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Cooperative Game Theory for Distributed Spectrum Sharing

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Abstract— **There is a need for new spectrum access protocols that are opportunistic, flexible and efficient, yet fair. Game theory provides a framework for analyzing spectrum access, a problem that involves complex distributed decisions by independent spectrum users. We develop a cooperative game theory model to analyze a scenario where nodes in a multi-hop wireless network need to agree on a fair allocation of spectrum. We show that in high interference environments, the utility space of the game is non-convex, which may make some optimal allocations unachievable with pure strategies. However, we show that as the number of channels available increases, the utility space becomes close to convex and thus optimal allocations become achievable with pure strategies. We propose the use of the Nash Bargaining Solution and show that it achieves a good compromise between fairness and efficiency, using a small number of channels. Finally, we propose a distributed algorithm for spectrum sharing and show that it achieves allocations reasonably close to the Nash Bargaining Solution.**

I. Introduction

Opportunistic spectrum access has become a high priority research area, since the limited spectrum available is inefficiently utilized. We study the problem where nodes in a wireless network try to gain access to bandwidth by competitively allocating their own transmission power across multiple channels. We specifically study the case where no coordinating entity exists in the network, and nodes need to arrive in a distributed fashion at a fair and efficient sharing of available channels. However, we assume the presence of an entity (such as the FCC) that can enforce agreements between players, as this is a requirement for cooperative games.

Game theory studies mathematical models of conflict and cooperation among intelligent and rational decision makers. Non-cooperative game theory has been used to analyze wireless networks in [1]–[4]. Iterative water-filling [5] and interference avoidance [6] can also be viewed as types of non-cooperative games. In cooperative game models, players coordinate to achieve a mutually desirable solution. Cooperative game theory for analysis of networks and spectrum sharing has been studied in [7]–[9]. Different fairness definitions have been proposed in the literature, including proportional fairness [10] and max-min fairness [11].

Our primary contribution is to analyze the utility space of the spectrum sharing game as the number of channels increases and show that the Nash Bargaining Solution achieves a fair and efficient spectrum allocation. We begin by showing that in a high interference environment and a finite number of channels, the utility space of the spectrum sharing game is non-convex. Non-convexity can lead to optimal points that are mixed strategies, possibly not achievable with pure strategies[1]. We show that by increasing the number of channels available, the utility space becomes closer to convex and more optimal points can be achieved with pure strategies. Next, we show that the NBS allocation provides a reasonable compromise between efficiency and fairness. We argue what the fair spectrum allocation should be and show that the NBS achieves it. In our simulation results we show that maximum NBS efficiency can be achieved even with a small number of channels. Another contribution of our work is to propose an algorithm that can achieve the proposed allocation in a distributed manner, using only local information. The algorithm focuses not on implementing a bargaining process between players, but on arriving at the NBS, since that is the expected outcome of the bargaining process. To find the NBS point requires the solution to a non-linear, non-convex optimization problem. The algorithm implements a distributed approximation to the optimization problem. In our simulation results, we show that the algorithm reasonably approximates the NBS allocation using only information about nodes within approximately two hops.

The remainder of the paper is organized as follows. Section II gives a brief description of cooperative game theory and the NBS. Section III discusses the spectrum sharing problem, including the game model, the utility space and the NBS in the context of the spectrum sharing game. Section IV presents the distributed algorithm and show that it converges. Section V provides our simulation results. Finally, section VI provides some concluding remarks.

II. Cooperative Game Theory and Nash Bargaining

Game theory provides a set of mathematical tools that are useful in analyzing complex decision problems with interactions between self-interested decision makers, called players. The basic component of game theory is a game, $G = \langle M, A, \{u_i\}\rangle$. $M = \{1, \ldots N\}$ is the set of players, A_i is the set of actions for player i, $A = A_1 \times ... \times A_N$, and u_i is the objective function, sometimes called utility function, which player i wishes to maximize. For convenience, the set of actions for all players except player i is denoted as $A_{-i} = A_1 \times \ldots \times A_{i-1} \times A_{i+1} \times \ldots \times A_N$.

In a cooperative game, players bargain with each other before the game is played. If an agreement is reached, players act according to the agreement reached, otherwise players act in a non-cooperative way. Note that the agreements reached must be binding, so players are not allowed to deviate from

[1] A mixed strategy is composed of a set of possible actions and a probability distribution over this action space. A pure strategy is a mixed strategy that consists of a single possible action.

what is agreed upon. John Nash wrote in his seminal paper on cooperative games [12] that to understand the outcome of a bargaining game, we should not focus on trying to model the bargaining process itself, but instead, we should list the properties, or axioms, that we expect the outcome of the bargaining process to exhibit. This way of analyzing cooperative games is called *axiomatic bargaining game theory* [13].

Before we proceed, we need to introduce some terminology. An *agreement point* is any action vector $\mathbf{a} \in A$ that is a possible outcome of the bargaining process. A *disagreement point* is an action vector $\mathbf{a} \in A$ that is expected to be the result of non-cooperative play given a failure of the bargaining process (i.e., what will happen if players cannot come to an agreement). Clearly, the utility achieved by every player at any agreement point has to be at least as much as the utility achieved at the disagreement point. A *bargaining solution* is a map that assigns a solution to a given cooperative game. Following is the bargaining solution proposed by Nash in [12].

Definition 1: **Nash Bargaining Solution (NBS)**. Let $U = \{(u_i(a))|a \in A\}$ be a convex, closed and upper bounded subset of $\mathbb{R}^N$, a_0 be the disagreement point, $u_i^0 = u_i(a_0)$ be the utility of player i achieved at the disagreement point, and $U_0 = \{\mathbf{u} \in U | \mathbf{u} \geq \mathbf{u}^0\}$ be the set of achievable utilities. Then $\mathbf{u}^* = \phi(U, \mathbf{u}^0)$ is a NBS if it meets the following conditions:

1) Individual rationality (IR): $u_i^* \geq u_i^0$. That is, $\mathbf{u}^* \in U_0$.
2) Pareto optimality (PO): If there exists $\mathbf{u}' \in U_0$ such that $u_i' \geq u_i^*, \forall i$ then $u_i' = u_i^*, \forall i$.
3) Invariance to affine transformations (INV): if $\psi : \mathbb{R}^N \rightarrow \mathbb{R}^N$, $\psi(\mathbf{u}) = \mathbf{u}'$ with $u_i' = c_i u_i + d_i, c_i, d_i \in \mathbb{R}, c_i > 0, \forall i$, then $\phi(\psi(U), \psi(\mathbf{u}^0)) = \psi(\phi(U, \mathbf{u}^0))$.
4) Independence of irrelevant alternatives (IIA): if $\mathbf{u}' \in V \subset U$ and $\mathbf{u}' = \phi(U, \mathbf{u}^0)$ then $\phi(V, \mathbf{u}^0) = \mathbf{u}'$.
5) Symmetry (SYM): if U is symmetric with respect to i and j, $u_i^0 = u_j^0$, and $\mathbf{u}' = \phi(U, \mathbf{u}^0)$, then $u_i' = u_j'$.

Conditions 3-5 are the so called fairness axioms. The INV axiom assures that the solution is invariant if affinely scaled. The IIA axiom states that if the domain is reduced to a subset of the domain that contains the NBS, then the NBS is not changed. The SYM axiom states that the NBS does not depend on the labels, i.e. if two players have the same disagreement utility and the same set of feasible utility, then they will achieve the same NBS utility.

The following theorem, first proposed by Nash for two-player games [12], and later extended for more than two players [13], shows how we can find the unique NBS for convex utility spaces.

Theorem 1: Let $I = \{i \in \{1, \dots N\} | \exists \mathbf{u} \in U_0, u_i > u_i^0\}$ be the set of players that can achieve a utility strictly greater than the disagreement utility. The maximizer of the Nash Product (NP), $\mathbf{u}^*$, is the unique NBS:

$$\mathbf{u}^* = \arg \max_{\mathbf{u} \in U_0} \prod_{i \in I} (u_i - u_i^0). \quad (1)$$

Theorem 1 states that the convexity of the utility space (U) is a sufficient condition to guarantee that the maximizer of the NP is the unique NBS. This condition is sufficient but not necessary. In other words, there may exist non-convex utility spaces where the maximizer of the NP is a unique NBS. Following is a definition of a $\mathbf{u}^0$-comprehensive set as well as a theorem, proposed by [14], that shows that $\mathbf{u}^0$-comprehensive sets are guaranteed to satisfy some of the NBS axioms.

Definition 2: **$\mathbf{u}^0$-comprehensive**. A set $S \subset \mathbb{R}^N$ is said to be $\mathbf{u}^0$-comprehensive if $\mathbf{x}, \mathbf{y} \in \mathbb{R}^N$ such that $\mathbf{u}^0 \leq \mathbf{y} \leq \mathbf{x}$, $\forall i$, then $\mathbf{x} \in S$ implies $\mathbf{y} \in S$.

Theorem 2: Let U be a non-convex, closed, bounded, and $\mathbf{u}^0$-comprehensive utility space. The maximizer of the NP for set U satisfies the NBS axioms: INV, IR and IIA.

This result shows that a maximizer of the NP for a $\mathbf{u}^0$-comprehensive set satisfies most of the NBS axioms. Noteworthy is the fact that the PO axiom is not guaranteed to be satisfied. This is because a $\mathbf{u}^0$-comprehensive set may be non-convex and in game theory, all possible mixed strategies (which we may think of as convex combinations of pure strategies) are considered to be available strategies to the players. So, it is possible for a mixed strategy to obtain an expected utility that Pareto dominates the NP maximizer of the $\mathbf{u}^0$-comprehensive set. In the remainder of this manuscript, when we refer to Pareto optimality with respect to only pure strategies, we call it *Limited Pareto Optimality* (LPO). In the next theorem, we show that under certain conditions, the maximizer of the NP for a non-convex set can be a unique NBS.

Theorem 3: Let U be a closed and bounded utility space and U_c be the smallest convex set that contains U (the convex hull of U). If U has a unique maximizer of the NP, u^*, which coincides with the NP maximizer of U_c, u_c^*, then u^* is the unique NBS for U.

Proof: We know that $U \subset U_c$. Since the unique maximizer of the NP for both U and U_c are the same, this implies $u^* = u_c^* \in U$. Thus, by the IIA axiom, u^* is the unique NBS for U. ■

Theorem 3 tells us that if a non-convex set coincides with its convex hull on the maximizer of the NP, then the NP maximizer of the non-convex set is a NBS. We will subsequently show that the utility space for the spectrum sharing problem is $\mathbf{u}^0$-comprehensive and not always convex, but given the right conditions, can be nearly convex. Thus, the NP maximizer for the spectrum sharing utility space satisfies most of the NBS axioms and can satisfy all axioms under the conditions where it is nearly convex.

III. Spectrum Sharing

The spectrum sharing problem addresses the issue of how to allocate the limited available spectrum among multiple wireless devices. The problem has two important, orthogonal goals: efficiency and fairness. The allocation of spectrum should utilize as much of the resource as possible. However, when utilization is maximized, fairness can be compromised.

Following is a detailed discussion of the spectrum sharing problem. We propose a game model and discuss our assumptions. We then analyze the resulting utility space of the game, examine its properties and show that when the available spectrum is divided into a large enough number of channels, efficient spectrum allocation is achieved with a pure strategy.

A. Game Model

The spectrum sharing problem can be modeled as follows. The available bandwidth is divided equally into multiple channels. Each wireless device (referred to as node) can transmit in any combination of channels at any time and can set its transmit power on each channel. Each transmitting node is only interested in communicating to a single receiver node. Receiver nodes do not transmit and thus are not considered players in the game (since they will act in coordination with the transmitter).

Let $\chi = \{1, \ldots, K\}$ be the set of available channels, B be the aggregate bandwidth, with each channel having bandwidth $\frac{B}{K}$, and N be the number of transmitter nodes in the network. We formulate the spectrum sharing game as follows: $M = \{1, \ldots, N\}$, $P_i^\chi = \{(p_i^k)_{k\in\chi} | p_i^k \geq 0, \sum_{k\in\chi} p_i^k \leq P_{max}\}$ and $P^\chi = P_1^\chi \times \ldots \times P_N^\chi$. Let $\mathbf{p} \in P^\chi$ and $u_i(\mathbf{p}) = C_i(\mathbf{p})$, where $C_i(\mathbf{p})$ is the Shannon capacity:

$$C_i(\mathbf{p}) = \frac{B}{K}\sum_{k=1}^{K} \log_2\left(1 + \frac{H_{ii}^k p_i^k}{\frac{\sigma^2}{K} + \sum_{j\neq i} H_{ji}^k p_j^k}\right) \qquad (2)$$

where p_i^k is the power transmitted by node i on channel k, P_{max} is the maximum transmit power, H_{ji}^k is the channel gain from j to the receiver of i on channel k, and σ^2 is the thermal noise for the entire bandwidth B.

B. Utility Space

This section discusses the properties of the utility space for the spectrum sharing game. Specifically, we explore the effect of increasing the number of channels on the utility space. We show that given enough channels, for any mixed strategy we can find a pure strategy that achieves a utility at least as high as the mixed strategy. This result implies that to achieve efficient spectrum use we need not employ mixed strategies. This also implies that in the cases where the utility space is not convex (some mixed strategies are not included in the space), increasing the number of channels increases the number of mixed strategies that are included in the set (the space becomes closer to being convex).

Theorem 4: For some χ and finite subset $S^\chi \subset P^\chi$, consider a mixed strategy defined by probability distribution π, such that $\pi(s)$ is rational for all $s \in S^\chi$. For any such mixed strategy, there exists a pure strategy $t \in P^{\chi'}$, with $P^{\chi'}$ associated with a set of channels χ', that yields the same utility as the mixed strategy. The proof is in Appendix A.

Theorem 4 says that we can replicate any mixed strategy with a pure strategy[2]. This result implies that, given enough channels, we do not need to employ mixed strategies to achieve efficient spectrum utilization. Another less evident, yet important, implication is that we can make the utility space closer to convex by increasing the number of channels. If we can replicate mixed strategies with pure strategies, we reduce the number of convex combinations not included in the utility space and thus make it closer to convex.

[2]We can only replicate mixed strategies with rational probabilities, but since rationals are dense in the reals, we can approximate any mixed strategy arbitrarily closely.

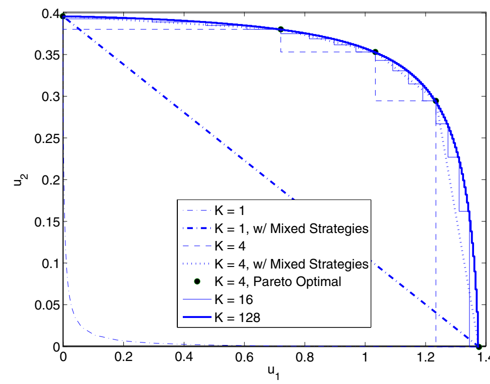

Fig. 1. Example utility space for 2 player game under high interference

In the remainder of this section, we investigate the behavior of the utility space as the number of channels increases. For purposes of illustration, we assume a game with only two players. For readability, when we graph a utility space, we only graph the upper boundary of the set (that is, the set includes all the points enclosed by the upper boundary, the x axis and the y axis), as the LPO points are all in this boundary.

Figure 1 shows the utility space for the case where both nodes experience interference stronger than their received signal strength ($H_{21}^k \gg H_{11}^k, H_{12}^k \gg H_{22}^k, \forall k$), for increasing values of K. The figure also shows the utility space when all possible mixed strategies are included (convexified space) for some values of K.

By examining the utility space for $K = 1$, we can clearly see that the boundary of the mixed strategies dominates the boundary of the pure strategies. This shows that under the high interference case for $K = 1$, using mixed strategies can be more efficient than adopting pure strategies. Now examine the utility space for $K = 4$. We notice that the boundary of this case dominates most of the boundary for the $K = 1$ case with mixed strategies. Specifically, the LPO points for $K = 4$ overwhelmingly dominate the mixed strategies for $K = 1$. These points are not dominated by the mixed strategies for the case $K = 4$, which implies that these points are PO. From the results of Theorem 4 we would expect the boundary for $K = 4$ to be at least as efficient as some mixed strategies for the $K = 1$ case. The overwhelming dominance of the $K = 4$ over $K = 1$case is due also in part to an effect that is most noticeable in high interference environments. When interference is high, optimal mixed strategies usually involve only a single node transmitting at a time. By increasing the number of channels, we allow for frequency separation of transmissions and thus decrease interference. The decrease in available bandwidth to each player is more than offset by the decrease in interference. Corollary 1 formally presents this effect.

Corollary 1: Consider a mixed strategy as Theorem 4, such that $s_i^k > 0$ for some $k \in \chi$ implies $s_j^k = 0$ for all $j \neq i$ and $k \in \chi$. Also, for every i there exists $s \in S^\chi$ such that $s_i^k > 0$ for some k. For any such mixed strategy, there exists a pure strategy $t \in P^{\chi'}$, with $P^{\chi'}$ associated with a set of channels χ', that yields utility greater than the mixed strategy. The proof is in Appendix B.

Now compare the boundaries for $K = 4$ with and without mixed strategies. We can see that using mixed strategies can

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achieve PO points that are not achievable with pure strategies. Although these points do not dominate the pure strategy PO points, it may be desirable to achieve them to meet fairness objectives. Finally, examine the utility spaces for $K > 4$. We can see that as K increases, the pure strategy utility space has more PO points. A more subtle, yet important effect of increasing channels is that the utility space becomes closer to its convex hull. As the utility space becomes convex, we can achieve more PO points with pure strategies.

IV. Distributed Algorithm

Our goal is to design a distributed algorithm that achieves the NBS for the spectrum sharing game. We need the algorithm to operate only with local information and no centralized control. In this section we show that nodes can be aggregated into overlapping groups, which we can then leverage to distribute the computation of the NBS. Nodes within each group are in close proximity, which allows nodes to only use local information. Finally, we propose an algorithm for computing an approximation to the NBS and prove its convergence.

We make the following assumptions:

1) There is an underlying method for information exchange such that nodes within two hops can communicate within a time scale shorter than the time scale for updates to channel allocation.
2) Nodes run the algorithm at random intervals such that the probability that two or more nodes (within two hops of each other) run the algorithm simultaneously is small.
3) The execution time of the algorithm is small relative to the interval between executions of the algorithm.
4) The initial agreement point is **0**.

The NBS solution is based on the assumption that all players in the game bargain as a group to reach a cooperative solution to the game. This means that in the spectrum sharing game a node cooperates with all nodes in the network. That is, we must consider the utility achieved by all nodes in the network in order to implement the NBS. However, we know that a node's effect is limited to other nodes within close proximity. This allows us to limit the scope of a node's bargaining to a subset of the network. Consider the following concept:

Definition 3: Let $R > 0$ and $Rx(i)$ be the set of receiver nodes of node i. The *interference zone* (IZ) of node j, with interference radius R, is defined as $IZ_R(j) = \{i | \exists k \in Rx(i), \mathtt{distance}(j,k) < R, j \neq k\}$

The interference zone for node j is the set of transmitter nodes such that one of their receivers is within distance R of node j. If we set R to a large enough value, then node $i \notin IZ_R(j)$ can ignore node j's actions. Thus, we can approximate the utility function of node i as follows:

$$\tilde{u}_i(\mathbf{p}) = \frac{B}{K} \sum_{k=1}^{K} \log_2 \left(1 + \frac{H_{ii}^k p_i^k}{\frac{\sigma^2}{K} + \sum_{l \in J_i} H_{li}^k p_l^k} \right) \quad (3)$$

where $J_i = \{j | i \in IZ_R(j)\}$. This approximation drops the interference terms from nodes that are far enough away from node i's receivers such that they cause negligible interference. Then, the utility function of node $i \notin IZ_R(j)$ is independent of node j's actions.

Algorithm 1 Distributed NBS Computation

```
IZ = i ∪ {j | distance(Rx(j), i) < R, u_j > 0}
oldNP = ∏_{j∈IZ} u_j(p)
p̂ = MaximizeNP(i, IZ, δ)
newNP = ∏_{j∈IZ} u_j(p̂)
if newNP > (1 + tol) * oldNP then
    p_i^k = p̂_i^k
end if
```

Consider node j maximizing the NP while the other nodes' actions remain constant. Let $\mathbf{p}' \in P^K$ be the current strategy employed in the network, $\rho_j = \{\mathbf{p} | \mathbf{p} \in P^K, p_i^k = {p'}_i^k, \forall i \neq j\}$ be the set of strategies such that only node j has an action different from $\mathbf{p}'$, and $IZ_R^+(j) = IZ_R(j) \cup \{j\}$. Then,

$$\max_{\mathbf{p} \in \rho_j} \prod_i \tilde{u}_i(\mathbf{p}) = \prod_{l \notin IZ_R(j)} \tilde{u}_l(\mathbf{p}') \times \max_{\mathbf{p} \in \rho_j} \prod_{i \in IZ_R^+(j)} \tilde{u}_i(\mathbf{p}). \quad (4)$$

Equation 4 demonstrates that, by using the approximation for the utility function, nodes need only consider nodes in their IZ when maximizing the NP. This result allows us to design an algorithm to calculate the NP for the entire network only using local information at each node. We propose Algorithm 1 for calculating the NP, which consists of nodes choosing their actions so as to maximize the NP of their interference zone. The algorithm only updates the node's actions if the value of the NP is increased by at least tol percent. The algorithm calls the function $\mathtt{MaximizeNP}(i, IZ, \delta)$, which calculates the maximum of the NP for all nodes in the IZ with respect to node i's actions, within a neighborhood of the current operating point of size δ. Following is a proof of convergence for the algorithm. The proof shows that the algorithm converges, but not to what value it converges to. In the simulation section, we show that the algorithm indeed converges to a value close to the NBS.

Proof: **Convergence of Algorithm 1**. Since P is compact, $\tilde{u}_i(\mathbf{p})$ is upper-bounded. Since $\tilde{u}_i(\mathbf{p}) \geq 0$, then the NP is also upper-bounded. Eq. 4 shows that node i, by maximizing the NP for nodes in its IZ, actually maximizes the NP for the entire network. So, after a node executes the algorithm, the value of the NP does not decrease. The value of the NP is upper-bounded and non-decreasing and thus converges. ■

V. Simulation Results

In this section we present our simulation results. All simulations consist of the following setup. There are N transmitter-receiver pairs of nodes placed randomly in an R_a meter by R_a meter square. A receiver is no more than 100m away from its transmitter. The total bandwidth, B, is evenly divided into K channels. The propagation loss exponent is 4 and the root-mean-square (RMS) delay spread is $1\mu s$. The antenna gain is 0.01, the maximum transmission power is 100mW, and the noise level is -80dBm for the entire bandwidth. All capacity numbers are reported as multiples of B and are the actual capacity achieved, not the approximation used by the algorithm. Unless otherwise stated, all simulations are averaged over 100 randomly chosen network topologies, $N = 10$ and $tol = 2\%$.

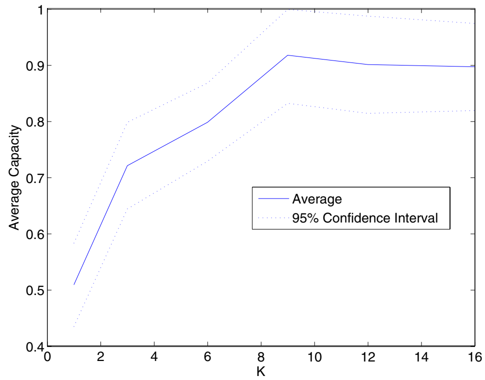

Fig. 2. Average capacity per node achieved by the NBS. ($R_a = 200m$)

TABLE I

SPECTRUM ALLOCATION SCHEME COMPARISON

$N = 5, R_a = 200m$

	NBS	MaxSum	MaxMin	WF
Avg u_i	1.18	1.31	0.69	1.16
Min u_i	0.21	0.02	0.25	0.13
Fairness Score	1.00	0.11	0.69	0.73

In our simulations, we aim to show that the NBS obtains a fair and efficient spectrum allocation. To achieve this, we compare the NBS to three other spectrum allocation schemes: MaxMin, MaxSum and Water-filling. Water-filling is an iterative scheme where each node maximizes its own utility without regard to the utility other nodes achieve. The NBS and Water-filling algorithms are simulated in a random round robin fashion.

To measure the relative fairness of a spectrum allocation, we need a metric that captures how close the allocation obtained by the algorithm is to the value of the NBS. We propose a metric, which we call the *fairness score*, based on the NBS. The metric attains values between 0 and 1.0, where a value of 1.0 indicates a utility allocation equal to the NBS. As the value decreases, the allocation becomes skewed and thus, less fair.

Definition 4: Let $\mathbf{u}^*$ be the utility achieved by the nodes at the true NBS and $\mathbf{u}$ be some other utility allocation. The fairness score of $\mathbf{u}$ is: $\left(\prod_i \frac{u_i - u_i^0}{u_i^* - u_i^0}\right)^{\frac{1}{N}}$

A. NBS Efficiency and Fairness

In this section we investigate the spectrum allocation achieved by the NBS and compare its fairness and efficiency to those of the other spectrum allocation schemes. In this section, all NBS values presented are the true NBS points, not the approximations by the proposed algorithm.

Table I shows the average and minimum of the capacity achieved for all nodes and the average fairness score, with $R_a = 200m$ and $K = 8$. We can observe that the average capacity achieved by the NBS is close to that by MaxSum and much more than by MaxMin. The NBS also achieves a minimum capacity very similar to MaxMin and significantly higher than MaxSum. This shows that NBS balances efficiency and fairness effectively and significantly better than MaxSum or MaxMin. Water-filling achieves average utility similar to NBS, but a much lower minimum capacity and fairness score. All schemes achieve fairness scores significantly lower than the optimal. Notable is the score of MaxSum, which is near zero, as in many cases one player is assigned zero utility.

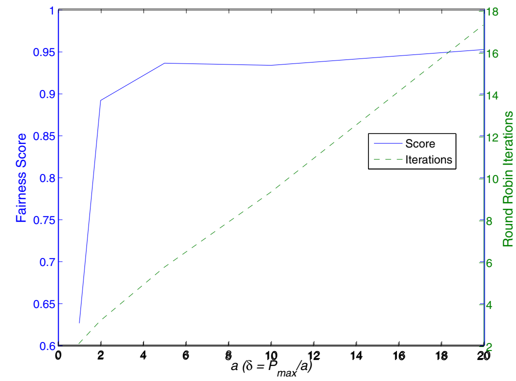

Fig. 3. Effect of delta on average score. ($K = 16, R_a = 200m$)

Figure 2 shows the average capacity per node achieved by the NBS, for several values of K, with $R_a = 200m$. Since R_a is small relative to the transmission range, we expect significant contention for the spectrum. As we showed in the previous section, we see that under a high interference environment, the average capacity per node increases as the number of channels increases. We see that after a certain point ($K = 9$), the average capacity stops increasing. This is not surprising, as $K = 9$ is close to the point where every node can utilize a single channel exclusively and thus avoid significant interference.

B. Algorithm Performance

Figure 3 examines the effect of the parameter δ, which controls the amount of the space the algorithm can search at a given iteration. We can see that as $\delta \to \infty$ the algorithm performs rather poorly. This is because the search space is unconstrained and the first node to execute the algorithm will skew the allocation in its favor. By limiting the space each node can explore at any iteration, it limits the skewing of the allocation. The figure shows how making δ smaller, the algorithm converges to a higher fairness score. However, this comes at a cost of slower convergence. The figure shows the average number of round robin iterations required for the algorithm to converge. It clearly shows the algorithm requires more iterations to converge, as δ decreases. For the rest of the simulations we set this parameter to $\delta = P_{max}/5$, as that is the point where we have the best compromise between fairness score and convergence speed.

In Figure 4, we show the performance of the algorithm as a function of the interference radius. Clearly, we would like this radius to be as small as possible, so as to minimize the information exchange required. As expected, the fairness score increases as R increases. However, we notice that even when $R = 100$, the fairness score achieved by the algorithm is greater than 0.94. This result is encouraging as it tells us that we can significantly limit the number of nodes involved in the bargaining process and still achieve a reasonable outcome.

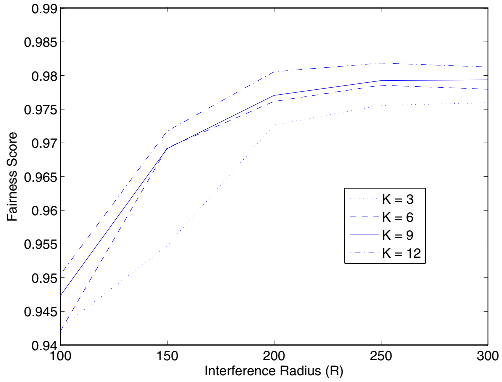

Fig. 4. Performance for various values of Interference Radius (R)

VI. Conclusions

The spectrum sharing problem consists of dividing a given amount of spectrum among many nodes in a way that is efficient and fair. In this manuscript we have addressed this problem by formulating a cooperative game model of the spectrum sharing problem. We analyzed the utility space of the spectrum sharing game and showed that efficiency is maximized by increasing the number of channels. We also showed that the utility space convexifies and thus maximizing the NP gives us an allocation that approximately satisfies the NBS axioms. Consequently, we show that the NBS allocation provides a reasonable compromise between efficiency and fairness, as it achieves allocations with minimums close to MinMax and efficiency close to the MaxSum. Finally, we proposed an algorithm that, with only local information, approximates the maximization of the NP and show that it converges quickly to a value close to the true NBS.

Appendix

A. Proof of Theorem 4

By construction, for any $\mathbf{s} \in S^{\chi}$ and mixed strategy π, there exists positive numbers $a_{\mathbf{s}}, \beta$ such that $\pi(\mathbf{s}) = \frac{a_{\mathbf{s}}}{\beta}$ ($\sum_{\mathbf{s} \in S^{\chi}} a_{\mathbf{s}} = \beta$). We will now find a pure strategy that yields the same utility as this pure strategy. Let $\chi' = \{1, \ldots, \beta K\}$. Let us partition χ' into disjoint sets $\phi_{\mathbf{s},k}$, $\mathbf{s} \in S^{\chi}$ and $k \in \chi$, with $\bigcup_{\mathbf{s} \in S^{\chi}, k \in \chi} \phi_{\mathbf{s},k} = \chi'$ and with $|\phi_{\mathbf{s},k}| = a_{\mathbf{s}}$. It is assumed that for all $k' \in \phi_{\mathbf{s},k}, H_{ij}^{k'} = H_{ij}^{k}$ for all $i, j \in M$. Let us construct a pure strategy, $\mathbf{t} \in P^{\chi'}$, as follows: for each $k' \in \chi'$, find $\mathbf{s}, k$ such that $k' \in \phi_{\mathbf{s},k}$ and for each $i \in M$ set $t_i^{k'} = \frac{s_i^k}{\beta}$.

Note that t is a valid strategy, as:

$$\sum_{k'=1}^{\beta K} t_i^{k'} = \sum_{\mathbf{s} \in S^{\chi}} \sum_{k=1}^{K} \sum_{k' \in \phi_{\mathbf{s},k}} t_i^{k'} = \sum_{\mathbf{s} \in S^{\chi}} \pi(\mathbf{s}) \sum_{k=1}^{K} s_i^k \leq P_{max}$$

The utility achieved by player i for strategy $\mathbf{t}$, $u_i(\mathbf{t})$, is,

$$\sum_{\mathbf{s} \in S^{\chi}} \sum_{k=1}^{K} \sum_{k' \in \phi_{\mathbf{s},k}} \frac{B}{\beta K} \log_2 \left(1 + \frac{H_{ii}^{k'} t_i^{k'}}{\frac{\sigma^2}{\beta K} + \sum_{l \neq i} H_{li}^{k'} t_l^{k'}} \right)$$

$$= \sum_{\mathbf{s} \in S^{\chi}} \sum_{k=1}^{K} \sum_{k' \in \phi_{\mathbf{s},k}} \frac{B}{\beta K} \log_2 \left(1 + \frac{H_{ii}^{k} s_i^{k}}{\frac{\sigma^2}{K} + \sum_{l \neq i} H_{li}^{k} s_l^{k}} \right)$$

$$= \sum_{\mathbf{s} \in S^{\chi}} \frac{a_{\mathbf{s}}}{\beta} \sum_{k=1}^{K} \frac{B}{K} \log_2 \left(1 + \frac{H_{ii}^{k} s_i^{k}}{\frac{\sigma^2}{K} + \sum_{l \neq i} H_{li}^{k} s_l^{k}} \right)$$

$$= \sum_{\mathbf{s} \in S^{\chi}} \pi(\mathbf{s}) u_i(\mathbf{s}) = E_{\pi}[u_i(\mathbf{s})]$$

B. Proof of Corollary 1

Let S_i^{χ} be the set of strategies such that for some $k \in \chi$, $s_i^k > 0$. By construction, $|S_i^{\chi}| > 0$ and $\bigcup_{i \in M} S_i^{\chi} = S^{\chi}$, and thus S_i^{χ} is a proper subset of S^{χ}. Now consider the pure strategy $\mathbf{t} \in P^{\chi'}$ as defined in the above proof. Let $\beta_i = \sum_{\mathbf{s} \in S_i^{\chi}} a_{\mathbf{s}} < \beta$ and let us construct another pure strategy, $\mathbf{v} \in P^{\chi'}$, such that $v_i^{k'} = \frac{t_i^{k'} \beta}{\beta_i} > t_i^{k'}$.

Note that $\mathbf{v}$ is a valid strategy, as:

$$\sum_{k'=1}^{\beta K} v_i^{k'} = \sum_{\mathbf{s} \in S_i^{\chi}} \sum_{k=1}^{K} \sum_{k' \in \phi_{\mathbf{s},k}} v_i^{k'} = \sum_{\mathbf{s} \in S_i^{\chi}} \frac{a_{\mathbf{s}}}{\beta_i} \sum_{k=1}^{K} t_i^k \leq P_{max}$$

The utility achieved by player i for strategy $\mathbf{v}$, $u_i(\mathbf{v})$, is,

$$\sum_{\mathbf{s} \in S^{\chi}} \sum_{k=1}^{K} \sum_{k' \in \phi_{\mathbf{s},k}} \frac{B}{\beta K} \log_2 \left(1 + \frac{H_{ii}^{k'} v_i^{k'}}{\frac{\sigma^2}{\beta K} + \sum_{l \neq i} H_{li}^{k'} v_l^{k'}} \right)$$

$$= \sum_{\mathbf{s} \in S^{\chi}} \sum_{k=1}^{K} \sum_{k' \in \phi_{\mathbf{s},k}} \frac{B}{\beta K} \log_2 \left(1 + \frac{H_{ii}^{k} \frac{s_i^k \beta}{\beta_i}}{\frac{\sigma^2}{K}} \right)$$

$$> \sum_{\mathbf{s} \in S^{\chi}} \frac{a_{\mathbf{s}}}{\beta} \sum_{k=1}^{K} \frac{B}{K} \log_2 \left(1 + \frac{H_{ii}^{k} s_i^{k}}{\frac{\sigma^2}{K}} \right) = E_{\pi}[u_i(\mathbf{s})]$$

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