx + \sum_{Link2} \frac{1}{2} \int_{element} EI_2 \left(\frac{\partial^2 v}{\partial x^2} \right) dx \quad (3)$$

In order to fulfill the integration, according to the Euler-Bernoulli beam theory, a matrix of Hermite shape functions, $N(x)$, is adopted which relates the continuous function $v(x)$ of an arbitrary element i to the nodal coordinates as follow

$$v(x,t) = [N(x)]\{v_i, \varphi_i, v_{i+1}, \varphi_{i+1}\}^T \quad (4)$$

Now, using equation (4) in equations (2) and (3), and considering the physical parameters of the system as given in

Table I, the overall energies are obtained in terms of the generalized coordinate vector of equation (1).

TABLE I.
PHYSICAL PARAMETERS OF THE MANIPULATOR

Parameter	***Upper arm***	***Forearm***	***Unit***
Length	300	300	mm
Thickness	1.5	1	mm
Width	30	25	mm
Mas per unit length	1.993	1.107	g/cm
Elasticity Modulus	113.8	113.8	GPa
Initial angle	$\pi/6$	$\pi/3$	Rad

Defining the inertia and stiffness matrices *M* and *K*, the overall kinetic and potential energies can be rearranged in a matrix form as

$$T=\frac{1}{2}\dot{\vec{q}}^{T}M\dot{\vec{q}} \quad ; \quad U=\frac{1}{2}\vec{q}^{T}K\vec{q} \tag{5}$$

Having measured the system energies, the Lagrange's equations may be used to yield the relation between $k=1,2,3,\ldots,n$ generalized coordinates and the generalized forces Q_k as

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_k}\right)-\frac{\partial L}{\partial q_k}=Q_k \tag{6}$$

In which $L=T-U$ is the Lagrangian of the system. With a '*n* by *2'* matrix *F* known as the input matrix given by

$$\sum_{k=1}^{n} Q_k.\delta q_k = \delta W_{nc} = \sum_{k=1}^{n} (F_{k1}\tau_1 + F_{k2}\tau_2).\delta q_k \tag{7}$$

The generalized forces Q_k are related to the virtual work done by non-conservative forces and the torques applied by the first and the second motors (τ_1, τ_2). Then, the linearized form of the dynamic equations is represented in the following matrix equation

$$M\,\ddot{\vec{q}}+K\,\vec{q}=F\,\{\tau_1,\tau_2\}^T \tag{8}$$

Equation (8) needs to be supplemented with the relevant boundary conditions (BCs). For the manipulator, the BCs include pinned BC at the joints, and free BC at the tip. Therefore, for the first node of each link just rotational DOF are possible; and so some of the columns and rows of *M*, *K*, and *F* can be eliminated, and the matrices are reduced to $\tilde{M}$, $\tilde{K}$, and $\tilde{F}$ and equation (8) is rewritten as follow

$$\tilde{M}\ddot{\vec{q}}+\tilde{K}\vec{q}=\tilde{F}\{\tau_1,\tau_2\}^T \tag{9}$$

Then, with zero and identity matrices *O* and *I*, a state space representation of the system is obtained as

$$\dot{X}=AX+Bu,$$
$$X=\begin{Bmatrix}\vec{q}\\ \dot{\vec{q}}\end{Bmatrix},\ A=\begin{bmatrix}O & I\\ -\tilde{M}^{-1}\tilde{K} & O\end{bmatrix},\ B=\begin{bmatrix}O\\ \tilde{M}^{-1}\tilde{F}\end{bmatrix},\ u=\begin{Bmatrix}\tau_1\\ \tau_2\end{Bmatrix} \tag{10}$$

The transfer matrix of the system is also achieved by selecting the proper output vector of the joint angles and displacement of the tip. Then, the system is represented as

$$G=\begin{bmatrix}G_{11} & G_{12}\\ G_{21} & G_{22}\\ G_{31} & G_{32}\end{bmatrix}$$

so that

$$\begin{Bmatrix}\theta_1\\ \theta_2\\ v_{Tip}\end{Bmatrix}=\begin{bmatrix}G_{11} & G_{12}\\ G_{21} & G_{22}\\ G_{31} & G_{32}\end{bmatrix}\begin{Bmatrix}\tau_1\\ \tau_2\end{Bmatrix} \tag{11}$$

where θ_1 and θ_2 are angular motion of the shoulder and elbow joints, and v_{Tip} is the deflection of the tip.

III. THE METHOD OF H_2/H_∞ CONTROLLER DEIGN USING LMI SOLVER

The method of mixed H_2/H_∞ synthesis using Linear Matrix Inequality (LMI) [29] (also developed in [30], and [31]) performs multi-objective output-feedback synthesis to design a suboptimal LTI controller *K*(*s*) that minimizes a mixed H_2/H_∞ criterion using convex optimization. First, the system equation is rearranged in the form of a generalized plant, *P(s)*, with state vector *x*, exogenous input *w*, and control input *u* as follow

$$P(s):\begin{cases}\dot{x}=Ax+B_1w+B_2u\\ z_\infty=C_\infty x+D_{\infty1}w+D_{\infty2}u\\ z_2=C_2x+D_{21}w+D_{22}u\\ y=C_yx+D_{y1}w\end{cases} \tag{12}$$

Here y is the feedback signal, and z_2 and z_∞ are the output signals used as performance index. The matrices *A*, *B*, *C*, and *D* are the system matrices. Then, a linear controller is considered as

$$K(s):\begin{cases}\dot{\zeta}=A_K\zeta+B_Ky\\ u=C_K\zeta+D_Ky\end{cases} \tag{13}$$

Note that *K(s)* has the measured outputs of the plant as its input, and the input vector of the plant as its output. The corresponding closed-loop system can be measured and rearranged in the form of an LTI system as

$$\begin{cases} \dot{x}_{cl} = A_{cl}x_{cl} + B_{cl}w \\ z_\infty = C_{cl1}x + D_{cl1}w \\ z_2 = C_{cl2}x + D_{cl2}w \end{cases} \tag{14}$$

The general form of the closed-loop system is sketched in Figure (2)

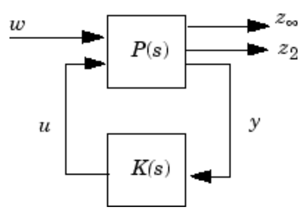

Fig. 2. General representation of the control problem

Note that with this configuration, the system has two output and one exogenous input vector. Therefore, two transfer functions are required to relate the outputs to the input. The closed-loop transfer functions from w to z_∞ and z_2 are denoted by $T_\infty(s)$ and $T_2(s)$ respectively.

That is $\{z_\infty, z_2\}^T = [T_\infty, T_2]^T \{w\}$. Then, denoting by $\|.\|_\infty$ and $\|.\|_2$ the H_∞ norm, and H_2 norm of the transfer functions, the control objective of the H_2/H_∞ is to minimize γ, ν so that

$$\|T_\infty\|_\infty < \gamma, \tag{15 a}$$

and

$$\|T_2\|_2 < \nu. \tag{15 b}$$

The mixed H_2/H_∞ method [31], seeks for a common Lyapunov matrix $\chi = \chi_2 = \chi_\infty$ which satisfies a set of the multiple constraints using convex optimization of an LMI. The method relies on two theorems; the first stating that the closed-loop RMS gain from w to z_∞ does not exceed γ, if and only if there exist a symmetric positive definite matrix $\chi_\infty > 0$ such that

$$\begin{pmatrix} A_{cl}\chi_\infty + \chi_\infty A_{cl}^T & B_{cl} & \chi_\infty C_{cl1}^T \\ B_{cl}^T & -I & D_{cl1}^T \\ C_{cl1}\chi_\infty & D_{cl1} & -\gamma^2 I \end{pmatrix} < 0 \tag{16}$$

The second theorem, for satisfying the H_2 performance requirement, states the transfer function from w to z_2 does not exceed ν if and only if $D_{cl2}=0$ and there exist symmetric matrices χ_2 and Q such that

$$\begin{pmatrix} A_{cl}\chi_2 + \chi_2 A_{cl}^T & B_{cl} \\ B_{cl}^T & -I \end{pmatrix} < 0 \tag{17}$$

$$\begin{pmatrix} Q & C_{cl2}\chi_2 \\ \chi_2 C_{cl2}^T & \chi_2 \end{pmatrix} < 0 \tag{18}$$

$$trace(Q) < \nu^2 \tag{19}$$

In this work, first one H_2 control and one H_∞ control are designed using the algorithm. Then, the final mixed H_2/H_∞ controller is achieved by solving a constrained H_∞ optimization problem to make a trade-off between the H_2 and the H_∞ control.

The architecture of the control system and the outputs considered for optimization is shown in Figure 3. In order to conform to the standard format given in Figure 2, the configuration of the controlled system is rearranged to the linear fractional transformation (LFT) as shown in Figure 4. These figures will be referred to in the next sections, to show the performance criterion.

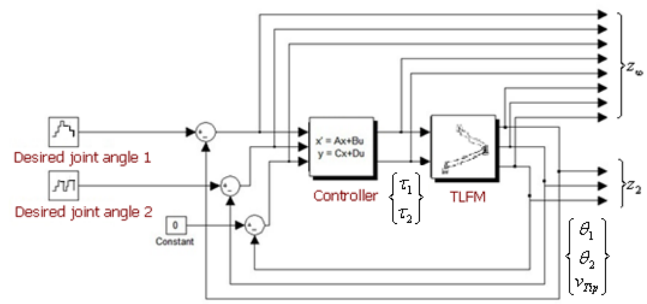

Fig. 3. Mixed sensitivity control configuration

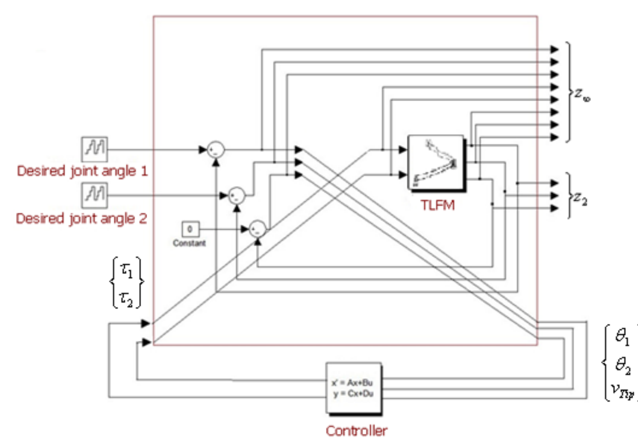

Fig. 4. Rearrangement of the feedback controlled system as LFT

IV. H_2 OPTIMAL DESIGN

One measure for vibration level is provided by the H_2 norm. The H_2 norm of a stable continuous system is related to the root-mean-square (RMS) of its impulse response. The H_2 norm also represents the steady-state covariance (or power) of the output response to unit white noise inputs. Therefore the objective of vibration suppression can be translated to minimizing the H_2 norm of the output signals of a mechanical system. The controller design method with the objective of minimizing a H_2 norm is known as the H_2 optimal design. In this section an H_2 control design is performed for the two-link FLM. The design will be used as a reference for evaluation of the next H_2/H_∞ controllers.

Let T_2 be the transfer function from the vector of desired joint angles to the output vector, including the actual joint angles as well as the transversal displacement of the tip due to elastic deformation.

$$\begin{Bmatrix} \theta_1 \\ \theta_2 \\ v_{Tip} \end{Bmatrix} = T_2 \begin{Bmatrix} \theta_1^{desired} \\ \theta_2^{desired} \end{Bmatrix} \tag{20}$$

The H_2 controller is designed targeting at the following optimization problem

$$Minimize: \; \nu_0 = \|T_2\|_2 \tag{21}$$

The step response of the closed-loop system is shown in Figure 5. As the system has two inputs, two unit step commands are applied separately; and the response of three outputs are plotted. Note that the time range of the third output, that is the normal acceleration of the tip due to deflection, is different. The simulation result shows that the response of the joint angles is very slow; and the overshoot is more than 20%. However, the vibration of the tip of the manipulator has a very small value, in better words an optimal value in the sense of H_2 norm. The achieved optimal value for Equation 21 was: $\nu_0 = 0.3930$. This value will be used in next sections, as a reference point in our H_2/H_∞ controller design.

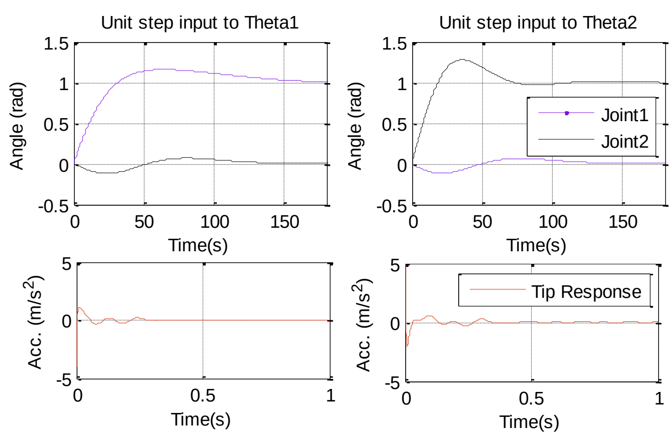

Fig. 5. Step response of the H_2 optimal design

V. THE MIXED SENSITIVITY H_∞ OPTIMAL DESIGN

In addition to the H_2 norm, a well-known standard measure for evaluation of vibration levels is provided by the H_∞ norm which can be thought of as a measure for the peak amplitude of the FRF of the system. This norm is even more common in vibration measurement and control as it shows the resonant characteristics of vibration modes. The controller design methodology targeting at the minimization of the H_∞ norm is known as H_∞ optimization design.

As mentioned before, different performance requirements of the controlled system can be translated to conditions on the H_∞ norm of the S, T, and KS, where S and T are the sensitivity and complementary sensitivity transfer matrices, and K is the controller. For example good tracking, noise attenuation and robust stability (with respect to multiplicative output uncertainties) can be achieved by small $T(s)$. On the other hand, the performance of the system in terms of command tracking and disturbance attenuation can be translated to requirements for $S(s)$ since it relates the error signals with references and disturbances. Additionally, the optimization of the control effort within a limited bandwidth (constraining actuator saturation) is equivalent to minimizing $KS(s)$. Thus, all in all, the optimization problem is summarized as a mixed-sensitivity synthesis aiming at minimization of the H_∞ norm of

$$T_\infty = \begin{Bmatrix} w_1 S \\ w_2 KS \\ w_3 T \end{Bmatrix} \tag{22}$$

With weighting filters $w_{1,2,3}$, this objective is known as the weighted *S/KS/T* (read S over KS over T) mixed-sensitivity. In this section the pure H_∞ optimal design of the controller is considered. The controller is designed to

$$Minimize: \left\| \begin{matrix} S \\ K\,S \\ T \end{matrix} \right\|_\infty \tag{23}$$

The step response of the resulting closed-loop system with the H_∞ controller is represented in Figure 6. It is obsered that the H_∞ provides better performance in terms of the rise time and overshoot of the step response, compared with the H_2 controller.

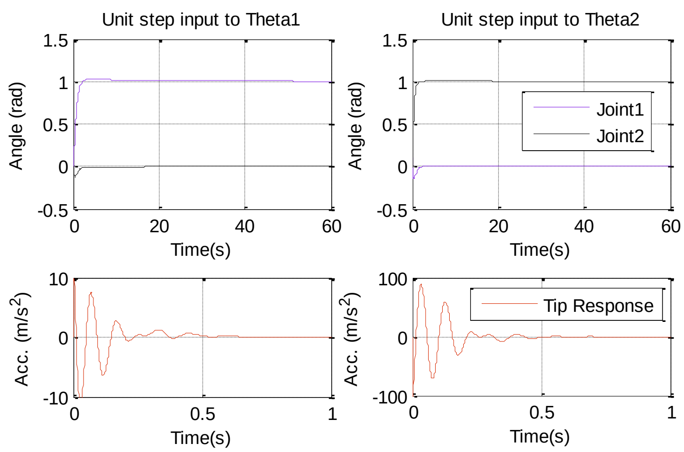

Fig. 6. Step response of the H_∞ optimal design

Comparison of Figure 5 with Figure 6 reveals that the H_2 control results in less vibration at the tip. Therefore, it is inferred that including an H_2 condition in the H_∞ norm optimization may provide better vibration suppression. The method of designing controllers with minimizing both the H_2 and H_∞ norms are known as mixed H_2/H_∞ design.

VI. The Mixed H_2/H_∞ Design

In this section, a mixed H_2/H_∞ design is proposed through the following optimization problem:

$$\textit{Minimize:}\quad \gamma = \left\| \begin{matrix} S \\ KS \\ T \end{matrix} \right\|_\infty , \qquad (24)$$

$$\textit{Subject to:}\quad \|T\|_2 \le n\,\nu_0 .$$

The design parameter n is used as a tool for tuning the controller. By increasing n the controller performs similar to the H_∞ controller; and decreasing n will result a performance like the H_2 controller. Recall that H_2 controller was slower but had less vibration. Figure 7 shows the response of the controlled system with different values of n. The results show that with increasing the n, the response of the joint angles become more desirable. Particularly in an approximate range of $2<n<10$ the mixed control method is effective. With higher values of n, more overshoot and more vibration of the end-effector is resulted. Therefore, the final design is selected with $n=5$. Figure 8 show the step response of the closed-loop system with the final controller.

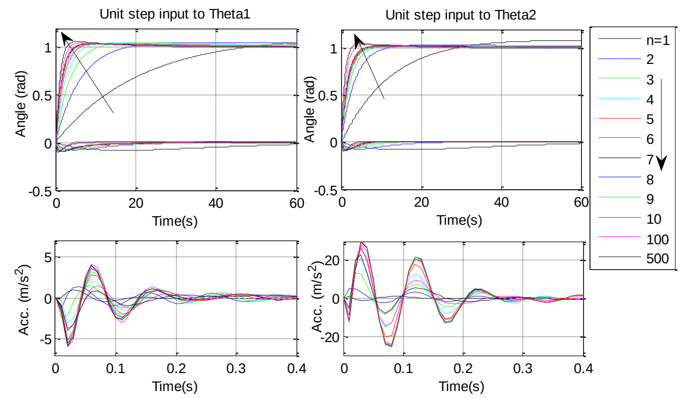

Fig. 7. Step response of the mixed H_2/H_∞ design

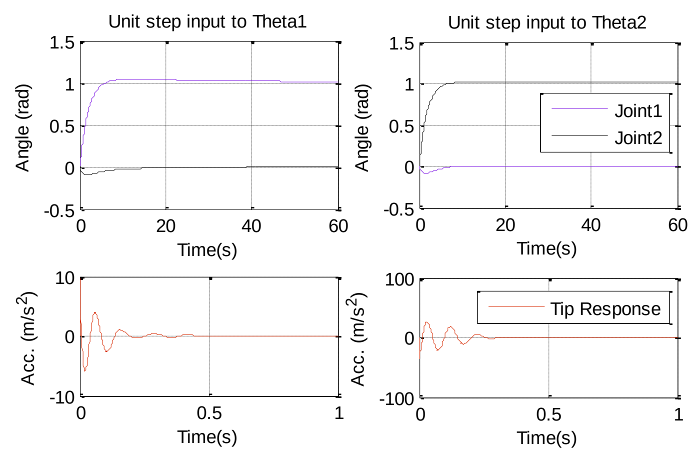

Fig. 8. Step response of the mixed H_2/H_∞ design (with $n=5$)

VII. Conclusion

In this paper a set of H_2/H_∞ controllers was developed for simultaneous motion and vibration control of a two-link manipulator with flexible links. A multi-element FEM model of the manipulator was considered and the control problem was casted into standard configuration of linear fractional transformation. The simulation results showed that although the response of the closed-loop system under the H_2 control was very slow compared with that of the H_∞ controller; the H_2 controller was more successful in vibration suppression. It was inferred that attacking at the H_2 norm is very effective in vibration control of the manipulator. Therefore, a mixed H_2/H_∞ control was designed to include the advantage of the H_2 control in the H_∞ design. It was shown that the mixed H_2/H_∞ method provides a trade-off among the advantages of the H_2 and H_∞ control for the complex dynamic system.

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