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Wavelet Transform for Structural Health Monitoring: A Compendium of Uses and Features

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The strategic and monetary value of the civil infrastructure worldwide necessitates the development of structural health monitoring (SHM) systems that can accurately monitor structural response due to real-time loading conditions, detect damage in the structure, and report the location and nature of this damage. In the last decade, extensive research has been carried out for developing vibration-based damage detection algorithms that can relate structural dynamics changes to damage occurrence in a structure. In the mean time, the wavelet transform (WT), a signal processing technique based on a windowing approach of dilated 'scaled' and shifted wavelets, is being applied to a broad range of engineering applications. Wavelet transform has proven its ability to overcome many of the limitations of the widely used Fourier transform (FT); hence, it has gained popularity as an efficient means of signal processing in SHM systems. This increasing interest in WT for SHM in diverse applications motivates the authors to write an exposition on the current WT technologies.

This article presents a utilitarian view of WT and its technologies. By reviewing the state-of-the-art in WT for SHM, the article discusses specific needs of SHM addressed by WT, classifies WT for damage detection into various fields, and describes features unique to WT that lends itself to SHM. The ultimate intent of this article is to provide the readers with a background on the various aspects of WT that might appeal to their need and sector of interest in SHM. Additionally, the comprehensive literature review that comprises this study will provide the interested reader a focused search to investigate using wavelets in SHM.

Keywords structural health monitoring (SHM) · wavelet transform (WT) · wavelet multi-resolution analysis (WMRA) · signal processing · damage detection

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Figures 1–3 and 5–8 appear in color online: http://shm.sagepub.com

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1 Introduction

Structural health monitoring (SHM) aims at real-time characterization of structural performance to enhance structural safety and to significantly reduce lifetime operating costs by early detection for maintenance. Thus, the basis for implementing SHM systems in critical structures (airplanes, rotorcraft, vehicles, and bridges) is to detect damage occurrence and inform the responsible stakeholders about the location, nature, and severity of this damage [1,2]. Repeated incidents in the airline industry have resulted in growing uncertainty regarding the reliability of routine maintenance methods and inspection techniques [3]. The difficulty of thorough visual inspection of structures has increased the need for complex SHM systems that can provide early and reliable damage detection in critical and historical structures [4,5].

Vibration-based analysis has evolved in the past decade as a promising method for SHM. The premise of vibration-based SHM is that dynamic characteristics of a structure are a function of its mechanical properties. Thus changes in these mechanical properties as a result of localized structural damage will result in observable changes in the dynamic characteristics vibrations of the structure. Research on vibration-based damage identification goes back to the late 1970s in the study of offshore oil and gas platforms as well as in the aerospace industry. The civil engineering industry captured this interest much later due to the increased rate of infrastructure deterioration observed late in the twentieth century. A review of SHM techniques for detecting changes in structural dynamics due to damage was provided by Doebling et al. [6] and more recently by Yong [7].

Currently, most SHM systems depend on measuring structural dynamics characteristics and analyzing these data in the frequency domain by performing modal analysis [6–10]. Damage detection algorithms from modal analysis depend on observing signals from sensors distributed over the structure and developing accurate structural models (e.g., finite element (FE) models) to identify observable mode shapes. Problems associated with FE modeling such as, discretization, configuration errors, and modeling errors, as discussed by Yang et al. [11], proved that modal testing might not be sufficiently practical. Moreover, experimental verification of damage detection algorithms using modal data from relatively large structures showed that modal characteristics might be insensitive to localized damage [12,13]. Kim et al. [14] suggested using modal strain energy to estimate the severity of damage and pointed out that changes in the natural frequencies are difficult to measure due to the limited change in frequencies due to the uneven mass distribution in large structures. It has also been proven that changes in the mode shapes due to damage might not be identifiable due to signal corruption by noise and/or to the limited number of identifiable mode shapes (because of the low frequency of vibration of relatively large structures) [15]. Ren and De Roeck [8] examined the effect of noise on the reliability of damage identification from modal analysis and reported that the effect of noise is highly dependent on damage severity such that limited damage states will be more challenging to detect at high levels of noise than severe ones.

As modal analysis is performed in the frequency domain, most current SHM systems implement fast Fourier transform (FFT) [6,8,16]. Fast Fourier transform is used to decompose a time-domain sequence in terms of a set of basis functions. The set of complex sinusoids $\{e^{i\omega n}, -\infty < f < \infty\}$ forms the set of these basis functions where i is the complex number ($i = \sqrt{-1}$), ($\omega = 2\pi f$), f is the frequency and n is the discrete time variable [17]. The discrete Fourier transform (DFT) of a discrete time signal $x(n)$ is given by:

$$\mathrm{X}(\omega) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-i\omega n} \tag{1}$$

A major problem in using the FFT results from the fact that the transform is the result of a summation (or integration in the continuous time domain) over the entire signal length. This means that the signal decomposition cannot indicate the time of occurrence for a transient signal. Therefore, the DFT can provide good frequency resolution, but no time resolution [18]. Therefore, DFT-based SHM systems might recognize damage occurrence if they are used to observe frequency spikes [16] but this damage recognition would be based on frequency information only while all possible time information will be lost. Unless a time resolution analysis is used, the value of such time information in structural vibration signals cannot be assessed. It has been shown that good time localization can be accomplished by applying the short time Fourier transform (STFT), which utilizes a window function that is multiplied by the input signal before computing the FFT [19]. Although STFT provides a time–frequency representation of a signal, there is a major drawback with respect to utilizing STFT in SHM applications; namely that the width of the window is fixed. The STFT of a signal can be represented in a two-dimensional grid as shown in Figure 1(a) where the divisions in the horizontal direction represent the time extent for each window; the divisions in the vertical direction represent the frequencies; and the shade of each rectangle is proportional to the frequency of the monitored signal component. The darker the shade of the rectangle means the lower the frequency of the signal component that can be observed. As the width of the window function increases, more accurate information about the different frequencies within the window is obtained; but the ability to determine when those frequencies occur is lost [20]. Thus, there remains a need for multiple resolution analysis that can provide fine time resolution for long duration signals and fine frequency resolution for high frequency signals [21]. A thorough review of various time–frequency techniques for structural vibration analysis is provided by Neild et al. [22]. Strengths and weaknesses of each technique were examined through examining a group of synthetic signals representing possible structural dynamics. Although the review did not address the issue of damage diagnosis it shed light on similarities between these techniques [22].

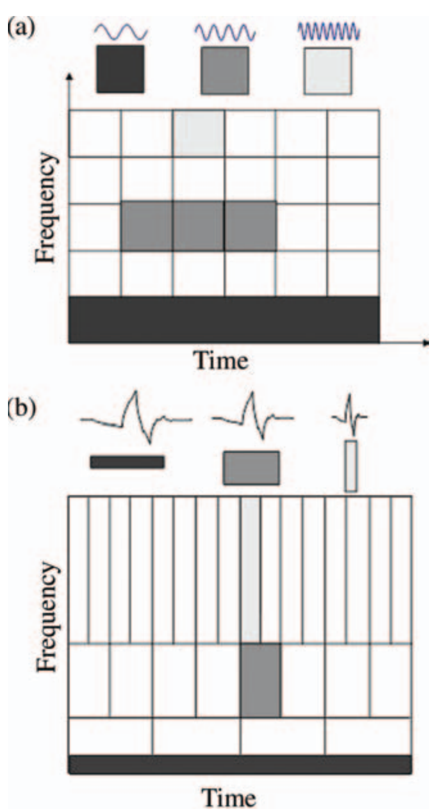

Figure 1 Time–frequency representation of signals: (a) Time–frequency representation using (STFT) for wide window (low frequency), medium window (medium frequency), and narrow window (high frequency) and (b) time–frequency representation of wavelet transform (WT).

2 Prologue to Wavelet Transform for Structural Health Monitoring

The subsections that follow will recapitulate the motivation for and the concepts of wavelet theory. An efficient digital signal processing algorithm that is capable of analyzing continuous and transient signals must provide multiple resolutions in time and frequency domains. More precisely, such an algorithm should consider fine time resolution for long duration signals and fine frequency resolution for high frequency signals. Wavelet transform (WT) represents the next logical step in the evolution of digital signal processing algorithms since it is based on a windowing technique with variable-sized regions. In fact, wavelet analysis is capable of utilizing long time intervals (large window) where precise low frequency information is needed, and short time intervals (small window) where high frequency information is considered [23]. The WT, therefore, will provide accurate location of the transient signals while simultaneously reporting the fundamental frequency and its low-order harmonics. This inherent feature of the WT (the ability to provide good time and frequency resolutions of a signal) contributed to its widespread use in engineering applications [24–28]. Before going into further discussion on WT and its features and uses in SHM, it is important to emphasize that WT shall not be regarded as a competitive technique to FT but simply an extension. The two techniques have fundamentally the same approach. As stated by Hubbard [23] 'The goal is to turn the information of a signal into numbers – coefficients – that can be manipulated, stored, transmitted, analyzed, or used to reconstruct the original signal'.

As described by Mallat [29], two types of wavelet functions might be used in performing WT: the real and the analytic wavelets. The real wavelets are those wavelets used to detect sharp signal transitions [21,23,29]. The analytic wavelets are those wavelets used to identify instantaneous frequency evolution. A wavelet is named analytic if its FT is zero for all negative frequencies [29]. The distinction between the two types of wavelets might be important for SHM applications as damage detection might be captured through observing sharp transitions of signals or evolution of new frequencies in the structural dynamics. The WT output can be represented in a two-dimensional grid similar to the STFT but with very different divisions in time and frequency as shown in Figure 1(b). The rectangles in Figure 1(b) have equal area or constant time-bandwidth product (following the Heisenberg uncertainty principle [21,23,29]) such that they narrow at the low scales (high frequencies) and widen at the high scales (low frequencies). In contrast to the STFT, the WT isolates the transient high frequency components in the top frequency band at the time of their occurrence while the long lasting low frequency components are presented as a continuous magnitude. The wavelet coefficients can thus be useful in analyzing nonstationary events.

The WT of a time-domain signal is defined in terms of the projections of this signal on to a family of functions that are all normalized dilations and translations of a wavelet function [21,23,29,30]. The wavelet (basis) functions $\psi(t)$ are not limited to exponential (or sinusoidal) basis functions as in the case of the FT. The choice of the wavelet function strives to produce the maximum number of wavelet coefficients within the full time span of the original signal to be close to zero to guarantee good time localization [29]. This can be achieved by restricting $\psi(t)$ to be short and oscillatory to ensure that the integration (summation) of the transform is finite. Furthermore, it must have an average of zero and decay quickly at both ends. These admissibility conditions ensure that the integration in the WT transform equation is finite [21,29]. The function $\psi(t)$ has been given the name wavelet or 'small wave' and is referred to as the 'mother wavelet' and its dilates (scaled) and translates (shifted) simply as 'wavelets' or 'daughter wavelets' [29–31]. A schematic representation of a few mother wavelet functions is presented in Figure 2. The major WT techniques that are currently used in SHM systems are discussed in the following sections.

2.1 Continuous and Discrete Wavelet Transform

Continuous wavelet transform (CWT) of a time-domain signal $x(t)$ is determined as:

$$\mathrm{CWT}(a,\tau) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} x(t)\ \psi\left(\frac{t-\tau}{a}\right) \mathrm{d}t \qquad (2)$$

where a and τ are the scaling and shift (position) parameters of the wavelet function $\psi(t)$, respectively. For each scale 'a' and position 'τ' the time-domain signal is multiplied by shifted and scaled versions of the wavelet function. The discrete wavelet transform (DWT) of a discrete time sequence $x(n)$ is given as:

$$C_{j,k} = 2^{(-j/2)} \sum_{n} x(n)\psi\left(2^{-j}n - k\right) \qquad (3)$$

where $\psi(n)$ is the wavelet function (the basis function utilized in the WT) and $2^{(-j/2)}$

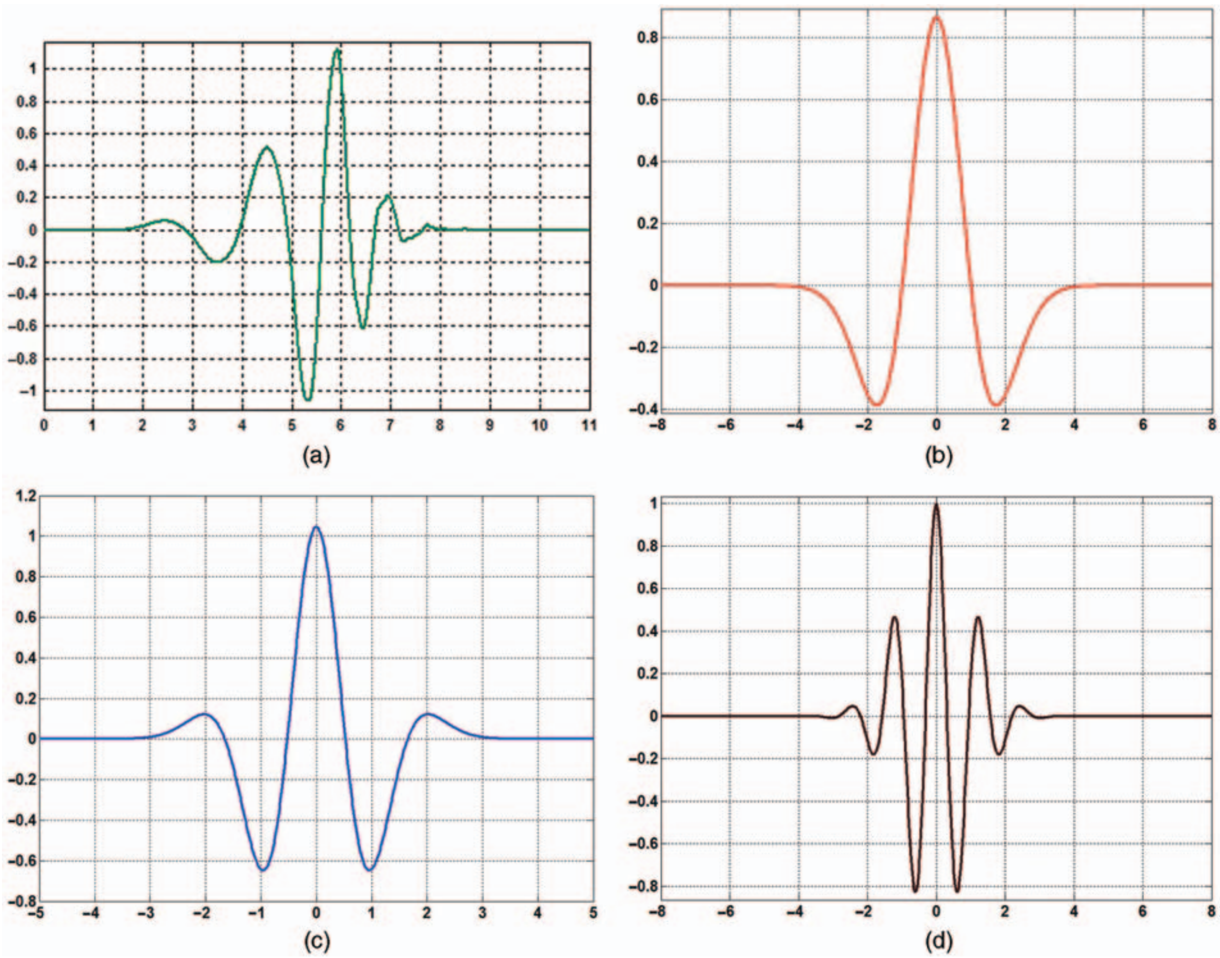

Figure 2 Example representation of some mother wavelets: (a) Daubechies wavelet (db6), (b) Mexican hat wavelet, (c) Gaussian wavelet, and (d) Morlet wavelet.

$\psi(2^{-j}\,n-k)$ are scaled and shifted versions of $\psi(n)$ based on the values of j (scaling coefficient), and k (shifting coefficient) and is usually written as $\psi_{j,k}\,(n)$. The j and k coefficients take integer values for different scaling and shifted versions of $\psi(n)$. $C_{j,k}$ represents the corresponding wavelet coefficients. It is worth noting that the scale in wavelet analysis is analogous to frequency in Fourier analysis [23,29]. A graphical representation of the wavelet coefficients plotted on a time-scale grid is named 'the scalogram'. It is a graphical representation of the amplitude of wavelet coefficients ($C_{j,k}$) with respect to time and scale variables. It is usually represented in a two-dimensional fashion with the wavelet coefficients corresponding to the color intensity or the gray intensity of the grid. The gray intensity is commonly used to represent the change in the wavelet coefficients with black, gray, and white corresponding to positive, zero, and negative wavelet coefficients, respectively [29]. The scalogram reveals the wavelet coefficients at each time step and allows determination of the occurrence of transient signals [29,32,33]. This is because of the fact that the wavelet coefficients measure the correlation between a segment of the signal of interest and the wavelet. As stated by Meyer [34] '*strong correlation means that there is a little piece of the signal that looks like the wavelet*'. The square of amplitude of the wavelet coefficients can also be graphically represented with respect to time and scale in what is called 'wavelet maps' which can be interpreted as the time-scale distributions of the signal energy [2,35].

2.2 Wavelet Multi-resolution Analysis (WMRA)

Wavelet multi-resolution analysis (WMRA) is a technique used to implement the DWT with filters. It allows the decomposition of signals into various resolution scales. Originally the multi-resolution technique came after the work by Mallat [29] to speed the computation and to make the computation possible for two-dimensional image analysis. The concept is to study the signal at different resolutions: coarse resolution to get the big picture and fine resolution to get the details [23]. Thus, the data with coarse resolution contains information about low frequency components and retains the main features of the original signal. The data with fine resolution retain information about the high frequency components.

Scaling a wavelet simply means stretching or compressing it in the time domain. The smaller the scale the more compressed the wavelet will be, while the larger the scale the more stretched the wavelet will be [21,29,31]. Therefore, frequency scales allow the analysis of rapidly changing details (high frequency components) and low frequency scales allow the analysis of slowly changing features (low frequency components) [36]. The low frequency component of the signal usually identifies the long-term variation of the signal [35,36]. The approximations correspond to the low scale, low frequency components of the signal while the details correspond to the high scale, high frequency components of the signal. The WMRA, therefore, decomposes the signal into various resolution levels: the data with coarse resolution (approximations) contain information about low frequency components and data with fine resolution (details) contain information about the high frequency components [35].

Consider j and k to be the scaling (dilation) index and the translation (shifting) index, respectively. Each value of j corresponds to analyzing a different resolution level of the signal. The WMRA of a digital signal $x(n)$ can be described as follows [21,35,36]:

For an input signal $x(n)$, the approximation coefficient $a_{j,k}$ at the jth resolution can be computed as follows:

$$a_{j,k} = 2^{(-j/2)} \sum_n x(n)\, \phi\left(2^{-j}n - k\right) \tag{4}$$

where ϕ is called the scaling function. It might be worth noting that the approximation coefficient $a_{j,k}$ computed using Equation (4) is obtained in a similar way to the wavelet coefficient $C_{j,k}$ computed using Equation (3). The major difference is the use of the scaling function ϕ to compute the approximation coefficient $a_{j,k}$ instead of the wavelet function ψ to compute the wavelet coefficient $C_{j,k}$. Scaling functions are similar to wavelet functions except that they have only positive values [29]. They are designed to smooth the input signal, thus operating in a manner equivalent to a low pass filter which rejects high frequency components of the signal [23,29,36]. The approximation signal $x_j(n)$ at the jth resolution level is then computed as

$$x_j(n) = \sum_{k=-\infty}^{\infty} a_{j,k}\, \phi_{j,k}(n) \tag{5}$$

The detail coefficient $d_{j,k}$ at the jth resolution level and the detail signal $g_j(n)$ are then computed as

$$d_{j,k} = 2^{(-j/2)} \sum_n x(n) \psi\left(2^{-j}n - k\right) \tag{6}$$

$$g_j(n) = \sum_{k=-\infty}^{\infty} d_{j,k} \cdot \psi_{j,k}(n) \tag{7}$$

where $\psi_{j,k}(n)$ is the wavelet basis function. The above three steps are repeated for the $j+1$ resolution level but using the approximation $x_j(n)$ obtained in Step 2. The original signal $x(n)$ can be reconstructed using an infinite number of details obtained after decomposing the signal at infinite resolution levels as per the equation [23,29]

$$x(n) = \sum_{j=-\infty}^{\infty} g_j(n) = \sum_{j=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} d_{j,k}\ \psi_{j,k}(n). \quad (8)$$

The above equation implies that one has to break the original signal to an infinite number of details, which is impractical. Alternatively, the analysis can stop at the Mth resolution level and the signal can be reconstructed using the approximation at the Mth level and all the details starting from the first level until the Mth level [21,29]. The following equation presents this procedure

$$x(n) = \sum_{k=-\infty}^{\infty} a_{M,k} \cdot \phi_{M,k}(n) + \sum_{j=1}^{M} \sum_{k=-\infty}^{\infty} \mathrm{d}_{j,k} \cdot \psi_{j,k}(n). \quad (9)$$

The first term represents the approximation at level M and the second term represents the details at level M and lower. Therefore, multi-resolution analysis (MRA) builds a pyramidal structure that requires an iterative application of the scaling and the wavelet functions, respectively [21,35]. A schematic representation of the WMRA pyramid structure is presented in Figure 3. The high frequency band outputs are taken as the detail coefficients (D1, D2, D3), and the low frequency band outputs are taken as the approximation coefficients (A1, A2, A3). It is worth mentioning that a down-sampling process is performed at every decomposition stage. If the decomposition operation is performed on the entire range of the signal, the number of samples considered in the analysis will be doubled. For instance, if the original signal consists of 100 samples, the approximation and the detail signals will each have 100 samples (total of 200 samples). In order to overcome this problem which affects computation time and data storage, the down-sampling process is carried out by ignoring the second sample of each sampling pair.

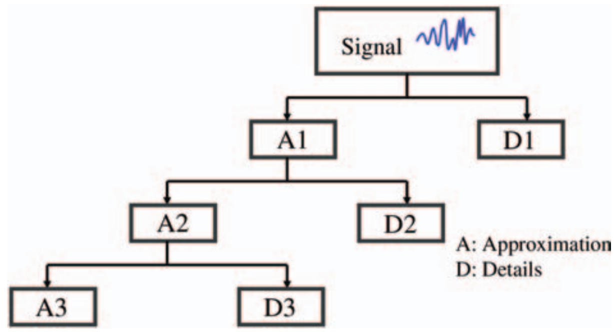

Figure 3 Schematic representation of the pyramid structure representing WMRA.

2.3 Wavelet Packet Transform

Wavelet packet transform (WPT), like WMRA, is a technique to decompose a signal repeatedly into successive low and high frequency components. However, it differs from WMRA in that not only is the approximation at a given level decomposed further, but so are the details. This results in a more flexible and wider base for the analysis of monitored signals [29,37,38].

A wavelet packet is a family of scaling functions and wavelet functions constructed by following a binary tree of dilations/translations. Thus, wavelet packets inherit properties, such as orthonormality and time–frequency localization from their corresponding wavelet functions. A wavelet packet, $\psi^i_{j,k}(t)$, is a function of three indices, where i, j, and k are integers representing the modulation, the scale, and the translation parameters, respectively. It can be written as

$$\psi^i_{j,k}(t) = 2^{j/2}\psi^i\left(2^j t - k\right), \quad i = 1, 2, 3, \ldots. \quad (10)$$

where the wavelets ψ^i are obtained from the following recursive relationships in Equations (11) and (12)

$$\psi^{2i}(t) = \sqrt{2} \sum_{k=-\infty}^{\infty} h(k)\psi^i(2t-k) \quad (11)$$

$$\psi^{2i+1}(t) = \sqrt{2} \sum_{k=-\infty}^{\infty} g(k)\psi^i(2t-k) \quad (12)$$

where $h(k)$ and $g(k)$ are filters associated with the scaling function and the mother wavelet

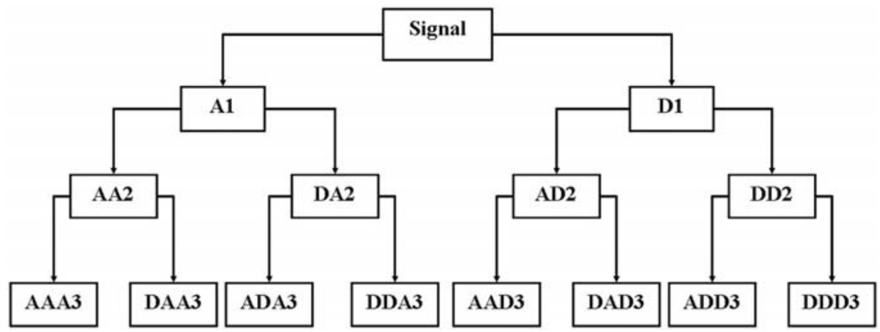

Figure 4 Schematic representation of wavelet packet transform (WPT) of a signal.

function, respectively. It should be emphasized that in the above two equations, $\psi^i(t)$ is the mother wavelet function. The analysis procedure starts by computing the wavelet packet decomposition of the monitored dynamic signal at level N using a given wavelet. This means there are 2^N components at the Nth level which gives the flexibility in choosing the manner in which one wants to encode the original signal (this is specifically useful for de-noising) such that the reconstruction error 'distortion' is minimum. Research is actively trying to determine the best encoding scheme of the original signal that shows important features without inducing distortion, for example, an entropy-based criterion [37–40]. This will be discussed later in further detail in Section 3.2.

For each packet (except for the approximations), a threshold value is selected and the thresholding procedure is applied to the coefficients. The wavelet packet reconstruction is performed based on the original approximation coefficients at level N and the threshold coefficients. Schematic representation of the wavelet packet transform (WPT) of a signal is presented in Figure 4. For instance, reconstruction of the signal shown in Figure 4 can be achieved by summing the signal components A1, AAD3, DAD3, and DD2.

3 Wavelet Transform for Structural Health Monitoring

The ability of the WT to provide time and frequency information of the signal without the limitations of the FFT paved the way for using WT in many engineering applications including SHM. In order to fully appreciate and understand the value that WT can add to the SHM system, it is necessary to discuss the requirements of SHM. Structural health monitoring is somewhat precarious and seeks to balance what one can measure with the technology to decipher the implications of the response values.

3.1 Structural Health Monitoring Needs

The basic needs for SHM with respect to damage identification comprise acquisition, processing, detection, localization, assessment, and prediction. Specifically, the need for an efficient signal de-noising technique is one fulfilled by WT even though signal de-noising techniques captured very little interest in SHM research. On the contrary, as many researchers described SHM systems in the context of anomaly detection [e.g., 41–44], major SHM research using wavelets was focused on feature extraction and pattern recognition. Such hierarchical structures of SHM systems are described by Worden and Dulieu-Barton [41] as enhancing the ability of WT to provide good time and frequency resolutions of the signal. This reconfigured structure motivated researchers to investigate the use of WT to provide a relevant feature extraction for damage detection [e.g., 41–45].

Moreover, Farrar et al. [42] provided an in-depth analysis of the status and needs for damage-prognosis as an estimate of a system's remaining useful life. Essential damage prognosis research demonstrated that a critical issue to sensing and data acquisition is the need to capture the response on varying length and time scales. Patsias and Staszewski [46] showed the possible use of the WT in developing a damage detection method for optically observed mode shapes. Companion information is produced from the WT of mode shapes: the displacement mode shape comes from the approximation signals and the damage location comes from the detail signals. This emphasizes the usefulness of WT to address one of the challenges of sensing and data acquisition, i.e., providing localized information about damage.

The following sections are designed to give specific accounts of the usefulness of WT in SHM systems. This article first introduces a survey about the use of WT for damage detection and diagnosis, and then provides an in-depth discussion of specific features of WT applicable to SHM. It is followed by conferring the important issue of choosing the appropriate wavelets for an analysis. The last section provides a discussion on wavelet-aided analysis where artificial intelligence techniques (e.g., artificial neural networks (ANN) and fuzzy systems) were combined with WT to provide system learning for pattern recognition. Furthermore, a summary of the variety of wavelet features, their corresponding tools and their application in different SHM applications are presented in Table 1. More importantly, this table is provided to give a consistent and lucid flow of knowledge for practitioners on using specific wavelet features and WT tools in SHM.

3.2 A Survey of Wavelet Transform for Damage Detection

This section presents a survey of WT applications in four areas: composite plates, large-scale structures like bridges, civil infrastructure and mechanical systems. It is important to realize that WT is not used in one particular field: it is being used across a variety of fields showing its generality. WT's generality is facilitated by its many aspects and features, all of which are based on the same utility of WT as a signal processing technique.

Composite materials are becoming essential for industrial applications like aircraft design because of its light weight and strength. However, it is notorious for concealing damage in the way of delamination. Kumara et al. [47] and Sohn et al. [48] suggested using CWT to detect delamination of composite structures. The system is designed to analyze the responses of an active SHM system using piezoelectric sensors. Damage detection is performed by observing the signal energy in a wavelet scalogram. Rus et al. [49] showed that the CWT can be used to discriminate between degraded and intact composite. Damage occurrence in thin-walled composite structures, 'sandwich panels' can be detected using a combined WT and finite element algorithm. Qi et al. [50] showed that WMRA can not only detect damage but also detect particular levels of damage. They demonstrated that the energy computed from decomposed signals at three specific frequency ranges was related to three different damage modes in a carbon fiber reinforced polymer (CFRP) composite. Dawood et al. [51] showed successful WMRA signal de-noising of signals generated by Bragg grating sensors that were contaminated with noise generated by thermal effects of a structural composite. Yan and Yam [52] used the energy spectrum and wavelet packet analysis with an index vector to detect small structural damage. Their index vector is a non-dimensional comparison of intact and delaminated plates. The maximum value of the index vector at a particular wavelet decomposition level indicates damage due to an excited and damaged mode. Finally damage location and severity were identified in a composite structure in [53]. Here ANN were used to relate the damage energy detected by WT. Using WT, one can now detect an otherwise unobservable delamination damage in composite plates and structures.

Detecting damage in large-scale structures like bridges can be an especially daunting challenge because of the bridge's complexity and its monitoring requirements; but WT has been successful for SHM in bridge applications.

Table 1 Summary of literature review on WT applications, tools and features for structural health monitoring.

SHM application	*Wavelet tool*	*Feature used*	*Comments*	*References*
Damage detection in civil engineering structures	WPT	Energy	Analysis of energy	[50,85,86]
			Statistical analysis of WPT energy components	[37–39,87,88]
	WT	Coefficients	Changes in coefficients	[58,59,90,91]
			Extracting modal parameters	[107]
			Extracting eigenvectors	[57]
			Discontinuity in coefficients	[89]
		Energy		[84]
		Wavelet ridges	Number of ridges relate to level of damage	[60,76]
		Wavelet connection coefficient	Solving inverse problem	[92]
	WMRA	Spikes and changes in decomposed signals		[78–81,95,98]
	Wavelet aided AI	Anomaly detection in structural dynamic response	Wavelet Neural Networks are used for detecting abnormalities	[121,122]
		Single discontinuity	WT and PCA for identifying damage	[119]
		Signal energy	WMRA is combined with ANN to learn healthy structural response	[43]
			Fuzzy sets used to identify damage in the wavelet domain	[104,129]
		Signal singularity and discontinuity	Hölder exponent in the wavelet domain	[101,102]
		Spectrum analysis	WMRA combined with ANN	[126]
		WPT decomposed signals	WPT aided with ANN	[124]
		WPT coefficients	WPT aided with PNNs	[112]

Damage detection in composites and aerospace structures	WT	Wavelet coherence maps		[33,66]
		Wavelet coefficients	Observe changes in lamb waves	[49,93,94]
		Signal energy	Signals observed by piezoelectric sensors	[47,48]
	WMRA	Wavelet coefficients		[96,97]
		Signal energy		[50]
		Spatial variation of wavelet decomposed signals		[95,98]
	Wavelet aided AI	Energy	WT and ANN	[53,127,128]
Damage detection in mechanical systems	WT	Wavelet coefficients		[59,62–65,67,69]
		Spatial wavelet coefficients		[68]
		Time–frequency localizations	Defects in bearing faults	[72]
	WMRA	MRA	Using orthogonal wavelets	[63,70]
	Wavelet aided AI	Wavelet coefficients	WT and pattern recognition	[120]
			WT and ANN	[11,107,121]
			WT, ANN, and simulated annealing to determine forces	[123]
			WT and Neuro-fuzzy learning	[130]
		Energy of wavelet packets	Combined RNN with WPT	[125]
			Fuzzy clustering of WPT energy components	[88]
Signal de-noising for damage diagnosis	WMRA		Signal decomposition and extraction of noise signals	[51,99,109,126]

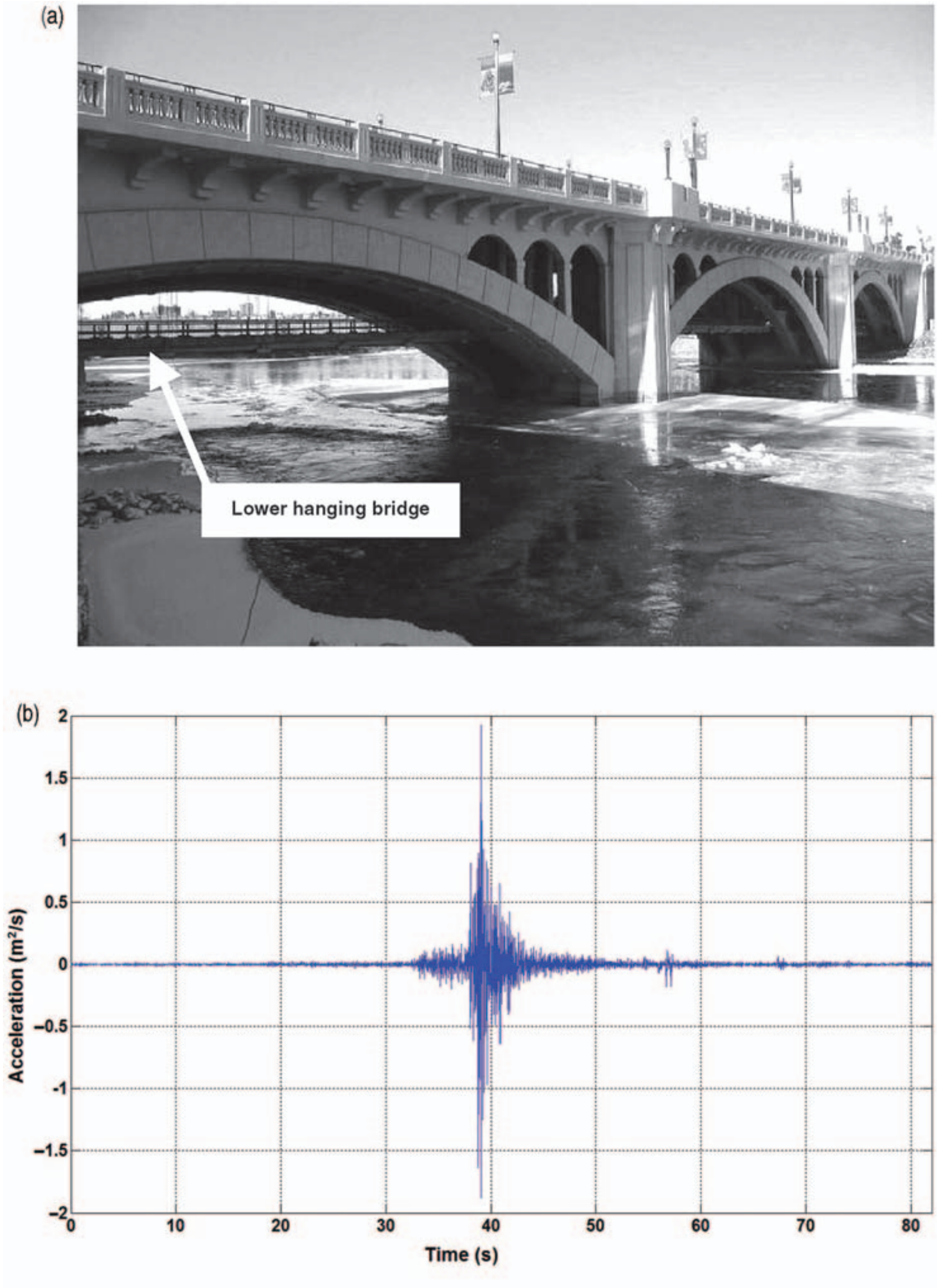

Figure 5 Health monitoring of the Centre Street Bridge in Calgary, Canada [54]: (a) the bridge, (b) measured acceleration of the bridge due to a moving truck 'Time-domain', (c) Fourier transform of the acceleration measured of the bridge due to a moving truck 'frequency-domain', and (d) wavelet transform of the acceleration measured of the bridge due to a moving truck 'scalogram'.

The Centre Street Bridge in Calgary, Canada is monitored by ISIS Canada after its rehabilitation to ensure its performance [54]. The lower level bridge is hung from the concrete arched girders as shown in Figure 5(a). The accelerometers were installed on the deck slab of the lower level bridge to monitor its vertical acceleration. The dynamic activity of the bridge was measured as the truck crossed over the bridge [54]. The time-domain and the frequency-domain

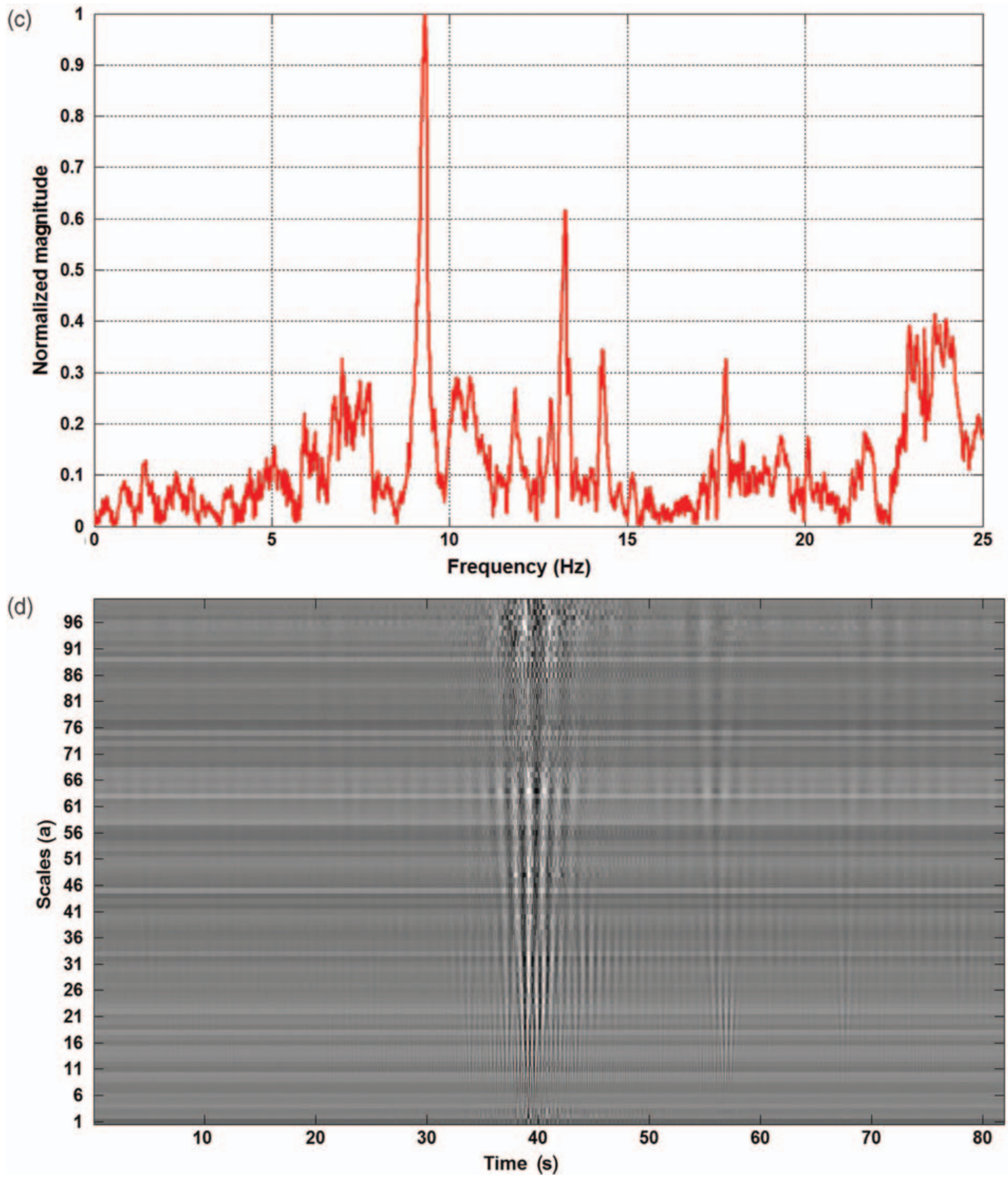

Figure 5 Continued.

representation of the recorded bridge dynamics are shown in Figure 5(b) and (c), respectively. Figure 5(d) presents a typical scalogram of the WT of the dynamic response acceleration signal of the bridge. The WT is performed using the Morlet mother wavelet showing the variation of the amplitude of wavelet coefficients as variation of the gray color intensities with respect to both time and scale. It is obvious that Figure 5(d) is more capable of describing the changes in the

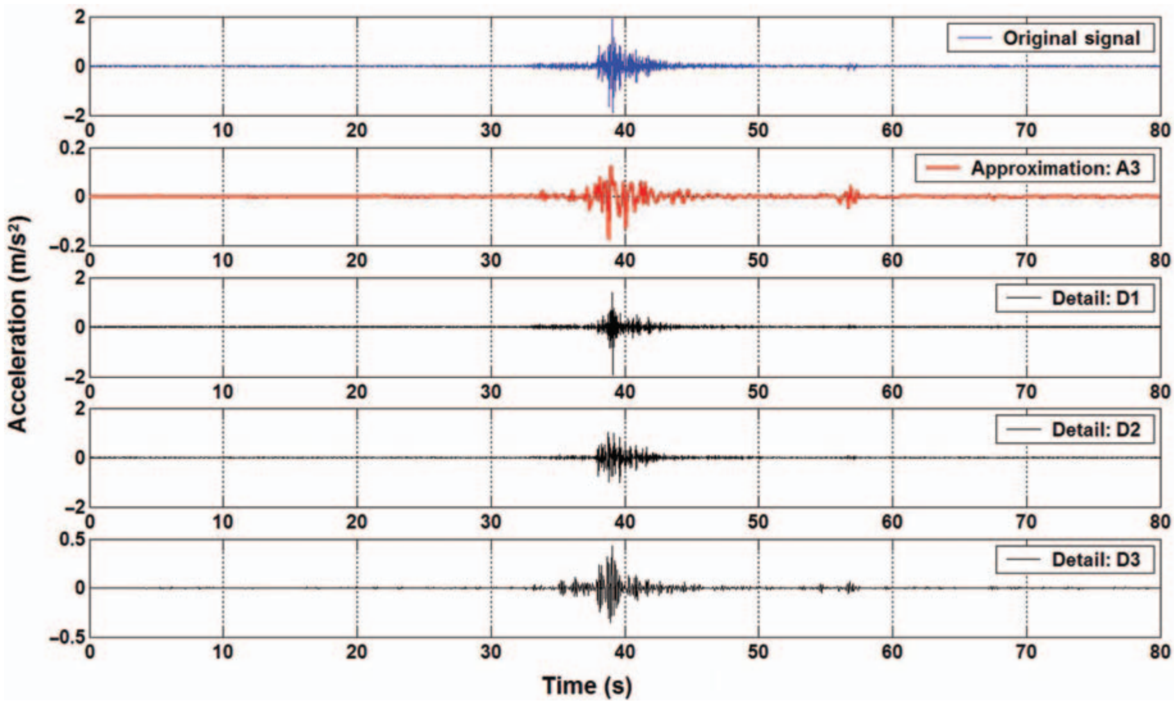

Figure 6 Original acceleration signal from the Centre Street Bridge in Calgary and its decomposed approximation and details using Morlet wavelet at three levels of decomposition.

system dynamics in both time and frequency domains than Figure 5(b) and (c) individually. While a peak acceleration in the structural response is indicated close to 40 s, this peak intensifies at relatively high scale values (relatively stretched wavelets) indicating the existence of high frequency components at this time instant.

The specific analysis uses WMRA to decompose the dynamic signal (see Figure 6). The dynamic response of the Centre Street Bridge in Calgary, Canada that was shown earlier in Figure 5 is decomposed here using the Morlet wavelet at three decomposition levels. Figure 6 presents the components that constitute this decomposition, including the third level approximation (A3) and the first, second, and third level details (D1, D2, and D3). This process of decomposition can be useful for de-noising or for damage detection in SHM systems. However, this analysis in not limited to one form of WT technology.

The structural dynamic analysis of the Centre Street Bridge can also make use of the wavelet packet decomposition. Figure 7(a) and (b) demonstrates the wavelet packet decomposition of the dynamic response of the Centre Street Bridge this time using the Daubechies wavelet and not the Morlet wavelet as in the case with WMRA at three levels of decomposition. Here the flexibility of WT to glean new information is shown by its use of two different wavelet functions for two different WT approaches. This flexibility of use avails itself to the more general use of structural damage detection in the civil infrastructure.

Civil infrastructure damage detection expands on the complexity of damage detection in bridge structures and would be even more daunting if it were not for WT's manifold signal processing capability. For example, the WPT can lead to a variety of decompositions if a selection procedure is not used. Entropy-based criterion has been nominated by many researchers to be the most successful method for selecting the optimal wavelet packet tree for efficient damage diagnosis [37–39]. Coifman et al. [40] and Bukkapatnam et al. [44] explained the basic principles of using entropy-based analysis for signal processing. An entropy-based criterion would try to establish

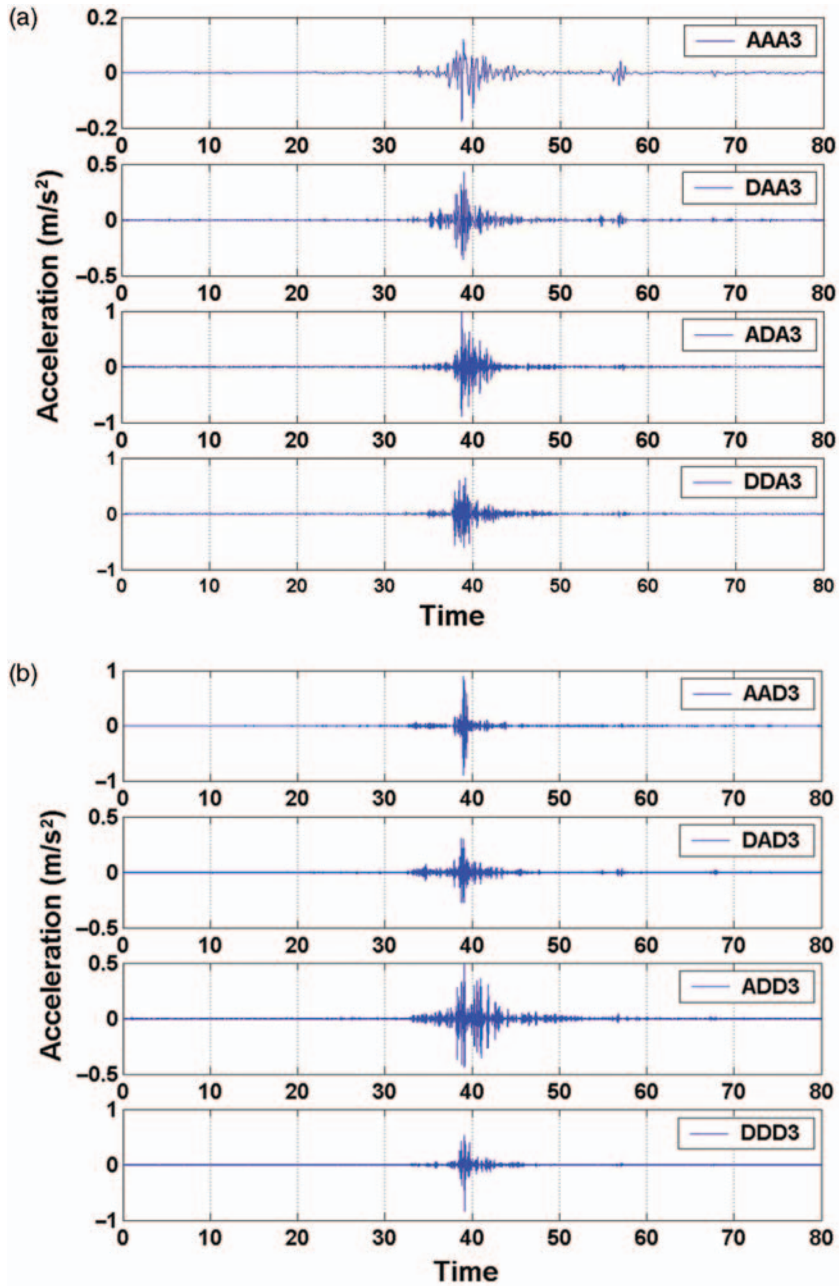

Figure 7 Wavelet packet decomposed structural dynamic signal using Daubechies mother wavelet. Three levels of decomposition. Original signal observed from the Centre Street Bridge in Canada (Figure 6 (b)): (a) WPT components [AAA3, DAA3, ADA3, and DDA3] see Figure 5 and (b) WPT components [AAD3, DAD3, ADD3 and DDD3] see Figure 5.

a crisp division between systematic signals and noise using means of Shannon's information theory [40,44,55]. Ross [55] explained that the entropy of a set of possible outcomes (here systematic signals and noise) where one and only one outcome is true is defined by the summation of probability and logarithm of the probability for all outcomes. The minimization of the entropy targets quantifying the quantity of information in the given data set [55,56]. Bukkapatnam et al. [44] showed that a pure noise signal has the largest entropy while a systematic signal has zero entropy. Thus, by minimizing the entropy of the WPT signal components, an optimal selection of the wavelet packet tree becomes possible. This process permits extraction of the original structural response for damage diagnosis [37–39].

Liew and Wang [57] showed that crack identification of a non-propagating crack in structural systems, such as simply supported beams using WT is much more efficient than using eigenvalues analysis. Douka et al. [58] used CWT to determine the location and size of the crack in a beam using the fundamental mode of vibration. The size of the crack was related to the wavelet coefficients. Similar work was also reported by Gentile and Messina [59] who showed that the CWT can detect damage location and crack size from both noisy and clean data. Melhem and Kim [60] used CWT and the wavelet ridges to detect structural beam damage. It has been shown that using a scalogram, damage occurrence was detectable in an asphalt pavement as well as in a prestressed concrete beam. The number of ridges on the scalogram was shown to be directly proportional to the number of cracks and inversely proportional to the crack growth. Features like wavelet coefficients and wavelet ridges and their uses will be further described in Section 3.3. Gurley and Kareem [61] proved the effectiveness of monitoring an offshore platform's response to wind and waves using a DWT. They also pointed out features that would have been missed by using time or frequency domain methods like distinguishing between responses due to large waves impacting below deck or topside of the platform. In their work [61], it was shown that WT can provide insights into the transfer of energy by plotting the squared coefficients on a time-scale grid which is called the 'mean square map'.

The use of WT for detecting structural damage in mechanical systems is not new and was proposed by many researchers in the last decade [62–64]. It was argued that damage of machinery parts can be predicted by observing the changes in the wavelet coefficients of the wavelet-transformed vibration signal [64,65]. Recently, Giurgiutiu et al. [66] reviewed some different damage identification in helicopter components and identified the WT as one of the most efficient methods for mechanical components health monitoring specially for early identification of incipient damage. Similarly, Rubini and Menegheiti [67] showed that WT can be used for real-time monitoring of fatigue crack growth in rotating machines. Chang and Chen [68] discussed the use of 'spatial wavelet' coefficients for damage detection of a rotating blade. Instead of working in the time domain, the spatial wavelet based approach replaces the time variable with a spatial coordinate allowing for the detection and the positioning of a crack. The crack is detected based on the distribution of the wavelet coefficients. In addition to detecting and locating the crack, the distribution can also indicate crack size. The method was applied to a FEM model of a cracked rotating cantilever blade. Discrete wavelet transform using the Gabor wavelet function with dyadic dilation and translation parameter was applied to the simulated mode shapes. A peak in the distribution of wavelet coefficients indicates a geometric discontinuity (crack) and locates its position [68]. Boltežer et al. [69] compared the ability of CWT to that of FFT in detecting faults in DC electric motors. The analysis proved CWT has superior abilities compared to FFT especially when analyzing data with considerable noise. Three criteria were used for fault detection using CWT, these include the highest coefficient magnitude, the frequency of the first and second harmonics, and the third period between the magnitude pulses at high frequencies.

What distinguishes the use of WT for mechanical systems and highlights its strength is that mechanical systems are of smaller scale than discussed earlier and thus have several dynamic features. For example, Wang and McFadden [63] showed that orthogonal wavelets can be used to detect gear damage in a gear box casing. Similarly, Seker and Ayaz [70] presented an algorithm to detect the effect of aging on bearings using WMRA. Kim and Ewins [71] showed, using simulated rotor system data, that efficient signal noise removal is a major advantage of WT over FT. Yiakopoulos and

Antoniadis [72] proposed using WT for detection of defects in rolling element bearings. The vibration response of the faulty bearings is analyzed using time–frequency localization from the WT. What makes all of these methods under WT work so well is the effective use of certain features of WT. The following section discusses some of the most helpful features that are useful from WT.

3.3 Features of Wavelet Transform

Ridges, spikes, scalogram, energy, coefficients, and thresholding are terms that are often encountered in the vocabulary of the WT practitioner and the latest research literature. This section discusses each feature and cite their respective reference to further allow the reader to understand the WT tools and features as they relate to SHM.

The wavelet ridges are simply the maxima points of the normalized scalogram representing the wavelet coefficients with respect to the different time and scale values [23,29]. The method of wavelet ridges depends on the fact that the instantaneous frequencies determined at the ridges can be used to characterize the signal. For instance, if a slice of the signal is considered at a given time, the peak of the instantaneous spectrum of the signal representing the 'frequency spectrum at this time instant' defines the ridge of this frequency band [73]. As the energy distribution of the monitored signal might be concentrated at more than one frequency, classifying these frequency components requires acquiring the relevant information of the signal by extracting the main frequencies along the ridges [74,75]. Hou et al. [76] suggested the use of WT for instantaneous identification of modal parameters of time-varying systems. The modal parameters can be observed as wavelet ridges in the wavelet scalogram. Hasse and Widjajkusuma [77] suggested that fundamental information about structural dynamics is contained in the maxima lines or ridges of the CWT. It was shown that using analytic wavelets (explained in Section 2) it was possible to extract the corresponding frequencies and damping parameters from the maxima of the wavelet coefficients and the wavelet ridges, respectively. It appears that the use of the wavelet ridges approach might be useful for monitoring progressively developing damage and its effect on the structural response.

Spikes on the other hand represent more discrete signatures of a decomposed signal and come with their own uses. Hou and Hera [78] reported that spikes observed in wavelet transformed signals from damaged structures are difficult to identify at high noise levels but are recognizable with low levels of noise. Corbin et al. [79], Hou et al. [80], and Hera and Hou [81] reported that spikes within high level details (high resolution signals) were observable when damage was introduced to the ASCE Benchmark structure [81] at different levels and types of damage. The benefit of using the spike feature is the potential cost savings if continous monitoring can be avoided. The need to provide continuous monitoring of the structure might have a significant cost burden on the SHM system.

Recently, Gurely et al. [82] introduced the concept to develop wavelet coherence maps to represent correlation between signals. The method utilized the theory of ridges and the concept of hard and smart thresholding to isolate meaningful coherence between signals. The proposed plot is called the 'co-scalogram'. The wavelet coherence maps are described in both frequency and time domains to represent the dual nature of the wavelet analysis leading to the definition of the bi-coherence. Amaravadi et al. [33] suggested analyzing a two-dimensional wavelet scalogram obtained from individual curvature mode shapes. It was shown that the use of wavelet coherence maps is very useful especially in determining damage location of composite materials. The premise of this technique is that if the damage location has a high energy content, it can be observed using the wavelet maps. However, energy has its own benefit when it is the feature of interest.

Karim and Adeli [83] demonstrated that the wavelet energy algorithm can be used efficiently to provide automatic incident detection on urban and rural freeways. Batko and Mikulski [84] proposed the use of signal energy in the wavelet domain to identify the failure of steel ropes using a nondestructive magnetic method. Damage is identified as local energy maxima that can be detected if the energy distribution of the wavelet-transformed signal is observed continuously on the time-scale plots. Damage identification using the energy extracted from WPT has been suggested [85,86]. Moreover, damage diagnosis by statistical analysis of the wavelet packet component energies using means of statistical process control was also suggested [37,38,87,88]. In this method, the wavelet packet energy component was selected as the feature to recognize damage. The wavelet packet component energy can be defined as

$$E^i_{f_j} = \int_{-\infty}^{\infty} f^i_j(t)^2 \mathrm{d}t \tag{13}$$

where $f^i_j(t)$ is the ith level component signal decomposed at the jth level by the wavelet function and translated in the time domain [87]. It was also argued that the WPT component energy is more sensitive than the energy computed using WMRA. These energy indices were reported capable of detecting changes in signal characteristics due to damage.

Next, the wavelet coefficients themselves have been shown to be beneficial. An early application of a damage index based on wavelet coefficients was first reported by Kumara et al. [47]. In this work indices based on wavelet coefficients derived from continuous and discrete wavelet transforms were used for gear fault diagnosis and prognosis. Ovanesova and Suarez [89] discussed the possible extraction of cracking (damage) of frame structures by observing the discontinuity in the coefficients of the WT of the deflection signal of the structure. Pittner and Kamarthi [90] suggested that clustered wavelet coefficients can be used to provide the necessary feature for pattern recognition for damage detection. The Euclidian norm of the detected clusters is used as a sensitive indicator of the change in the structure in response to damage. Wimmer and De Giorgi [91] suggested another technique where the summed wavelet coefficients derived from the CWT of the signal representing the structural dynamic response (e.g., strain) are used to detect the occurrence of damage. Zabel [92] suggested using wavelet connection coefficients to determine the decomposed displacement and velocity signals using experimentally measured accelerations to avoid numerical integration of the acceleration data that is usually noisy. The decomposed displacements and velocities were then used to solve the inverse problem and determine the structure stiffness matrix to detect damage occurrence. The fundamental connection coefficients for a set of linear operators need to be computed for each specific mother wavelet function used in this method [92]. Paget [93] and Paget et al. [94] showed that the change of the amplitude of the wavelet coefficients can be used to detect damage in aerospace composites using Lamb waves generated and received by piezoceramic transducers.

Wang and Deng [95] showed that using WMRA crack propagation can be detected by observing sudden changes in spatial variation of the decomposed signals. It was possible to relate the scaling factor (a) from the wavelet coefficients of WMRA to the frequency of the structure and therefore to represent damage in structures and degradation in heterogeneous materials [96–98]. Analysis of experimental data where damage was induced on a four-degree of freedom system showed spikes in the wavelet decomposed signals. It has been suggested that the magnitude of the spike in the wavelet analysis will be the maximum if the measurement point is next to the damage location [98].

The next feature, thresholding, makes wavelet de-noising a more efficient method than conventional de-noising methods as described in [99]. The procedure starts by choosing an appropriate level for decomposition and decomposing the dynamics signal up to this level. For each level of decomposition, a certain threshold is selected and an appropriate thresholding criterion is applied. The threshold criterion can either be hard or soft thresholding [21,100]. The major challenge in this case is that the maximum level of signal decomposition and the level of noisy signals need to be known *a priori*.

Other tools have also been combined with WT for damage feature extraction. It was suggested that using the magnitude of the

Lipschitz exponent of the WT would be a meaningful signal feature for damage detection. The Lipschitz exponent (also known as the Hölder exponent (HE)) is a measure of how smooth the signal is at a certain point. Mathematically, the Lipschitz (Hölder) exponent represents how many times the signal is differentiable at a certain time [101]. Do et al. [102] recommended using the HE as a measure for signal singularity and discontinuity to detect damage occurrence. The HE of the signal is calculated after it is transformed into the wavelet domain [29]. Although reported to be useful to detect damage for inputs at low frequencies, the HE was reported to be more sensitive to local dynamic responses than to global responses due to damage. The HE was also found to be sensor location dependent which makes it less useful in real applications. Robertson et al. [103] used the HE to detect singularity and determine a structural vibration response signal's regularity. The singularity is defined as an abrupt change (or impulse) that indicates a sudden shift in a structural response and damage. The change in the HE provides a good identification for singularities and the time of damage occurrence.

3.4 Choosing the Appropriate Wavelet

Now that the flexibility of WT is demonstrated, the practitioner is cautioned against a cavalier approach to using WT for signal decomposition. The fundamental aspect in using WT is the correct selection of the wavelet function (that is known as the mother wavelet). While the choice of the wavelet function in many engineering applications has been based on trial and error, when choosing the wavelet function for SHM applications it is important to consider the ability of the chosen wavelet to perform the DWT or the fast wavelet transform (FWT). This is because such transforms are capable of handling discrete digital signals which is necessary for practical SHM applications. Ovanesova and Saurez [89] argued that wavelets with explicit mathematical expression, such as Gaussian, Mexican Hat, Shannon, and Morlet wavelets do not have a scaling function and therefore cannot be used in performing DWT. Successful use of other wavelets, such as the Meyer, the Haar, and the Daubechies wavelets which have a scaling function for damage diagnosis was reported by many researchers [e.g., 89,104,105].

Example research in the literature on choosing the appropriate wavelet in SHM demonstrates the fact that there is no unique wavelet that can be used to satisfy all SHM needs. However, the choice is usually performed with the objective of maximizing the success rate of the damage pattern classifier. For example, Ogaja and Rizos [105] suggested that the Harr wavelet transform (HWT) is an efficient preprocessing technique for deformation monitoring. It was demonstrated using GPS observations that significant events in real-time monitoring can be detected by utilizing the principal component analysis (PCA) on data preprocessed using the HWT. Statistical analysis represented by means of chi-square and F-distributions was able to identify these events with a relatively high probability. A similar result on the efficiency of Harr wavelets over other types of wavelets in detecting signal discontinuity was reported by Alarcon-Aquino and Barria [106]. Conversely, it was demonstrated that most wavelet functions rather than the Morlet wavelets were able to successfully detect unbalance forces in flexible rotors based on measured vibration responses by combining WT and ANN [107]. Furthermore, the best results of combining the Lipschitz exponent with WT were reported when the Mexican Hat wavelet was used in the CWT [101]. Nishikawa et al. [26], Valente and Spina [108], and Ahmad and Kundu [109] showed that it might be useful to choose a mother wavelet that has a complex exponential function (e.g., Gabor mother wavelet) similar to the basis function of FFT when previous experiences with FFT showed promising results. The use of the Gabor filtering technique has also gained wide acceptance in other engineering fields, such as image processing [110]. On the other hand, Yiakopoulos and Antoniadis [72] argued that the choice of the mother wavelet has much less impact on the efficiency of the damage detection in rolling element bearings using WT compared to the choice of the level of decomposition. Moreover, Wu and Wang [111] showed that acoustic mother wavelets that are constructed based on wave equations rather than pure mathematics were capable of decomposing and analyzing seismic signals more efficiently than the known wavelet functions.

Considering another class of wavelets, Ovanesova and Suárez [89] showed that deflection signal discontinuity was better extracted using biorthogonal wavelets than orthogonal wavelets. In biorthogonal wavelets, two wavelets and scaling functions are used rather than one wavelet and scaling function in orthogonal wavelets [29,89]. Duan et al. [112] also the reported successful use of WPT using a biorthogonal wavelet 'bior3.9' mother wavelet.

This discussion was clearly not exhaustive on the use of a particular wavelet function. It is simply meant to show that there is no unique choice that can be recommended for all SHM applications using wavelets and that some functions might be better used in certain situations than others. The choice of wavelet function is simply application dependent and requires careful scrutiny in its use and its results.

3.5 Wavelet Aided Analysis for SHM

This section describes some of the research that has been accomplished using WT with another technique that brings forth the strengths of each method for a more effective means to SHM. For example, Staszewski [2] discussed two approaches for using the WT for SHM applications within the context of statistical pattern recognition. The first method considers classifying all operating modes of the structures and recognizing any structural response as one of the pre-classified modes. The second method compared any unknown structural response to *a priori*-known healthy response subsequently to distinguish the difference between the two responses. While almost all SHM methods utilize one of these two approaches, two major differences between these methods can be recognized. The first is the choice of the feature for building the patterns, and the second is the approach used to aid the wavelet analysis in performing further feature extractions. Significant research utilized artificial intelligence combined with modal analysis to develop the response patterns and to perform efficient feature extraction using ANNs [113–117], genetic algorithms [8], or fuzzy systems [118]. First WT is discussed with a more traditional pattern recognition method, probabilistic analysis.

3.5.1 Wavelet Transform with Probabilistic Analysis Alarcon-Aquino and Barria [106] reported a successful use of the WT with Bayesian analysis in monitoring communication networks and the ability of the wavelet-Bayesian algorithm to detect the presence of abnormal network performance. Moreover, Browne et al. [119] demonstrated that cracking introduces a discontinuity to curvatures and deformation time history. In this method, the displacement of nodes during a fixed time period was analyzed using WT. It was also shown that signal discontinuity detection is dependent on the regularity of the mother wavelets chosen for analysis. A combination of WT and principal component analysis (PCA) was developed to detect crack occurrence in a sewer pipe by a maintenance robot. The proposed method transforms the pipe response into a set of basis vectors. The basis vectors for the WT are determined from an *a priori*-known time–frequency relationship, while the basis vectors for the PCA are determined empirically using variance maximization criterion [106]. Yan et al. [107] examined combining WT and modal analysis to extract a damage index. Modal parameters are extracted from WT coefficients. A statistical approach is proposed to provide confidence intervals to assess the uncertainty in the measured structural damage.

3.5.2 Wavelet Transform with Artificial Neural Networks Thomas et al. [120] showed that engine knock can be detected if engine vibration signal is analyzed using WT and a pattern recognition algorithm is used to classify the time-scale analysis of the transform. Zhao et al. [121] suggested using a multi-dimensional wavelet-neural network for fault diagnosis using dynamic response information. The proposed system utilized a non-orthogonal wavelet sigmoid basis function to perform the transform. Moreover, a wavelet neural network model is used for determining the remaining service lifetime of a structure (known as fault prognosis). The model combines WT with recurrent neural networks (RNNs). The system uses the intrinsic learning abilities of the neural network to predict crack growth extracted from wavelet transformed structural response signals. The crack growth is used to provide information about the structural health and the residual service life of the structure [122].

Lepore et al. [123] combined the use of WT, ANN, and simulated annealing to identify the excitation forces in a rotary machine. A dynamic model for the rotary machine was developed using neural networks and was optimized using simulated annealing algorithms. The method proved successful in identifying the unbalanced forces in flexible rotors based on measured vibration responses. Furthermore, an ANN was used to relate the energy computed using WT from a monitored composite structure to identify damage location and severity for the structure [53]. Reda Taha et al. [43] suggested augmenting WMRA with ANN to evaluate the wavelet norm index (WNI) that represents the energy of the signals in the wavelet domain. The suggested index can describe the change in system dynamics due to damage. Wavelet packet transform was also suggested to address the poor resolution of the signals in the high-frequency region when WT is used. Dynamic signals measured from the structure are transformed into wavelet packet components. The approach was further aided by using an ANN which relates energies computed from the wavelet packet components to the damage status of the structure [124]. Kim and Parlos [125] showed successful diagnosis of induction motor faults by combining recurrent dynamic neural networks with WPT. Moreover, Duan et al. [112] utilized a combination of WPT and probabilistic neural networks (PNNs) for damage identification. Wavelet packet transform coefficients of the cross-correlation functions between the structural responses are extracted. The coefficients are then used to calculate normalized energy coefficients that are fed as input to the PNN with the output being different damage scenarios. The system requires *a priori* knowledge of damaged structure's response which reduces its practical use in detecting damage in civil structures.

Al-Khalidy and Dragomir-Daescu [126] suggested that the signals received from the structure be decomposed using WMRA and have signal noise removed. The signals are reconstructed afterward and a signal envelope is constructed using a digital envelope detector. The spectrum analysis of the signal envelope is then used to detect damage. Another method was also suggested that combined WT and ANN for detecting bearing faults in motor rolling elements. Results better than those reported by the technique explained above were achieved when the summed bi-spectrum signatures instead of wavelets were combined with the ANN in the analyses. The work also shed light on some noteworthy limitations of the WT for fault detection [11]. Combining WT and ANN, Su and Ye [127,128] showed how to concisely quantify signals for structural damage identification of composite structures. Wavelets were used to pre-process the signals and characteristic points were defined as a damage feature vectors named 'digital damage fingerprints (DDFs)'. These DDFs were then used as training data for ANN to recognize a damage pattern. The method was shown to successfully recognize hole and delamination damage in composite structures.

3.5.3 WT with Fuzzy Logic Reda Taha et al. [104] showed that damage detection can be performed by means of wavelet-aided fuzzy pattern recognition. A neuro-wavelet algorithm is used to compute an energy index that is related to damage occurrence. Damage is then recognized using means of fuzzy pattern recognition [104]. The system was proved capable of recognizing damage with an acceptable level of accuracy using finite element data simulated from a prestressed concrete bridge. *A priori* knowledge is needed using finite element (FE) analysis to establish the fuzzy sets representing the different status of structural health. Figure 8 provides a pictorial representation of the fuzzy sets that were used to classify structural health conditions. An alternative wavelet aided approach for damage pattern recognition that utilizes evidence

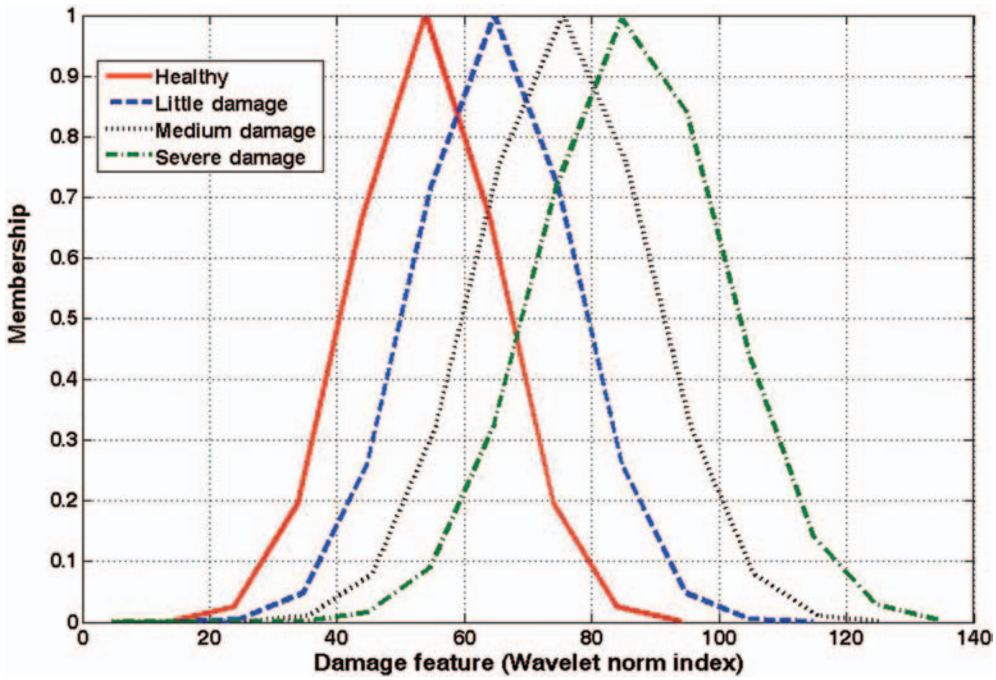

Figure 8 A fuzzy structural patterns representing the different state of structural health based on a wavelet norm index using a neural-wavelet algorithm for structural health monitoring [104].

theory has been recently developed [129]. This approach can perform fuzzy damage pattern recognition without *a priori* knowledge based on FE analysis.

Wang et al. [130] showed that WT aided with a neuro-fuzzy learning algorithm were capable of enhancing a localized damage diagnosis in gear systems. Liu et al. [88] showed that WPT-aided with fuzzy clustering can be used to provide damage pattern recognition for identifying thermal damage during material machining. The WPT was used to extract features from acoustic emission signals used to monitoring the machining process. It was shown that WPT can capture features that are sensitive to thermal damage during machining and that fuzzy clustering of these features was an efficient means of recognizing this damage.

3.6 Additional Comments

A compendium of uses and features of wavelets in SHM have been demonstrated. The different WTs might provide means for fulfilling several needs of SHM systems. However, it needs to be emphasized that it is not the intention of this article to suggest replacing all Fourier-based transforms and analysis in SHM by wavelet ones. On the contrary and as mentioned by many researchers on wavelets (e.g. [23,29]), the use of wavelets shall be made from the scope of complementary and not competitive analysis to FT. It also needs to be emphasized that whereas wavelets are useful tools for SHM, they are not always necessary and shall not be used blindly. The decision to use WT as an alternative means of digital signal processing in SHM shall be based on the value of the analysis and the computation expense of using the transform.

4 Conclusions

The fast growing multi-disciplinary structural health monitoring (SHM) has adopted multiple formulations of the wavelet transfer (WT) in order to overcome many of the limitations of fast Fourier transfer (FFT) in signal processing of structural dynamics. A substantial amount of research was developed in the last decade trying to solve the damage detection problem with the help of the wavelet transform (WT). This research review examined the use of continuous and discrete wavelet transforms, wavelet multi-resolution analysis (WMRA), and wavelet packet transform (WPT) to analyze signals observed from the structure.

This article also provides a review of wavelet theory within the context of its use in SHM systems. The article also spanned across the use of WT in damage diagnosis of different scales of structures ranging from considerably large structures (e.g., bridges) to relatively small structures (e.g., mechanical gear boxes). This work also discussed the different features of the WT and how researchers used these features for damage detection. While, some stand-alone wavelet models proved efficient by being able to provide a damage identification feature, it became obvious that better results were attainable when the WT, WMRA, or WPT were combined with some means of artificial intelligence. Such combination made it possible to extract features from the structural dynamics using the WT while providing means of pattern recognition of these features under different performance environments. Although substantial research work has been developed in the area, there is still a significant need for further research if wavelet theory is to be used as a widespread tool for intelligent SHM.

Nomenclature

A = approximation signal decomposed using WMRA
a = scaling parameter
$a_{J,k}$ = approximation coefficients for WMRA at Jth level of decomposition
ANNs = artificial neural networks
C_{ac} = wavelet coefficients of the acquired signal at different levels of decomposition
C_{ec} = wavelet coefficients of the error signal at different levels of decomposition
CFRPs = carbon fiber reinforced polymers
$C_{j,k}$ = corresponding wavelet coefficients
C_{pc} = wavelet coefficients of the predicted signal at different levels of decomposition
CWT = continuous wavelet transform
D = detail signal decomposed using WMRA
DDFs = digital damage fingerprints
DFT = discrete Fourier transform
$d_{j,k}$ = details coefficients of WMRA at j-level of decomposition
DOF = degrees of freedom
DWT = discrete wavelet transform
$E^i_{f_j}$ = energy of the ith wavelet packet component at the jth level of decomposition
$f^i_j(t)$ = ith component signal superpositioned by the wavelet function and translated in the time-domain at the jth level of decomposition
FE = finite element
FFT = fast Fourier transform
FWT = fast wavelet transform
FT = Fourier transform
$g(k)$ = filter associated with mother wavelet function
$g_j(n)$ = detail signal at j-level of decomposition
$h(k)$ = filter associated with scaling function
HE = Hölder exponent
HWT = Harr wavelet transform
i = ith wavelet packet component signal (Equation 13)
i = complex number
j = jth level of wavelet packet decomposition (Equation 13)
j = scaling coefficient/index
k = shifting coefficient/index
MRA = multi-resolution analysis
n = discrete time variable
N = level of decomposition using wavelet packet transform
M = the level at which WMRA stopped (total number of levels of decomposition)
PCA = principal component analysis
PNNs = probabilistic neural networks
RNNs = recurrent neural networks
SHM = structural health monitoring
STFT = short time Fourier transform
t = time

WMRA = wavelet multi-resolution analysis
WPT = wavelet packet transform
WT = wavelet transform
$x(n)$ = discrete time signal sampled at every T seconds
$x(t)$ = continuous time signal
$X(\omega)$ = Fourier transformed continuous time signal
$\phi(n)$ = scaling function
τ = shifting parameter
ω = angular frequency
$\psi(n)$ = wavelet basis function (discrete)
$\psi(t)$ = wavelet basis function (continuous)
$\psi^i_{j,k}(t)$ = a wavelet packet: a function of three indices, where i, j, and k are integers representing the modulation, the scale, and the translation parameters, respectively.

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