Availability Analysis of Satellite Constellation

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***Abstract*—This paper firstly analyzes the concept of availability of satellite constellation. Then the method of constellation availability analysis is proposed on the basis of Markov chain model, outage analysis, as well as MTBF and MTTR of satellite constellations. Finally, an illustrative example is presented to validate the method.**

Keywords-satellite constellations; availability analysis; outage analysis

I. INTRODUCTION

A satellite constellation consists of many satellites located in different orbits. The constellations can expand the functions which single satellite can not perform, like global navigation, multi-coverage and continuous coverage [1].

Four specifications are important on the evaluation of satellite constellations systems, including accuracy, availability, continuity and integrity. Among them, availability is the core specification. Availability during mission lifetime for a specified service is currently used for missions where a "steady-state" nominal service is planned, and for which a percentage of the mission time can be specified as an availability performance measure [2]. However, this concept of availability is so simple that lots of important information can not be captured in the expression, like the outage rate and duration of the constellation. The availability of satellite constellation has two different meanings, one is associated with the reliability of the constellation systems' performance, and the other is associated with the failure rates and restoration rates of the constellation.

The concept of availability of satellite constellation is firstly analyzed regarding MTBF and MTTR of the constellation system in this paper. Then the method of availability analysis of the satellite constellations is proposed on the basis of Markov chain model, outage analysis, as well as MTBF and MTTR of satellite constellations. At the end of the paper, an illustrative example is presented to validate the availability analysis method.

II. CONCEPT OF AVAILABILITY OF CONSTELLATION

"Availability" is the key factor on the evaluation of the performance of satellite constellation. However, the definition of Availability and its targets have evolved over years. In the early days of GPS, availability of a simple requirement (minimum of 4 visible satellites or maximum PDOP of 6) of an ideal constellation (with 18 or 21 or 24 satellites) was adequate [3]. Then the availability of same requirement was applied the degraded constellations with outages of one or more satellites. Today, availability has become more complex involving satellite's Markov state probability with different satellite failure and restoration models. For navigation, availability of continuity of accuracy, not accuracy itself, is the dominant requirement. The system performance requirement has also evolved from a simple number of visible satellites to Dilution of Precision (where satellites' error are assumed to be identical) to more complex horizontal and vertical user navigation accuracy with satellites' errors changing with time as the satellites' position changes.

As mentioned before, the constellation availability is associated with the failure rates and restoration rates of the satellite constellation. Here, the concept of maintenance of satellite constellation is of great importance. As we know, satellites in the constellation may fail during their operation period. Spare satellite strategy is adopted in constellation design to deal with the problem. These measures can be regarded as maintenance of the constellation.

Under these considerations, the constellation availability may be regarded as the probability that the slots in the satellite constellation will be occupied by satellites transmitting a trackable and healthy SIS (signal in space) [4].

III. AVAILAVILITY ANALYSIS OF CONSTELLATIONS

A. Expected availability

Availability analysis or simulations shall be performed in order to assess the availability of the constellation system. The results are used:

* to optimize the system concept with respect to design, operations and maintenance,
* to verify conformance to availability requirements, and
* to provide inputs to estimate the overall cost of operating the system.

The availability analysis (predictions/assessments) shall be carried out at system level using the system reliability and maintainability models as well as the data from the outage analysis.

The expected availability of satellite constellation can be expressed as:

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$$A_k = \sum_n P_{k,n} \cdot \alpha_{k,n} \qquad (1)$$

A_k denotes the expected availability of the k scheme.

$P_{k,n}$ denotes the state probabilities associated with the state of *n* satellites are normally working.

$\alpha_{k,n}$ denotes probabilities of these *n* satellites meeting the requirement of navigation performance.

B. Availability analysis based on Markov chain and outage analysis

In formula (1), $P_{k,n}$ can be calculated through Markov chain model and the input data such as MTBF and MTTR of the constellation.

Markov chain model is the popular model to deal with the constellation state probabilities [5]. Here we choose 5 IGSO satellites to illustrate the model, as shown in Figure 1. This model is a Markov chain with the maximum of 5 satellites, and a minimum of zero.

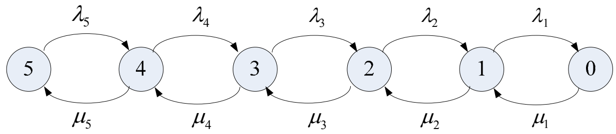

Figure 1. Markov chain for 5 IGSO model

Each state is associated with four types of outages.

According to IEC Multilingual Dictionary (2001 edition), outage is defined as the state of an item of being unable to perform its required function [6].

The relations between the various values such as MTBF MTTR, and MTTF, which characterize the reliability, maintainability and availability of equipment are shown as Figure 2.

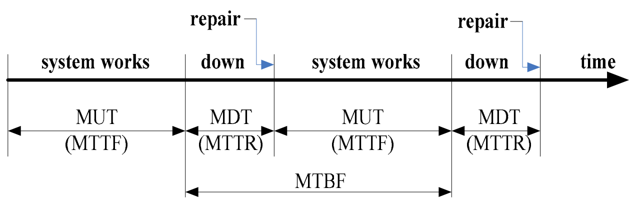

Figure 2. Relations between the various values that characterize the reliability, maintainability and availability of constellation

The outage analysis shall be performed in order to supply input data for availability analysis. The outage analysis output includes a list of all potential outages, their causes, probabilities of occurrence and duration. Instead of outage probabilities, failure rates associated with outages can be provided. Furthermore, the means of outage detection and the recovery methods shall be identified in the analysis.

Satellites outages are divided into scheduled and unscheduled outages, which are then each further divided into short term and long-term. Thus, there are four categories of satellites outages: short-term scheduled (STS), long-term scheduled (LTS), short-term unscheduled (STU), and long-term unscheduled (LTU). Scheduled outages, both short- and long-term, are under the control of the Control Segment and can be deferred for a length of time and rescheduled as needed. STS outages are for routine maintenance actions such as clock ion pumping, station keeping maneuvers, and switching between redundant subsystems. LTS outages are commonly referred to as End of Life (EOL) events. STU outages result from sudden failures that cannot be predicted or scheduled, but they are problems that can typically be fixed on orbit by switching to a redundant subsystem on board the satellites. LTU outages also result from sudden failures, but the nature of the failure is such that the satellite cannot be fixed on orbit and must be replaced [7].

The four failure mechanisms can be combined into a single aggregate Markov chain, in which the failure and restoration probabilities are:

$$\lambda_i = i(\lambda_s + \lambda_U + \lambda_P + P_E\lambda_E) \qquad (2)$$

$$\mu_i = \frac{\lambda_s + \lambda_U + \lambda_P + P_E\lambda_E}{\frac{\lambda_s}{K_{Si}\mu_s} + \frac{\lambda_U}{K_{Ui}\mu_U} + \frac{\lambda_P}{K_{Pi}\mu_P} + \frac{P_E\lambda_E}{K_{Ei}\mu_E}} \qquad (3)$$

where $_i$ is the probability that any failure will occur from the state of *i* satellites,

$_i$ is the probability that any restoration will occur returning the system to the state of *i* satellites,

$_S$ is the probability of a STS failure,

$_U$ is the probability of an STU failure,

$_P$ is the probability of a LTU failure,

$_E$ is the probability of a LTS failure on a weak satellite,

$_S$ is the probability of a restoration of a STS failure,

$_U$ is the probability of restoration of an STU failure,

$_P$ is the probability of restoration of a LTU failure,

and $_E$ is the probability of the restoration of a LTS failure. When there is no surplus satellite, P_E defines the percentage of weak satellites in the constellation. When there are surplus satellites, P_E is the probability of a surplus satellite not being launched before an LTS failure in the baseline system.

C. Performance analysis of the constellations

In formula (1), $\alpha_{k,n}$ denotes probabilities of these *n* satellites meeting the requirement of navigation performance. To calculate these parameters, positioning dilution of precision (PDOP) of different satellite constellation must be considered. PDOP of navigation constellation is one of the important specifications in the phase of general system design. Effects of different distributions of fault satellites on PDOP availability of navigation constellation can be studied by the simulation tools, like STK or GSSF.

IV. AN ILLUSTRATIVE EXAMPLE

A. *Availability analysis of regional navigation systems*

Continuous and stable coverage in the specified regions are required for the regional navigation constellations, whereas in the unspecified regions the coverage is not required. Three orbits are available in the regional navigation constellations, i.e. GEO, IGSO, and MEO. We focus on the regional navigation constellations having GEO and IGSO satellites. The availability of three schemes can be calculated by the methods proposed in section III. These three schemes are:

(1) 5 GEO+3 IGSO

(2) 5 GEO+4 IGSO

(3) 5 GEO+5 IGSO

The outage data of the regional navigation constellation are illustrated in table 1. By virtue of the Markov chain model and the data in table 1, the state probabilities of the constellation can be calculated, as shown in table 2.

TABLE I. OUTAGE DATA OF THE REGIONAL NAVIGATION CONSTELLATION

Outage Type	MTBF(hrs)	MTTR(hrs)
STS	360	7.5
STU	4380	21
LTS	70080	4380
LTU	420000	5100

$$P_5=(1+\sum_{j=0}^{4}(\prod_{k=5}^{5-j}\frac{\lambda_k}{\mu_k}))^{-1}$$

$$=(1+(\frac{\lambda_5}{\mu_5}+\frac{\lambda_5}{\mu_5}\cdot\frac{\lambda_4}{\mu_4}+\frac{\lambda_5}{\mu_5}\cdot\frac{\lambda_4}{\mu_4}\cdot\frac{\lambda_3}{\mu_3}+\frac{\lambda_5}{\mu_5}\cdot\frac{\lambda_4}{\mu_4}\cdot\frac{\lambda_3}{\mu_3}\cdot\frac{\lambda_2}{\mu_2}+\frac{\lambda_5}{\mu_5}\cdot\frac{\lambda_4}{\mu_4}\cdot\frac{\lambda_3}{\mu_3}\cdot\frac{\lambda_2}{\mu_2}\cdot\frac{\lambda_1}{\mu_1}))^{-1}$$

$$=(1+(\frac{0.015}{0.125}+\frac{0.015\times0.012}{0.125^2}+\frac{0.015\times0.012\times0.009}{0.125^3}+\frac{0.015\times0.012\times0.009\times0.006}{0.125^4}$$

$$+\frac{0.015\times0.012\times0.009\times0.006\times0.003}{0.125^5}))^{-1}=0.8831$$

$$P_4=(\prod_{k=5}^{5}\frac{\lambda_k}{\mu_k})\cdot P_5=\frac{\lambda_5}{\mu_5}\cdot P_5=\frac{0.015}{0.125}\times0.8831=0.1060$$

$P_3=0.0102$, $P_2=0.0007$, $P_1=0$, $P_0=0$

B. *Comparision of the schemes*

Table 3 shows the availabilities of different schemes. From the table, we can see the availability of scheme 3 is highest, but not significant with comparison to scheme 2. If the costs of the satellites are considered, maybe scheme 2 is the best one.

TABLE II. THE STATE PROBABILITIES OF IGSO SATELLITES

Numbers of the normal satellites	State probability	Cumulative probability
5	0.8831	0.8831
4	0.1060	0.9891
3	0.0102	0.9993
2	0.0007	1
1	0	1
0	0	1

TABLE III. AVAILABILITIES OF DIFFERENT SCHEMES

Operation Schemes			State Probabilities $P_{k,n}$	Probabilities of different configurations meeting the performance requirement $\alpha_{k,n}$	Constellation availability A_k
1	**5 GEO+3 IGSO** (The baseline configuration meeting the performance requirement)	No IGSO satellite fails (totally 1 kind of configuration)	$P_{1,3}$=0.9301	$\alpha_{1,3}=1$	A_1=0.9301
2	**5 GEO+4 IGSO**	Only one IGSO satellite fails,(totally $C_4^3=4$ kinds of configurations)	$P_{2,3}$=0.0755	$\alpha_{2,3}=0.75$	A_2=0.9689
		No IGSO satellite fails (totally 1 kind of configuration)	$P_{2,4}$=0.9123	$\alpha_{2,4}=1$	
3	**5 GEO+5 IGSO**	Two IGSO satellites fail, (totally $C_5^3=20$ kinds of configurations)	$P_{3,3}$=0.0102	$\alpha_{3,3}=0.65$	A_3=0.9745
		Only one IGSO satellite fails,(totally $C_5^4=5$ kinds of configurations)	$P_{3,4}$=0.1060	$\alpha_{3,4}=0.8$	
		No IGSO satellite fails totally 1 kind of configuration)	$P_{3,5}$=0.8831	$\alpha_{3,5}=1$	

V. CONCLUSION

The concept of availability of satellite constellations can be established based on the concept of the maintainability of satellite constellations. By means of Markov chain model, PDOP analysis, as well as data from the outage analysis, the availability of constellations can be analyzed quantitatively. This availability analysis method is validated through the example of the availability computing of the regional navigation systems, which results may effectively support the relevant decision-makings.

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