

# Uncertain Currency Model and Currency Option Pricing

Yuhan Liu<sup>1</sup>, Xiaowei Chen<sup>2</sup> and Dan A. Ralescu<sup>1\*</sup>

<sup>1</sup>Department of Mathematical Sciences, University of Cincinnati, Cincinnati, OH 45221-00025, USA

<sup>2</sup>Department of Risk Management and Insurance, Nankai University, Tianjin 300071, China

\*Corresponding author: dan.alescu@math.uc.edu

**Abstract:** Liu process is a new tool to deal with the noise process based on uncertainty theory. In this paper, we view the foreign exchange rate as an uncertain process, described by an uncertain differential equation driven by Liu process, and build an uncertain currency model. Then the uncertain currency option processes are discussed. Moreover, European and American currency option pricing formulas are derived for the proposed uncertain currency model and some mathematical properties are studied. Finally, two numerical examples are described.

**keywords:** Finance, foreign currency, uncertainty theory, uncertain differential equation, option pricing

## 1 Introduction

In the early 1970s, Black and Scholes [1] and, independently, Merton [31] constructed a theory for determining the stock option price which resulted in the famous Black-Scholes formula. From then on, stochastic financial mathematics was founded based on probability theory. However, when we applied it large samples to obtain the distribution function are desired. There are many cases in practice when small samples or no samples at all are available to estimate a probability distribution. In such case, we have to invite some domain experts to evaluate the degree of belief that each event may occur. In order to deal with this, uncertainty theory was founded by Liu [16] and subsequently studied by several researchers. Uncertainty theory has become a branch of axiomatic mathematics for modeling human uncertainty based on normality, duality, subadditivity and product axioms. This theory has been successfully applied to many fields such as uncertain programming ([19]), uncertain inference control ([22]) and uncertain optimal control (Zhu [41]) among others.

In order to describe dynamic uncertain systems, Liu [17] introduced uncertain process which can be seen as a sequence of uncertain variables indexed by time. Moreover, Liu [18] designed a Liu process that is a Lipschitz continuous uncertain process with stationary and independent increments. Following that, uncertain calculus was initiated in [18] to deal with differentiation and integration of functions of uncertain processes. Yao [39] introduced uncertain integral based on a renewal process. In addition, Liu [17] gave a definition of uncertain differential equation. After that, Chen and Liu [4] proved an existence and uniqueness theorem for uncertain differential equation. For exploring the recent developments of uncertainty theory, the readers may consult Liu [24].

Based on the assumption that the stock price follows a geometric Liu process, uncertainty theory was first introduced into finance by Liu [18] in 2009. Furthermore, Liu [17] proposed an uncertain stock

model and derived European option price formulas. Chen [5] studied American option pricing formulas for this stock model. In addition, Peng and Yao [34] proposed a different uncertain stock model and derived option price formula. Chen, Liu and Ralescu [7] introduced an uncertain stock model with dividends and derived option pricing formulas. Besides stock models, Chen and Gao [8] investigated an uncertain interest rate model and obtained bond pricing formula. Uncertainty theory was also introduced to insurance by Liu [21] based on the assumption that the claim process is an uncertain renewal reward process. Liu [28] gave a rational analysis of why an uncertain finance theory is needed. After Yao-Chen formula (Yao and Chen [38]) was proposed, the pricing models are recalculated by many researchers (see Chen [9]).

Foreign currency is an active instrument in capital market with a large percentage. Currency option is one of the most classic and useful options and one of the core content of modern finance. In this paper, an uncertain currency option model is introduced and the currency option pricing formula is derived for the proposed model and some mathematical properties are discussed. The rest of the paper is organized as follows: Some preliminary concepts of uncertain processes are recalled in Section 2. An uncertain currency model is proposed in Section 3. European and American currency option pricing formulas are derived and some properties are studied in Sections 4 and 5, respectively. Finally, a brief summary is given in Section 6.

## 2 Preliminary

An uncertain measure is a set function from a  $\sigma$ -algebra  $\mathcal{L}$  generated on  $\Gamma$  to the set of real numbers satisfying normality, duality, subadditivity and product axioms. The triplet  $(\Gamma, \mathcal{L}, \mathcal{M})$  is called an uncertainty space. An uncertain measure (Liu [16] and [18]) is a function  $\mathcal{M} : \mathcal{L} \rightarrow [0, 1]$  such that:

**Axiom 1.** (Normality Axiom)  $\mathcal{M}\{\Gamma\} = 1$  for the universal set  $\Gamma$ ;

**Axiom 2.** (Duality Axiom)  $\mathcal{M}\{\Lambda\} + \mathcal{M}\{\Lambda^c\} = 1$  for any event  $\Lambda$ ;

**Axiom 3.** (Subadditivity Axiom) For every countable sequence of events  $\Lambda_i$ , we have

$$\mathcal{M}\left\{\bigcup_{i=1}^{\infty} \Lambda_i\right\} \leq \sum_{i=1}^{\infty} \mathcal{M}\{\Lambda_i\};$$

**Axiom 4.** (Product Axiom) (Liu [18]) Let  $(\Gamma, \mathcal{L}, \mathcal{M})$  be uncertainty spaces for  $k = 1, 2, \dots, n$ . Then the product uncertain measure  $\mathcal{M}$  is an uncertain measure on the product  $\sigma$ -algebra  $\mathcal{L}_1 \times \mathcal{L}_2 \times \dots$  satisfying

$$\mathcal{M}\left\{\prod_{i=1}^{\infty} \Lambda_i\right\} = \bigwedge_{i=1}^{\infty} \mathcal{M}\{\Lambda_i\}.$$

**Definition 1.** (Liu [16]) An uncertain variable  $\xi$  is a measurable function from an uncertainty space  $(\Gamma, \mathcal{L}, \mathcal{M})$  to the set of real numbers, i.e., for any Borel set  $B$  of real numbers, the set

$$\{\xi \in B\} = \{\gamma \in \Gamma | \xi(\gamma) \in B\}$$

is an event.

The uncertainty distribution  $\Phi : \Re \rightarrow [0, 1]$  of an uncertain variable  $\xi$  is defined as  $\Phi(x) = \mathcal{M}\{\xi \leq x\}$ . The expected value of an uncertain variable is defined as follows.

**Definition 2.** (Liu [24]) An uncertainty distribution  $\Phi(x)$  is said to be regular if it is a continuous and strictly increasing function with respect to  $x$  at which  $0 < \Phi(x) < 1$ , and

$$\lim_{x \rightarrow -\infty} \Phi(x) = 0, \quad \lim_{x \rightarrow \infty} \Phi(x) = 1.$$

**Definition 3.** (Liu [24]) Let  $\xi$  be an uncertain variable with regular uncertainty distribution  $\Phi(x)$ . Then the inverse function  $\Phi^{-1}(\alpha)$  is called the inverse uncertainty distribution of  $\xi$ .

**Definition 4.** (Liu [16]) Let  $\xi$  be an uncertain variable. Then the expected value of  $\xi$  is defined by

$$E[\xi] = \int_0^{+\infty} \mathcal{M}\{\xi \geq r\} dr - \int_{-\infty}^0 \mathcal{M}\{\xi \leq r\} dr$$

provided that at least one of the two integrals is finite.

The expected value can also be written as

$$E[\xi] = \int_0^{+\infty} (1 - \Phi(r)) dr - \int_{-\infty}^0 \Phi(r) dr$$

where  $\Phi(r)$  is the uncertainty distribution of  $\xi$ . Suppose that  $\xi$  has a regular uncertainty distribution with an inverse uncertainty distribution  $\Phi^{-1}(\alpha)$ . Then the expected value is

$$E[\xi] = \int_0^1 \Phi^{-1}(\alpha) d\alpha.$$

If  $\xi$  is an uncertain variable with finite expected value  $e$ , then the variance of  $\xi$  is defined as  $Var[\xi] = E[(\xi - e)^2]$ .

Let  $\xi_1, \xi_2, \dots, \xi_n$  be independent uncertain variables with uncertainty distributions  $\Phi_1, \Phi_2, \dots, \Phi_n$ , respectively. Liu [24] showed that if the function  $f(x_1, x_2, \dots, x_n)$  is strictly increasing with respect to  $x_1, x_2, \dots, x_m$  and strictly decreasing with respect to  $x_{m+1}, x_{m+2}, \dots, x_n$ , then

$$\xi = f(\xi_1, \xi_2, \dots, \xi_n)$$

is an uncertain variable with inverse uncertainty distribution

$$\Psi^{-1}(\alpha) = f(\Phi_1^{-1}(\alpha), \dots, \Phi_m^{-1}(\alpha), \Phi_{m+1}^{-1}(1 - \alpha), \dots, \Phi_n^{-1}(1 - \alpha)). \quad (1)$$

Furthermore, Liu and Ha [29] proved that  $\xi$  has an expected value

$$E[\xi] = \int_0^1 f(\Phi_1^{-1}(\alpha), \dots, \Phi_m^{-1}(\alpha), \Phi_{m+1}^{-1}(1 - \alpha), \dots, \Phi_n^{-1}(1 - \alpha)) d\alpha. \quad (2)$$

An uncertain process is essentially a sequence of uncertain variables indexed by time or space. The study of uncertain process was started by Liu [17] in 2008.

**Definition 5.** (Liu [17]) Let  $T$  be an index set and let  $(\Gamma, \mathcal{L}, \mathcal{M})$  be an uncertainty space. An uncertain process  $X_t(\gamma)$  is a measurable function from  $T \times (\Gamma, \mathcal{L}, \mathcal{M})$  to the set of real numbers, i.e., for each  $t \in T$  and any Borel set  $B$  of real numbers, the set

$$\{X_t \in B\} = \{\gamma \in \Gamma | X_t(\gamma) \in B\}$$

is an event.

An uncertain process  $X_t$  is said to have independent increments if

$$X_{t_0}, X_{t_1} - X_{t_0}, X_{t_2} - X_{t_1}, \dots, X_{t_k} - X_{t_{k-1}}$$

are independent uncertain variables where  $t_0$  is the initial time and  $t_1, t_2, \dots, t_k$  are any times with  $t_0 < t_1 < \dots < t_k$ .

**Theorem 1.** (Liu [21], Extreme Value Theorem) Let  $X_t$  be a sample-continuous independent increment process with uncertainty distribution  $\Phi_t(x)$ . Then the supremum

$$\sup_{0 \leq t \leq s} X_t$$

has an uncertainty distribution

$$\Psi_s(x) = \inf_{0 \leq t \leq s} \Phi_t(x)$$

and

$$\inf_{0 \leq t \leq s} X_t$$

has an uncertainty distribution

$$\Psi_s(x) = \sup_{0 \leq t \leq s} \Phi_t(x)$$

**Definition 6.** (Liu [18]) An uncertain process  $C_t$  is said to be a canonical Liu process if

- (i)  $C_0 = 0$  and almost all sample paths are Lipschitz continuous,
- (ii)  $C_t$  has stationary and independent increments,
- (iii) every increment  $C_{t+s} - C_s$  is a normal uncertain variable with expected value 0 and variance  $t^2$ , whose uncertainty distribution is

$$\Phi_t(x) = \left( 1 + \exp\left(-\frac{\pi x}{\sqrt{3}t}\right) \right)^{-1}, \quad x \in \mathfrak{R}.$$

If  $C_t$  is a canonical Liu process, then the uncertain process  $X_t = \exp(et + \sigma C_t)$  is called a geometric Liu process. Liu integral is a type of integral of an uncertain process with respect to a Liu process. A formal definition is given below.

**Definition 7.** (Liu [18]) Let  $X_t$  be an uncertain process and let  $C_t$  be a canonical Liu process. For any partition of the closed interval  $[a, b]$  with  $a = t_1 < t_2 < \dots < t_{m+1} = b$ , the mesh is written as

$$\Delta = \sup_{1 \leq i \leq k} |t_{i+1} - t_i|.$$

Then Liu integral of  $X_t$  is defined by

$$\int_a^b X_t dC_t = \lim_{\Delta \rightarrow 0} \sum_{i=1}^k X_{t_i} \cdot (C_{t_{i+1}} - C_{t_i})$$

provided that the limit exists almost surely and is an uncertain variable.

Let  $h(t, c)$  be a continuously differentiable function. Then  $X_t = h(t, C_t)$  is an uncertain process. Liu [18] proved a fundamental theorem of uncertain calculus that says  $X_t$  has an uncertain differential,

$$dX_t = \frac{\partial h}{\partial t}(t, C_t)dt + \frac{\partial h}{\partial c}(t, C_t)dC_t.$$

Under the assumption that the stock price follows a geometric Liu process, the first uncertain stock model was introduced by Liu [18]. In Liu's stock model, the bond price  $X_t$  and the stock price  $Y_t$  are determined by

$$\begin{cases} dX_t = rX_t dt \\ dY_t = eY_t dt + \sigma Y_t dC_t \end{cases} \quad (3)$$

where  $r$  is the riskless interest rate,  $e$  is the stock drift,  $\sigma$  is the stock diffusion, and  $C_t$  is a canonical Liu process. The option pricing problem is a fundamental problem in financial market. Liu [18] derived the European option pricing formulae, and Chen [5] studied American option pricing formulae for Liu's stock model. In particular, Peng-Yao's stock model (Peng and Yao [34]) has the following form

$$\begin{cases} dX_t = rX_t dt \\ dY_t = (m - \alpha Y_t) dt + \sigma Y_t dC_t. \end{cases} \quad (4)$$

The European and American option pricing formulae for Peng-Yao's stock model were also derived by [34]. Chen, Liu and Ralescu [7] built an uncertain stock model with periodic dividends.

### 3 Uncertain Currency Model

In foreign exchange market, the exchange rate changes everyday. A good forecast in the changes of foreign exchange rate can help us to avoid risks and obtain profits. In this section, we assume that the exchange rate follows a geometric Liu process. Based on this assumption, an uncertain currency model with uncertain exchange rate is defined as follows,

$$\begin{cases} dX_t = uX_t dt & \text{(Domestic Currency)} \\ dY_t = v dt & \text{(Foreign Currency)} \\ dZ_t = eZ_t dt + \sigma Z_t dC_t & \text{(Exchange Rate)} \end{cases} \quad (5)$$

where  $X_t$  represents the riskless domestic currency with domestic interest rate  $u$ ,  $Y_t$  represents the riskless foreign currency with foreign interest rate  $v$ , and  $Z_t$  represents the exchange rate that is domestic currency price of one unit of foreign currency at time  $t$ . Here  $Z_t$  follows a geometric Liu process with log-drift  $e$  and log-diffusion. Note that the domestic currency price is  $X_t = X_0 \exp(ut)$ , the foreign currency price is  $Y_t = Y_0 \exp(vt)$  and the exchange rate is  $Z_t = Z_0 \exp(et + \sigma C_t)$  whose inverse uncertainty distribution is

$$\Phi_t^{-1}(\alpha) = Z_0 \exp \left( et + \frac{\sqrt{3}\sigma t}{\pi} \ln \frac{\alpha}{1-\alpha} \right).$$

### 4 European Currency Option Pricing

The foreign exchange options market is the deepest, largest and most liquid market for options of any kinds. In this section, we will discuss the European currency option model in this section. In financial markets, a *European foreign currency option* is a contract that gives the investor the right (not obligation) to buy one unit of foreign currency with  $K$  (or a pre-agreed exchange rate) units of domestic currency

at time  $T$ . Then, we will discuss European currency option pricing formula of our proposed uncertain currency model (5).

In the pricing process, both the buyer's and bank's profits are taken into account and an equilibrium are computed between them. Suppose that the price of this contract is  $f$ . First let us take the viewpoint of the buyer. The buyer pays  $f$  for buying one contract at time 0, and receives  $(Z_s - K)^+$  at time  $T$ . Then the expected return in view of the buyer equals to the present value of the return  $E[(Z_T - K)^+]$  minus the payment  $f$ , namely

$$-f + \exp(-uT)E[(Z_T - K)^+]. \quad (6)$$

On the other hand, the bank receives  $f$  for selling the contract at time 0, and pays  $(1 - K/Z_T)^+$  at time  $T$ . The expected profit in the view of the bank equals to the gained currency option price minus the present value of potential loss  $E[(1 - K/Z_T)^+]$ , namely

$$f - Z_0 \exp(-vT)E[(1 - K/Z_T)^+]. \quad (7)$$

The fair price of this contract should make the buyer and the bank have an identical expected return, i.e.,

$$-f + \exp(-uT)E[(Z_T - K)^+] = f - Z_0 \exp(-vT)E[(1 - K/Z_T)^+].$$

**Definition 8.** Assume a European currency option has a strike price  $K$  and an expiration time  $T$ . Then the European currency option price is

$$f = \frac{1}{2} \left( \exp(-uT)E[(Z_T - K)^+] + Z_0 \exp(-vT)E[(1 - K/Z_T)^+] \right). \quad (8)$$

In order to get this currency option price for the uncertain currency model (5), we should give a formula for the value  $f$ .

**Theorem 2.** Assume a European currency call option for the uncertain currency model (5) has a strike price  $K$  and an expiration time  $T$ . Then the European currency option price is

$$\begin{aligned} f &= \frac{1}{2} \exp(-uT) \int_0^1 \left( Z_0 \exp \left( eT + \frac{\sqrt{3}\sigma T}{\pi} \ln \frac{\alpha}{1-\alpha} \right) - K \right)^+ d\alpha \\ &\quad + \frac{1}{2} \exp(-vT) \int_0^1 \left( Z_0 - K / \exp \left( eT + \frac{\sqrt{3}\sigma T}{\pi} \ln \frac{\alpha}{1-\alpha} \right) \right)^+ d\alpha. \end{aligned}$$

**Proof:** Since  $(Z_s - K)^+$  and  $Z_0(1 - K/Z_s)^+$  are increasing functions with respect to  $Z_s$ , they have inverse uncertainty distributions

$$\Phi^{-1}(\alpha) = \left( Z_0 \exp \left( eT + \frac{\sqrt{3}\sigma T}{\pi} \ln \frac{\alpha}{1-\alpha} \right) - K \right)^+$$

and

$$\Psi^{-1}(\alpha) = \left( Z_0 - K / \exp \left( eT + \frac{\sqrt{3}\sigma T}{\pi} \ln \frac{\alpha}{1-\alpha} \right) \right)^+,$$

respectively. Thus the European currency option price formula is proved since  $f = f_1 + f_2$ .

**Theorem 3.** Let  $f$  be the European currency option price of the uncertain currency model (5). Then

- (i).  $f$  is a decreasing function of  $K$ ;
- (ii).  $f$  is a decreasing function of  $u$ ;
- (iii).  $f$  is a decreasing function of  $v$ .

**Proof:** (i). It is obvious that  $(Z_T - K)^+$  and  $(1 - K/Z_T)^+$  are decreasing with respect to  $K$ . Therefore the function  $f$  is decreasing with respect to  $K$ .

(ii) Since  $\exp(-ut)$  is decreasing in  $u$ , the European currency option price is decreasing with respect to  $u$ .

(iii) Since  $\exp(-vt)$  is decreasing in  $v$ , the European currency option price is decreasing with respect to  $v$ .

**Example 1.** Assume the domestic interest rate  $u = 8\%$ , the foreign interest  $v = 5\%$ , the log-drift  $e = 0.06$ , the log-diffusion  $\sigma = 0.32$ , the initial exchange rate  $Z_0 = 6.2$ , the strike price  $K = 6.5$  and expiration time  $T = 2$ . Then we get the European currency option price is  $f = 0.988$ .

## 5 American Currency Option Pricing

In this section, we will discuss American currency option problem. An American currency option is a contract that gives the buyer the right to exchange one unit of foreign currency at any time prior to an expiration time  $T$  for  $K$  (a pre-agreed exchange rate) units of domestic currency. Next, we will consider a currency option for our proposed uncertain currency model (5).

In the pricing process, we will consider the profits between buyer and the bank and compute an equilibrium between them. Suppose that the price of this contract is  $f$ . First let us take the viewpoint of the buyer. The buyer pays  $f$  for buying one contract at time 0, and receives

$$\sup_{0 \leq s \leq T} \exp(-rs)(Z_s - K)^+$$

at time before  $T$ . Then the expected return in view of the buyer equals to the present value of the return

$$E \left[ \sup_{0 \leq s \leq T} \exp(-rs)(Z_s - K)^+ \right]$$

minus the payment  $f$ , namely

$$-f + E \left[ \sup_{0 \leq s \leq T} \exp(-rs)(Z_s - K)^+ \right]. \quad (9)$$

On the other hand, the bank receives  $f$  for selling the contract at time 0, and pays

$$\sup_{0 \leq s \leq T} \exp(-rs)Z_0(1 - K/Z_s)^+$$

up to time  $T$ . The expected profit in the view of the bank equals to the gained currency option price minus the present value of potential loss  $E \left[ \sup_{0 \leq s \leq T} \exp(-rs)Z_0(1 - K/Z_s)^+ \right]$ , namely

$$f - E \left[ \sup_{0 \leq s \leq T} \exp(-rs)Z_0(1 - K/Z_s)^+ \right]. \quad (10)$$

The fair price of this contract should make the buyer and the bank have an identical expected return, i.e.,

$$-f + E \left[ \sup_{0 \leq s \leq T} \exp(-us)(Z_s - K)^+ \right] = f - E \left[ \sup_{0 \leq s \leq T} Z_0 \exp(-vs)(1 - K/Z_s)^+ \right].$$

**Definition 9.** Assume an American currency option has a strike price  $K$  and an expiration time  $T$ . Then the American currency option price is

$$f = \frac{1}{2} \left( E \left[ \sup_{0 \leq s \leq T} \exp(-us)(Z_s - K)^+ \right] + E \left[ \sup_{0 \leq s \leq T} Z_0 \exp(-vs)(1 - K/Z_s)^+ \right] \right). \quad (11)$$

In order to get this currency option price for the uncertain currency model (5), we should give a formula for the value  $f$ .

**Theorem 4.** Assume an American currency option for the uncertain currency model (5) has a strike price  $K$  and an expiration time  $T$ . Then the American currency option price is

$$f = \frac{1}{2} \int_0^1 \sup_{0 \leq s \leq T} \exp(-us) \left( Z_0 \exp \left( es + \frac{\sqrt{3}\sigma s}{\pi} \ln \frac{\alpha}{1-\alpha} \right) - K \right)^+ d\alpha \\ + \frac{1}{2} \int_0^1 \sup_{0 \leq s \leq T} \exp(-vs) \left( Z_0 - K / \exp \left( es + \frac{\sqrt{3}\sigma s}{\pi} \ln \frac{\alpha}{1-\alpha} \right) \right)^+ d\alpha.$$

**Proof:** It follows from the extreme value theorem that  $\sup_{0 \leq s \leq T} \exp(-us)(Z_s - K)^+$  and  $\sup_{0 \leq s \leq T} \exp(-vt)Z_0(1 - K/Z_T)^+$  have inverse uncertainty distributions

$$\Phi^{-1}(\alpha) = \sup_{0 \leq s \leq T} \exp(-us) \left( Z_0 \exp \left( es + \frac{\sqrt{3}\sigma s}{\pi} \ln \frac{\alpha}{1-\alpha} \right) - K \right)^+$$

and

$$\Psi^{-1}(\alpha) = \sup_{0 \leq s \leq T} \exp(-vs) \left( Z_0 - K / \exp \left( es + \frac{\sqrt{3}\sigma s}{\pi} \ln \frac{\alpha}{1-\alpha} \right) \right)^+,$$

respectively. Thus the American currency option price formula follows from the definition of American currency option immediately.

**Theorem 5.** Let  $f$  be the American currency option price of the uncertain currency model (5). Then

- (i).  $f$  is a decreasing function of  $K$ ;
- (ii).  $f$  is a decreasing function of  $u$ ;
- (iii).  $f$  is a decreasing function of  $v$ .

**Proof:** (i). It is obvious that  $\sup_{0 \leq s \leq T} \exp(-rs)Z_0(Z_s - K)^+$  and  $\sup_{0 \leq s \leq T} \exp(-rs)(1 - K/Z_s)^+$  are decreasing with respect to  $K$ .

(ii) Since  $\exp(-ut)$  is decreasing in  $u$ , the American currency option price is decreasing with respect to  $u$ .

(iii) Since  $\exp(-vt)$  is decreasing in  $v$ , the American currency option price is decreasing with respect to  $v$ .

**Example 2.** Assume the domestic interest rate  $u = 8\%$ , the foreign interest  $v = 5\%$ , the log-drift  $e = 0.06$ , the log-diffusion  $\sigma = 0.32$ , the initial exchange rate  $Z_0 = 6.2$ , the strike price  $K = 6.5$  and expiration time  $T = 2$ . Then we get the American currency option price is  $f = 0.990$ .

## 6 Conclusion

In this paper, we introduced an uncertain currency model in uncertain financial markets based on the uncertain calculus theory and we derived both the European option pricing formula and American currency option formula for this uncertain currency model. Some mathematical properties of this formula were studied. Finally, two numerical examples were discussed.

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